Pearson

Topic 1: Algebraic Methods

Bronze, Silver and Gold Worksheets for A Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel A Level Mathematics: Pure Mathematics Year 2' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Non-calculator

The total mark for this section is 25

Q1

(a) Simplify fully $\frac{x^2+3x-4}{2x^2-5x+3}$.

(3)

(b) Write $\frac{4}{x+2}+\frac{3}{x-2}$ as a single fraction in its simplest form.

(3)

(Total for Question 1 is 6 marks)

Q2

Express

$$\frac{2(3x+2)}{9x^2-4} - \frac{2}{3x+1}$$

as a single fraction in its simplest form.

(4)

(Total for Question 2 is 4 marks)

Q3

$$f(x) = 3x^3 - 5x^2 - 16x + 12.$$

Given that $(x + 2)$ is a factor of $f(x)$,

Factorise $f(x)$ completely.

(4)

(Total for Question 3 is 4 marks)

Q4

$$f(x) = 4x^3 - 12x^2 + 2x - 6$$

Hence show that 3 is the only real root of the equation $f(x) = 0$

(4)

(Total for Question 4 is 4 marks)

Q5

Express $\dfrac{5}{(x-1)(3x+2)}$ in partial fractions.

(3)

(Total for Question 5 is 3 marks)

Q6

Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4.

(4)

(Total for Question 6 is 4 marks)

End of Questions

Bronze Mark Scheme

Q1

Question		Working	Answer	Mark	Notes
	(a)	$\frac{(x+4)(x-1)}{(2x-3)(x-1)}$	$\frac{x+4}{2x-3}$	3	M1 for $(x + 4)(x - 1)$ M1 for $(2x - 3)(x - 1)$ A1 cao
	(b)	$\frac{4(x-2)}{(x+2)(x-2)}+\frac{3(x+2)}{(x+2)(x-2)}$	$\frac{7x-2}{(x+2)(x-2)}$	3	M1 for denominator $(x + 2)(x - 2)$ oe **or** $x^2 - 4$ M1 for $\frac{4(x-2)}{(x+2)(x-2)}$ oe **or** $\frac{3(x+2)}{(x+2)(x-2)}$ oe (NB. The denominator must be $(x + 2)(x - 2)$ **or** $x^2 - 4$ **or** another suitable common denominator) A1 for $\frac{7x-2}{(x+2)(x-2)}$ **or** $\frac{7x-2}{x^2-4}$ SC: If no marks awarded then award B1 for $\frac{4(x-2)}{x^2-2}+\frac{3(x+2)}{x^2-2}$ oe

Q2

Question Number	Scheme	Marks
	$9x^2-4=(3x-2)(3x+2)$ At any stage	B1
	Eliminating the common factor of $(3x+2)$ at any stage $\dfrac{2(3x+2)}{(3x-2)(3x+2)}=\dfrac{2}{3x-2}$	B1
	Use of a common denominator $\dfrac{2(3x+2)(3x+1)}{(9x^2-4)(3x+1)}-\dfrac{2(9x^2-4)}{(9x^2-4)(3x+1)}$ or $\dfrac{2(3x+1)}{(3x-2)(3x+1)}-\dfrac{2(3x-2)}{(3x+1)(3x-2)}$	M1
	$\dfrac{6}{(3x-2)(3x+1)}$ or $\dfrac{6}{9x^2-3x-2}$	A1
		(4 marks)

Notes

B1 For factorising $9x^2-4=(3x-2)(3x+2)$ using difference of two squares. It can be awarded at any stage of the answer but it must be scored on E pen as the first mark

B1 For eliminating/cancelling out a factor of $(3x+2)$ at any stage of the answer.

M1 For combining two fractions to form a single fraction with a common denominator. Allow slips on the numerator but at least one must have been adapted. Condone invisible brackets. Accept two separate fractions with the same denominator as shown in the mark scheme. Amongst possible (incorrect) options scoring method marks are

$\dfrac{2(3x+2)}{(9x^2-4)(3x+1)}-\dfrac{2(9x^2-4)}{(9x^2-4)(3x+1)}$ Only one numerator adapted, separate fractions

$\dfrac{2\times 3x+1-2\times 3x-2}{(3x-2)(3x+1)}$ Invisible brackets, single fraction

A1 $\dfrac{6}{(3x-2)(3x+1)}$

This is not a given answer so you can allow recovery from 'invisible' brackets.

