Bronze Questions

41 Marks

Calculator

The total mark for this section is 41

Q1

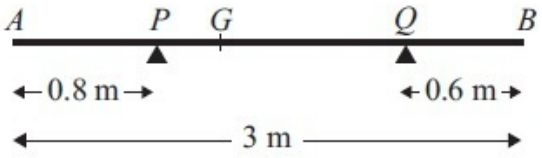


Figure 1

A non-uniform rod AB has length 3 m and mass 4.5 kg. The rod rests in equilibrium, in a horizontal position, on two smooth supports at P and at Q, where $AP = 0.8$ m and $QB = 0.6$ m, as shown in Figure 1. The centre of mass of the rod is at G. Given that the magnitude of the reaction of the support at P on the rod is twice the magnitude of the reaction of the support at Q on the rod, find

(a) the magnitude of the reaction of the support at Q on the rod,

(3)

(b) the distance AG.

(4)

(Total for Question 1 is 7 marks)

Q2

A plank AB has length 6m and mass 30kg. The point C is on the plank with $CB = 2$m. The plank rests in equilibrium in a horizontal position on supports at A and C. Two people, each of mass 75kg, stand on the plank. One person stands at the point P of the plank, where $AP = x$ metres, and the other person stands at the point Q of the plank, where $AQ = 2x$ metres. The plank remains horizontal and in equilibrium with the magnitude of the reaction at C five times the magnitude of the reaction at A. The plank is modelled as a uniform rod and each person is modelled as a particle.

(a) Find the value of x.

(7)

(b) State two ways in which you have used the assumptions made in modelling the plank as a uniform rod.

(2)

(Total for Question 2 is 9 marks)

Q3

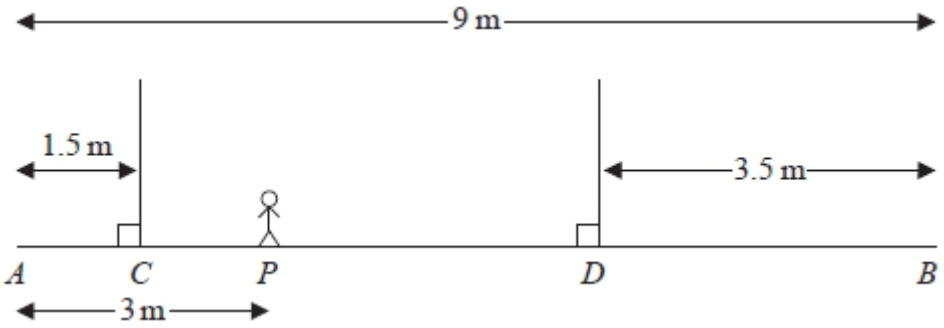


Figure 2

A wooden beam AB, of mass 150 kg and length 9 m, rests in a horizontal position supported by two vertical ropes. The ropes are attached to the beam at C and D, where $AC = 1.5$ m and $BD = 3.5$ m. A gymnast of mass 60 kg stands on the beam at the point P, where $AP = 3$ m, as shown in Figure 2. The beam remains horizontal and in equilibrium.

By modelling the gymnast as a particle, the beam as a uniform rod and the ropes as light inextensible strings,

(a) find the tension in the rope attached to the beam at C.

(3)

The gymnast at P remains on the beam at P and another gymnast, who is also modelled as a particle, stands on the beam at B. The beam remains horizontal and in equilibrium. The mass of the gymnast at B is the largest possible for which the beam remains horizontal and in equilibrium.

(b) Find the tension in the rope attached to the beam at D.

(4)

(Total for Question 3 is 7 marks)

Q4

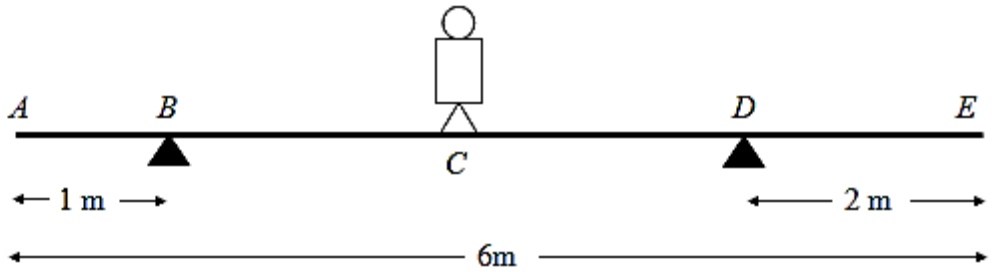
A beam AB has length 6 m and weight 200 N. The beam rests in a horizontal position on two supports at the points C and D, where $AC = 1$ m and $DB = 1$ m. Two children, Sophie and Tom, each of weight 500 N, stand on the beam with Sophie standing twice as far from the end B as Tom. The beam remains horizontal and in equilibrium and the magnitude of the reaction at D is three times the magnitude of the reaction at C. By modelling the beam as a uniform rod and the two children as particles, find how far Tom is standing from the end B.

(7)

(Total for Question 4 is 7 marks)

Q5

Figure 1



A plank *AE,* of length 6 m and mass 10 kg, rests in a horizontal position on supports at *B* and *D*, where *AB* = 1 m and *DE* = 2 m. A child of mass 20 kg stands at *C*, the mid-point of *BD*, as shown in Figure 1. The child is modelled as a particle and the plank as a uniform rod. The child and the plank are in equilibrium. Calculate

(a) the magnitude of the force exerted by the support on the plank at *B*, **(4)**
(b) the magnitude of the force exerted by the support on the plank at *D*. **(3)**

The child now stands at a point *F* on the plank. The plank is in equilibrium and on the point of tilting about *D*.

(c) Calculate the distance *DF*. **(4)**

(Total for Question 5 is 11 marks)

End of questions

Silver Questions

Calculator

The total mark for this section is 30

Q1

A steel girder AB, of mass 200 kg and length 12 m, rests horizontally in equilibrium on two smooth supports at C and at D, where $AC = 2$ m and $DB = 2$ m. A man of mass 80 kg stands on the girder at the point P, where $AP = 4$ m, as shown in Figure 1.

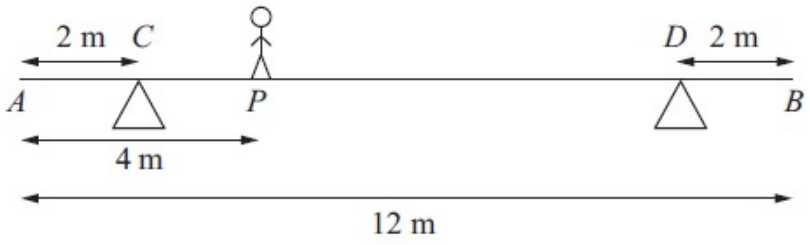


Figure 1

The man is modelled as a particle and the girder is modelled as a uniform rod.

(a) Find the magnitude of the reaction on the girder at the support at C.

(3)

The support at D is now moved to the point X on the girder, where $XB = x$ metres. The man remains on the girder at P, as shown in Figure 2.

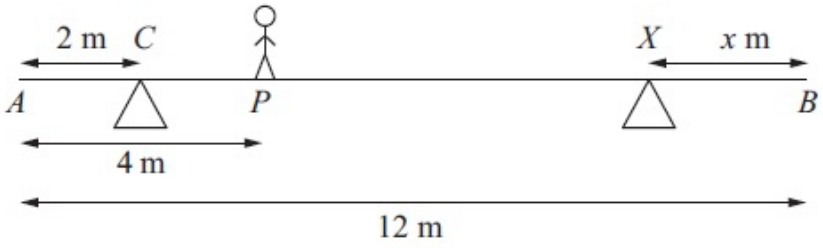


Figure 2

Given that the magnitudes of the reactions at the two supports are now equal and that the girder again rests horizontally in equilibrium, find

(b) the magnitude of the reaction at the support at X,

(2)

(c) the value of x.

(4)

(Total for Question 1 is 9 marks)

Q2

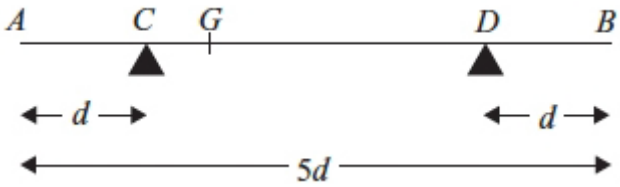


Figure 1

A non-uniform rod AB, of mass m and length $5d$, rests horizontally in equilibrium on two supports at C and D, where $AC = DB = d$, as shown in Figure 1. The centre of mass of the rod is at the point G. A particle of mass $\frac{5}{2}m$ is placed on the rod at B and the rod is on the point of tipping about D.

