Pearson

Topic 4 - Moments

Bronze, Silver and Gold Worksheets for

A Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis.

They are drawn from the latest specification questions and legacy questions. The papers are between approximately 25 and 45 marks.

The topic number on this worksheet relates to the corresponding chapter in the 'Pearson Edexcel A Level Mathematics: Statistics and Mechanics Year 2' textbook.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

41 Marks

Calculator

The total mark for this section is 41

Q1

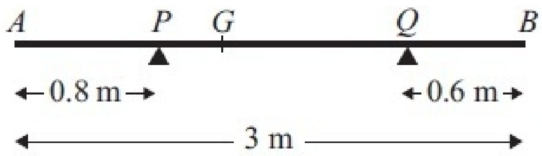


Figure 1

A non-uniform rod AB has length 3 m and mass 4.5 kg. The rod rests in equilibrium, in a horizontal position, on two smooth supports at P and at Q, where $AP = 0.8$ m and $QB = 0.6$ m, as shown in Figure 1. The centre of mass of the rod is at G. Given that the magnitude of the reaction of the support at P on the rod is twice the magnitude of the reaction of the support at Q on the rod, find

(a) the magnitude of the reaction of the support at Q on the rod,

(3)

(b) the distance AG.

(4)

(Total for Question 1 is 7 marks)

Q2

A plank AB has length 6m and mass 30kg. The point C is on the plank with $CB = 2$m. The plank rests in equilibrium in a horizontal position on supports at A and C. Two people, each of mass 75kg, stand on the plank. One person stands at the point P of the plank, where $AP = x$ metres, and the other person stands at the point Q of the plank, where $AQ = 2x$ metres. The plank remains horizontal and in equilibrium with the magnitude of the reaction at C five times the magnitude of the reaction at A. The plank is modelled as a uniform rod and each person is modelled as a particle.

(a) Find the value of x.

(7)

(b) State two ways in which you have used the assumptions made in modelling the plank as a uniform rod.

(2)

(Total for Question 2 is 9 marks)

Q3

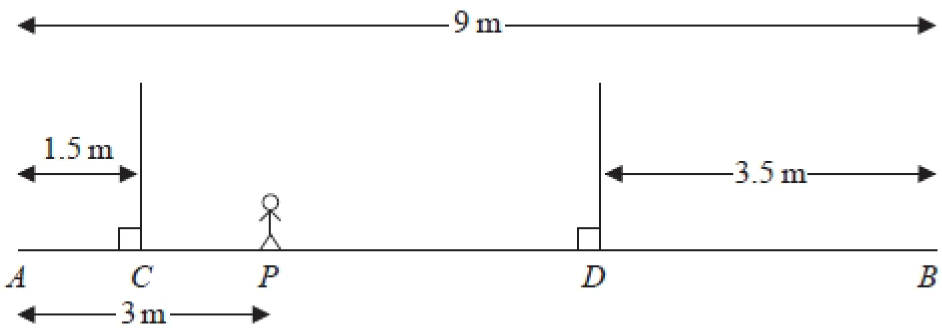


Figure 2

A wooden beam *AB*, of mass 150 kg and length 9 m, rests in a horizontal position supported by two vertical ropes. The ropes are attached to the beam at *C* and *D*, where *AC* = 1.5 m and *BD* = 3.5 m. A gymnast of mass 60 kg stands on the beam at the point *P*, where *AP* = 3 m, as shown in Figure 2. The beam remains horizontal and in equilibrium.

By modelling the gymnast as a particle, the beam as a uniform rod and the ropes as light inextensible strings,

(a) find the tension in the rope attached to the beam at *C*.

(3)

The gymnast at *P* remains on the beam at *P* and another gymnast, who is also modelled as a particle, stands on the beam at *B*. The beam remains horizontal and in equilibrium. The mass of the gymnast at *B* is the largest possible for which the beam remains horizontal and in equilibrium.

(b) Find the tension in the rope attached to the beam at *D*.

(4)

(Total for Question 3 is 7 marks)

Q4

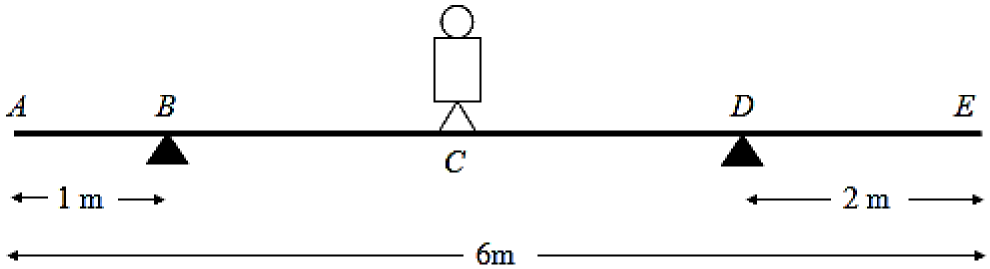
A beam *AB* has length 6 m and weight 200 N. The beam rests in a horizontal position on two supports at the points *C* and *D*, where *AC* =1 m and *DB* =1 m. Two children, Sophie and Tom, each of weight 500 N, stand on the beam with Sophie standing twice as far from the end *B* as Tom. The beam remains horizontal and in equilibrium and the magnitude of the reaction at *D* is three times the magnitude of the reaction at *C*. By modelling the beam as a uniform rod and the two children as particles, find how far Tom is standing from the end *B*.

(7)

(Total for Question 4 is 7 marks)

Q5

Figure 1



A plank *AE,* of length 6 m and mass 10 kg, rests in a horizontal position on supports at *B* and *D*, where *AB* = 1 m and *DE* = 2 m. A child of mass 20 kg stands at *C*, the mid-point of *BD*, as shown in Figure 1. The child is modelled as a particle and the plank as a uniform rod. The child and the plank are in equilibrium. Calculate

(a) the magnitude of the force exerted by the support on the plank at *B*, **(4)**

(b) the magnitude of the force exerted by the support on the plank at *D*. **(3)**

The child now stands at a point *F* on the plank. The plank is in equilibrium and on the point of tilting about *D*.

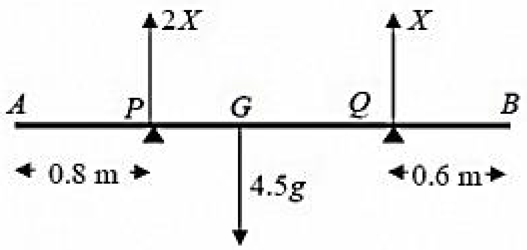
(c) Calculate the distance *DF*. **(4)**

(Total for Question 5 is 11 marks)

End of questions

Bronze Mark Scheme

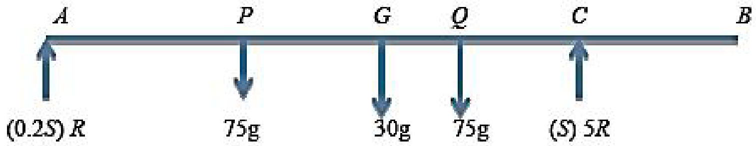
Q1

Question Number	Scheme	Marks
		
	(a) $\uparrow$ $2X + X = 4.5g$	M1 A1
	Leading to $X = \frac{3g}{2}$ or 14.7 or 15 (N)	A1 **(3)**
	(b) M(A) $4.5g \times AG = (2X) \times 0.8 + X \times 2.4$	M1 A2 ft (1,0)
	$AG = \frac{4}{3}$ (m), 1.3, 1.33,...	A1 **(4)**
		[7]

(a)
First M1 for a complete method for finding R_P, either by resolving vertically, or taking moments twice, with usual criteria (allow M1 even if $R_P = 2R_P$ not substituted)
First A1 for a correct equation in either R_P or R_P ONLY.
Second A1 for 1.5g or 14.7 or 15 (A0 for a negative answer)

(b)
First M1 for taking moments about any point, with usual criteria.
A2 ft for a correct equation (A1A0 one error, A0A0 for two or more errors, ignoring consistent omission of g's) in terms of X and their x (which may not be AG at this stage)
Third A1 for AG = 4/3, 1.3, 1.33,.....(any number of decimal places, since g cancels) need 'AG =' or x marked on diagram.
N.B. if $R_P = 2R_P$ throughout, mark as a misread as follows:
(a) M1A1A0 (resolution method) (b) M1A0A1A1, assuming all work follows through correctly.

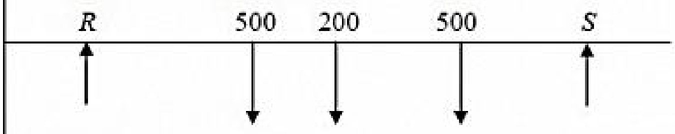
Q2

Question Number	Scheme	Marks
(a)	 $(\uparrow)\ R+5R=75g+30g+75g$ $M(A)\quad 75gx+75g2x+30g\times 3=5R\times 4$ $x=\frac{34}{15}=2.3$ or better (N.B. Or another Moments Equation)	M1 A2 M1 A2 A1 (M1 A2) (7)
(b)	uniform – mass is or acts at midpoint of plank; centre of mass is at middle of plank; weight acts at the middle of the plank, centre of gravity is at midpoint rod - plank does not bend, remains straight, is inflexible, is rigid	B1 B1 (2) **9**
	Notes	
(a)	First M1 for either a vertical resolution (with correct of terms) or a moments equation (all terms dim correct and correct no. of terms) First A1 and Second A1 for a correct equation in R (or S where $S=5R$) only or R and x only or S and x only. (-1 each error , A1A0 or A0A0) Second M1 for a moments equation (all terms dim correct and correct no. of terms) Third A1 and Fourth A1 for a correct equation in R (or S where $S=5R$) only or R and x only or S and x only. (-1 each error, A1A0 or A0A0) Fifth A1 for $x={}^{34}/_{15}$ oe or 2.3 (or better) (i) In a moments equation, if R and $5R$ (or S and $0.2S$) are interchanged, treat as 1 error. (ii) Ignore diagram if it helps the candidate. (iii) If an equation is correct but contains both R and S, *or $S = 5R$ is never used*, treat as 1 error. (iv) Full marks possible if all g's omitted. (v) For inconsistent omission of g, penalise each omission. $M(B),\ R\times 6+5R\times 2=75g(6-x)+75g(6-2x)+30g\times 3$ $M(C),\ 75g(4-x)+75g(4-2x)+30g\times 1=R\times 4$ $M(G),\ 75g(3-x)+5R\times 1=R\times 3+75g(2x-3)$ $M(P),\ Rx+30g(3-x)+75gx=5R(4-x)$ $M(Q),\ 75gx+30g(2x-3)+5R(4-2x)=R\times 2x$	
(b)	First B1 for first correct answer seen. Second B1 for the other answer, but only award this second mark if no extras given.	

Q3

Question Number	Scheme	Marks
(a)	$M(D)$, $(150g\times1)+(60g\times2.5)=Tc\times4$	M1 A1
	$Tc=75g$ or 735 N or 740 N Allow omission of N	A1 (3)
(b)	$M(B)$, $(150g\times4.5)+(60g\times6)=T_D\times3.5$	M1 A2
	$T_D=2900$ N or $\dfrac{2070g}{7}$ Allow omission of N	A1 (4)
		(7)
	Notes	
	(a) M1 for a complete method to find Tc (M0 if they assume $Tc=Td$) i.e. for producing an equation in Tc only. Each equation used must have correct no. of terms and be dimensionally correct. First A1 for correct equation. Second A1 for any of the 3 possible answers Other possible equations: $(\uparrow), Tc+T_D=60g+150g$ $M(A)$, $(150g\times4.5)+(60g\times3)=(Tc\times1.5)+(T_D\times5.5)$ $M(C)$, $(150g\times3)+(60g\times1.5)=T_D\times4$ $M(B)$, $(150g\times4.5)+(60g\times6)=(Tc\times7.5)+(T_D\times3.5)$ $M(G)$, $(T_D\times1)+(60g\times1.5)=T_C\times3$	
	(b) N.B. (M0 if T_C is never equated to 0) M1 for a complete method to obtain an equation in T_D only. If they use more than one equation, each equation used must have correct no. of terms and be dimensionally correct. First and second A1 for a correct equation in T_D only. A1A0 if one error.Consistent omission of g is one error except in *M(D)* where it's not an error. Third A1 for either answer Other possible equations: $(\uparrow), T_D=60g+150g+Mg$ $M(A)$, $(150g\times4.5)+(60g\times3)+9Mg=T_D\times5.5$ $M(C)$, $(150g\times3)+(60g\times1.5)+7.5Mg=T_D\times4$ $M(D)$, $(150g\times1)+(60g\times2.5)=3.5Mg$ $M(G)$, $(T_D\times1)+(60g\times1.5)=4.5Mg$	

Q4

Question Number	Scheme	Marks
		
	$M(B)$,	
	$500x + 500.2x + 200 \times 3 = R \times 5 + S \times 1$ (or any valid moments equation)	M1 A1 A1
	$(\downarrow)\ R + S = 500 + 500 + 200 = 1200$ (or a moments equation)	M1 A1
	solving for x; $x = 1.2$ m	M1 A1 cso **[7]**

Q5

Scheme	Marks
(a) M(D): $20g \times 1.5 + 10g \times 1 = R_B \times 3$	M1 A1
$\Rightarrow R_B = \underline{40g/3 \approx 131 \text{ or } 130 \text{ N}}$	M1 A1 **(4)**
[NB For moments about another point, allow M1 A1 for moments equation dimensionally correct and with correct number of terms; second M1 is for complete method to find R_B.]	
(b) R($\uparrow$): $R_D + 40g/3 = 20g + 10g$	M1 A1√
$\Rightarrow R_D = \underline{50g/3 \approx 163 \text{ or } 160 \text{ N}}$	A1 (3)
or M(B): $20g \times 1.5 + 10g \times 2 = R_D \times 3$	M1 A1
$\Rightarrow R_D = \underline{50g/3 \approx 163 \text{ or } 160 \text{ N}}$	A1 **(3)**
[NB For moments about another point, allow M1 for a complete method to find R_D, A1 for a correct equation for R_D.]	
(c) $R_B = 0$	M1
M(D): $20g \times x = 10g \times 1$	M1 A1
$x = DF = \underline{0.5 \text{ m}}$	A1 **(4)**
For weight/mass confusion, A0 A0 in (a) but allow f.t. in (b) (ans 50/3 = 16.7) *General rule of deducting max. 1 per question for > 3 s.f* (c) *2nd M1: must have correct no. of non=zero terms, and equation in x only* *If use value(s) of R's from (a) or (b): M0.*	**Total 11 Marks**

Silver Questions

Calculator

The total mark for this section is 30

Q1

A steel girder AB, of mass 200 kg and length 12 m, rests horizontally in equilibrium on two smooth supports at C and at D, where $AC = 2$ m and $DB = 2$ m. A man of mass 80 kg stands on the girder at the point P, where $AP = 4$ m, as shown in Figure 1.

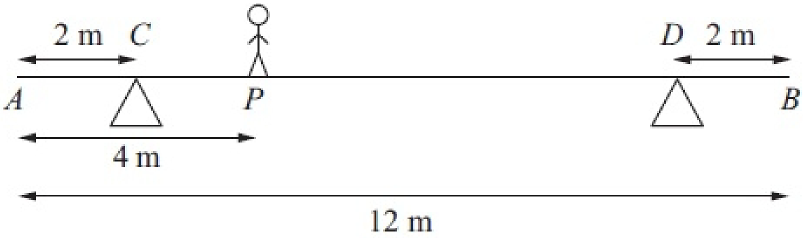


Figure 1

The man is modelled as a particle and the girder is modelled as a uniform rod.

(a) Find the magnitude of the reaction on the girder at the support at C.

(3)

The support at D is now moved to the point X on the girder, where $XB = x$ metres. The man remains on the girder at P, as shown in Figure 2.

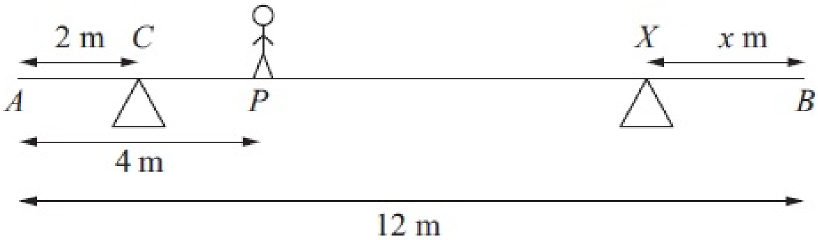


Figure 2

Given that the magnitudes of the reactions at the two supports are now equal and that the girder again rests horizontally in equilibrium, find

(b) the magnitude of the reaction at the support at X,

(2)

(c) the value of x.

(4)

(Total for Question 1 is 9 marks)

Q2

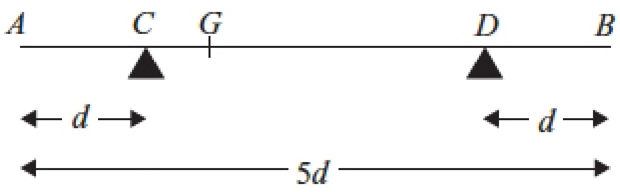


Figure 1

A non-uniform rod AB, of mass m and length $5d$, rests horizontally in equilibrium on two supports at C and D, where $AC = DB = d$, as shown in Figure 1. The centre of mass of the rod is at the point G. A particle of mass $\frac{5}{2}m$ is placed on the rod at B and the rod is on the point of tipping about D.

(a) Show that GD = $\frac{5}{2}d$.

(4)

The particle is moved from B to the mid-point of the rod and the rod remains in equilibrium.

(b) Find the magnitude of the normal reaction between the support at D and the rod.

(5)

(Total for Question 2 is 9 marks)

Q3.

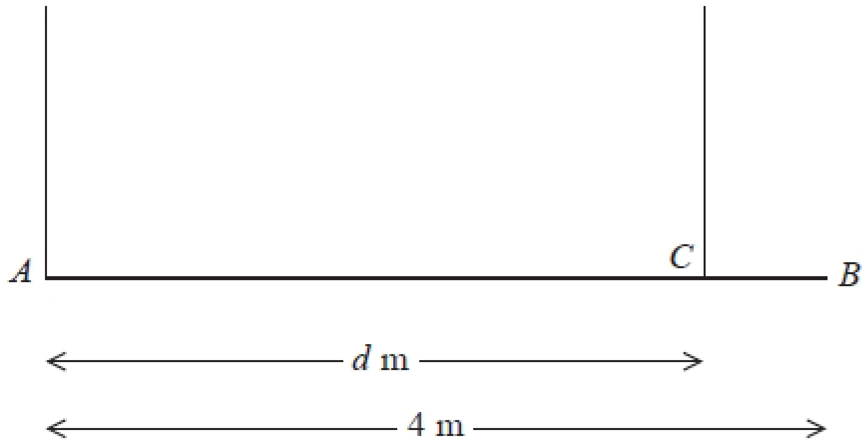


Figure 3

A beam AB has weight W Newtons and length 4 m. The beam is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. One rope is attached to A and the other rope is attached to the point C on the beam, where $AC = d$ metres, as shown in Figure 3. The beam is modelled as a uniform rod and the ropes as light inextensible strings. The tension in the rope attached at C is double the tension in the rope attached at A.