(a) Show that GD = $\frac{5}{2}d$.

(4)

The particle is moved from B to the mid-point of the rod and the rod remains in equilibrium.

(b) Find the magnitude of the normal reaction between the support at D and the rod.

(5)

(Total for Question 2 is 9 marks)

Q3.

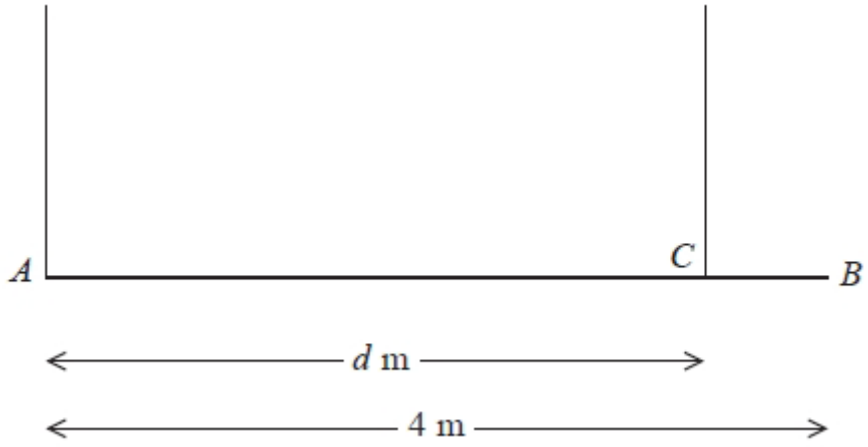


Figure 3

A beam AB has weight W Newtons and length 4 m. The beam is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. One rope is attached to A and the other rope is attached to the point C on the beam, where $AC = d$ metres, as shown in Figure 3. The beam is modelled as a uniform rod and the ropes as light inextensible strings. The tension in the rope attached at C is double the tension in the rope attached at A.

(a) Find the value of d.

(6)

A small load of weight kW Newtons is attached to the beam at B. The beam remains in equilibrium in a horizontal position. The load is modelled as a particle. The tension in the rope attached at C is now four times the tension in the rope attached at A.

(b) Find the value of k.

(6)

(Total for Question 3 is 12 marks)

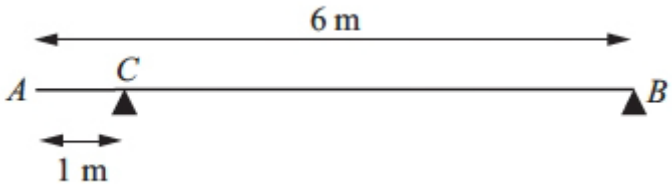
End of questions

Gold Questions

Calculator

The total mark for this section is 45

Q1



A uniform beam AB has mass 20 kg and length 6 m. The beam rests in equilibrium in a horizontal position on two smooth supports. One support is at C, where $AC = 1$ m, and the other is at the end B, as shown in the figure above. The beam is modelled as a rod.

(a) Find the magnitudes of the reactions on the beam at B and at C.

(5)

A boy of mass 30 kg stands on the beam at the point D. The beam remains in equilibrium. The magnitudes of the reactions on the beam at B and at C are now equal. The boy is modelled as a particle.

(b) Find the distance AD.

(5)

(Total for Question 1 is 10 marks)

Q2

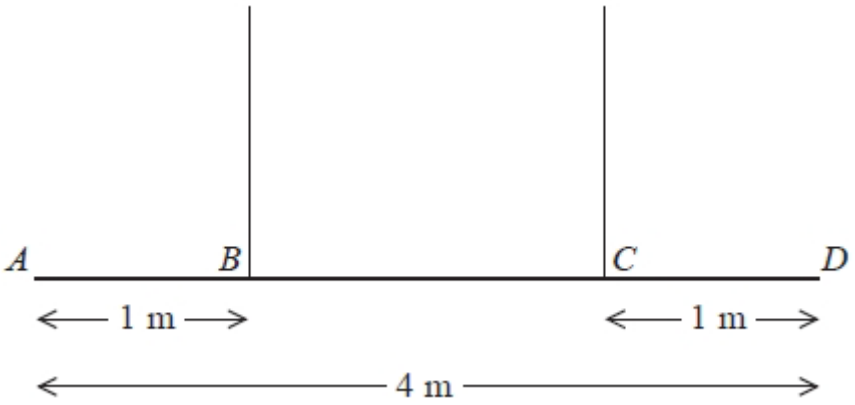


Figure 3

A non-uniform beam AD has weight W Newtons and length 4 m. It is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. The ropes are attached to two points B and C on the beam, where $AB = 1$ m and $CD = 1$ m, as shown in Figure 3. The tension in the rope attached to C is double the tension in the rope attached to B. The beam is modelled as a rod and the ropes are modelled as light inextensible strings.

(a) Find the distance of the centre of mass of the beam from A.

(6)

A small load of weight kW Newtons is attached to the beam at D. The beam remains in equilibrium in a horizontal position. The load is modelled as a particle.

Find

(b) an expression for the tension in the rope attached to B, giving your answer in terms of k and W,

(3)

(c) the set of possible values of k for which both ropes remain taut.

(2)

(Total for Question 2 is 11 marks)

Q3

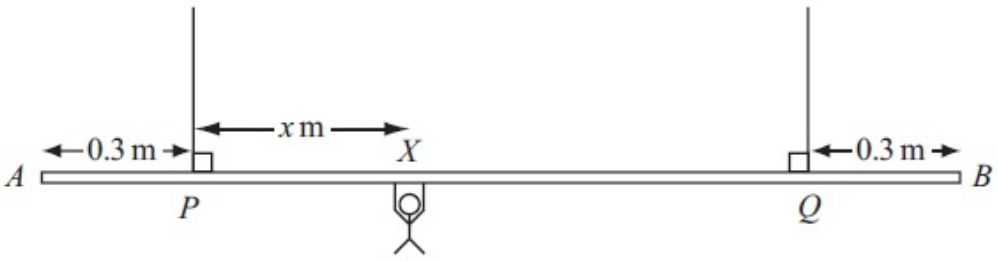


Figure 2

A beam AB is supported by two vertical ropes, which are attached to the beam at points P and Q, where $AP = 0.3$ m and $BQ = 0.3$ m. The beam is modelled as a uniform rod, of length 2 m and mass 20 kg. The ropes are modelled as light inextensible strings. A gymnast of mass 50 kg hangs on the beam between P and Q. The gymnast is modelled as a particle attached to the beam at the point X, where $PX = x$ m, $0 < x < 1.4$ as shown in Figure 2. The beam rests in equilibrium in a horizontal position.

(a) Show that the tension in the rope attached to the beam at P is $(588 - 350x)$ N.

(3)

(b) Find, in terms of x, the tension in the rope attached to the beam at Q.

(3)

(c) Hence find, justifying your answer carefully, the range of values of the tension which could occur in each rope.

(3)

Given that the tension in the rope attached at Q is three times the tension in the rope attached at P,

(d) find the value of x.

(3)

(Total for Question 3 is 12 marks)

End of questions

Bronze Questions

Calculator

The total mark for this section is 34

Q1

Three forces, $(15\mathbf{i} + \mathbf{j})$ N, $(5q\mathbf{i} - p\mathbf{j})$ N and $(-3p\mathbf{i} - q\mathbf{j})$ N, where p and q are constants, act on a particle. Given that the particle is in equilibrium, find the value of p and the value of q.

(Total for Question 1 is 5 marks)

Q2

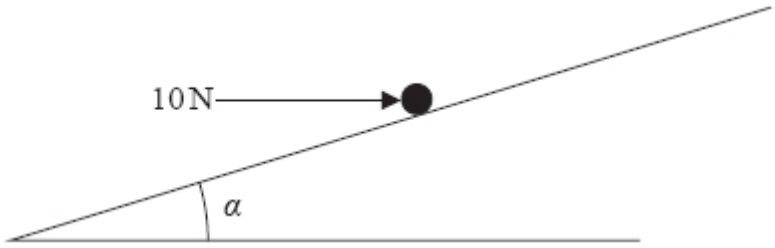


Figure 1

A particle P of mass 5kg is held at rest in equilibrium on a rough inclined plane by a horizontal force of magnitude 10N. The plane is inclined to the horizontal at an angle α where $\tan\alpha = \frac{3}{4}$, as shown in Figure 1. The line of action of the force lies in the vertical plane containing P and a line of greatest slope of the plane. The coefficient of friction between P and the plane is μ. Given that P is on the point of sliding down the plane, find the value of μ.