(a) Find the value of d.

(6)

A small load of weight kW Newtons is attached to the beam at B. The beam remains in equilibrium in a horizontal position. The load is modelled as a particle. The tension in the rope attached at C is now four times the tension in the rope attached at A.

(b) Find the value of k.

(6)

(Total for Question 3 is 12 marks)

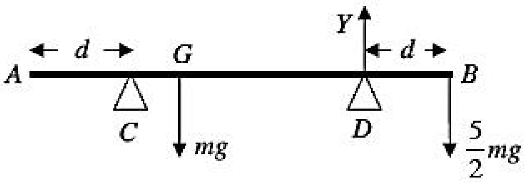
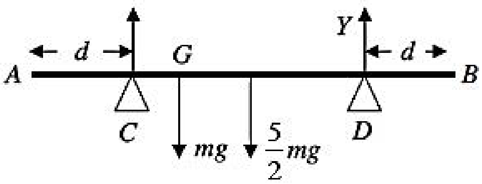
End of questions

Silver Mark Scheme

Q1

Question Number	Scheme	Marks
(a)	$M(D),\quad 8R = (80g \times 6) + (200g \times 4)$ $R = 160g,\ 1600,\ 1570$	M1 A1 A1 (3)
(b)	$(\uparrow),\quad 2S = 80g + 200g$ $S = 140g,\ 1400,\ 1370$	M1 A1 (2)
(c)	$M(B),\quad Sx + (S \times 10) = (80g \times 8) + (200g \times 6)$ $140x + 1400 = 640 + 1200$ $140x = 440$ $x = \frac{22}{7}$	M1 A2 A1 (4) **9**

Q2

Question Number	Scheme	Marks
(a)		
	M(D) $\quad mg \times GD = \frac{5}{2} mg \times d$	M1 A1
	$GD = \frac{5}{2} d$ *	DM1 A1 (4)
(b)		
	M(C) $\quad mg \times \frac{d}{2} + \frac{5}{2} mg \times \frac{3}{2} d = Y \times 3d$	M1 A2(1, 0)
	Leading to $\quad Y = \frac{17}{12} mg$	DM1 A1 (5) **9**

Q3

Question Number	Scheme	Marks
a	Resolving vertically: $T + 2T\,(= 3T) = W$	M1A1
	Moments about A: $2W = 2T \times d$	M1A1
	Substitute and solve: $2W = 2\frac{W}{3}d$	DM1
	$d = 3$	A1
		(6)
b	Resolving vertically: $T + 4T = W + kW \quad (5T = W(1+k))$	M1A1 ft
	Moments about A: $2W + 4kW = 3 \times 4T$	M1A1 ft
	Substitute and solve: $2W + 4kW = \frac{12}{5}W(1+k)$	DM1
	$2 + 4k = \frac{12}{5} + \frac{12}{5}k$	
	$\frac{8}{5}k = \frac{2}{5}, \qquad k = \frac{1}{4}$	A1
		(6)
		[12]

Notes for Question
N.B. In moments equations, for the M mark, all terms must be force x distance but take care in the cases when the distance is 1.
Question (a)
N.B. If Wg is used, mark as a misread. *If T and 2T are reversed, mark as per scheme NOT as a misread.*
First M1 for an equation in W and T and possibly d (either resolve vertically or moments about any point other than the mid-pt), with usual rules.
First A1 for a correct equation.
Second M1 for an equation in W and T and possibly d (either resolve vertically or moments about any point other than the mid-pt), with usual rules.
Second A1 for a correct equation.
Third M1, dependent on first and second M marks, for solving for d
Third A1 for $d = 3$ cso
N.B. If a single equation is used (see below) by taking moments about the mid-point of the rod, $2T = 2T(d - 2)$, this scores M2A2 (-1 each error)
Third M1, dependent on first and second M marks, for solving for d
Third A1 for $d = 3$ cso
Question (b)
N.B. If Wg and kWg are used, mark as a misread.
If they use any results from (a), can score max M1A1 in (b) for one equation.
If T and 4T are reversed, mark as per scheme NOT as a misread.
First M1 for an equation in W and a tension T_1 and possibly their d or their d and k (either resolve vertically or moments about any point), with usual rules.
First A1 ft on their d, for a correct equation.
Second M1 for an equation in W and the same tension T_1 and possibly their d or their d and k (either resolve vertically or moments about any point), with usual rules.
Second A1 ft on their d, for a correct equation.
Third M1, dependent on first and second M marks, for solving to give a numerical value of k
Third A1 for $k = 1/4$ *oe* cso

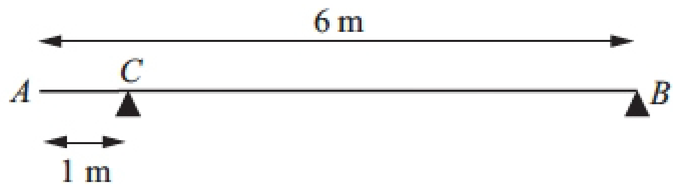
Pearson

Gold Questions

Calculator

The total mark for this section is 45

Q1



A uniform beam *AB* has mass 20 kg and length 6 m. The beam rests in equilibrium in a horizontal position on two smooth supports. One support is at *C*, where *AC* = 1 m, and the other is at the end *B*, as shown in the figure above. The beam is modelled as a rod.

(a) Find the magnitudes of the reactions on the beam at *B* and at *C*.

(5)

A boy of mass 30 kg stands on the beam at the point *D*. The beam remains in equilibrium. The magnitudes of the reactions on the beam at *B* and at *C* are now equal. The boy is modelled as a particle.

(b) Find the distance *AD*.

(5)

(Total for Question 1 is 10 marks)

Q2

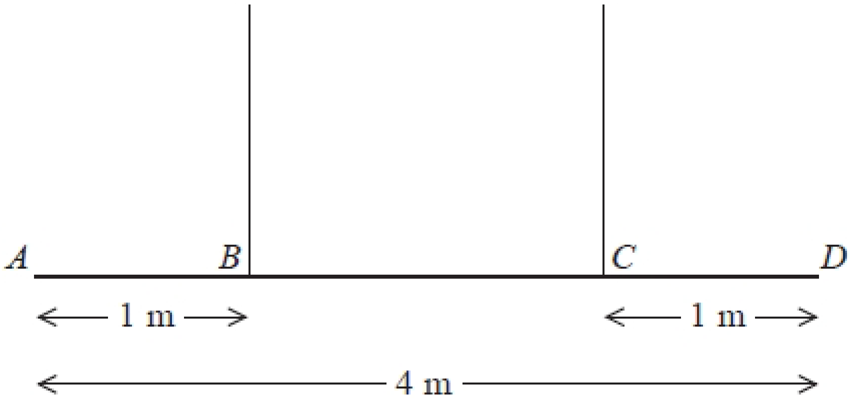


Figure 3

A non-uniform beam AD has weight W Newtons and length 4 m. It is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. The ropes are attached to two points B and C on the beam, where $AB = 1$ m and $CD = 1$ m, as shown in Figure 3. The tension in the rope attached to C is double the tension in the rope attached to B. The beam is modelled as a rod and the ropes are modelled as light inextensible strings.

(a) Find the distance of the centre of mass of the beam from A.

(6)

A small load of weight kW Newtons is attached to the beam at D. The beam remains in equilibrium in a horizontal position. The load is modelled as a particle.

Find

(b) an expression for the tension in the rope attached to B, giving your answer in terms of k and W,

(3)

(c) the set of possible values of k for which both ropes remain taut.

(2)

(Total for Question 2 is 11 marks)

Q3

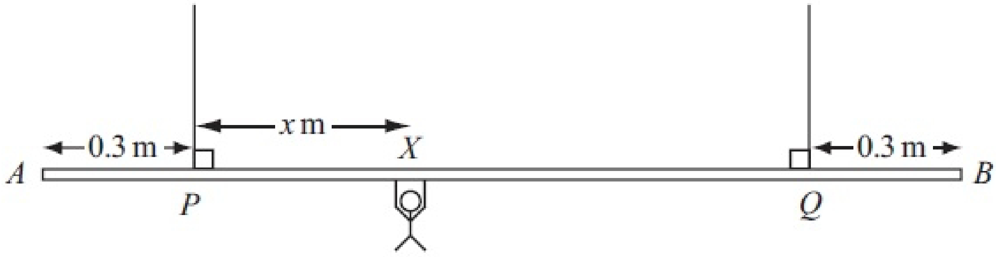


Figure 2

A beam AB is supported by two vertical ropes, which are attached to the beam at points P and Q, where $AP = 0.3$ m and $BQ = 0.3$ m. The beam is modelled as a uniform rod, of length 2 m and mass 20 kg. The ropes are modelled as light inextensible strings. A gymnast of mass 50 kg hangs on the beam between P and Q. The gymnast is modelled as a particle attached to the beam at the point X, where $PX = x$ m, $0 < x < 1.4$ as shown in Figure 2. The beam rests in equilibrium in a horizontal position.

(a) Show that the tension in the rope attached to the beam at P is $(588 - 350x)$ N.

(3)

(b) Find, in terms of x, the tension in the rope attached to the beam at Q.

(3)

(c) Hence find, justifying your answer carefully, the range of values of the tension which could occur in each rope.

(3)

Given that the tension in the rope attached at Q is three times the tension in the rope attached at P,

(d) find the value of x.

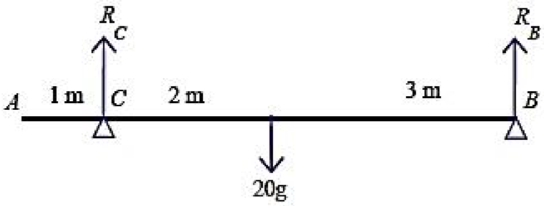
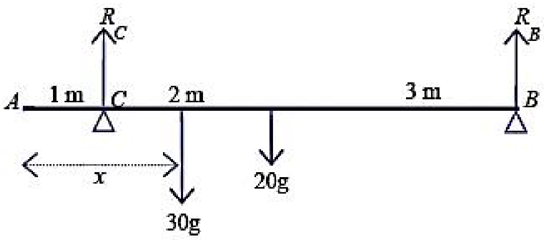
(3)

(Total for Question 3 is 12 marks)

End of questions

Gold Mark Scheme

Q1

Question Number	Scheme	Marks
(a)		
	Taking moments about B: $5 \times R_C = 20g \times 3$	M1A1
	$R_C = 12g$ or $60g/5$ or 118 or 120	A1
	Resolving vertically: $R_C + R_B = 20g$	M1
	$R_B = 8g$ or 78.4 or 78	A1
		(5)
(b)		
	Resolving vertically: $50g = R + R$	B1
	Taking moments about B:	
	$5 \times 25g = 3 \times 20g + (6 - x) \times 30g$	M1 A1 A1
	$30x = 115$	
	$x = 3.8$ or better or 23/6 oe	A1
		(5) [10]

Q2

Question Number	Scheme	Marks
a	Resolving vertically: $T+2T(=3T)=W$	M1A1
	Moments about B: $2\times 2T=(d-1)W$	M1A1
	Substitute and solve for d : $2\times 2T=(d-1)3T$	DM1
	$d=\frac{7}{3}$ (m)	A1 (6)
b	Moments about C: $(T_B\times 2)+(kW\times 1)=W\times\frac{2}{3}$	M1A1
	$T_B=W\frac{(2-3k)}{6}$ or equivalent	A1 (3)
c	solving $T_B\geq 0$ or $T_B>0$ for k.	M1
	$0<k\leq 2/3$ or $0<k<2/3$ only	A1 (2)
		[11]

Notes for Question

Question (a)

N.B. If Wg is used, mark as a misread.
First M1 for an equation in W and T and possibly d (either resolve vertically or moments about any point other than the centre of mass of the rod), with usual rules.
First A1 for a correct equation.
Second M1 for an equation in W and T and possibly d (either resolve vertically or moments about any point other than the centre of mass of the rod), with usual rules.
Second A1 for a correct equation.
N.B. The above 4 marks can be scored if their d is measured from a different point
Third M1, dependent on first and second M marks, for solving for d
Third A1 for $d = 7/3$, 2.3 (m) or better
N.B. Alternative
If a single equation is used (see below) by taking moments about the centre of mass of the rod, $2T(3 - d) = T(d - 1)$, this scores M2A2 (-1 each error)
Third M1, dependent on first and second M marks, for solving for d
Third A1 for $d = 7/3$

Question (b)

First M1 for producing an equation in T_B and W *only*, either by taking moments about C, or using two equations and eliminating
First A1 for a correct equation
Second A1 for $W(2 - 3k)/6$ oe.

N.B. **M0** if they use any information about the tension(s) from part (a).

Question (c)

M1 for solving $T_B \geq 0$ or $T_B > 0$ for k.
A1 for $0 < k \leq 2/3$ or $0 < k < 2/3$ only.
N.B.
$T = 0 \Rightarrow k = 2/3$ then answer is M0.
If they also solve $T_C \geq 0$ or $T_C > 0$, can still score M1 and possibly A1.

Q3

Question Number	Scheme	Marks
(a)	$M(Q)$, $50g(1.4-x)+20g \times 0.7 = T_P \times 1.4$	M1 A1
	$T_P = 588 - 350x$ Printed answer	A1 (3)
(b)	$M(P)$, $50gx + 20g \times 0.7 = T_Q \times 1.4$ or R(↑), $T_P + T_Q = 70g$	M1 A1
	$T_Q = 98 + 350x$	A1 (3)
(c)	Since $0 < x < 1.4$, $98 < T_P < 588$ and $98 < T_Q < 588$	M1 A1 A1 (3)
(d)	$98 + 350x = 3(588 - 350x)$	M1
	$x = 1.19$	DM1 A1 (3) [12]

Topic 5 - Forces and Friction

Bronze, Silver and Gold Worksheets for A Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis.

They are drawn from the latest specification questions and legacy questions. The papers are between approximately 25 and 45 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel A Level Mathematics: Statistics and Mechanics Year 2' textbook.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculator

The total mark for this section is 34

Q1

Three forces, $(15\mathbf{i} + \mathbf{j})$ N, $(5q\mathbf{i} - p\mathbf{j})$ N and $(-3p\mathbf{i} - q\mathbf{j})$ N, where p and q are constants, act on a particle. Given that the particle is in equilibrium, find the value of p and the value of q.

(Total for Question 1 is 5 marks)

Q2

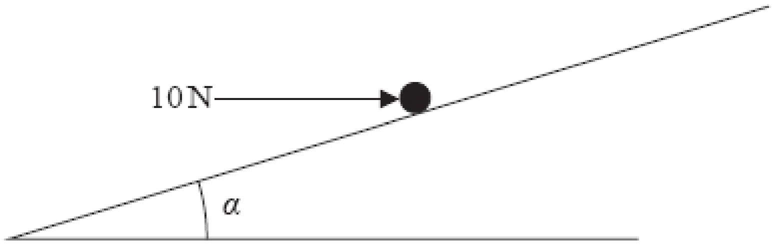


Figure 1

A particle P of mass 5kg is held at rest in equilibrium on a rough inclined plane by a horizontal force of magnitude 10N. The plane is inclined to the horizontal at an angle α where $\tan\alpha = \frac{3}{4}$, as shown in Figure 1. The line of action of the force lies in the vertical plane containing P and a line of greatest slope of the plane. The coefficient of friction between P and the plane is μ. Given that P is on the point of sliding down the plane, find the value of μ.

(Total for Question 2 is 9 marks)

Q3

Two forces $(4\mathbf{i} - 2\mathbf{j})$ N and $(2\mathbf{i} + q\mathbf{j})$ N act on a particle P of mass 1.5 kg. The resultant of these two forces is parallel to the vector $(2\mathbf{i} + \mathbf{j})$.

(a) Find the value of q.

(4)

At time $t = 0$, P is moving with velocity $(-2\mathbf{i} + 4\mathbf{j})\text{m s}^{-1}$.

(b) Find the speed of P at time $t = 2$ seconds.

(6)

(Total for Question 3 is 10 marks)

Q4

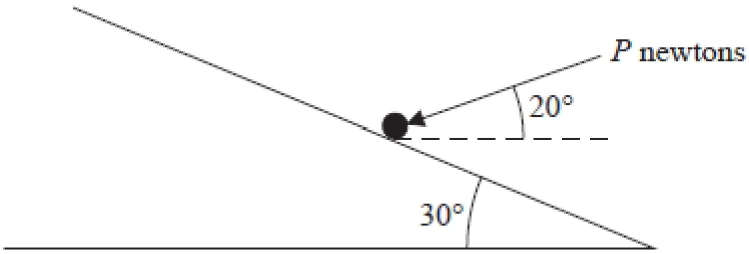


Figure 1

A particle of mass 2 kg lies on a rough plane. The plane is inclined to the horizontal at 30°.

The coefficient of friction between the particle and the plane is $\frac{1}{4}$. The particle is held in equilibrium by a force of magnitude P newtons. The force makes an angle of 20° with the horizontal and acts in a vertical plane containing a line of greatest slope of the plane, as shown in Figure 1. Find the least possible value of P.

(Total for Question 3 is 10 marks)

End of questions

Bronze Mark Scheme

Q1

Question Number	Scheme	Marks
	$(15\mathbf{i} + \mathbf{j}) + (5q\mathbf{i} - p\mathbf{j}) + (-3p\mathbf{i} - q\mathbf{j}) = \mathbf{0}$ $3p - 5q = 15$ $p + q = 1$ $p = 2.5 \quad q = -1.5$	M1 M1 A1 M1 A1 A1 **6**
	Notes	
	First M1 for equating the sum of the three forces to zero (can be implied by subsequent working) Second M1 for equating the sum of the **i** components to zero AND the sum of the **j** components to zero oe to produce TWO equations, each one being in p and q ONLY. First A1 for TWO correct equations (in any form) **N.B.** It is possible to obtain TWO equations by using $\lambda(3p - 5q - 15) = \mu(p + q - 1)$with TWO different pairs of values for $\lambda$and μ, with one pair not a multiple of the other e.g λ=1, μ=1 AND λ=1, μ=2. Third M1(independent) for attempt (either by substitution or elimination) to produce an equation in either p ONLY or q ONLY. Second A1 for p = 2.5 (any equivalent form, fractions do not need to be in lowest terms) Third A1 for q = - 1.5 (any equivalent form, fractions do not need to be in lowest terms)	

Q2

Question Number	Scheme	Marks
	$F = \mu R$ $(\nwarrow)$, $R = 10\sin\alpha + 5g\cos\alpha$ (45.2) $(\nearrow)$, $F = 5g\sin\alpha - 10\cos\alpha$ (21.4) $\mu = \dfrac{g\sin\alpha - 2\cos\alpha}{2\sin\alpha + g\cos\alpha} = 0.47$ or 0.473	B1 M1 A2 M1 A2 M1 A1 **9**
	Notes	
	B1 for $F = \mu R$ seen or implied First M1 for resolving perpendicular to the plane with usual rules First and second A1's for a correct equation. A1A0 if one error. Second M1 for resolving parallel to the plane with usual rules Third and fourth A1's for a correct equation. A1A0 if one error. If m is used instead of 5, penalise once in each equation. Third M1 <u>independent</u> for eliminating R to produce an equation in μ only. Does not need to be $\mu = ...$ Fifth A1 for 0.47 or 0.473.	

Q3

Question Number	Scheme	Marks
(a)	$(4\mathbf{i} - 2\mathbf{j}) + (2\mathbf{i} + q\mathbf{j}) = (6\mathbf{i} + (q - 2)\mathbf{j})$ $6 = 2(q-2)$ ratio 2:1 $q = 5$	M1A1 **DM1** A1 (4)
(b)	$6\mathbf{i} + 3\mathbf{j} = 1.5\mathbf{a}$ $\mathbf{a} = (4\mathbf{i} + 2\mathbf{j})$ m s^{-2} $\mathbf{v} = \mathbf{u} + \mathbf{a}t = (-2\mathbf{i} + 4\mathbf{j}) + 2(4\mathbf{i} + 2\mathbf{j})$ $= 6\mathbf{i} + 8\mathbf{j}$ speed $= \sqrt{6^2 + 8^2}$ $= 10$ m s^{-1}	M1 A1 M1 A1ft M1 A1 (6) **[10]**

Notes for Question

<u>Question (a)</u>

First M1 for $(4\mathbf{i} - 2\mathbf{j}) + (2\mathbf{i} + q\mathbf{j})$

First A1 for $(6\mathbf{i} + (q - 2)\mathbf{j})$ (seen or implied)

Second M1, **dependent on first M1**, for using 'parallel to $(2\mathbf{i} + \mathbf{j})$' to obtain an equation in q *only*.

Second A1 for $q = 5$

<u>Question (b)</u>

First M1 for their resultant force $= 1.5\mathbf{a}$

First A1 for $\mathbf{a} = 4\mathbf{i} + 2\mathbf{j}$

Second M1 for $(-2\mathbf{i} + 4\mathbf{j}) + 2$ x (their **a**) **(M0 if force is used instead of a)**

Second A1 **ft** for their velocity at $t = 2$

Third M1for finding the magnitude of <u>their velocity at $t = 2$</u>

Third A1 for 10 (ms^{-1})

N.B. In (b), if they use scalars throughout, M0A0M0A0M0A0

Q4

Question Number	Scheme	Marks
	(Parallel to plane): $P\cos 50 + F = 2g\cos 60$	M1 A2
	(Perp to plane): $R - P\sin 50 = 2g\cos 30$	M1 A2
	Other possible equations:	
	$(\rightarrow)$: $R\cos 60 - F\cos 30 = P\cos 20$ M1 A2	
	$(\uparrow)$: $R\cos 30 + F\cos 60 = P\cos 70 + 2g$ M1 A2	
	$F = \frac{1}{4}R$	B1
	Attempt to eliminate F and R to give an equation in P only	M1
	Solve for P	**DM1**
	$P = 6.7$ (2 SF) or 6.66 (3SF)	A1
		(10)
	Notes	
	First M1 for resolving parallel to the plane with usual rules. $2g$ term must be using 30° or 60° angle but allow sin/cos confusion. First and second A1's for a correct equation. A1A0 if one error. Second M1 for resolving perpendicular to the plane with usual rules. $2g$ term must be using 30° or 60° angle but allow sin/cos confusion. Third and fourth A1's for a correct equation. A1A0 if one error. B1 for $F = ¼ R$ seen or implied Third M1, independent but must have two 3 (or 4) term equations, for attempt to eliminate F and R to give an equation in P only. Fourth DM1, dependent on third M1, for solving for P. Fifth A1 for 6.7 or 6.66 **Other possible equations**: First M1 for resolving horizontally with usual rules. R term must be using 30° or 60° angle and F term must be using 30° or 60° angle but allow sin/cos confusion. First and second A1's for a correct equation. A1A0 if one error. Second M1 for resolving vertically with usual rules. R term must be using 30° or 60° angle and F term must be using 30° or 60° angle but allow sin/cos confusion. Third and fourth A1's for a correct equation. A1A0 if one error.	

Silver Questions

Calculator

The total mark for this section is 27

Q1

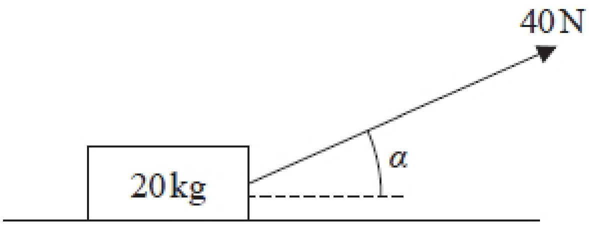


Figure 1

A wooden crate of mass 20 kg is pulled in a straight line along a rough horizontal floor using a handle attached to the crate.

The handle is inclined at an angle α to the floor, as shown in Figure 1, where $\tan\alpha = \frac{3}{4}$.

The tension in the handle is 40 N.
The coefficient of friction between the crate and the floor is 0.14.
The crate is modelled as a particle and the handle is modelled as a light rod.

Using the model,

(a) find the acceleration of the crate.

(6)

The crate is now pushed along the same floor using the handle. The handle is again inclined at the same angle α to the floor, and the thrust in the handle is 40 N as shown in Figure 2 below.

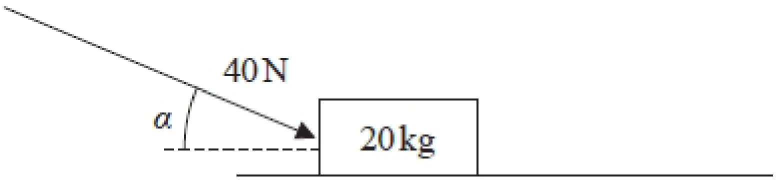


Figure 2

(b) Explain briefly why the acceleration of the crate would now be less than the acceleration of the crate found in part (a).

(2)

(Total for Question 1 is 8 marks)

Q2

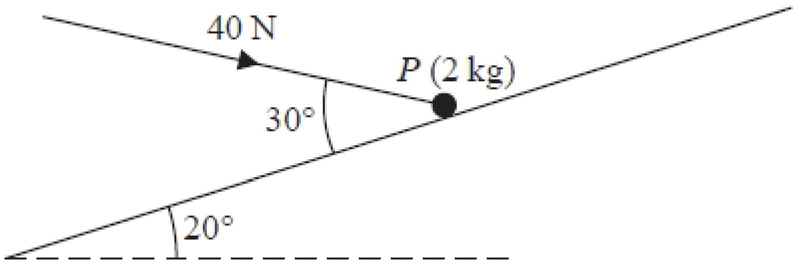


Figure 2

A particle P of mass 2 kg is held at rest in equilibrium on a rough plane by a constant force of magnitude 40 N. The direction of the force is inclined to the plane at an angle of 30°. The plane is inclined to the horizontal at an angle of 20°, as shown in Figure 2. The line of action of the force lies in the vertical plane containing P and a line of greatest slope of the plane. The coefficient of friction between P and the plane is μ.

Given that P is on the point of sliding up the plane, find the value of μ.

(Total for Question 2 is 10 marks)

Q3

A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$.

A brick P of mass m is placed on the plane.

The coefficient of friction between P and the plane is μ.

Brick P is in equilibrium and on the point of sliding down the plane.

Brick P is modelled as a particle.

Using the model,

(a) find, in terms of m and g, the magnitude of the normal reaction of the plane on brick P,

(2)

(b) show that $\mu = \frac{3}{4}$.

(4)

For parts (c) and (d), you are not required to do any further calculations.

Brick P is now removed from the plane and a much heavier brick Q is placed on the plane.

The coefficient of friction between Q and the plane is also $\frac{3}{4}$.

(c) Explain briefly why brick Q will remain at rest on the plane.

(1)

Brick Q is now projected with speed 0.5 m s^{-1} down a line of greatest slope of the plane.

Brick Q is modelled as a particle.

Using the model,

(d) describe the motion of brick Q, giving a reason for your answer.

(2)

(Total for Question 3 is 9 marks)

End of questions

Silver Mark Scheme

Q1

Question	Scheme	Marks	AOs
(a)	Resolve vertically	M1	3.1b
	$R + 40\sin\alpha = 20g$	A1	1.1b
	Resolve horizontally	M1	3.1b
	$40\cos\alpha - F = 20a$	A1	1.1b
	$F = 0.14R$	B1	1.2
	$a = 0.396$ or 0.40 (m s^{-2})	A1	2.2a
		(6)	
(b)	Pushing will increase R which will increase available F	B1	2.4
	Increasing F will <u>decrease a</u> * GIVEN ANSWER	B1*	2.4
		(2)	
			(8 marks)

Notes:

(a)

M1: Resolve vertically with usual rules applying

A1: Correct equation. Neither g nor $\sin\alpha$ need to be substituted

M1: Apply $F = ma$ horizontally, with usual rules

A1: Neither F nor $\cos\alpha$ need to be substituted

B1: $F = 0.14R$ seen (e.g. on a diagram)

A1: Either answer

(b)

B1: Pushing increases R which produces an increase in available (limiting) friction

B1: F increase produces an <u>a decrease (need to see this)</u>

N.B. It is possible to score B0 B1 but for the B1, some "explanation" is needed to say why friction is increased e.g. by pushing into the ground.

Q2

Question Number	Scheme	Marks
	μR $R = 2g\cos 20° + 40\cos 60°$ $F = 40\cos 30° - 2g\cos 70°$ $\mu = \dfrac{40\cos 30° - 2g\cos 70°}{2g\cos 20° + 40\cos 60°}$ $= 0.73$ or 0.727	B1 M1 A2 M1 A2 M1 M1 A1 **10**
	Notes	
	B1 for μR seen or implied.	
	First M1 for resolving perpendicular to the plane with usual rules (must be using $2(g)$ with 20^0 or 70^0 and 40 with 30^0 or 60^0)	
	First and second A1's for a correct equation. A1A0 if one error	
	Second M1 for resolving parallel to the plane with usual rules (must be using $2(g)$ with 20^0 or 70^0 and 40 with 30^0 or 60^0)	
	Third and fourth A1's for a correct equation. A1A0 if one error	
	Third M1 <u>independent</u> for eliminating R to produce an equation in μ only. Does not need to be $\mu = \ldots$.	
	Fourth M1 <u>independent</u> for solving for μ	
	Fifth A1 for 0.727 or 0.73	
	N.B. They may choose to resolve in 2 other directions e.g. horizontally and vertically.	
	N.B. If F is replaced by $-F$ in the second equ^n, treat this as an error unless they subsequently explain that they have their F acting in the wrong direction, in which case they could score full marks for the question.	

Q3

Question	Scheme	Marks	AOs
(a)	Resolve perpendicular to the plane	M1	3.4
	$R = mg\cos\alpha = \frac{4}{5}mg$	A1	1.1b
		(2)	
(b)	Resolve parallel to the plane or horizontally or vertically	M1	3.4
	$F = mg\sin\alpha$ or $R\sin\alpha = F\cos\alpha$	A1	1.1b
	Use $F = \mu R$ and solve for μ	M1	2.1
	$\mu = \frac{3}{4}$ *	A1*	2.2a
		(4)	
(c)	The forces acting on Q will still balance as the m's cancel oe Other possibilities: e.g. the friction will increase in the same proportion as the weight component or force down the plane. The force pulling the brick down the plane increases by the same amount as the friction oe This mark can be scored if they do the calculation.	B1	2.4
		(1)	
(d)	Brick Q slides down the plane with **constant** speed.	B1	2.4
	No resultant force down the plane (so no acceleration) oe	B1	2.4
	These marks can be scored if they do the calculation.	(2)	
		(9 marks)	

Notes:		
a	M1	Correct no. of terms, condone sin/cos confusion
	A1	cao with no wrong working seen. $mg\cos 36.86$ is A0
b	M1	Correct no. of terms, condone sin/cos confusion
	A1	Correct equation
	M1	Must use $F = \mu R$ (not merely state it) to obtain a numerical value for μ. This is an independent M mark.
	A1*	Given answer correctly obtained
c	B1	Must have the 3 underlined phrases/word oe
d	B1	Must say **constant** speed.
	B1	Any appropriate equivalent statement

Gold Questions

Calculator

The total mark for this section is 38

Q1

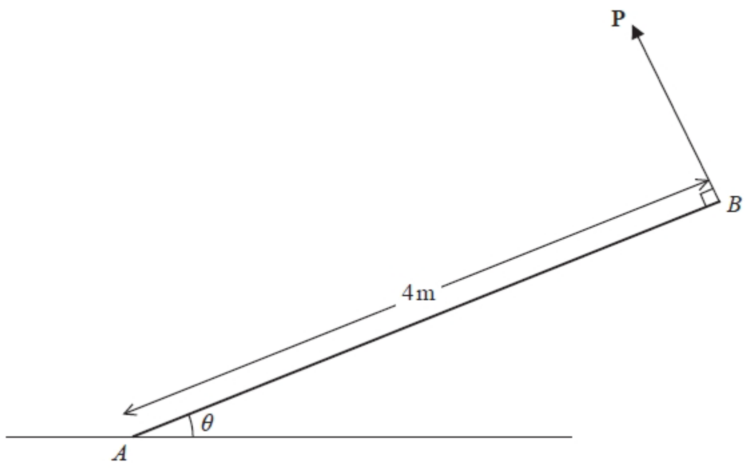


Figure 2

A non-uniform rod *AB*, of mass 5kg and length 4m, rests with one end *A* on rough horizontal ground. The centre of mass of the rod is *d* metres from *A*. The rod is held in limiting equilibrium at an angle θ to the horizontal by a force **P**, which acts in a direction perpendicular to the rod at *B*, as shown in Figure 2. The line of action of **P** lies in the same vertical plane as the rod.

(a) Find, in terms of *d*, *g* and θ,

(i) the magnitude of the vertical component of the force exerted on the rod by the ground,
(ii) the magnitude of the friction force acting on the rod at *A*.

(8)

Given that $\tan\theta = \frac{5}{12}$ and that the coefficient of friction between the rod and the ground is $\frac{1}{2}$,

(b) find the value of *d*.

(4)

(Total for Question 1 is 12 marks)

Q2

A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$.

A particle of mass m is placed on the plane and then projected up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is μ.

The particle moves up the plane with a constant deceleration of $\frac{4}{5}g$.

(a) Find the value of μ.

(6)

The particle comes to rest at the point A on the plane.

(b) Determine whether the particle will remain at A, carefully justifying your answer.

(2)

(Total for Question 2 is 8 marks)

Q3

A particle P moves with acceleration $(4\mathbf{i} - 5\mathbf{j})$ m s^{-2}

At time $t = 0$, P is moving with velocity $(-2\mathbf{i} + 2\mathbf{j})$ m s^{-1}

(a) Find the velocity of P at time $t = 2$ seconds.

(2)

At time $t = 0$, P passes through the origin O.

At time $t = T$ seconds, where $T > 0$, the particle P passes through the point A.

The position vector of A is $(\lambda\mathbf{i} - 4.5\mathbf{j})$m relative to O, where λ is a constant.

(b) Find the value of T.

(4)

(c) Hence find the value of λ

(2)

(Total for Question 3 is 8 marks)

Q4

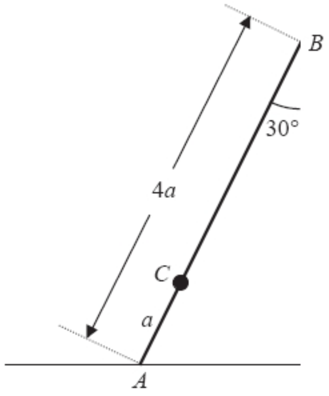


Figure 2

A ladder AB, of mass m and length $4a$, has one end A resting on rough horizontal ground. The other end B rests against a smooth vertical wall. A load of mass $3m$ is fixed on the ladder at the point C, where $AC = a$. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle. The ladder rests in limiting equilibrium making an angle of 30° with the wall, as shown in Figure 2.

Find the coefficient of friction between the ladder and the ground.