(Total for Question 2 is 9 marks)

Q3

Two forces $(4\mathbf{i} - 2\mathbf{j})$ N and $(2\mathbf{i} + q\mathbf{j})$ N act on a particle P of mass 1.5 kg. The resultant of these two forces is parallel to the vector $(2\mathbf{i} + \mathbf{j})$.

(a) Find the value of q.

(4)

At time $t = 0$, P is moving with velocity $(-2\mathbf{i} + 4\mathbf{j})$m s^{-1}.

(b) Find the speed of P at time $t = 2$ seconds.

(6)

(Total for Question 3 is 10 marks)

Q4

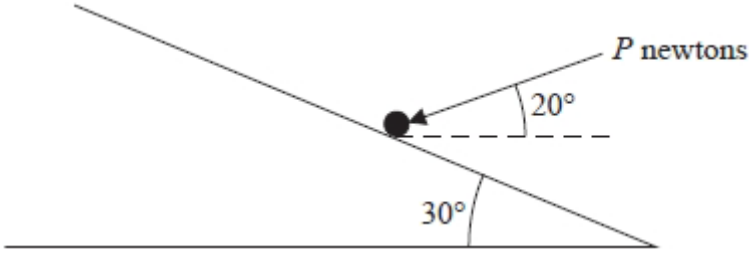


Figure 1

A particle of mass 2 kg lies on a rough plane. The plane is inclined to the horizontal at 30°.

The coefficient of friction between the particle and the plane is $\frac{1}{4}$. The particle is held in equilibrium by a force of magnitude P newtons. The force makes an angle of 20° with the horizontal and acts in a vertical plane containing a line of greatest slope of the plane, as shown in Figure 1. Find the least possible value of P.

(Total for Question 3 is 10 marks)

End of questions

Silver Questions

Calculator

The total mark for this section is 27

Q1

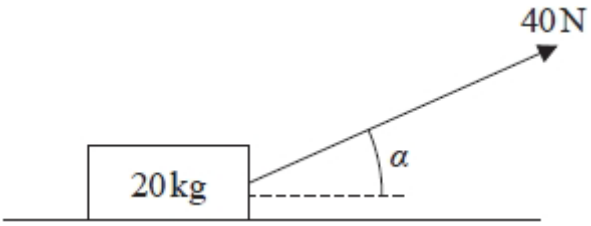


Figure 1

A wooden crate of mass 20 kg is pulled in a straight line along a rough horizontal floor using a handle attached to the crate.

The handle is inclined at an angle α to the floor, as shown in Figure 1, where $\tan\alpha = \frac{3}{4}$.

The tension in the handle is 40 N.
The coefficient of friction between the crate and the floor is 0.14.
The crate is modelled as a particle and the handle is modelled as a light rod.

Using the model,

(a) find the acceleration of the crate.

(6)

The crate is now pushed along the same floor using the handle. The handle is again inclined at the same angle α to the floor, and the thrust in the handle is 40 N as shown in Figure 2 below.

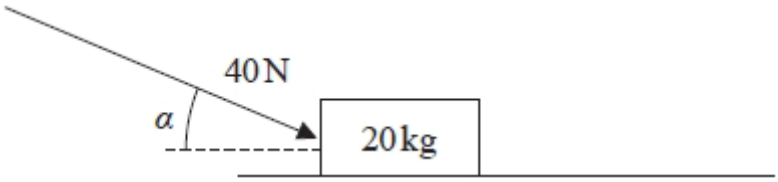


Figure 2

(b) Explain briefly why the acceleration of the crate would now be less than the acceleration of the crate found in part (a).

(2)

(Total for Question 1 is 8 marks)

Q2

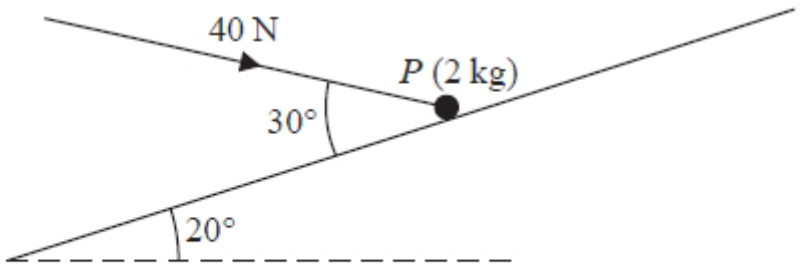


Figure 2

A particle P of mass 2 kg is held at rest in equilibrium on a rough plane by a constant force of magnitude 40 N. The direction of the force is inclined to the plane at an angle of 30°. The plane is inclined to the horizontal at an angle of 20°, as shown in Figure 2. The line of action of the force lies in the vertical plane containing P and a line of greatest slope of the plane. The coefficient of friction between P and the plane is μ.

Given that P is on the point of sliding up the plane, find the value of μ.

(Total for Question 2 is 10 marks)

Q3

A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$.

A brick P of mass m is placed on the plane.

The coefficient of friction between P and the plane is μ.

Brick P is in equilibrium and on the point of sliding down the plane.

Brick P is modelled as a particle.

Using the model,

(a) find, in terms of m and g, the magnitude of the normal reaction of the plane on brick P,

(2)

(b) show that $\mu = \frac{3}{4}$.

(4)

For parts (c) and (d), you are not required to do any further calculations.

Brick P is now removed from the plane and a much heavier brick Q is placed on the plane.

The coefficient of friction between Q and the plane is also $\frac{3}{4}$.

(c) Explain briefly why brick Q will remain at rest on the plane.

(1)

Brick Q is now projected with speed 0.5 m s^{-1} down a line of greatest slope of the plane.

Brick Q is modelled as a particle.

Using the model,

(d) describe the motion of brick Q, giving a reason for your answer.

(2)

(Total for Question 3 is 9 marks)

End of questions

Gold Questions

Calculator

The total mark for this section is 38

Q1

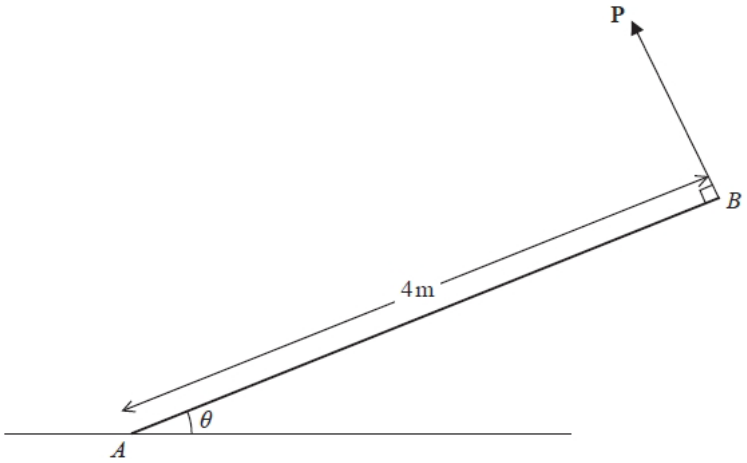


Figure 2

A non-uniform rod AB, of mass 5kg and length 4m, rests with one end A on rough horizontal ground. The centre of mass of the rod is d metres from A. The rod is held in limiting equilibrium at an angle θ to the horizontal by a force **P**, which acts in a direction perpendicular to the rod at B, as shown in Figure 2. The line of action of **P** lies in the same vertical plane as the rod.

(a) Find, in terms of d, g and θ,

(i) the magnitude of the vertical component of the force exerted on the rod by the ground,
(ii) the magnitude of the friction force acting on the rod at A.

(8)

Given that $\tan\theta = \frac{5}{12}$ and that the coefficient of friction between the rod and the ground is $\frac{1}{2}$,

(b) find the value of d.

(4)

(Total for Question 1 is 12 marks)

Q2

A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$.

A particle of mass m is placed on the plane and then projected up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is μ.

The particle moves up the plane with a constant deceleration of $\frac{4}{5}g$.

(a) Find the value of μ.

(6)

The particle comes to rest at the point A on the plane.

(b) Determine whether the particle will remain at A, carefully justifying your answer.