(Total for Question 4 is 10 marks)

Gold Mark Scheme
Q1

Q	Scheme	Marks	Notes
a	M(A): $d\cos\theta\times 5g = 4P$	M1	Terms must be dimensionally correct. Condone trig confusion
		A1	
	Resolving horizontally: $P\sin\theta = F$	B1	
	Resolving vertically: $P\cos\theta + R = 5g$	M1	Requires all 3 terms. Condone trig confusion and sign errors
		A1	Correct equation
		DM1	Substitute for P to find R or F Dependent on both previous M marks
	$R = 5g - \dfrac{5gd\cos^2\theta}{4}$	A1	One force correct. Accept equivalent forms e.g. $R = \dfrac{20g - 5gd + 20g\tan^2\theta}{4\left(1+\tan^2\theta\right)}$
	$F = \dfrac{5gd\cos\theta\sin\theta}{4}$	A1	Both forces correct. Accept equivalent forms e.g. $F = \dfrac{5gd\tan\theta}{4\sec^2\theta}$
		(8)	
a alt	M(B): $5g\cos\theta\times(4-d) + F\sin\theta\times 4 = R\cos\theta\times 4$	M1	Needs all three terms. Terms must be dimensionally correct. Condone trig confusion
		A1	At most one error
	Resolve parallel to the rod: $5g\sin\theta = R\sin\theta + F\cos\theta$	M1	Requires all 3 terms. Condone trig confusion and sign errors
		B1	At most one error
		A1	Correct equation
	$\Rightarrow R = 5g - \dfrac{F\cos\theta}{\sin\theta}$		
	$5g\cos\theta\times(4-d) + F\sin\theta\times 4 = 4\cos\theta\left(5g - \dfrac{F\cos\theta}{\sin\theta}\right)$	DM1	Eliminate one variable to find F or R Dependent on both previous M marks
	$4F\left(\sin\theta + \dfrac{\cos^2\theta}{\sin\theta}\right) = 20g\cos\theta - 20g\cos\theta + 5gd\cos\theta$		
	$F = \dfrac{5gd\cos\theta\sin\theta}{4}$	A1	One force correct
	$R = 5g - \dfrac{5gd\cos^2\theta}{4}$	A1	Both forces correct
			See next page for part (b)

Pearson

b	$\mu = \dfrac{\dfrac{5gd\cos\theta\sin\theta}{4}}{5g - \dfrac{5gd\cos^2\theta}{4}}$	M1	Use of $F = \mu R$
	$\dfrac{1}{2}\left(5g - \dfrac{5gd\cos^2\theta}{4}\right) = \dfrac{5gd\cos\theta\sin\theta}{4}$	A1	$\left(4 - d\cos^2\theta = 2d\cos\theta\sin\theta\right)$
	$4\times169 = 120d + 144d$	M1	Use $\tan\theta = \dfrac{5}{12}$ and solve for d
	$d = \dfrac{169}{66}$	A1	(= 2.6 m or better)
		(4)	
balt	$F = 5gd\times\dfrac{12}{13}\times\dfrac{5}{13}\times\dfrac{1}{4}\left(=\dfrac{75gd}{169}\right)$	M1	Use $\tan\theta = \dfrac{5}{12}$
	$R = 5g - \dfrac{5gd}{4}\times\dfrac{144}{169}$	A1	Both unsimplified expressions
	$75gd = \dfrac{1}{2}(5\times169g - 180gd)$	M1	Use of $F = \mu R$ and solve for d
	$150gd + 180gd = 845g$, $d = \dfrac{169}{66}$	A1	(= 2.6 m or better)
		(4)	
balt	$R = 5g - \dfrac{12}{13}P$, $F = \dfrac{5}{13}P$	M1	Substitute trig in their equations from resolving.
	$\dfrac{5}{13}P = \dfrac{1}{2}\left(5g - \dfrac{12}{13}P\right)$	M1	use $F = \mu R$ and solve for d
	$\Rightarrow P = \dfrac{65}{22}g$	A1	
	$d = \dfrac{4P}{5g\cos\theta} = \dfrac{169}{66}$	A1	
		[12]	

Q2

Question	Scheme	Marks	AOs
(a)	$R = mg\cos\alpha$	B1	3.1b
	Resolve parallel to the plane	M1	3.1b
	$-F - mg\sin\alpha = -0.8mg$	A1	1.1b
	$F = \mu R$	M1	1.2
	Produce an equation in μ only and solve for μ	M1	2.2a
	$\mu = \frac{1}{4}$	A1	1.1b
		(6)	
(b)	Compare $\mu mg\cos\alpha$ with $mg\sin\alpha$	M1	3.1b
	Deduce an appropriate conclusion	A1 ft	2.2a
		(2)	
			(8 marks)

Notes:

(a)

B1: for $R = mg\cos\alpha$

1st **M1:** for resolving parallel to the plane

1st **A1:** for a correct equation

2nd **M1:** for use of $F = \mu R$

3rd **M1:** for eliminating F and R to give a value for μ

2nd **A1:** for $\mu = \frac{1}{4}$

(b)

M1: comparing size of limiting friction with weight component down the plane

A1ft: for an appropriate conclusion from their values

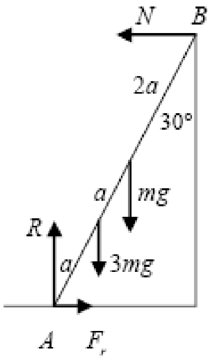
Q3

Question	Scheme	Marks	AOs
(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ or integrate to give: $\mathbf{v} = (-2\mathbf{i} + 2\mathbf{j}) + 2(4\mathbf{i} - 5\mathbf{j})$	M1	3.1a
	$(6\mathbf{i} - 8\mathbf{j})$ (m s^{-1})	A1	1.1b
		(2)	
(b)	Solve problem through use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ or integration (M0 if $\mathbf{u} = \mathbf{0}$) **Or** any other complete method e.g use $\mathbf{v} = \mathbf{u} + \mathbf{a}T$ and $\mathbf{r} = \frac{(\mathbf{u}+\mathbf{v})T}{2}$:	M1	3.1a
	$-4.5\mathbf{j} = 2t\mathbf{j} - \frac{1}{2}t^2 5\mathbf{j}$ (j terms only)	A1	1.1b
	The first two marks could be implied if they go straight to an algebraic equation.		
	Attempt to equate **j** components to give equation in T only $(-4.5 = 2T - \frac{5}{2}T^2)$	M1	2.1
	$T = 1.8$	A1	1.1b
		(4)	
(c)	Solve problem by substituting <u>their</u> T value (M0 if $T < 0$) into the **i** component equation to give an equation in λ only: $\lambda = -2T + \frac{1}{2}T^2 \times 4$	M1	3.1a
	$\lambda = 2.9$ or 2.88 or $\frac{72}{25}$ oe	A1	1.1b
		(2)	
Notes: Accept column vectors throughout			**(8 marks)**

Notes: Accept column vectors throughout **(8 marks)**

2a	M1	For any complete method to give a **v** expression with correct no. of terms with $t = 2$ used, so if integrating, must see the initial velocity as the constant. Allow sign errors.
	A1	Cao isw if they go on to find the speed.
2b	M1	For any complete method to give a vector expression for **j** component of displacement in t (or T) only, using $\mathbf{a} = (4\mathbf{i} - 5\mathbf{j})$, so if integrating, RHS of equation must have the correct structure. Allow sign errors.
	A1	Correct **j** vector equation in t or T. Ignore **i** terms.
	M1	Must have earned 1st M mark. Equate **j** components to give equation in T (allow t) only (no **j**'s) which has come from a displacement. Equation must be a 3 term quadratic in T.
	A1	cao
2c	M1	Must have earned 1st M mark in (b) Complete method - must have an equation in λ only (no **i**'s) which has come from an appropriate displacement.. (e.g M0 if $\mathbf{a} = \mathbf{0}$ has been used) Expression for λ must be a quadratic in T
	A1	cao

Q4

Question Number	Scheme	Marks
	(a) 	
	$\mathcal{M}(A)$ $\quad N\times 4a\cos 30° = 3mg\times a\sin 30° + mg\times 2a\sin 30°$	M1 A2(1,0)
	$N = \frac{5}{4}mg\tan 30°\ \left(= \frac{5}{4\sqrt{3}}mg\ = 7.07\ldots m\right)$	DM1 A1
	$\rightarrow\ F_r = N\ ,\quad \uparrow\ R = 4mg$	B1, B1
	Using $F_r = \mu R$	B1
	$\frac{5}{4\sqrt{3}}mg = \mu R$ for their R	M1
	$\mu = \frac{5}{16\sqrt{3}}$ awrt 0.18	A1 **(10)**
		[10]
	Alternative method: M(B): $mg\times 2a\sin 30 + 3mg\times 3a\sin 30 + F\times 4a\cos 30 = R\times 4a\sin 30$ $11mga\sin 30 + F\times 4a\cos 30 = R\times 4a\sin 30$	M1A3(2,1,0)
	$\frac{11mg}{2} + F\frac{4\sqrt{3}}{2} = 2R$	DM1A1
	$\uparrow\ R = 4mg$,	B1
	Using $F_r = \mu R$	B1
	$8\mu\sqrt{3} = \frac{5}{2}$, $\quad \mu = \frac{5}{16\sqrt{3}}$	M1 A1

Pearson

Topic 6 - Projectiles

Bronze, Silver and Gold Worksheets for A Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis.

They are drawn from the latest specification questions and legacy questions. The papers are between approximately 25 and 45 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel A Level Mathematics: Statistics and Mechanics Year 2' textbook.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculator

The total mark for this section is 38

Q1

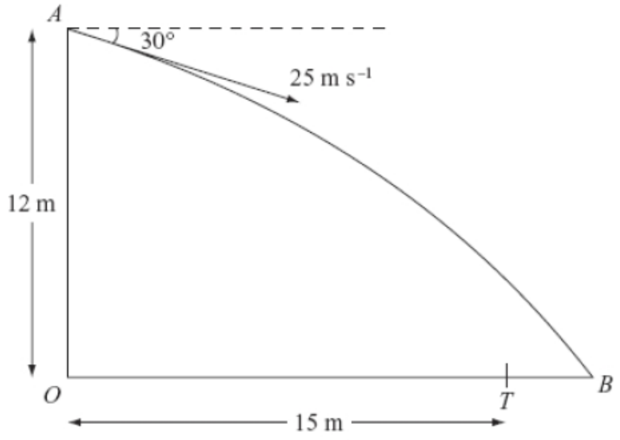


Figure 4

A ball is thrown from a point A at a target, which is on horizontal ground. The point A is 12 m above the point O on the ground. The ball is thrown from A with speed 25 m s^{-1} at an angle of 30° below the horizontal. The ball is modelled as a particle and the target as a point T. The distance OT is 15 m. The ball misses the target and hits the ground at the point B, where OTB is a straight line, as shown in Figure 4. Find

(a) the time taken by the ball to travel from A to B,

(5)

(b) the distance TB.

(4)

The point X is on the path of the ball vertically above T.

(c) Find the speed of the ball at X.

(5)

(Total for Question 1 is 14 marks)

Q2

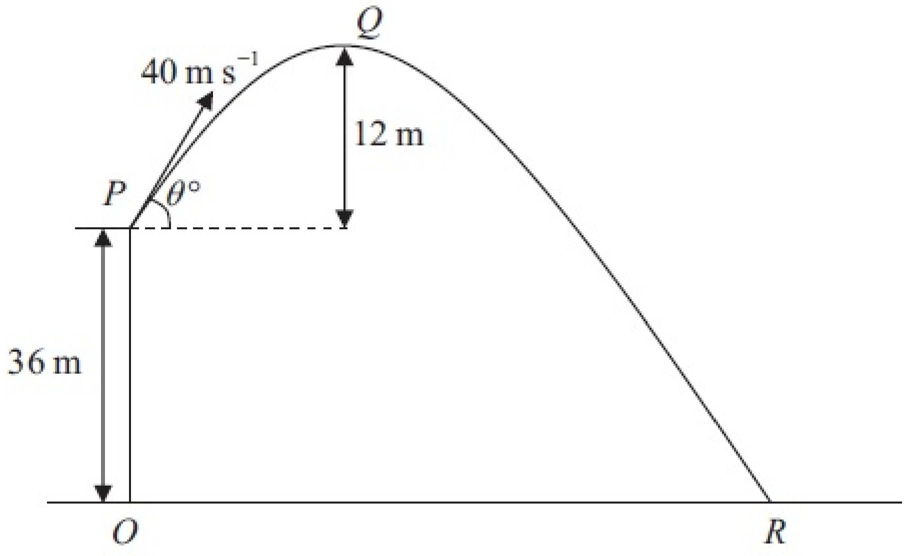


Figure 3

A ball is projected with speed 40 m s^{-1} from a point *P* on a cliff above horizontal ground. The point *O* on the ground is vertically below *P* and *OP* is 36 m. The ball is projected at an angle θ° to the horizontal. The point *Q* is the highest point of the path of the ball and is 12 m above the level of *P*. The ball moves freely under gravity and hits the ground at the point *R*, as shown in Figure 3. Find

(a) the value of θ,

(3)

(b) the distance *OR*.

(6)

(Total for Question 2 is 9 marks)

Q3

[In this question, the unit vectors **i** *and* **j** *are horizontal and vertical respectively.]*

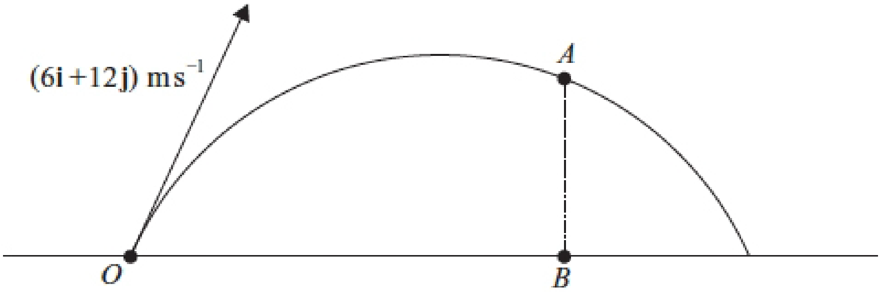


Figure 3

The point O is a fixed point on a horizontal plane. A ball is projected from O with velocity $(6\mathbf{i} + 12\mathbf{j})$ m s^{-1}, and passes through the point A at time t seconds after projection. The point B is on the horizontal plane vertically below A, as shown in Figure 3. It is given that $OB = 2AB$.

Find

(a) the value of t,

(7)

(b) the speed, V m s^{-1}, of the ball at the instant when it passes through A.

(5)

At another point C on the path the speed of the ball is also V m s^{-1}.

(c) Find the time taken for the ball to travel from O to C.

(3)

(Total for Question 3 is 15 marks)

End of questions

Bronze Mark Scheme

Q1

Question Number	Scheme		Marks
(a)	$(\downarrow)$ $u_y = 25\sin 30° \ (=12.5)$		B1
	$12 = 12.5t + 4.9t^2$	−1 each error	M1 A2, 1, 0
	Leading to $t = 0.743$, 0.74		A1 (5)
(b)	$(\rightarrow)$ $u_x = 25\cos 30° \left(= \frac{25\sqrt{3}}{2} \approx 21.65\right)$		B1
	$OB = 25\cos 30° \times t \ (\approx 16.094\,58)$	ft their (a)	M1 A1ft
	$TB \approx 1.1$ (m)	awrt 1.09	A1 (4)
(c)	$(\rightarrow)$ $15 = u_x \times t \Rightarrow t = \frac{15}{u_x} (= \frac{2\sqrt{3}}{5} \approx 0.693 \text{ or } 0.69)$		M1 A1
	either $(\downarrow)$ $v_y = 12.5 + 9.8t \ (\approx 19.2896)$		M1
	$V^2 = u_x^2 + v_y^2 \ (\approx 840.840)$		
	$V \approx 29 \ (\text{ms}^{-1})$, 29.0		M1 A1 (5)
			(14 marks)

Q2

Question Number	Scheme	Marks
(a)	Vertical motion: $v^2 = u^2 + 2as$	M1
	$(40\sin\theta)^2 = 2 \times g \times 12$	A1
	$(\sin\theta)^2 = \frac{2 \times g \times 12}{40^2}$	
	$\theta = 22.54 = 22.5°$ (accept 23)	A1 (3)
(b)	Vert motion $P \rightarrow R$: $s = ut + \frac{1}{2}at^2$	
	$-36 = 40\sin\theta t - \frac{g}{2}t^2$	M1
	$\frac{g}{2}t^2 - 40\sin\theta t - 36 = 0$	A1 A1
	$t = \frac{40\sin 22.54 \pm \sqrt{(40\sin 22.54)^2 + 4 \times 4.9 \times 36}}{9.8}$	
	$t = 4.694...$	A1
	Horizontal P to R: $s = 40\cos\theta t$	M1
	$= 173$ m (or 170 m)	A1 (6)

(9 marks)

Pearson

Q3

Question Number	Scheme	Marks
(a)	$\mathbf{i} \rightarrow$ distance $= 6t$	B1
	$\mathbf{j} \uparrow$ distance $= 12t - \frac{1}{2}gt^2$	M1 A1
	At B, $2\left(12t - \frac{1}{2}gt^2\right) = 6t$	M1 A1
	$(24-6)t = gt^2$	DM1
	$18 = gt$, $t = \frac{18}{g}(=1.84\text{s})$	A1
		(7)
(b)	$\mathbf{i} \rightarrow$ speed $= 6$	B1
	$\mathbf{j} \uparrow$ velocity $= 12 - gt = -6$	M1 A1
	$\therefore$ speed at A	
	$= \sqrt{6^2 + 6^2} = \sqrt{72} = 6\sqrt{2}(=8.49)(\text{ms}^{-1})$	M1 A1
		(5)
(c)	$\uparrow$ speed $= 12 - gt = +6$	M1 A1 ft
	$t = \frac{6}{g}(=0.61\text{s})$	A1
		(3)
		15

Silver Questions

Calculator

The total mark for this section is 45

Q1

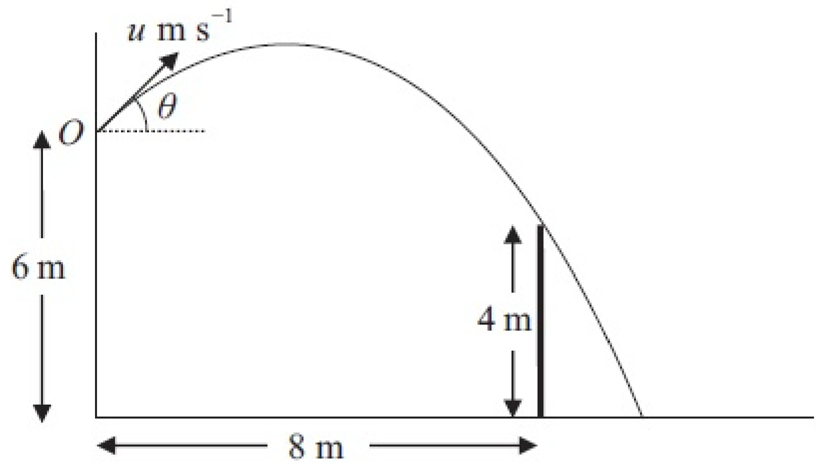


Figure 2

A ball is thrown from a point O, which is 6 m above horizontal ground. The ball is projected with speed u m s^{-1} at an angle θ above the horizontal. There is a thin vertical post which is 4 m high and 8 m horizontally away from the vertical through O, as shown in Figure 2. The ball passes just above the top of the post 2 s after projection. The ball is modelled as a particle.

(a) Show that $\tan \theta = 2.2$

(5)

(b) Find the value of u.

(2)

The ball hits the ground T seconds after projection.

(c) Find the value of T.

(3)

Immediately before the ball hits the ground the direction of motion of the ball makes an angle α with the horizontal.

(d) Find α.

(5)

(Total for Question 1 is 15 marks)

Q2

A particle is projected from a point O with speed u at an angle of elevation α above the horizontal and moves freely under gravity. When the particle has moved a horizontal distance x, its height above O is y.

(a) Show that

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

(4)

A girl throws a ball from a point A at the top of a cliff. The point A is 8 m above a horizontal beach. The ball is projected with speed 7 m s^{-1} at an angle of elevation of 45º. By modelling the ball as a particle moving freely under gravity,

(b) find the horizontal distance of the ball from A when the ball is 1 m above the beach.

(5)

A boy is standing on the beach at the point B vertically below A. He starts to run in a straight line with speed v m s^{-1}, leaving B 0.4 seconds after the ball is thrown.

He catches the ball when it is 1 m above the beach.

(c) Find the value of v.

(4)

(Total for Question 2 is 13 marks)

Q3

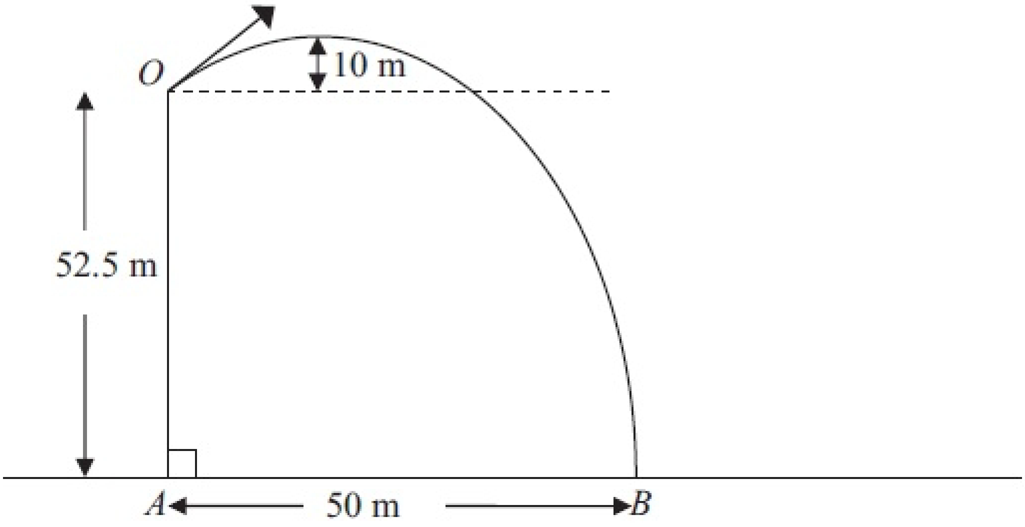


Figure 4

A small stone is projected from a point O at the top of a vertical cliff OA. The point O is 52.5 m above the sea. The stone rises to a maximum height of 10 m above the level of O before hitting the sea at the point B, where $AB = 50$ m, as shown in Figure 4. The stone is modelled as a particle moving freely under gravity.

(a) Show that the vertical component of the velocity of projection of the stone is 14 m s^{-1}.

(3)

(b) Find the speed of projection.

(9)

(c) Find the time after projection when the stone is moving parallel to OB.

(5)

(Total for Question 3 is 17 marks)

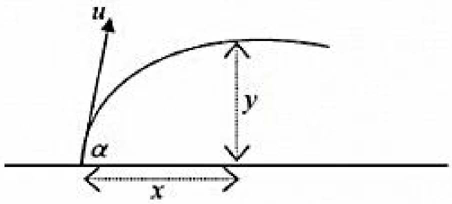
End of questions

Silver Mark Scheme

Q1

Q.	Scheme	Marks	
(a)	$2 = -2u\sin\theta + \frac{1}{2}g \times 4$	M1	Vertical distance. Condone sign errors. Must have used $t = 2$, but could be using $u_y = u\sin\theta$
	$\left(-2 = u\sin\theta t - \frac{1}{2}gt^2\right)$	A1	All correct
	$u\sin\theta = g - 1$		
	$2u\cos\theta = 8 \quad (u\cos\theta = 4)$ $(u\cos\theta t = 8)$	B1	Horizontal distance. Accept $u_x = 4$ o.e.
	$\tan\theta = \frac{g-1}{4} = 2.2$ *	M1	Divide to obtain expression for $\tan\theta$
		A1	**Given answer** It is acceptable to quote and use the equation for the projectile path. Incorrect equation is 0/5.
(b)	$u\cos\theta = 4$	M1	Use the horizontal distance and θ to find u
	$u = \frac{4}{\cos\theta} = 9.66... = 9.7$	A1	9.67 or 9.7 NB $\theta = 65.6°$ leading to 9.68 is an accuracy penalty.
	OR use components from (a) and Pythagoras.		
(c)	$6 = (1-g)T + \frac{1}{2} \times 9.8T^2$	M1	Equation for vertical distance $= \pm 6$ to give a quadratic in T. Allow their u_y
	$4.9T^2 - 8.8T - 6 = 0$		
	$T = \frac{8.8 \pm \sqrt{[(-)8.8]^2 + 24 \times 4.9}}{9.8}$	DM1	Solve a 3 term quadratic
	$T = 2.323... = 2.32$ or 2.3	A1	2.3 or 2.32 only
(d)	$v^2 = 8.8^2 + 2g \times 6$ or $v = -8.8 + gT$	M1	Use *suvat* to find vertical speed
		A1	Correct equation their u_y, T
	$v = 13.96...$		
	Horiz speed = 4		
	$\tan\alpha = \frac{v}{4}$	DM1	Correct trig. with their vertical speed to find the required angle.
		A1	Correct equation
	$\alpha = 74.01... = 74°$	A1	$74°$ or $74.0°$. Allow 106.

Q2

Question Number	Scheme	Marks
(a)		
	Horiz: $x = u\cos\alpha t$	B1
	Vert: $y = u\sin\alpha t - \frac{1}{2}gt^2$	M1
	$y = u\sin\alpha \times \frac{x}{u\cos\alpha} - \frac{1}{2}g \times \frac{x^2}{u^2\cos^2\alpha}$	DM1
	$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$ **	A1
		(4)
(b)	$y = -7:\quad -7 = \tan 45x - \frac{gx^2}{2\times 7^2\cos^2 45}$	M1 A1
	$-7 = x - \frac{9.8x^2}{7^2}$	
	$-7 = x - \frac{x^2}{5}$	M1
	$x^2 - 5x - 35 = 0$	
	$x = \frac{5 \pm \sqrt{25 + 4\times 35}}{2}$	M1
	$x = 8.92$ or 8.9	A1
		(5)
(c)	Time to travel 8.922 m horizontally $= \frac{8.922}{7\cos 45} = 1.802...\text{s}$	M1
	$v = \frac{8.922}{1.402}$	M1 A1 ft
	$= 6.36$ or 6.4 (m s^{-1})	A1
		(4)
		13

Q3

Question Number	Scheme	Marks	
(a)	$0^2 = u_V^2 - 2 \times 9.8 \times 10$	M1	Complete method using *suvat* to form an equation in u_V.
		A1	Correct equation e.g. $0 = u^2 - 20g$
	$u_V = 14$ *	A1	*Answer given* requires equation and working, including 196, seen.
		(3)	
(b)	$(\uparrow), \; -52.5 = 14t - \frac{1}{2}gt^2$	M1	Use the vertical distance travelled to find the total time taken.
		A1	At most one error
		A1	Correct equation
	$49t^2 - 140t - 525 = 0$	DM1	Solve for t. Dependent on the preceding M mark
	$(t-5)(49t+105) = 0 \quad t = 5$	A1	only
	$(\rightarrow), \quad 50 = 5u_H$	M1	Use their time of flight to form an equation in u_H
	$u_H = 10$	A1	only
	$u = \sqrt{10^2 + 14^2}$	M1	Use of Pythagoras with two non-zero components, or solution of a pair of simultaneous equations in u and α.
	$= \sqrt{296}$; 17.2 m s^{-1}	A1	17.2 or 17 (method involves use of g = 9.8 so an exact surd answer is not acceptable)
		(9)	
OR			First 3 marks for the quadratic as above.
	$50 = u\cos\alpha t$ or $50 = u_H t$	M1	Used in their quadratic
	$49\left(\frac{50}{u_H}\right)^2 - 140\left(\frac{50}{u_H}\right) - 525 = 0$	A1	Correct quadratic in u_H
	$525(u_H)^2 + 140(u_H) - 122500 = 0$		
	Solve for u_H	DM1	Dependent on the M mark for setting up the initial quadratic equation in t.
	$u_H = 10$	A1	only
	etc.		Complete as above.
(c)	$\tan OBA = \frac{52.5}{50} = 1.05$	B1	Correct direction o.e. (accept reciprocal)
	$v_V = 1.05 \times 10 = 10.5$	M1	Use trig. with their u_H and correct interpretation of direction to find the vertical component of speed. Working with distances is M0. (condone $10 \div 1.05$)
		DM1	Use suvat to form an equation in t. Dependent on the preceding M.
	$(\uparrow), \; -10.5 = 14 - gt$	A1	Correct equation for their u_H. For incorrect direction give A0 here.
	$t = 2.5$	A1	only
		(5)	
		17	

Gold Questions

40 Marks

Calculator

The total mark for this section is 40

Q1.

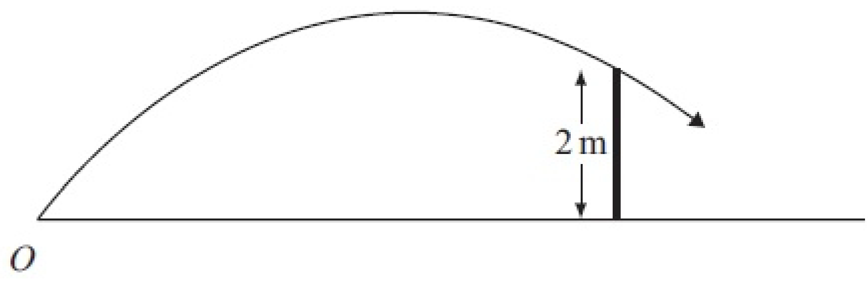


Figure 3

A child playing cricket on horizontal ground hits the ball towards a fence 10 m away. The ball moves in a vertical plane which is perpendicular to the fence. The ball just passes over the top of the fence, which is 2 m above the ground, as shown in Figure 3.

The ball is modelled as a particle projected with initial speed u m s^{-1} from point O on the ground at an angle α to the ground.

(a) By writing down expressions for the horizontal and vertical distances, from O of the ball t seconds after it was hit, show that

$$2 = 10\tan\alpha - \frac{50g}{u^2\cos^2\alpha}.$$

(6)

Given that $\alpha = 45°$,

(b) find the speed of the ball as it passes over the fence.

(6)

(Total for Question 1 is 12 marks)

Q2

[In this question, the unit vectors **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertically upwards.]*

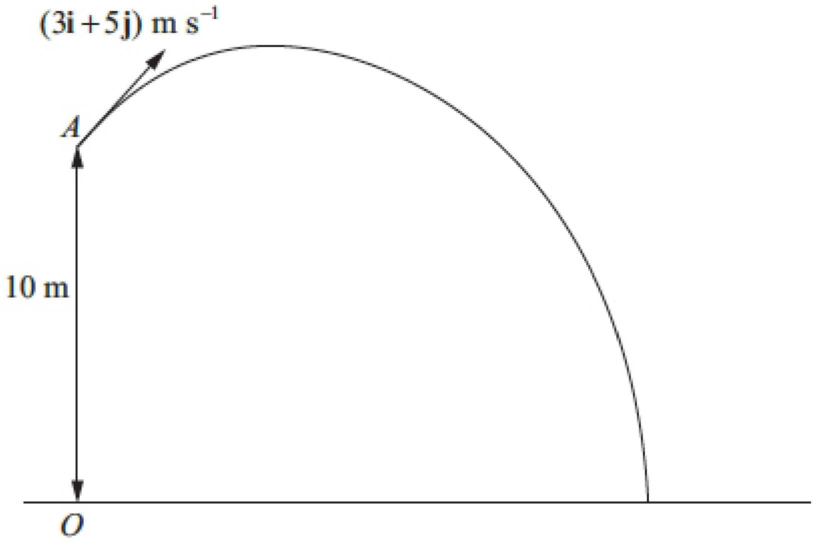


Figure 1

At time $t = 0$, a particle P is projected from the point A which has position vector 10**j** metres with respect to a fixed origin O at ground level. The ground is horizontal. The velocity of projection of P is $(3\mathbf{i} + 5\mathbf{j})$ m s^{-1}, as shown in Figure 1. The particle moves freely under gravity and reaches the ground after T seconds.

(a) For $0 \leqslant t \leqslant T$, show that, with respect to O, the position vector, **r** metres, of P at time t seconds is given by

$$\mathbf{r} = (3t)\mathbf{i} + (10 + 5t - 4.9t^2)\mathbf{j}\ .$$

(3)

(b) Find the value of T.

(3)

(c) Find the velocity of P at time t seconds ($0 \leqslant t \leqslant T$).

(2)

When P is at the point B, the direction of motion of P is 45° below the horizontal.

(d) Find the time taken for P to move from A to B.

(2)

(e) Find the speed of P as it passes through B.

(2)

(Total for Question 2 is 12 marks)

Q3

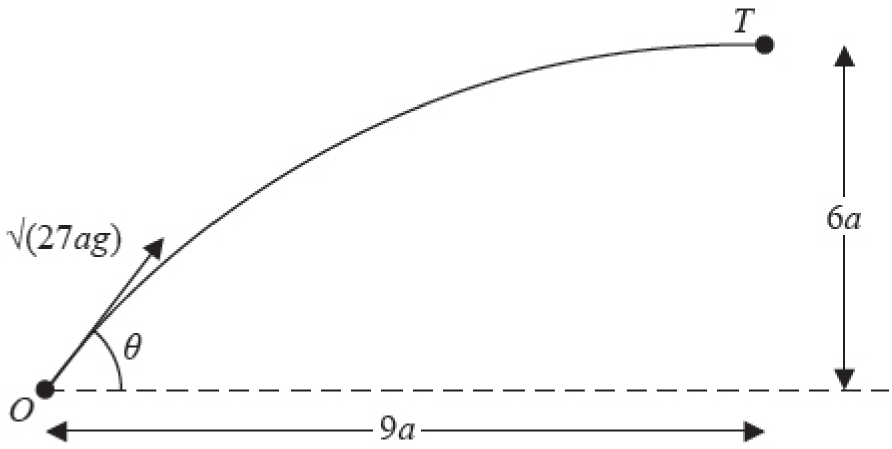


Figure 4

A small ball is projected from a fixed point O so as to hit a target T which is at a horizontal distance $9a$ from O and at a height $6a$ above the level of O. The ball is projected with speed $\sqrt{(27ag)}$ at an angle θ to the horizontal, as shown in Figure 4. The ball is modelled as a particle moving freely under gravity.

(a) Show that $\tan^2\theta - 6\tan\theta + 5 = 0$

(7)

The two possible angles of projection are θ_1 and θ_2, where $\theta_1 > \theta_2$.

(b) Find $\tan\theta_1$ and $\tan\theta_2$.

(3)

The particle is projected at the larger angle θ_1.

(c) Show that the time of flight from O to T is $\sqrt{\left(\frac{78a}{g}\right)}$.

(3)

(d) Find the speed of the particle immediately before it hits T.

(3)

(Total for Question 3 is 16 marks)

End of questions

Gold Mark Scheme

Q1

Question Number	Scheme	Marks
(a)	$\rightarrow \quad x = u\cos\alpha t = 10$	M1A1
	$\uparrow \quad y = u\sin\alpha t - \frac{1}{2}gt^2 = 2$	M1A1
	$\Rightarrow t = \frac{10}{u\cos\alpha}$	
	$2 = u\sin\alpha \times \frac{10}{u\cos\alpha} - \frac{g}{2} \times \frac{100}{u^2\cos^2\alpha}$	M1
	$= 10\tan\alpha - \frac{50g}{u^2\cos^2\alpha}$ (given answer)	A1
		(6)
(b)	$2 = 10\times1 - \frac{100g\times2}{2u^2\times1}$	M1A1
	$u^2 = \frac{100g}{8},\ u = \sqrt{\frac{100g}{8}} = 11.1$ (m s^{-1})	A1
	$\frac{1}{2}mu^2 = m\times9.8\times2 + \frac{1}{2}mv^2$	M1A1
	$v = 9.1ms^{-1}$	A1
		(6)
		[12]

Q2

(a)	Using $s = ut + \frac{1}{2}at^2$ Method must be clear	M1
	$\mathbf{r} = (3t)\mathbf{i} + (10 + 5t - 4.9t^2)\mathbf{j}$ Answer given	A1 A1
		(3)
(b)	$\mathbf{j}$ component $= 0$: $10 + 5t - 4.9t^2$	M1
	quadratic formula: $t = \dfrac{5 \pm \sqrt{25+196}}{9.8} = \dfrac{5 \pm \sqrt{221}}{9.8}$	DM1
	$T = 2.03$(s), 2.0 (s) positive solution only.	A1
		(3)
(c)	Differentiating the position vector (or working from first principles)	M1
	$\mathbf{v} = 3\mathbf{i} + (5 - 9.8t)\mathbf{j}$ (ms^{-1})	A1
		(2)
(d)	At B the $\mathbf{j}$ component of the velocity is the negative of the $\mathbf{i}$ component: $5 - 9.8t = -3$, $8 = 9.8t$,	M1
	$t = 0.82$	A1
		(2)
(e)	$\mathbf{v} = 3\mathbf{i} - 3\mathbf{j}$, speed $= \sqrt{3^2 + 3^2} = \sqrt{18} = 4.24$ (m s^{-1})	M1A1
		(2)
		[12]

Q3

Question Number	Scheme	Marks	Notes
(a)	$(\rightarrow)\sqrt{27ag}\cos\theta.\ t = 9a$	M1	Horizontal motion. Condone trig confusion.
		A1	
	$(\uparrow)\sqrt{27ag}\sin\theta.\ t - \frac{1}{2}gt^2 = 6a$	M1	Vertical motion. Condone sign errors and trig confusion.
		A1	
	$(\uparrow)\sqrt{27ag}\sin\theta.\ \frac{9a}{\sqrt{27ag}\cos\theta} - \frac{1}{2}g\left(\frac{9a}{\sqrt{27ag}\cos\theta}\right)^2 = 6a$	DM1	Substitute for t (unsimplified). Dependent on both previous M marks
	$9a\tan\theta - \frac{1}{2}g.81a^2\frac{(1+\tan^2\theta)}{27ag} = 6a$	DM1	Express all trig terms in terms of tan. Dependent on preceding M.
	$\tan^2\theta - 6\tan\theta + 5 = 0$	A1 (7)	
(b)	$\tan^2\theta - 6\tan\theta + 5 = 0$		
	$(\tan\theta - 1)(\tan\theta - 5) = 0$	M1	Method to find one root of the quadratic
	$\tan\theta_2 = 1$ or $\tan\theta_1 = 5$	A1 A1 (3)	
(c)	$t = \frac{9a}{\sqrt{27ag}\cos\theta} = \frac{9a}{\sqrt{27ag}} \times \frac{\sqrt{26}}{1}$	M1 A1ft	Use $\tan\theta =$ their 5 to find t. Correct unsimplified. Correct $\cos\theta$ for their $\tan\theta$
	$= \sqrt{\frac{81a^2.26}{27a}} = \sqrt{\frac{78a}{g}}$ ***Answer given***	A1 (3)	Given answer →evidence of working is required

Question Number	Scheme	Marks	Notes
Question continued...			
(d)	$\frac{1}{2}m(27ag - v^2) = mg6a$	M1 A1	Conservation of energy. Requires all 3 terms. Condone sign error Correct equation
	$v = \sqrt{15ag}$	A1 (3)	
Or (d)	$v^2 = \left(\sqrt{27ag}\cos\theta\right)^2 + \left(\sqrt{27ag}\sin\theta - g.\sqrt{\frac{78a}{g}}\right)^2$	M1	Horizontal and vertical components and Pythagoras. Condone trig confusion.
	$= \left(\frac{27ag}{26}\right) + \left(5\sqrt{\frac{27ag}{26}} - \sqrt{78ag}\right)^2 \left(= ag\left(\frac{27}{26} + \frac{363}{26}\right)\right)$	A1	Correctly substituted
	$v = \sqrt{15ag}$	A1 (3)	
		[16]	

Topic 7 - Applications of Forces

Bronze, Silver and Gold Worksheets for A Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis.