(2)

(Total for Question 2 is 8 marks)

Q3

A particle P moves with acceleration $(4\mathbf{i} - 5\mathbf{j})$ m s^{-2}

At time $t = 0$, P is moving with velocity $(-2\mathbf{i} + 2\mathbf{j})$ m s^{-1}

(a) Find the velocity of P at time $t = 2$ seconds.

(2)

At time $t = 0$, P passes through the origin O.

At time $t = T$ seconds, where $T > 0$, the particle P passes through the point A.

The position vector of A is $(\lambda\mathbf{i} - 4.5\mathbf{j})$m relative to O, where λ is a constant.

(b) Find the value of T.

(4)

(c) Hence find the value of λ

(2)

(Total for Question 3 is 8 marks)

Q4

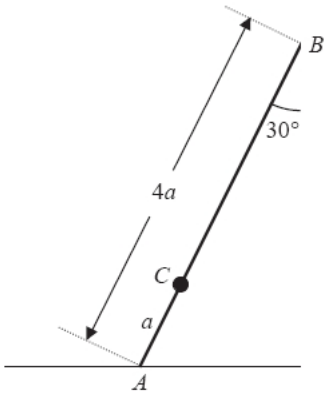


Figure 2

A ladder AB, of mass m and length $4a$, has one end A resting on rough horizontal ground. The other end B rests against a smooth vertical wall. A load of mass $3m$ is fixed on the ladder at the point C, where $AC = a$. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle. The ladder rests in limiting equilibrium making an angle of 30° with the wall, as shown in Figure 2.

Find the coefficient of friction between the ladder and the ground.

(Total for Question 4 is 10 marks)

Bronze Questions

Calculator

The total mark for this section is 38

Q1

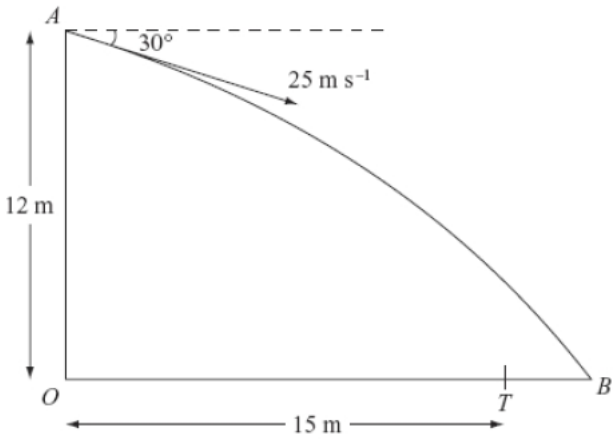


Figure 4

A ball is thrown from a point A at a target, which is on horizontal ground. The point A is 12 m above the point O on the ground. The ball is thrown from A with speed 25 m s^{-1} at an angle of 30° below the horizontal. The ball is modelled as a particle and the target as a point T. The distance OT is 15 m. The ball misses the target and hits the ground at the point B, where OTB is a straight line, as shown in Figure 4. Find

(a) the time taken by the ball to travel from A to B,

(5)

(b) the distance TB.

(4)

The point X is on the path of the ball vertically above T.

(c) Find the speed of the ball at X.

(5)

(Total for Question 1 is 14 marks)

Q2

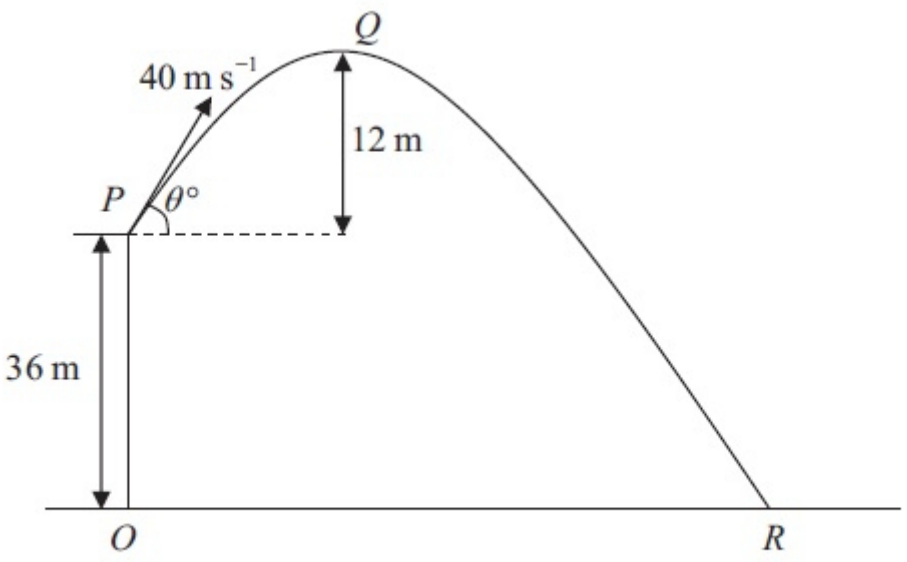


Figure 3

A ball is projected with speed 40 m s^{-1} from a point P on a cliff above horizontal ground. The point O on the ground is vertically below P and OP is 36 m. The ball is projected at an angle θ^o to the horizontal. The point Q is the highest point of the path of the ball and is 12 m above the level of P. The ball moves freely under gravity and hits the ground at the point R, as shown in Figure 3. Find

(a) the value of θ,

(3)

(b) the distance OR.

(6)

(Total for Question 2 is 9 marks)

Q3

[In this question, the unit vectors **i** *and* **j** *are horizontal and vertical respectively.]*

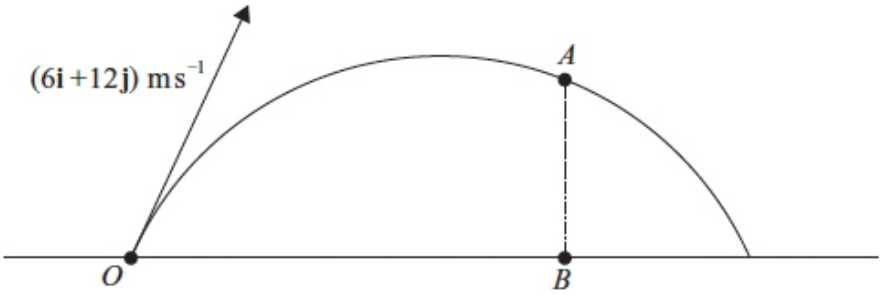


Figure 3

The point O is a fixed point on a horizontal plane. A ball is projected from O with velocity $(6\mathbf{i} + 12\mathbf{j})$ m s^{-1}, and passes through the point A at time t seconds after projection. The point B is on the horizontal plane vertically below A, as shown in Figure 3. It is given that $OB = 2AB$.

Find

(a) the value of t,

(7)

(b) the speed, V m s^{-1}, of the ball at the instant when it passes through A.

(5)

At another point C on the path the speed of the ball is also V m s^{-1}.

(c) Find the time taken for the ball to travel from O to C.

(3)

(Total for Question 3 is 15 marks)

End of questions

Silver Questions

Calculator

The total mark for this section is 45

Q1

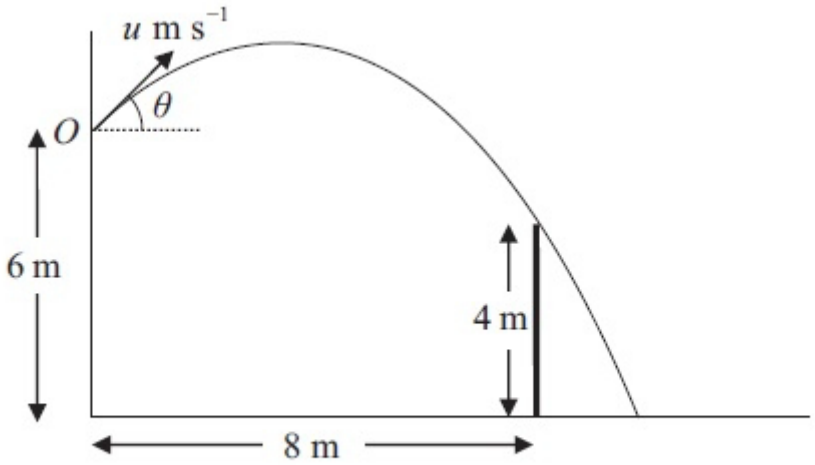


Figure 2

A ball is thrown from a point O, which is 6 m above horizontal ground. The ball is projected with speed u m s^{-1} at an angle θ above the horizontal. There is a thin vertical post which is 4 m high and 8 m horizontally away from the vertical through O, as shown in Figure 2. The ball passes just above the top of the post 2 s after projection. The ball is modelled as a particle.