They are drawn from the latest specification questions and legacy questions. The papers are between approximately 25 and 45 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel A Level Mathematics: Statistics and Mechanics Year 2' textbook.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculator

The total mark for this section is 38

Q1

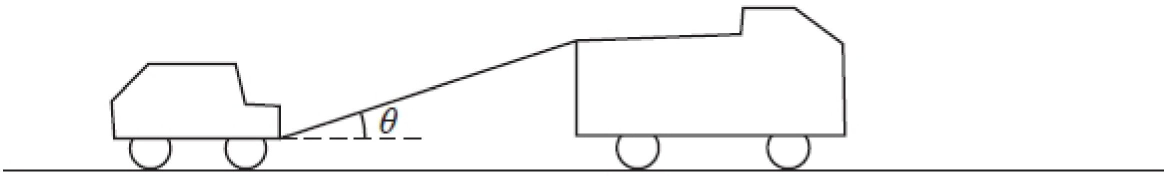


Figure 4

A truck of mass 1750 kg is towing a car of mass 750 kg along a straight horizontal road. The two vehicles are joined by a light towbar which is inclined at an angle θ to the road, as shown in Figure 4. The vehicles are travelling at 20 m s^{-1} as they enter a zone where the speed limit is 14 m s^{-1}. The truck's brakes are applied to give a constant braking force on the truck. The distance travelled between the instant when the brakes are applied and the instant when the speed of each vehicle is 14 m s^{-1} is 100 m.

(a) Find the deceleration of the truck and the car.

(3)

The constant braking force on the truck has magnitude R Newtons. The truck and the car also experience constant resistances to motion of 500 N and 300 N respectively. Given that $\cos \theta = 0.9$, find

(b) the force in the towbar,

(4)

(c) the value of R.

(4)

(Total for Question 1 is 11 marks)

Q2

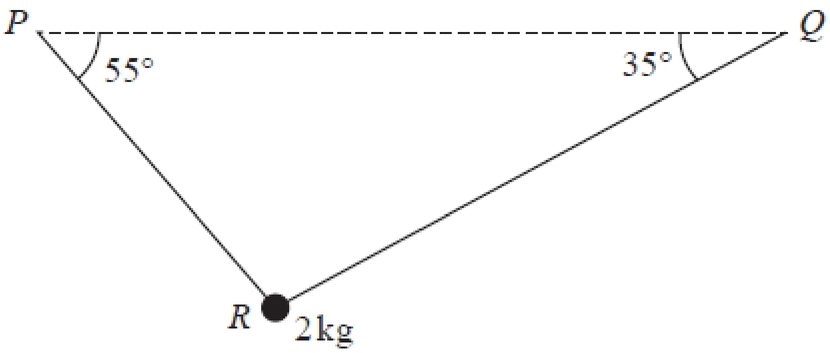


Figure 1

A particle of mass 2 kg is suspended from a horizontal ceiling by two light inextensible strings, *PR* and *QR*. The particle hangs at *R* in equilibrium, with the strings in a vertical plane. The string *PR* is inclined at 55° to the horizontal and the string *QR* is inclined at 35° to the horizontal, as shown in Figure 1.

Find

(i) the tension in the string *PR*,

(ii) the tension in the string *QR*.

(Total for Question 2 is 7 marks)

Q3

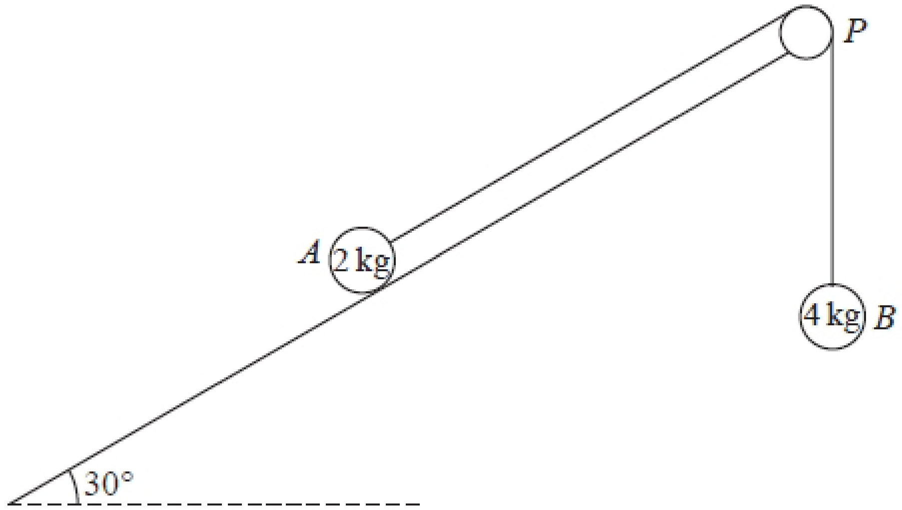


Figure 2

A fixed rough plane is inclined at 30° to the horizontal. A small smooth pulley P is fixed at the top of the plane. Two particles A and B, of mass 2 kg and 4 kg respectively, are attached to the ends of a light inextensible string which passes over the pulley P. The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs freely below P, as shown in Figure 2. The coefficient of friction between A and the plane is $\frac{1}{\sqrt{3}}$. Initially A is held at rest on the plane. The particles are released from rest with the string taut and A moves up the plane.

Find the tension in the string immediately after the particles are released.

(9)

(Total for Question 3 is 9 marks)

Q4

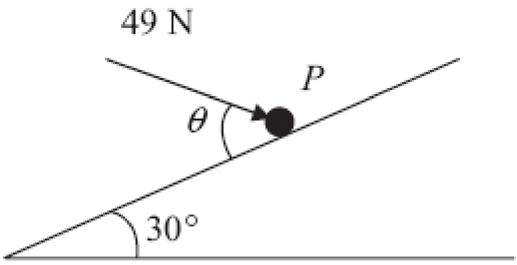


Figure 1

A particle P of mass 6 kg lies on the surface of a smooth plane. The plane is inclined at an angle of 30° to the horizontal. The particle is held in equilibrium by a force of magnitude 49 N, acting at an angle θ to the plane, as shown in Figure 1. The force acts in a vertical plane through a line of greatest slope of the plane.

(a) Show that $\cos\theta = \frac{3}{5}$.

(3)

(b) Find the normal reaction between P and the plane.

(4)

The direction of the force of magnitude 49 N is now changed. It is now applied horizontally to P so that P moves up the plane. The force again acts in a vertical plane through a line of greatest slope of the plane.

(c) Find the initial acceleration of P.

(4)

(Total for Question 4 is 11 marks)

End of questions

Bronze Mark Scheme

Q1

Question Number	Scheme	Marks
(a)	Use of $v^2 = u^2 + 2as$ $14^2 = 20^2 - 2a \times 100$ Deceleration is 1.02(m s^{-2})	M1 A1 A1 **(3)**
(b)	Horizontal forces on the car: $\pm T\cos\theta - 300 = 750 \times -1.02 = -765$ $T = -1550/3$ The force in the tow-bar is 1550/3, 520 (N) or better (allow –ve answer)	M1A2 **f.t.** A1 **(4)**
(c)	Horizontal forces on the truck: $\pm T\cos\theta - 500 - R = 1750 \times -1.02$ Braking force $R =$ 1750 (N)	M1A2 **f.t.** A1 **(4)** **[11]**
	ALT: Whole system: $800 + R = 2500 \times 1.02$ $R = 1750$	M1A2 **f.t.** A1

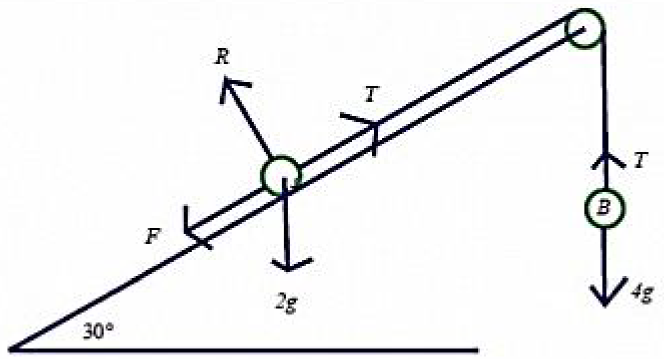
Notes for Question		
Q (a)	M1 for a complete method to produce an equation in *a* only. First A1 for a correct equation. Second A1 for 1.02 (ms^{-2}) oe. must be POSITIVE.	
Q (b)	M1 for considering *the car ONLY* horizontally to produce an equation in *T* only, with usual rules. i.e. correct no. of terms AND *T* resolved: $\pm T\cos\theta - 300 = 750$ x -1.02 A2 **ft** on their *a* for a correct equation (300 and *a* must have *same* sign); -1 each error (treat cos 0.9 as an A error) A1 for 1550/3 oe, 520 or better (N) N.B. Allow a negative answer.	
Q (c)	M1 for considering *the truck ONLY* horizontally to produce an equation, with usual rules. i.e. correct no. of terms AND *T* resolved: $\pm T\cos\theta - 500 - R = 1750$ x -1.02 A2 **ft** on their *T* and *a* for a correct equation (500, *a* and *R* must have *same* sign); -1 each error (treat cos 0.9 as an A error) A1 for 1750 (N). **OR** M1 for considering *the whole system* to produce an equation in *R* only, with usual rules. i.e. correct no. of terms. A2 **ft** on their *a* for a correct equation (*a* and *R* must have *same* sign) -1 each error A1 for 1750 (N). N.B. If 300 and 500 are given separately, penalise any sign errors only ONCE.	

Q2

Question Number	Scheme	Marks
	$T_P \cos 55 = T_Q \cos 35$ $T_P \sin 55 + T_Q \sin 35 = 2g$ Eliminating T_P or T_Q $T_P = 16\text{N}$ or 16.1N; $T_Q = 11\text{N}$ or 11.2N	M1 A1 M1 A1 M1 A1 A1 7
ALT 1	(Along RP) $T_P = 2g\cos 35° = 16\text{N}$ or 16.1N (Along RQ) $T_Q = 2g\cos 55° = 11\text{N}$ or 11.2N	M1 M1 A1 A1 M1 A1 A1
	Notes	
	First M1 for resolving horizontally with correct no. of terms and both T_P and T_Q terms resolved. (M0 if they assume $T_P = T_Q$) First A1 for a correct equation. Second M1 for resolving vertically with correct no. of terms and both T_P and T_Q terms resolved. (M0 if they assume $T_P = T_Q$) Second A1 for a correct equation. Third M1 (independent) for eliminating either T_P or T_Q <u>Third</u> A1 for T_P = 16 (N) or 16.1 (N) <u>Fourth</u> A1 for T_Q = 11 (N) or 11.2 (N) N.B. If both are given to more than 3SF, deduct the third A1.	
ALT 1	**<u>Alternative 1 (resolving along each string)</u>** First M2 for resolving along one of the strings (e.g. T_P=2gcos35°) First A1 for a correct equation (T_P=2gsin35° scores M2A0A0) <u>Third</u> A1 for T_P = 16 (N) or 16.1 (N) Third M1 for resolving along the other string (e.g. T_Q=2gcos55°) Second A1 for a correct equation (T_Q=2gsin55° scores M1A0A0) <u>Fourth</u> A1 for T_Q = 11 (N) or 11.2 (N)	

Question Number	Scheme	Marks
	$T_P \cos 55 = T_Q \cos 35$ $T_P \sin 55 + T_Q \sin 35 = 2g$ Eliminating T_P or T_Q $T_P = 16\text{N}$ or 16.1N; $T_Q = 11\text{N}$ or 11.2N	M1 A1 M1 A1 M1 A1 A1 7
ALT 1	$(\text{Along } RP)\ T_P = 2g\cos 35° = 16\text{N}$ or 16.1N $(\text{Along } RQ)\ T_Q = 2g\cos 55° = 11\text{N}$ or 11.2N	M1 M1 A1 A1 M1 A1 A1
	Notes	
	First M1 for resolving horizontally with correct no. of terms and both T_P and T_Q terms resolved. (M0 if they assume $T_P = T_Q$) First A1 for a correct equation. Second M1 for resolving vertically with correct no. of terms and both T_P and T_Q terms resolved. (M0 if they assume $T_P = T_Q$) Second A1 for a correct equation. Third M1 (independent) for eliminating either T_P or T_Q Third A1 for T_P = 16 (N) or 16.1 (N) Fourth A1 for T_Q = 11 (N) or 11.2 (N) N.B. If both are given to more than 3SF, deduct the third A1.	
ALT 1	**Alternative 1 (resolving along each string)** First M2 for resolving along one of the strings (e.g. $T_P = 2g\cos 35°$) First A1 for a correct equation ($T_P = 2g\sin 35°$ scores M2A0A0) Third A1 for T_P = 16 (N) or 16.1 (N) Third M1 for resolving along the other string (e.g. $T_Q = 2g\cos 55°$) Second A1 for a correct equation ($T_Q = 2g\sin 55°$ scores M1A0A0) Fourth A1 for T_Q = 11 (N) or 11.2 (N)	

Q3

Question Number	Scheme	Marks
		
	Equation of motion of B: $4g - T = 4a$	M1A1
	Equation of motion of A: $T - F - 2g\sin 30 = 2a$	M1A2
	OR: $4g - F - 2g\sin 30 = 6a$	
	Resolve perpendicular to the plane at A: $R = 2g\cos 30$	B1
	Use of $F = \mu R$: $F = \frac{1}{\sqrt{3}} \times 2g\cos 30 (= g)$	M1
	$T - g - g = T - 2g = 2a$	
	$2T - 4g = 4g - T$, $3T = 8g$, $T = \frac{8g}{3} (\approx 26)$ 26.1(N)	DM1A1
		(9) [9]

Notes for Question

	First M1 for resolving vertically (up or down) for *B*, with correct no. of terms. First A1 for a correct equation. Second M1 for resolving parallel to the plane (up or down) for *A*, with correct no. of terms. A2 for a correct equation (-1 each error) **OR**: M2 A3 for the whole system equation - any method error loses all the marks. B1 for perpendicular resolution Third M1 for sub for R in $F = \mu R$ Fourth DM1, dependent on first and second M marks, for eliminating a. Fourth A1 for 8g/3, 26.1 or 26 (N). (392/15 oe is A0)	

Q4

Question Number	Scheme	Marks
(a)	R (// plane): $49\cos\theta = 6g\sin 30$	M1 A1
	$\Rightarrow \cos\theta = 3/5$ *	A1 (3)
(b)	R (perp to plane): $R = 6g\cos 30 + 49\sin\theta$	M1 A1
	$R \approx$ <u>90.1 or 90 N</u>	DM1 A1 (4)
(c)	R (// to plane): $49\cos 30 - 6g\sin 30 = 6a$	M1 A2,1,0
	$\Rightarrow a \approx 2.17$ or $2.2\ \text{m s}^{-2}$	A1 (4)
		11

Silver Questions

Calculator

The total mark for this section is 35

Q1

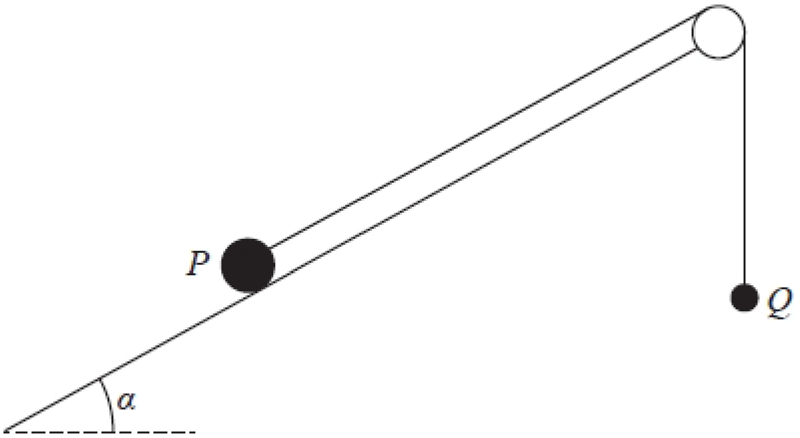


Figure 4

Two particles P and Q have mass 4 kg and 0.5 kg respectively. The particles are attached to the ends of a light inextensible string. Particle P is held at rest on a fixed rough plane, which is inclined to the horizontal at an angle α where $\tan\alpha = \frac{4}{3}$. The coefficient of friction between P and the plane is 0.5. The string lies along the plane and passes over a small smooth light pulley which is fixed at the top of the plane. Particle Q hangs freely at rest vertically below the pulley. The string lies in the vertical plane which contains the pulley and a line of greatest slope of the inclined plane, as shown in Figure 4. Particle P is released from rest with the string taut and slides down the plane.

Given that Q has not hit the pulley, find

(a) the tension in the string during the motion,

(11)

(b) the magnitude of the resultant force exerted by the string on the pulley.

(4)

(Total for Question 1 is 15 marks)

Q2

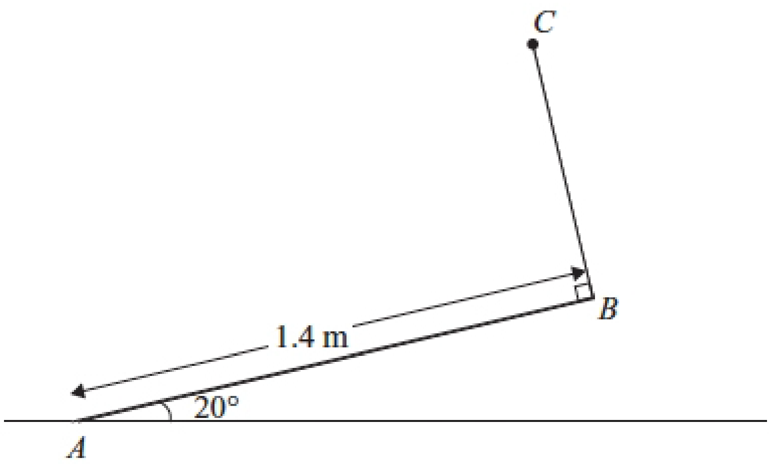


Figure 2

A uniform rod *AB* has mass 4 kg and length 1.4 m. The end *A* is resting on rough horizontal ground. A light string *BC* has one end attached to *B* and the other end attached to a fixed point *C*. The string is perpendicular to the rod and lies in the same vertical plane as the rod. The rod is in equilibrium, inclined at 20° to the ground, as shown in Figure 2.

(a) Find the tension in the string.

(4)

Given that the rod is about to slip,

(b) find the coefficient of friction between the rod and the ground.