(a) Show that $\tan\theta = 2.2$

(5)

(b) Find the value of u.

(2)

The ball hits the ground T seconds after projection.

(c) Find the value of T.

(3)

Immediately before the ball hits the ground the direction of motion of the ball makes an angle α with the horizontal.

(d) Find α.

(5)

(Total for Question 1 is 15 marks)

Q2

A particle is projected from a point O with speed u at an angle of elevation α above the horizontal and moves freely under gravity. When the particle has moved a horizontal distance x, its height above O is y.

(a) Show that

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

(4)

A girl throws a ball from a point A at the top of a cliff. The point A is 8 m above a horizontal beach. The ball is projected with speed 7 m s^{-1} at an angle of elevation of 45°. By modelling the ball as a particle moving freely under gravity,

(b) find the horizontal distance of the ball from A when the ball is 1 m above the beach.

(5)

A boy is standing on the beach at the point B vertically below A. He starts to run in a straight line with speed v m s^{-1}, leaving B 0.4 seconds after the ball is thrown.

He catches the ball when it is 1 m above the beach.

(c) Find the value of v.

(4)

(Total for Question 2 is 13 marks)

Q3

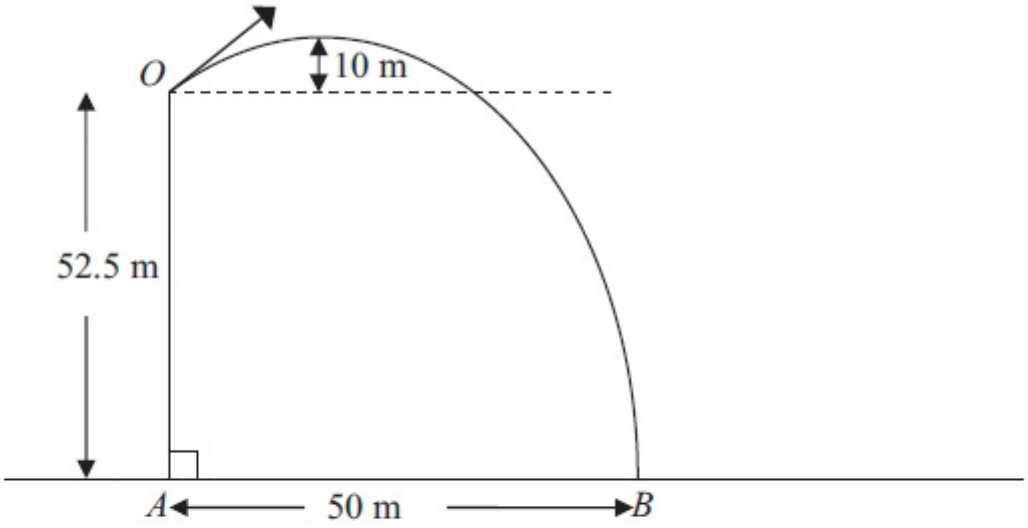


Figure 4

A small stone is projected from a point O at the top of a vertical cliff OA. The point O is 52.5 m above the sea. The stone rises to a maximum height of 10 m above the level of O before hitting the sea at the point B, where $AB = 50$ m, as shown in Figure 4. The stone is modelled as a particle moving freely under gravity.

(a) Show that the vertical component of the velocity of projection of the stone is 14 m s^{-1}.

(3)

(b) Find the speed of projection.

(9)

(c) Find the time after projection when the stone is moving parallel to OB.

(5)

(Total for Question 3 is 17 marks)

End of questions

Gold Questions

40 Marks

Calculator

The total mark for this section is 40

Q1.

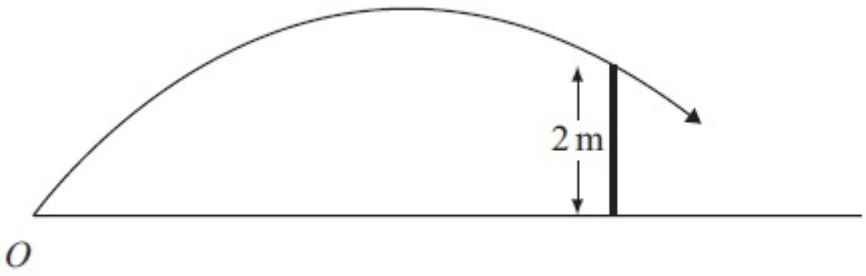


Figure 3

A child playing cricket on horizontal ground hits the ball towards a fence 10 m away. The ball moves in a vertical plane which is perpendicular to the fence. The ball just passes over the top of the fence, which is 2 m above the ground, as shown in Figure 3.

The ball is modelled as a particle projected with initial speed u m s^{-1} from point O on the ground at an angle α to the ground.

(a) By writing down expressions for the horizontal and vertical distances, from O of the ball t seconds after it was hit, show that

$$2 = 10\tan\alpha - \frac{50g}{u^2\cos^2\alpha}.$$

(6)

Given that $\alpha = 45^\circ$,

(b) find the speed of the ball as it passes over the fence.

(6)

(Total for Question 1 is 12 marks)

Q2

*[In this question, the unit vectors **i** and **j** are in a vertical plane, **i** being horizontal and **j** being vertically upwards.]*

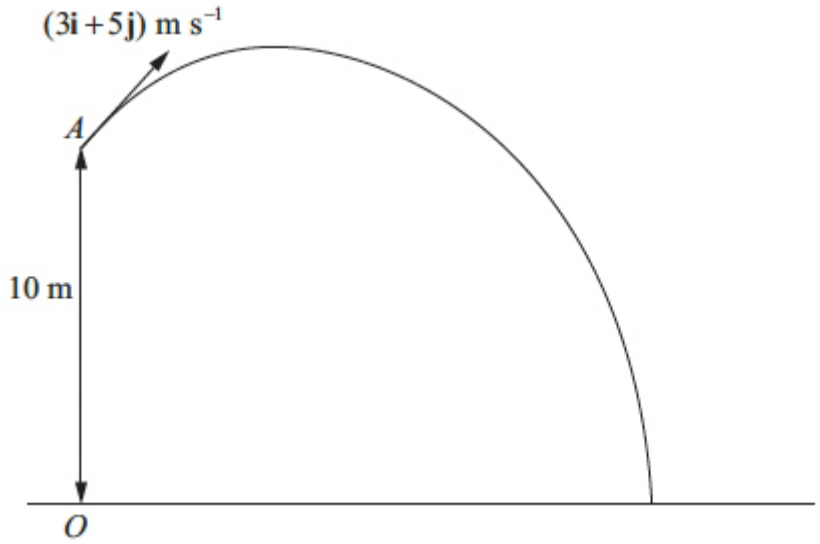


Figure 1

At time $t = 0$, a particle P is projected from the point A which has position vector $10\mathbf{j}$ metres with respect to a fixed origin O at ground level. The ground is horizontal. The velocity of projection of P is $(3\mathbf{i} + 5\mathbf{j})$ m s^{-1}, as shown in Figure 1. The particle moves freely under gravity and reaches the ground after T seconds.

(a) For $0 \leqslant t \leqslant T$, show that, with respect to O, the position vector, $\mathbf{r}$ metres, of P at time t seconds is given by

$$\mathbf{r} = (3t)\mathbf{i} + (10 + 5t - 4.9t^2)\mathbf{j} .$$

(3)

(b) Find the value of T.

(3)

(c) Find the velocity of P at time t seconds ($0 \leqslant t \leqslant T$).

(2)

When P is at the point B, the direction of motion of P is 45° below the horizontal.

(d) Find the time taken for P to move from A to B.

(2)

(e) Find the speed of P as it passes through B.

(2)

(Total for Question 2 is 12 marks)

Q3

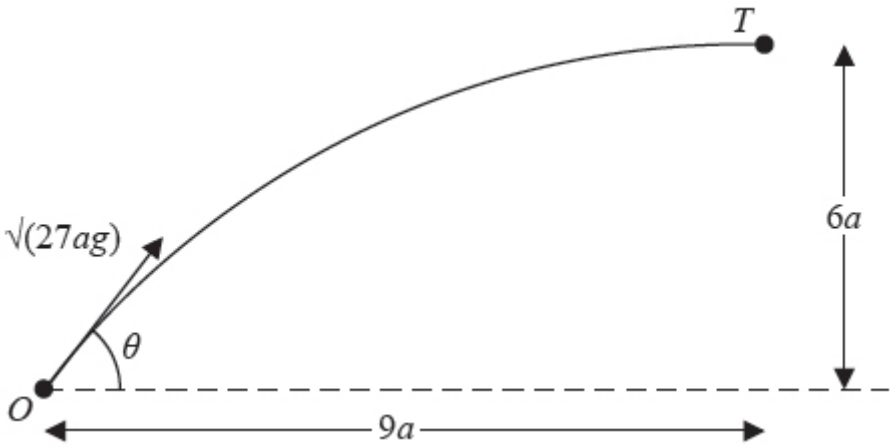


Figure 4

A small ball is projected from a fixed point O so as to hit a target T which is at a horizontal distance $9a$ from O and at a height $6a$ above the level of O. The ball is projected with speed $\sqrt{(27ag)}$ at an angle θ to the horizontal, as shown in Figure 4. The ball is modelled as a particle moving freely under gravity.

(a) Show that $\tan^2\theta - 6\tan\theta + 5 = 0$

(7)

The two possible angles of projection are θ_1 and θ_2, where $\theta_1 > \theta_2$.

(b) Find $\tan\theta_1$ and $\tan\theta_2$.

(3)

The particle is projected at the larger angle θ_1.

(c) Show that the time of flight from O to T is $\sqrt{\left(\frac{78a}{g}\right)}$.

(3)

(d) Find the speed of the particle immediately before it hits T.

(3)

(Total for Question 3 is 16 marks)

End of questions

Bronze Questions

Calculator

The total mark for this section is 38

Q1

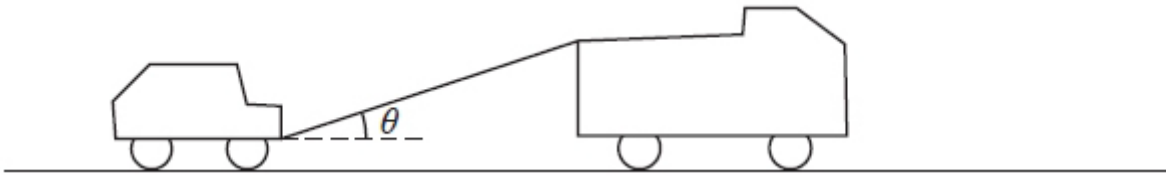


Figure 4

A truck of mass 1750 kg is towing a car of mass 750 kg along a straight horizontal road. The two vehicles are joined by a light towbar which is inclined at an angle θ to the road, as shown in Figure 4. The vehicles are travelling at 20 m s^{-1} as they enter a zone where the speed limit is 14 m s^{-1}. The truck's brakes are applied to give a constant braking force on the truck. The distance travelled between the instant when the brakes are applied and the instant when the speed of each vehicle is 14 m s^{-1} is 100 m.

(a) Find the deceleration of the truck and the car.

(3)

The constant braking force on the truck has magnitude R Newtons. The truck and the car also experience constant resistances to motion of 500 N and 300 N respectively. Given that $\cos\theta = 0.9$, find

(b) the force in the towbar,

(4)

(c) the value of R.

(4)

(Total for Question 1 is 11 marks)

Q2

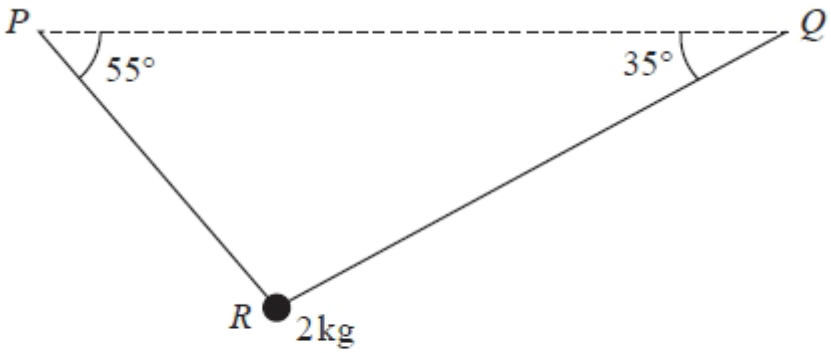


Figure 1

A particle of mass 2 kg is suspended from a horizontal ceiling by two light inextensible strings, *PR* and *QR*. The particle hangs at *R* in equilibrium, with the strings in a vertical plane. The string *PR* is inclined at 55° to the horizontal and the string *QR* is inclined at 35° to the horizontal, as shown in Figure 1.

Find

(i) the tension in the string *PR*,

(ii) the tension in the string *QR*.

(Total for Question 2 is 7 marks)

Q3

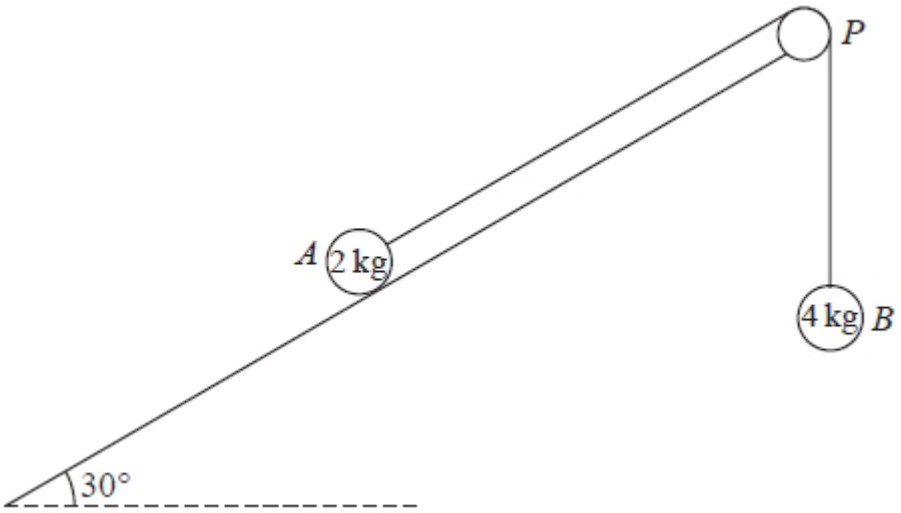


Figure 2

A fixed rough plane is inclined at 30° to the horizontal. A small smooth pulley P is fixed at the top of the plane. Two particles A and B, of mass 2 kg and 4 kg respectively, are attached to the ends of a light inextensible string which passes over the pulley P. The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs freely below P, as shown in Figure 2. The coefficient of friction between A and the plane is $\frac{1}{\sqrt{3}}$. Initially A is held at rest on the plane. The particles are released from rest with the string taut and A moves up the plane.

Find the tension in the string immediately after the particles are released.

(9)

(Total for Question 3 is 9 marks)

Q4

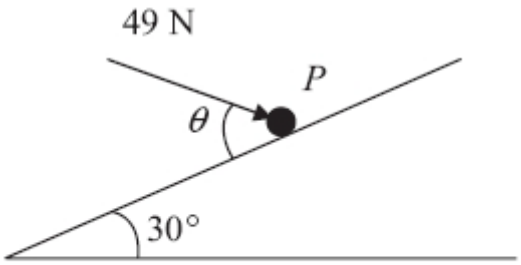


Figure 1

A particle P of mass 6 kg lies on the surface of a smooth plane. The plane is inclined at an angle of 30° to the horizontal. The particle is held in equilibrium by a force of magnitude 49 N, acting at an angle θ to the plane, as shown in Figure 1. The force acts in a vertical plane through a line of greatest slope of the plane.

(a) Show that $\cos\theta = \frac{3}{5}$.

(3)

(b) Find the normal reaction between P and the plane.

(4)

The direction of the force of magnitude 49 N is now changed. It is now applied horizontally to P so that P moves up the plane. The force again acts in a vertical plane through a line of greatest slope of the plane.

(c) Find the initial acceleration of P.