(7)

(Total for Question 2 is 11 marks)

Q3

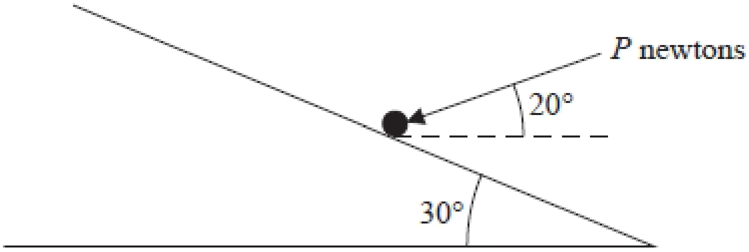


Figure 1

A particle of mass 2 kg lies on a rough plane. The plane is inclined to the horizontal at 30°.

The coefficient of friction between the particle and the plane is $\frac{1}{4}$. The particle is held in equilibrium by a force of magnitude P Newtons. The force makes an angle of 20° with the horizontal and acts in a vertical plane containing a line of greatest slope of the plane, as shown in Figure 1. Find the least possible value of P.

(Total for Question 3 is 10 marks)

Q4

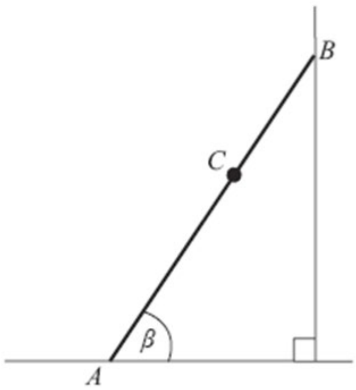


Figure 1

Figure 1 shows a ladder AB, of mass 25 kg and length 4 m, resting in equilibrium with one end A on rough horizontal ground and the other end B against a smooth vertical wall. The ladder is in a vertical plane perpendicular to the wall. The coefficient of friction between the ladder and the ground is $\frac{11}{25}$. The ladder makes an angle β with the ground. When Reece, who has mass 75 kg, stands at the point C on the ladder, where $AC = 2.8$ m, the ladder is on the point of slipping. The ladder is modelled as a uniform rod and Reece is modelled as a particle.

(a) Find the magnitude of the frictional force of the ground on the ladder. **(3)**

(b) Find, to the nearest degree, the value of β. **(6)**

(Total for Question 4 is 9 marks)

Silver Mark Scheme

Q1

Question Number	Scheme	Marks
(a)	$R = 4g\cos\alpha$	M1 A1
	$T - 0.5g = 0.5a$	M1 A1
	$4g\sin\alpha - T - F = 4a$	M1 A1
	(OR: $4g\sin\alpha - F - 0.5g = 4.5a$)	
	$F = \frac{1}{2}R$; $\sin\alpha = \frac{4}{5}$ or $\cos\alpha = \frac{3}{5}$	B1; B1
	Eliminating a or finding a	M1
	Solving for T (must have had an a)	M1
	$T = \frac{2g}{3}$N or 6.5N or 6.53N	A1
		(11)
(b)	Magnitude $= 2T\cos\left(\frac{90-\alpha}{2}\right)$	M1 A1
	$= 2 \text{ x } \frac{2g}{3} \text{ x } \frac{3}{\sqrt{10}}$ (0.94868..)	**A1 ft on T**
	$= 12N$ or $12.4N$ $\left(\frac{4g}{\sqrt{10}}\right)$	A1 (4)
		15

	Notes	
(a)	First M1 for resolving perp to plane, with usual criteria First A1 for a correct equation Second M1 for resolving vertically, with usual criteria Second A1 for a correct equation, in terms of a and T Third M1 for resolving parallel to the slope, with usual criteria. Third A1 for a correct equation , in terms of a, F and T N.B. Their a could be UP the slope in which case all 4 marks for the 2 equations are available with $-a$ replacing a, provided they are consistent. If they are inconsistent, then assume the vertical resolution is the correct one and mark accordingly. Either of the above two equations can be replaced by the 'whole system' equation **N.B. If they use a = 0, in any of the above 3 equations, and they use the equation to find T, they lose both marks for that equation, and they lose the two M marks for eliminating and solving.** First B1 for $F = \frac{1}{2}R$ seen or implied; Second B1 for $\sin\alpha = 0.8$ or $\cos\alpha = 0.6$ seen or implied. Allow close approximations if $\alpha = 53.1°\ldots$ used. Fourth M1 independent for eliminating a or finding a. Fifth M1 for solving for T but must have had an a. Fourth A1 for 2g/3, 6.5 or 6.53.	

(b)	First M1 for a complete method for finding the magnitude of the resultant (**N.B.** M0 if same tensions used) $2T\cos\left(\frac{90°-\alpha}{2}\right)$.Allow sin/cos confusion and allow $2T\cos\left(\frac{\alpha}{2}\right)$ **OR** $\sqrt{(T+T\sin\alpha)^2+(T\cos\alpha)^2}$. Allow sin/cos confusion and allow omission of √ sign, but only if $R^2 =$ ……. is included **OR** $\sqrt{T^2+T^2-2T^2\cos(90°+\alpha)}$. Allow $(90°-\alpha)$ but must be cos and and allow omission of √ sign, but only if $R^2 =$…….is included **OR** $\frac{T\sin(90+\alpha)}{\sin\left(\frac{90°-\alpha}{2}\right)}$. **(Sine Rule)** Allow sign errors in angles but must be sin First A1 for correct expression in terms of T and α Second A1, **ft** on their T, for a 'correct' **single numerical** answer Third A1 cao for 12 (N) or 12.4 (N)	

Q2

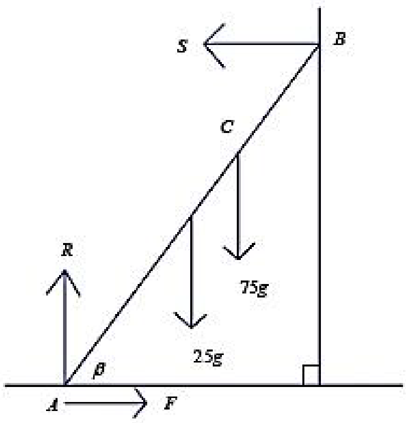
(a)	Taking moments about A: $4g\times0.7\times\cos20° = 1.4T$ $T = 18.4$ N	M1 A1 A1 A1 (4)
(b)	$\uparrow\ R + T\cos20 = 4g$ $R = 4g - T\cos20°$ $\rightarrow\ F = T\sin20$ $F = \mu R \Rightarrow T\sin20° = \mu(4g - T\cos20°)$ $\mu = \frac{T\sin20°}{4g - T\cos20°} = 0.29$	 M1 A1 M1 A1 DM1 A1 A1 (7) **11**

Q3

Question Number	Scheme	Marks
	(Parallel to plane): $P\cos 50 + F = 2g\cos 60$	M1 A2
	(Perp to plane): $R - P\sin 50 = 2g\cos 30$	M1 A2
	Other possible equations:	
	$(\rightarrow)$: $R\cos 60 - F\cos 30 = P\cos 20$ M1 A2	
	$(\uparrow)$: $R\cos 30 + F\cos 60 = P\cos 70 + 2g$ M1 A2	
	$F = \frac{1}{4}R$	B1
	Attempt to eliminate F and R to give an equation in P only	M1
	Solve for P	DM1
	$P = 6.7$ (2 SF) or 6.66 (3SF)	A1
		(10)
	Notes	
	First M1 for resolving parallel to the plane with usual rules. $2g$ term must be using 30° or 60° angle but allow sin/cos confusion. First and second A1's for a correct equation. A1A0 if one error. Second M1 for resolving perpendicular to the plane with usual rules. $2g$ term must be using 30° or 60° angle but allow sin/cos confusion. Third and fourth A1's for a correct equation. A1A0 if one error. B1 for $F = \frac{1}{4}R$ seen or implied Third M1, independent but must have two 3 (or 4) term equations, for attempt to eliminate F and R to give an equation in P only. Fourth DM1, dependent on third M1, for solving for P. Fifth A1 for 6.7 or 6.66 **Other possible equations:** First M1 for resolving horizontally with usual rules. R term must be using 30° or 60° angle and F term must be using 30° or 60° angle but allow sin/cos confusion. First and second A1's for a correct equation. A1A0 if one error. Second M1 for resolving vertically with usual rules. R term must be using 30° or 60° angle and F term must be using 30° or 60° angle but allow sin/cos confusion. Third and fourth A1's for a correct equation. A1A0 if one error.	

Q4

(a)	$R(\uparrow): R = 25g + 75g(=100g)$	B1	
	$F = \mu R \Rightarrow F = \frac{11}{25} \times 100g$	M1	
	$= 44g\ (=431)$	A1	
			(3)
(b)	M(A): $25g \times 2\cos\beta + 75g \times 2.8\cos\beta = S \times 4\sin\beta$	M1 A2,1,0	
	$R(\leftrightarrow): F = S$		
	$176g\sin\beta = 260g\cos\beta$	M1A1	
	$\beta = 56(°)$	A1	
			(6)



Gold Questions

Calculator

The total mark for this section is 37

Q1

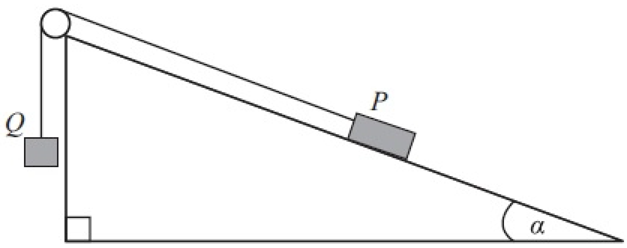


Figure 2

Two particles P and Q have masses 0.3 kg and m kg respectively. The particles are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a fixed rough plane. The plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between P and the plane is $\frac{1}{2}$.

The string lies in a vertical plane through a line of greatest slope of the inclined plane. The particle P is held at rest on the inclined plane and the particle Q hangs freely below the pulley with the string taut, as shown in Figure 2.

The system is released from rest and Q accelerates vertically downwards at 1.4 m s^{-2}.
Find

(a) the magnitude of the normal reaction of the inclined plane on P,

(2)

(b) the value of m.

(8)

When the particles have been moving for 0.5 s, the string breaks.
Assuming that P does not reach the pulley,

(c) find the further time that elapses until P comes to instantaneous rest.

(6)

(Total for Question 1 is 16 marks)

Q2

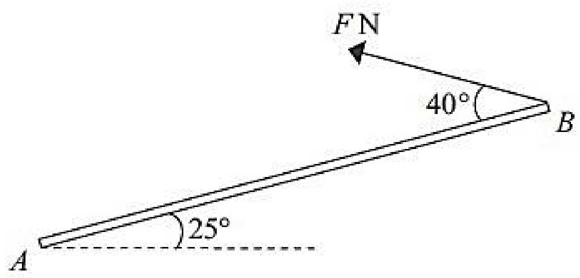


Figure 1

A uniform rod *AB*, of mass 5 kg and length 4 m, has its end *A* smoothly hinged at a fixed point. The rod is held in equilibrium at an angle of 25º above the horizontal by a force of magnitude *F* Newtons applied to its end *B*. The force acts in the vertical plane containing the rod and in a direction which makes an angle of 40º with the rod, as shown in Figure 1.

(a) Find the value of *F*.

(4)

(b) Find the magnitude and direction of the vertical component of the force acting on the rod at *A*.

(4)

(Total for Question 2 is 8 marks)

Q3

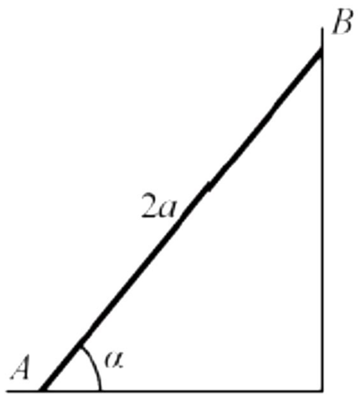


Figure 1

A uniform ladder *AB*, of length 2*a* and weight *W*, has its end *A* on rough horizontal ground.

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$. The end *B* of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight $7W$ stands at the top of the ladder. To stop the ladder from slipping, the builder's assistant applies a horizontal force of magnitude *P* to the ladder at *A*, towards the wall. The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \frac{5}{2}$. The builder is modelled as a particle and the ladder is modelled as a uniform rod.

(a) Show that the reaction of the wall on the ladder at *B* has magnitude $3W$.

(5)

(b) Find, in terms of *W*, the range of possible values of *P* for which the ladder remains in equilibrium.

(5)

Often in practice, the builder's assistant will simply stand on the bottom of the ladder.

(c) Explain briefly how this helps to stop the ladder from slipping.

(3)

(Total for Question 3 is 13 marks)

End of questions

Gold Mark Scheme

Q1

Question Number	Scheme	Marks
(a)	$R = 0.3g\cos\alpha$ $= 0.24g = 2.35$ (3sf)=2.4 (2sf)	M1 A1 (2)
(b)	$mg - T = 1.4m$ $T - 0.3g\sin\alpha - F = 0.3 \times 1.4$ $F = 0.5R$ Eliminating R and T $m = 0.4$	M1 A1 M1 A2 M1 DM1 A1 (8)
(c)	$v = 1.4 \times 0.5$ $-0.3g\sin\alpha - F = 0.3a$ $a = -9.8$ $0 = 0.7 - 9.8t$ $t = 0.071$ s or 0.0714 s (1/14 A0)	B1 M1 A1 A1 M1 A1 (6) **16**

Q2

Question Number	Scheme	Marks	Notes
(a)	$M(A), F.4\sin 40° = 5g.2\cos 25°$ $F = 35$	M1 A1 A1 A1 (4)	A complete method to find F, e.g. take moments about A. Condone sin/cos confusion. Requires correct ratio of lengths. Correct terms with at most one slip All correct 35 or 34.5 (>3sf not acceptable due to use of 9.8, but only penalise once in a question)
(b)	$F\cos 75° \pm Y = 5g$ $Y = 40$; UP	M1 A1 A1 A1 (4) 8	Resolve vertically. Need all three terms but condone sign errors. Must be attempting to work with their 75° or 15°. Correct equation (their F) 40 or 40.1 Apply ISW if the candidate goes on to find R. cso (the Q does specifically ask for the direction, so this must be clearly stated)
(b)	OR1: $4m\cos 25 \times Y = 5g \times 2m\cos 25 + F\cos 15 \times 4m\sin 25$ etc. OR2: $R\cos\alpha = F\cos 40 + 5g\cos 65$ $R\sin\alpha + F\sin 40 = 5g\cos 25$ R=52.1, α=25.3° $Y = R\sin(25+\alpha)$ Etc.	M1 A1 M1A1	Taking moments about the point vertically below B and on the same horizontal level as A.(Their F) Resolve parallel & perpendicular to the rod Solve for R, α Need a complete strategy to find Y for M1.

Q3

Question	Scheme	Marks	AOs
(a)	Take moments about A (or any other complete method to produce an equation in S , W and α only)	M1	3.3
	$Wa\cos\alpha + 7W2a\cos\alpha = S\,2a\sin\alpha$	A1 A1	1.1b 1.1b
	Use of $\tan\alpha = \frac{5}{2}$ to obtain S	M1	2.1
	$S = 3W$ *	A1*	2.2a
		(5)	
(b)	$R = 8W$	B1	3.4
	$F = \frac{1}{4}R\ (= 2W)$	M1	3.4
	$P_{MAX} = 3W + F$ or $P_{MIN} = 3W - F$	M1	3.4
	$P_{MAX} = 5W$ or $P_{MIN} = W$	A1	1.1b
	$W \leq P \leq 5W$	A1	2.5
		(5)	
(c)	M(A) shows that the reaction on the ladder at B is unchanged	M1	2.4
	also R increases (resolving vertically)	M1	2.4
	which increases max F available	M1	2.4
		(3)	
			(13 marks)

Question 9 continued
Notes:
(a) 1st M1: for producing an equation in S, W and a only 1st A1: for an equation that is correct, or which has one error or omission 2nd A1: for a fully correct equation 2nd M1: for use of tana $= \frac{5}{2}$ to obtain S in terms of W only

3rd A1*:for given answer S = 3W correctly obtained
(b) B1: for R = 8W 1st M1: for use of $F = \frac{1}{4}R$ 2nd M1: for either P = (3W + their F) or P = (3W - their F) 1st A1: for a correct max or min value for a correct range for P 2nd A1: for a correct range for P
(c) 1st M1: for showing, by taking moments about A, that the reaction at B is unchanged by the builder's assistant standing on the bottom of the ladder 2nd M1: for showing, by resolving vertically, that R increases as a result of the builder's assistant standing on the bottom of the ladder 3rd M1: for concluding that this increases the limiting friction at A

Pearson

Topic 8 - Further Kinematics

Bronze, Silver and Gold Worksheets for

A Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis.

They are drawn from the latest specification questions and legacy questions. The papers are between approximately 25 and 45 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel A Level Mathematics: Statistics and Mechanics Year 2' textbook.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculator

The total mark for this section is 34

Q1

A particle P is moving in a plane. At time t seconds, P is moving with velocity $\mathbf{v}$ m s^{-1}, where $\mathbf{v} = 2t\mathbf{i} - 3t^2\mathbf{j}$.

Find

(a) the speed of P when $t = 4$,

(2)

(b) the acceleration of P when $t = 4$.

(3)

Given that P is at the point with position vector $(-4\mathbf{i} + \mathbf{j})$ m when $t = 1$,

(c) find the position vector of P when $t = 4$.

(5)

(Total for Question 1 is 10 marks)

Q2

A particle P of mass 2 kg is moving under the action of a constant force $\mathbf{F}$ Newtons.

When $t = 0$, P has velocity $(3\mathbf{i} + 2\mathbf{j})$ m s^{-1} and at time $t = 4$ s, P has velocity $(15\mathbf{i} - 4\mathbf{j})$ m s^{-1}.

Find

(a) the acceleration of P in terms of $\mathbf{i}$ and $\mathbf{j}$,

(2)

(b) the magnitude of $\mathbf{F}$,

(4)

(c) the velocity of P at time $t = 6$ s.

(3)

(Total for Question 2 is 9 marks)

Q3

[In this question, the unit vectors **i** *and* **j** *are due east and due north respectively. Position vectors are relative to a fixed origin* ***O****.]*

A boat P is moving with constant velocity $(-4\mathbf{i} + 8\mathbf{j})$ km h^{-1}.

(a) Calculate the speed of P.

(2)

When $t = 0$, the boat P has position vector $(2\mathbf{i} - 8\mathbf{j})$ km. At time t hours, the position vector of P is $\mathbf{p}$ km.

(b) Write down $\mathbf{p}$ in terms of t.

(1)

A second boat Q is also moving with constant velocity. At time t hours, the position vector of Q is $\mathbf{q}$ km, where

$$\mathbf{q} = 18\mathbf{i} + 12\mathbf{j} - t(6\mathbf{i} + 8\mathbf{j}) .$$

Find

(c) the value of t when P is due west of Q,

(3)

(d) the distance between P and Q when P is due west of Q.