(4)

(Total for Question 4 is 11 marks)

End of questions

Silver Questions

Calculator

The total mark for this section is 35

Q1

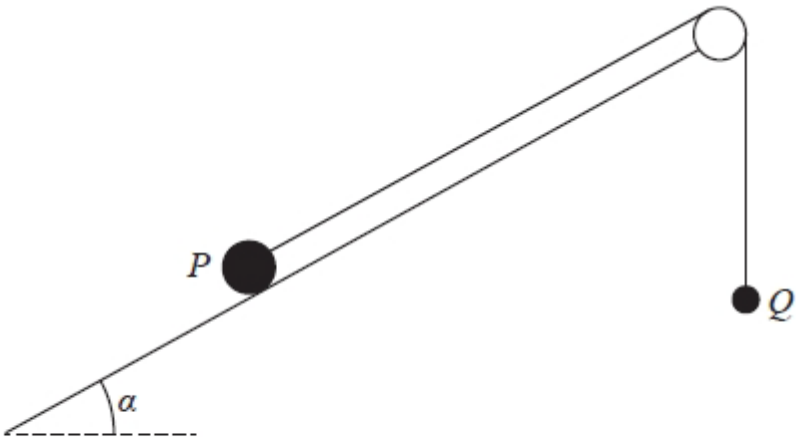


Figure 4

Two particles P and Q have mass 4 kg and 0.5 kg respectively. The particles are attached to the ends of a light inextensible string. Particle P is held at rest on a fixed rough plane, which is inclined to the horizontal at an angle α where $\tan\alpha = \frac{4}{3}$. The coefficient of friction between P and the plane is 0.5. The string lies along the plane and passes over a small smooth light pulley which is fixed at the top of the plane. Particle Q hangs freely at rest vertically below the pulley. The string lies in the vertical plane which contains the pulley and a line of greatest slope of the inclined plane, as shown in Figure 4. Particle P is released from rest with the string taut and slides down the plane.

Given that Q has not hit the pulley, find

(a) the tension in the string during the motion,

(11)

(b) the magnitude of the resultant force exerted by the string on the pulley.

(4)

(Total for Question 1 is 15 marks)

Q2

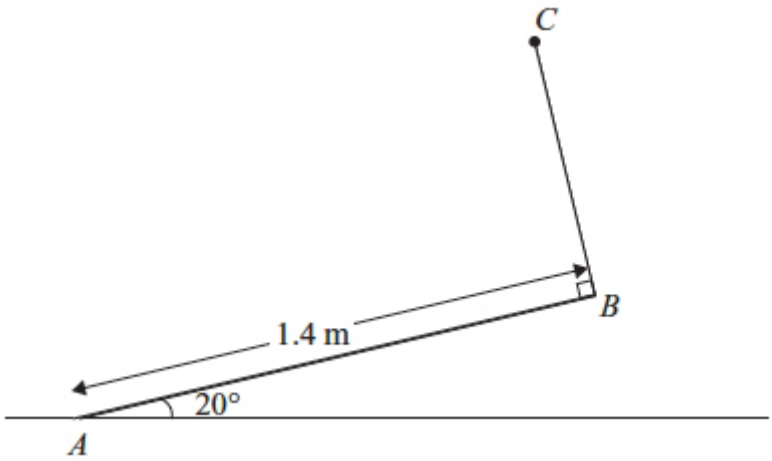


Figure 2

A uniform rod AB has mass 4 kg and length 1.4 m. The end A is resting on rough horizontal ground. A light string BC has one end attached to B and the other end attached to a fixed point C. The string is perpendicular to the rod and lies in the same vertical plane as the rod. The rod is in equilibrium, inclined at 20° to the ground, as shown in Figure 2.

(a) Find the tension in the string.

(4)

Given that the rod is about to slip,

(b) find the coefficient of friction between the rod and the ground.

(7)

(Total for Question 2 is 11 marks)

Q3

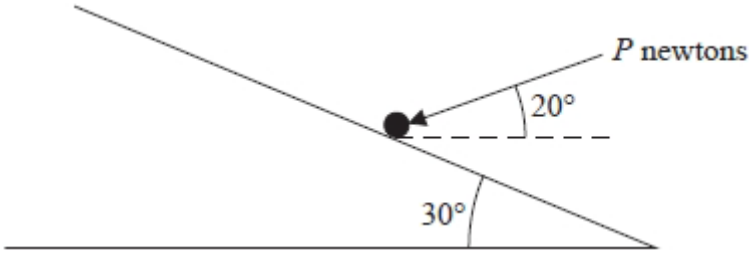


Figure 1

A particle of mass 2 kg lies on a rough plane. The plane is inclined to the horizontal at 30°. The coefficient of friction between the particle and the plane is $\frac{1}{4}$. The particle is held in equilibrium by a force of magnitude P Newtons. The force makes an angle of 20° with the horizontal and acts in a vertical plane containing a line of greatest slope of the plane, as shown in Figure 1. Find the least possible value of P.

(Total for Question 3 is 10 marks)

Q4

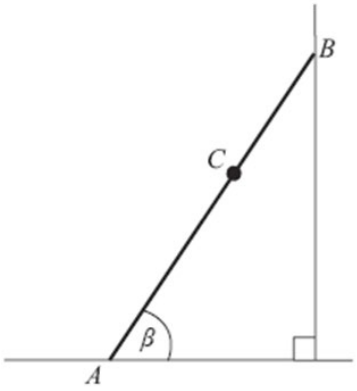


Figure 1

Figure 1 shows a ladder AB, of mass 25 kg and length 4 m, resting in equilibrium with one end A on rough horizontal ground and the other end B against a smooth vertical wall. The ladder is in a vertical plane perpendicular to the wall. The coefficient of friction between the ladder and the ground is $\frac{11}{25}$. The ladder makes an angle β with the ground. When Reece, who has mass 75 kg, stands at the point C on the ladder, where $AC = 2.8$ m, the ladder is on the point of slipping. The ladder is modelled as a uniform rod and Reece is modelled as a particle.

(a) Find the magnitude of the frictional force of the ground on the ladder. **(3)**

(b) Find, to the nearest degree, the value of β. **(6)**

(Total for Question 4 is 9 marks)

Gold Questions

Calculator

The total mark for this section is 37

Q1

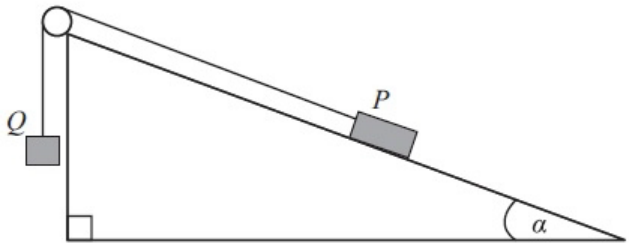


Figure 2

Two particles P and Q have masses 0.3 kg and m kg respectively. The particles are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a fixed rough plane. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$. The coefficient of friction between P and the plane is $\frac{1}{2}$.

The string lies in a vertical plane through a line of greatest slope of the inclined plane. The particle P is held at rest on the inclined plane and the particle Q hangs freely below the pulley with the string taut, as shown in Figure 2.

The system is released from rest and Q accelerates vertically downwards at 1.4 m s^{-2}.
Find

(a) the magnitude of the normal reaction of the inclined plane on P,

(2)

(b) the value of m.

(8)

When the particles have been moving for 0.5 s, the string breaks.
Assuming that P does not reach the pulley,

(c) find the further time that elapses until P comes to instantaneous rest.

(6)

(Total for Question 1 is 16 marks)

Q2

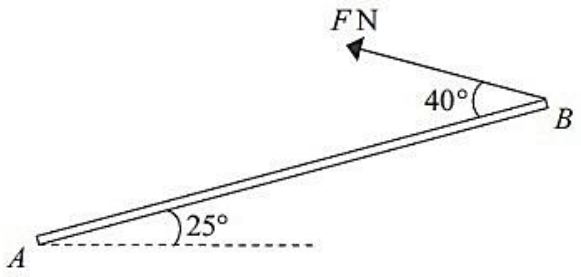


Figure 1

A uniform rod AB, of mass 5 kg and length 4 m, has its end A smoothly hinged at a fixed point. The rod is held in equilibrium at an angle of 25º above the horizontal by a force of magnitude F Newtons applied to its end B. The force acts in the vertical plane containing the rod and in a direction which makes an angle of 40º with the rod, as shown in Figure 1.

(a) Find the value of F.

(4)

(b) Find the magnitude and direction of the vertical component of the force acting on the rod at A.

(4)

(Total for Question 2 is 8 marks)

Q3

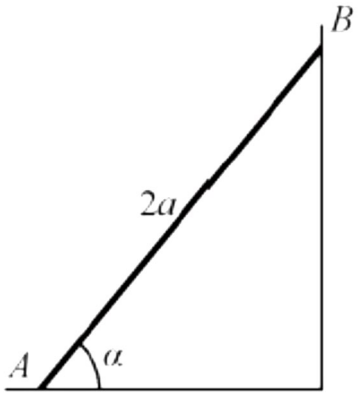


Figure 1

A uniform ladder AB, of length $2a$ and weight W, has its end A on rough horizontal ground.

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$. The end B of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight $7W$ stands at the top of the ladder. To stop the ladder from slipping, the builder's assistant applies a horizontal force of magnitude P to the ladder at A, towards the wall. The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \frac{5}{2}$. The builder is modelled as a particle and the ladder is modelled as a uniform rod.

(a) Show that the reaction of the wall on the ladder at B has magnitude $3W$.

(5)

(b) Find, in terms of W, the range of possible values of P for which the ladder remains in equilibrium.

(5)

Often in practice, the builder's assistant will simply stand on the bottom of the ladder.

(c) Explain briefly how this helps to stop the ladder from slipping.

(3)

(Total for Question 3 is 13 marks)

End of questions

Bronze Questions

Calculator

The total mark for this section is 34

Q1

A particle P is moving in a plane. At time t seconds, P is moving with velocity $\mathbf{v}$ m s^{-1}, where $\mathbf{v} = 2t\mathbf{i} - 3t^2\mathbf{j}$.

Find

(a) the speed of P when $t = 4$,

(2)

(b) the acceleration of P when $t = 4$.

(3)

Given that P is at the point with position vector $(-4\mathbf{i} + \mathbf{j})$ m when $t = 1$,

(c) find the position vector of P when $t = 4$.

(5)

(Total for Question 1 is 10 marks)

Q2

A particle P of mass 2 kg is moving under the action of a constant force $\mathbf{F}$ Newtons.

When $t = 0$, P has velocity $(3\mathbf{i} + 2\mathbf{j})$ m s^{-1} and at time $t = 4$ s, P has velocity $(15\mathbf{i} - 4\mathbf{j})$ m s^{-1}.

Find

(a) the acceleration of P in terms of $\mathbf{i}$ and $\mathbf{j}$,

(2)

(b) the magnitude of $\mathbf{F}$,

(4)

(c) the velocity of P at time $t = 6$ s.

(3)

(Total for Question 2 is 9 marks)

Q3

[In this question, the unit vectors **i** *and* **j** *are due east and due north respectively. Position vectors are relative to a fixed origin* ***O****.]*

A boat P is moving with constant velocity $(-4\mathbf{i} + 8\mathbf{j})$ km h^{-1}.

(a) Calculate the speed of P.

(2)

When $t = 0$, the boat P has position vector $(2\mathbf{i} - 8\mathbf{j})$ km. At time t hours, the position vector of P is $\mathbf{p}$ km.

(b) Write down $\mathbf{p}$ in terms of t.

(1)

A second boat Q is also moving with constant velocity. At time t hours, the position vector of Q is $\mathbf{q}$ km, where

$$\mathbf{q} = 18\mathbf{i} + 12\mathbf{j} - t(6\mathbf{i} + 8\mathbf{j}) .$$

Find

(c) the value of t when P is due west of Q,

(3)

(d) the distance between P and Q when P is due west of Q.

(3)

(Total for Question 3 is 9 marks)

Q4

At time t seconds, where $t \geq 0$, a particle P moves so that its acceleration $\mathbf{a}$ m s^{-2} is given by

$$\mathbf{a} = 5t\,\mathbf{i} - 15t^{\frac{1}{2}}\,\mathbf{j} .$$

When $t = 0$, the velocity of P is $20\mathbf{i}$ m s^{-1}.

Find the speed of P when $t = 4$.

(Total for Question 4 is 6 marks)

End of questions

Silver Questions

Calculator

The total mark for this section is 29

Q1

At time t seconds, where $t \geq 0$, a particle P is moving on a horizontal plane with acceleration $[(3t^2 - 4t)\mathbf{i} + (6t - 5)\mathbf{j}]$ m s^{-2}.

When $t = 3$ the velocity of P is $(11\mathbf{i} + 10\mathbf{j})$ m s^{-1}.

Find

(a) the velocity of P at time t seconds,

(5)

(b) the speed of P when it is moving parallel to the vector $\mathbf{i}$.

(4)

(Total for Question 1 is 9 marks)

Q2

[*In this question* **i** *and* **j** *are perpendicular horizontal unit vectors*.]

A particle P of mass 2 kg moves under the action of two forces, $(2\mathbf{i} + 3\mathbf{j})$ N and $(4\mathbf{i} - 5\mathbf{j})$ N.

(a) Find the magnitude of the acceleration of P.

(4)

At time $t = 0$, P has velocity $(-u\mathbf{i} + u\mathbf{j})$ m s^{-1}, where u is a positive constant.

At time $t = T$ seconds, P has velocity $(10\mathbf{i} + 2\mathbf{j})$ m s^{-1}.

(b) Find

(i) the value of T,
(ii) the value of u.

(5)

(Total for Question 2 is 9 marks)

Q3

[*In this question* **i** and **j** *are unit vectors due east and due north respectively. Position vectors are given relative to a fixed origin O.*]

Two ships P and Q are moving with constant velocities. Ship P moves with velocity $(2\mathbf{i} - 3\mathbf{j})$ km h^{-1} and ship Q moves with velocity $(3\mathbf{i} + 4\mathbf{j})$ km h^{-1}.

(a) Find, to the nearest degree, the bearing on which Q is moving.

(2)

At 2 pm, ship P is at the point with position vector $(\mathbf{i} + \mathbf{j})$ km and ship Q is at the point with position vector $(-2\mathbf{j})$ km.

At time t hours after 2 pm, the position vector of P is $\mathbf{p}$ km and the position vector of Q is $\mathbf{q}$ km.

(b) Write down expressions, in terms of t, for

(i) $\mathbf{p}$,
(ii) $\mathbf{q}$,
(iii) $\overrightarrow{PQ}$.

(5)

(c) Find the time when

(i) Q is due north of P,
(ii) Q is north-west of P.

(4)

(Total for Question 3 is 11 marks)

End of questions

Gold Questions

Calculator

The total mark for this section is 38

Q1

A particle P of mass 0.5 kg moves under the action of a single force **F** Newtons. At time t seconds, $t \geq 0$, P has velocity **v** m s^{-1}, where

$$\mathbf{v} = (4t - 3t^2)\mathbf{i} + (t^2 - 8t - 40)\mathbf{j} .$$

(a) Find

(i) the magnitude of **F** when $t = 3$,
(ii) the acceleration of P at the instant when it is moving in the direction of the $-\mathbf{i} - \mathbf{j}$.

(9)

When $t = 1$, P is at the point A. When $t = 2$, P is at the point B.

(b) Find, in terms of **i** and **j**, the vector $\overrightarrow{AB}$.

(5)

(Total for Question 1 is 14 marks)

Q2

[*In this question,* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship S is moving along a straight line with constant velocity. At time t hours the position vector of S is s km. When $t = 0$, s = $9\mathbf{i} - 6\mathbf{j}$. When $t = 4$, s = $21\mathbf{i} + 10\mathbf{j}$. Find

(a) the speed of S,

(4)

(b) the direction in which S is moving, giving your answer as a bearing.

(2)

(c) Show that s = $(3t + 9)\mathbf{i} + (4t - 6)\mathbf{j}$.

(2)

A lighthouse L is located at the point with position vector $(18\mathbf{i} + 6\mathbf{j})$ km.
When $t = T$, the ship S is 10 km from L.

(d) Find the possible values of T.

(6)

(Total for Question 2 is 14 marks)

Q3

[*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively.*]

A radio controlled model boat is placed on the surface of a large pond.
The boat is modelled as a particle.

At time $t = 0$, the boat is at the fixed point O and is moving due north with speed 0.6 m s^{-1}.

Relative to O, the position vector of the boat at time t seconds is **r** metres. At time $t = 15$, the velocity of the boat is $(10.5\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}. The acceleration of the boat is constant.

(*a*) Show that the acceleration of the boat is $(0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2}.

(2)

(*b*) Find **r** in terms of t.

(2)

(*c*) Find the value of t when the boat is north-east of O.

(3)

(*d*) Find the value of t when the boat is moving in a north-east direction.

(3)

(Total for Question 3 is 10 marks)

End of questions