(3)

(Total for Question 3 is 9 marks)

Q4

At time t seconds, where $t \geq 0$, a particle P moves so that its acceleration $\mathbf{a}$ m s^{-2} is given by

$$\mathbf{a} = 5t\,\mathbf{i} - 15t^{\frac{1}{2}}\,\mathbf{j} .$$

When $t = 0$, the velocity of P is $20\mathbf{i}$ m s^{-1}.

Find the speed of P when $t = 4$.

(Total for Question 4 is 6 marks)

End of questions

Bronze Mark Scheme

Q1

(a)	Speed = $\sqrt{8^2+48^2} = \sqrt{2368} = 48.7$ (ms^{-1})	M1 A1 (2)
(b)	$\mathbf{a} = 2\mathbf{i} - 6t\mathbf{j}$ When $t = 4$, $\mathbf{a} = 2\mathbf{i} - 24\mathbf{j}$ (ms^{-2})	M1 A1 A1 (3)
(c)	$\mathbf{r} = t^2\mathbf{i} - t^3\mathbf{j} + \mathbf{C}$ $t = 1$, $-4\mathbf{i} + \mathbf{j} = \mathbf{i} - \mathbf{j} + \mathbf{C}$, $\mathbf{C} = -5\mathbf{i} + 2\mathbf{j}$ $\mathbf{r} = (t^2 - 5)\mathbf{i} + (-t^3 + 2)\mathbf{j}$ When $t = 4$, $\mathbf{r} = (16-5)\mathbf{i} + (-64+2)\mathbf{j} = 11\mathbf{i} - 62\mathbf{j}$	M1 A1 DM1 DM1 A1 (5) **10**

Q2

3.	(a)	$\mathbf{a} = \frac{(15\mathbf{i} - 4\mathbf{j}) - (3\mathbf{i} + 2\mathbf{j})}{4} = 3\mathbf{i} - 1.5\mathbf{j}$	M1 A1 2
	(b)	N2L $\mathbf{F} = m\mathbf{a} = 6\mathbf{i} - 3\mathbf{j}$ ft their **a** $\lvert\mathbf{F}\rvert = \sqrt{(6^2+3^2)} \approx 6.71$ (N) accept $\sqrt{45}$, awrt 6.7	M1 A1 M1 A1 4
	(c)	$\mathbf{v}_6 = (3\mathbf{i} + 2\mathbf{j}) + (3\mathbf{i} - 1.5\mathbf{j})6$ ft their **a** $= 21\mathbf{i} - 7\mathbf{j}$ (m s^{-1})	M1 A1ft A1 1 9

Q3

Question Number	Scheme	Marks
(a)	$\sqrt{\left((-4)^2+8^2\right)}=\sqrt{80}\quad(\text{km h}^{-1})$ accept exact equivalents or 8.9 or better	M1 A1
		(2)
(b)	$\mathbf{p}=(2\mathbf{i}-8\mathbf{j})+t(-4\mathbf{i}+8\mathbf{j})$	B1
		(1)
(c)	Equating **j** components	
	$-8+8t=12-8t$	M1 A1
	$t=\frac{5}{4}$ oe	A1
		(3)
(d)	Using their t from (c) to find the **i**-cpts of **p** and **q** and subtract them	M1
	$10\frac{1}{2}-(-3)=13\frac{1}{2}\quad(\text{km})$	A1 ft A1
		(3)
		9

Q4

Question	Scheme	Marks	AOs
6	Integrate **a** w.r.t. time	M1	1.1a
	$\mathbf{v} = \frac{5t^2}{2}\mathbf{i}-10t^{\frac{3}{2}}\mathbf{j} + \mathbf{C}$ (allow omission of C)	A1	1.1b
	$\mathbf{v} = \frac{5t^2}{2}\mathbf{i}-10t^{\frac{3}{2}}\mathbf{j} + 20\mathbf{i}$	A1	1.1b
	When $t = 4$, $\mathbf{v} = 60\mathbf{i} - 80\mathbf{j}$	M1	1.1b
	Attempt to find magnitude: $\sqrt{(60^2 + 80^2)}$	M1	3.1a
	Speed = 100 m s^{-1}	A1ft	1.1b
			(6 marks)

Notes:

1st **M1:** for integrating **a** w.r.t. time (powers of t increasing by 1)
1st **A1:** for a correct **v** expression without C
2nd **A1:** for a correct **v** expression including C
2nd **M1:** for putting $t = 4$ into their **v** expression
3rd **M1:** for finding magnitude of their **v**
3rd **A1:** ft for 100 m s^{-1}, follow through on an incorrect **v**

Silver Questions

Calculator

The total mark for this section is 29

Q1

At time t seconds, where $t \geq 0$, a particle P is moving on a horizontal plane with acceleration $[(3t^2 - 4t)\mathbf{i} + (6t - 5)\mathbf{j}]$ m s^{-2}.

When $t = 3$ the velocity of P is $(11\mathbf{i} + 10\mathbf{j})$ m s^{-1}.

Find

(a) the velocity of P at time t seconds,

(5)

(b) the speed of P when it is moving parallel to the vector $\mathbf{i}$.

(4)

(Total for Question 1 is 9 marks)

Q2

[*In this question* $\mathbf{i}$ *and* $\mathbf{j}$ *are perpendicular horizontal unit vectors*.]

A particle P of mass 2 kg moves under the action of two forces, $(2\mathbf{i} + 3\mathbf{j})$ N and $(4\mathbf{i} - 5\mathbf{j})$ N.

(a) Find the magnitude of the acceleration of P.

(4)

At time $t = 0$, P has velocity $(-u\mathbf{i} + u\mathbf{j})$ m s^{-1}, where u is a positive constant.

At time $t = T$ seconds, P has velocity $(10\mathbf{i} + 2\mathbf{j})$ m s^{-1}.

(b) Find

(i) the value of T,
(ii) the value of u.

(5)

(Total for Question 2 is 9 marks)

Q3

[*In this question* **i** and **j** *are unit vectors due east and due north respectively. Position vectors are given relative to a fixed origin O.*]

Two ships P and Q are moving with constant velocities. Ship P moves with velocity $(2\mathbf{i} - 3\mathbf{j})$ km h^{-1} and ship Q moves with velocity $(3\mathbf{i} + 4\mathbf{j})$ km h^{-1}.

(a) Find, to the nearest degree, the bearing on which Q is moving.

(2)

At 2 pm, ship P is at the point with position vector $(\mathbf{i} + \mathbf{j})$ km and ship Q is at the point with position vector $(-2\mathbf{j})$ km.

At time t hours after 2 pm, the position vector of P is $\mathbf{p}$ km and the position vector of Q is $\mathbf{q}$ km.

(b) Write down expressions, in terms of t, for

(i) $\mathbf{p}$,
(ii) $\mathbf{q}$,
(iii) $\overrightarrow{PQ}$.

(5)

(c) Find the time when

(i) Q is due north of P,
(ii) Q is north-west of P.

(4)

(Total for Question 3 is 11 marks)

End of questions

Silver Mark Scheme

Q1

Question Number	Scheme	Marks	Notes
(a)	Integrate: $v = \left(t^3 - 2t^2\right)\mathbf{i} + \left(3t^2 - 5t\right)\mathbf{j} + \mathbf{C}$	M1 A2	At least 3 powers going up. Condone errors in constants. Must be two separate component equations if not in vector form. Could be in column vector form. Allow with no "+ C" -1 each integration error. i.e. All correct A1A1 1 error A1A0, 2 or more errors A0A0 Allow with no "+ C"
	$t = 3: \mathbf{v} = 9\mathbf{i} + 12\mathbf{j} + \mathbf{C} = 11\mathbf{i} + 10\mathbf{j}$ $\mathbf{C} = 2\mathbf{i} - 2\mathbf{j}$	DM1	Substitute given values to find **C**. Dependent on the previous M mark
	$\mathbf{v} = \left(t^3 - 2t^2 + 2\right)\mathbf{i} + \left(3t^2 - 5t - 2\right)\mathbf{j}$	A1 (5)	Correct velocity (any equivalent form)
(b)	Parallel to **i** $\Rightarrow 3t^2 - 5t - 2 = 0$	M1	Set **j** component of their **v** equal to zero and solve for t Correct answers imply method, but incorrect answers need to show method clearly.
	$(3t+1)(t-2) = 0$, $t = 2$	A1	Correct only. Ignore $-\frac{1}{3}$ if present.
	$\lvert\mathbf{v}\rvert = 8 - 8 + 2 = 2\ (\text{m s}^{-1})$	DM1 A1 (4)	Substitute their t to find **v**. Dependent on the previous M mark. **The answer must be a scalar** – the Q asks for speed. Results from negative t must be rejected.
		[9]	
	A candidate who has no "+C" can score at most M1A2M0A0 M1A0M1A0		

Q2

Question Number	Scheme	Marks
a	Resultant force $=(2\mathbf{i}+3\mathbf{j})+(4\mathbf{i}-5\mathbf{j})=6\mathbf{i}-2\mathbf{j}$ (N)	M1
	Use of $\mathbf{F}=m\mathbf{a}$: $6\mathbf{i}-2\mathbf{j}=2\mathbf{a}$, $\mathbf{a}=3\mathbf{i}-\mathbf{j}$	M1
	Magnitude: $\lvert a\rvert=\sqrt{3^2+1^2}=\sqrt{10}\,(=3.2\text{ or better})$ (ms^{-2})	M1A1
		(4)
b	$(10\mathbf{i}+2\mathbf{j})=(-u\mathbf{i}+u\mathbf{j})+T(3\mathbf{i}-\mathbf{j})$	M1
	$10=-u+3T$ and $2=u-T$	DM1A1ft
	$T=6$	A1
	(i) $u=8$	A1
	(ii)	(5)
		[9]
	Notes for question	
a	First M1 for adding forces – must collect **i**'s and **j**'s	
	Second M1 for use of $\mathbf{F}=m\mathbf{a}$ or $F=ma$	
	Third M1 for finding a magnitude	
	A1 for $\sqrt{10}\,(=3.2\text{ or better})$	
b	First M1 for use of $\mathbf{v}=\mathbf{u}+\mathbf{a}t$ with their **a** (M0 if clearly using **F** instead of **a**)	
	Second DM1, dependent on previous M, for equating cpts of **i** and **j**	
	First A1ft for two correct equations following their **a**	
	Second A1 for $T=6$	
	Third A1 for $u=8$	

Q3

Question Number	Scheme	Marks
(a)	$\tan\theta = \frac{3}{4}$; bearing is 37° **(nearest degree)**	M1; A1
		(2)
(b)		
(i)	$\mathbf{p} = (\mathbf{i} + \mathbf{j}) + t(2\mathbf{i} - 3\mathbf{j})$	M1 A1
(ii)	$\mathbf{q} = (-2\mathbf{j}) + t(3\mathbf{i} + 4\mathbf{j})$	A1
(iii)	$\mathbf{PQ} = \mathbf{q} - \mathbf{p} = (-\mathbf{i} - 3\mathbf{j}) + t(\mathbf{i} + 7\mathbf{j})$	M1 A1
		(5)
(c)		
(i)	$-1 + t = 0$	M1
	$t = 1$ or 3pm	A1
(ii)	$-1 + t = -(-3 + 7t)$	M1
	$t = \frac{1}{2}$ or 2.30 pm	A1
		(4)
		11

Pearson

Gold Questions

Calculator

The total mark for this section is 38

Q1

A particle P of mass 0.5 kg moves under the action of a single force **F** Newtons. At time t seconds, $t \geq 0$, P has velocity **v** m s^{-1}, where

$$\mathbf{v} = (4t - 3t^2)\mathbf{i} + (t^2 - 8t - 40)\mathbf{j} .$$

(a) Find

(i) the magnitude of **F** when $t = 3$,
(ii) the acceleration of P at the instant when it is moving in the direction of the $-\mathbf{i} - \mathbf{j}$.

(9)

When $t = 1$, P is at the point A. When $t = 2$, P is at the point B.

(b) Find, in terms of **i** and **j**, the vector $\overrightarrow{AB}$.

(5)

(Total for Question 1 is 14 marks)

Q2

[*In this question,* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin*.]

A ship S is moving along a straight line with constant velocity. At time t hours the position vector of S is s km. When $t = 0$, s = 9**i** − 6**j**. When $t = 4$, s = 21**i** + 10**j**. Find

(a) the speed of S,

(4)

(b) the direction in which S is moving, giving your answer as a bearing.

(2)

(c) Show that s = $(3t + 9)$**i** + $(4t - 6)$**j**.

(2)

A lighthouse L is located at the point with position vector (18**i** + 6**j**) km.
When $t = T$, the ship S is 10 km from L.

(d) Find the possible values of T.

(6)

(Total for Question 2 is 14 marks)

Q3

[*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively*.]

A radio controlled model boat is placed on the surface of a large pond.
The boat is modelled as a particle.

At time $t = 0$, the boat is at the fixed point O and is moving due north with speed 0.6 m s^{-1}.

Relative to O, the position vector of the boat at time t seconds is **r** metres. At time $t = 15$, the velocity of the boat is $(10.5\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}. The acceleration of the boat is constant.

(*a*) Show that the acceleration of the boat is $(0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2}.

(2)

(*b*) Find **r** in terms of t.

(2)

(*c*) Find the value of t when the boat is north-east of O.

(3)

(*d*) Find the value of t when the boat is moving in a north-east direction.

(3)

(Total for Question 3 is 10 marks)

End of questions

Gold Mark Scheme

Q1

Q	Scheme	Marks	Notes
a	Differentiate v: $\mathbf{a}=(4-6t)\mathbf{i}+(-8+2t)\mathbf{j}$	M1A1	Anywhere in (a)
	Use of $\mathbf{F}=m\mathbf{a}$ and substitute $t=3$: $\mathbf{F}=0.5((4-6\times3)\mathbf{i}+(-8+2\times3)\mathbf{j})=-7\mathbf{i}-\mathbf{j}$	DM1	Dependent on the first M1
	Use of Pythagoras' theorem:	DM1	Dependent on the first M1
			NB Could use Pythagoras and then use $\mathbf{F}=m\mathbf{a}$. 1st M1 – 1st step. 2nd M1 - 2nd step
	$\lvert\mathbf{F}\rvert=\sqrt{49+1}=\sqrt{50}\left(=5\sqrt{2}=7.07...\right)$	A1	7.1 or better
	For v, i component= j component: $(4t-3t^2)=(-40-8t+t^2)$	M1	With no incorrect equations in t seen
	Solve for t: $4t^2-12t-40=0,\ \Rightarrow\ t^2-3t-10=0$ $(t-5)(t+2)=0\ ,\ t=5$	DM1 A1	Dependent on the previous M, Must see method if solving an incorrect quadratic Only - could be implied by later rejection of -2
	$\mathbf{a}=(4-30)\mathbf{i}+(-8+10)\mathbf{j}=-26\mathbf{i}+2\mathbf{j}$ (ms^{-2})	A1	Only
		(9)	
b	Integrate v: $\mathbf{r}=(2t^2-t^3(+p))\mathbf{i}+\left(-40t-4t^2+\frac{1}{3}t^3(+q)\right)\mathbf{j}$	M1 A2	-1 ee
	$\mathbf{r}_1=\mathbf{i}-43\frac{2}{3}\mathbf{j}\ ,\ \mathbf{r}_2=-93\frac{1}{3}\mathbf{j}\qquad \overrightarrow{AB}=\mathbf{r}_2-\mathbf{r}_1$	DM1	$\left(\frac{131}{3},\ \frac{280}{3}\right)$ Use limits in a definite integral or to evaluate a constant of integration Dependent on the previous M1
	$\overrightarrow{AB}=-\mathbf{i}-49\frac{2}{3}\mathbf{j}\left(=-\mathbf{i}-\frac{149}{3}\mathbf{j}\right)$	A1	49.7 or better
		(5)	
		[14]	

Q2

Question Number	Scheme	Marks
	(a)	
	$\mathbf{v} = \dfrac{21\mathbf{i}+10\mathbf{j}-(9\mathbf{i}-6\mathbf{j})}{4} = 3\mathbf{i}+4\mathbf{j}$	M1 A1
	speed is $\sqrt{(3^2+4^2)} = 5\ (\text{km h}^{-1})$	M1 A1 (4)
	(b) $\tan\theta = \dfrac{3}{4}\ (\Rightarrow \theta \approx 36.9°)$	M1
	bearing is 37, 36.9, 36.87, …	A1 (2)
	(c) $\mathbf{s} = 9\mathbf{i}-6\mathbf{j}+t(3\mathbf{i}+4\mathbf{j})$	M1
	$= (3t+9)\mathbf{i}+(4t-6)\mathbf{j}$ * cso	A1 (2)
	(d) Position vector of S relative to L is	
	$(3T+9)\mathbf{i}+(4T-6)\mathbf{j}-(18\mathbf{i}+6\mathbf{j}) = (3T-9)\mathbf{i}+(4T-12)\mathbf{j}$	M1 A1
	$(3T-9)^2+(4T-12)^2 = 100$	M1
	$25T^2-150T+125=0$ or equivalent	DM1 A1
	$(T^2-6T+5=0)$	
	$T = 1, 5$	A1 (6)
		[14]

Q3

Question	Scheme	Marks	AOs
8(a)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $(10.5\mathbf{i} - 0.9\mathbf{j}) = 0.6\mathbf{j} + 15\mathbf{a}$	M1	3.1b
	$\mathbf{a} = (0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2} Given answer	A1	1.1b
		(2)	
(b)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$	M1	3.1b
	$\mathbf{r} = 0.6\mathbf{j}\,t + \frac{1}{2}(0.7\mathbf{i} - 0.1\mathbf{j})\,t^2$	A1	1.1b
		(2)	
(c)	Equating the **i** and **j** components of **r**	M1	3.1b
	$\frac{1}{2} \leftarrow 0.7\,t^2 = 0.6\,t - \frac{1}{2} \leftarrow 0.1\,t^2$	A1ft	1.1b
	$t = 1.5$	A1	1.1b
		(3)	
(d)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$: $\mathbf{v} = 0.6\mathbf{j} + (0.7\mathbf{i} - 0.1\mathbf{j})\,t$	M1	3.1b
	Equating the **i** and **j** components of **v**	M1	3.1b
	$t = 0.75$	A1 ft	1.1b
		(3)	
			(10 marks)

Notes:

(a)

M1: for use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$

A1: for given answer correctly obtained

(b)

M1: for use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

A1: for a correct expression for **r** in terms of t

(c)

M1: for equating the **i** and **j** components of their **r**

A1ft: for a correct equation following their **r**

A1: for $t = 1.5$

(d)

M1: for use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ for a general t

M1: for equating the **i** and **j** components of their **v**

A1ft: for $t = 0.75$, or a correct follow through answer from an incorrect equation