Alternative method

$\dfrac{2(3x+2)}{(9x^2-4)}-\dfrac{2}{(3x+1)}=\dfrac{2(3x+2)(3x+1)-2(9x^2-4)}{(9x^2-4)(3x+1)}=\dfrac{18x+12}{(9x^2-4)(3x+1)}$ has scored 0,0,1,0 so far

$=\dfrac{6(3x+2)}{(3x+2)(3x-2)(3x+1)}$ is now 1,1,1,0

$=\dfrac{6}{(3x-2)(3x+1)}$ and now 1,1,1,1

Q3

Question number	Scheme	Marks
	$(x+2)(3x^2-11x+6)$	M1 A1
	$(x+2)(3x-2)(x-3)$	M1 A1 (4)
	(If continues to 'solve an equation', isw)	

Q4

Question	Scheme	Marks	AOs
	Begins division or factorisation so $4x^3-12x^2+2x-6=(x-3)(4x^2+...)$	M1	2.1
	$4x^3-12x^2+2x-6=(x-3)(4x^2+2)$	A1	1.1b
	Considers the roots of their quadratic function using completion of square or discriminant	M1	2.1
	$(4x^2+2)=0$ has no real roots with a reason (e.g. negative number does not have a real square root, or $4x^2+2>0$ for all x) So $x=3$ is the only real root of $f(x)=0$ *	A1*	2.4
		(4)	

Q5

Question Number	Scheme	Marks
	$\frac{5}{(x-1)(3x+2)}=\frac{A}{x-1}+\frac{B}{3x+2}$	
	$5=A(3x+2)+B(x-1)$	
	$x\to 1$ $\quad 5=5A \Rightarrow A=1$	M1 A1
	$x\to -\frac{2}{3}$ $\quad 5=-\frac{5}{3}B \Rightarrow B=-3$	A1 (3)

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Q6

General points for marking question 6:

- Students who just try random numbers are not going to score any marks.
- Students can mix and match methods. Eg you may see odd numbers via logic and even via algebra
- Students who state $4m^2+2$ **cannot be divided** by (instead of is not divisible by) cannot be awarded credit for the accuracy/explanation marks, unless they state correctly that $4m^2+2$ **cannot be divided by 4 to give an integer.**
- Students who write $n^2+2=4k \Rightarrow k=\frac{1}{4}n^2+\frac{1}{2}$ which is not a whole number gains no credit unless they then start to look at odd and even numbers for instance
- Proofs via induction usually tend to go nowhere unless they proceed as in the main scheme
- Watch for unusual methods that are worthy of credit (See below)
- If the final conclusion is $n \in \mathbb{R}$ then the final mark is withheld. $n \in \mathbb{Z}^+$ is correct

Watch for methods that may not be in the scheme that you feel may deserve credit.

If you are uncertain of a method please refer these up to your team leader.

Eg 1. Solving by modulo arithmetic.

All $n \in \mathbb{N}$ mod 4	0	1	2	3
All $n^2 \in \mathbb{N}$ mod 4	0	1	0	1
All $n^2+2 \in \mathbb{N}$ mod 4	2	3	2	3

Hence for all n, n^2+2 is not divisible by 4.

Q6	Scheme	Marks	AOs

Notes: Note that M0 A0 M1 A1 and M0 A0 M1 A0 are not possible due to the way the scheme is set up

M1: Awarded for setting up the proof for either the even or odd numbers.

A1: Concludes correctly with a reason why n^2+2 cannot be divisible by 4 for either n odd or even.

dM1: Awarded for setting up the proof for both even and odd numbers

A1: Fully correct proof with valid explanation and conclusion for all n

Example of an algebraic proof

For $n=2m$, $n^2+2=4m^2+2$	M1	2.1

Concludes that this number is not divisible by 4 (as the explanation is trivial)	A1	1.1b
For $n = 2m+1$, $n^2 + 2 = (2m+1)^2 + 2 = ...$ FYI $(4m^2 + 4m + 3)$	dM1	2.1
Correct working and concludes that this is a number in the 4 times table add 3 so cannot be divisible by 4 or writes $4(m^2 + m) + 3$ ....AND states ......hence true for all	A1*	2.4
	(4)	

Example of a very similar algebraic proof

For $n = 2m$, $\dfrac{4m^2 + 2}{4} = m^2 + \dfrac{1}{2}$	M1	2.1
Concludes that this is not divisible by 4 due to the $\dfrac{1}{2}$ (A suitable reason is required)	A1	1.1b
For $n = 2m+1$, $\dfrac{n^2 + 2}{4} = \dfrac{4m^2 + 4m + 3}{4} = m^2 + m + \dfrac{3}{4}$	dM1	2.1
Concludes that this is not divisible by 4 due to the $\dfrac{3}{4}$ ...AND states ...... hence for all n, $n^2 + 2$ is not divisible by 4	A1*	2.4
	(4)	

Example of a proof via logic

When n is odd, "odd $\times$ odd" = odd	M1	2.1
so $n^2 + 2$ is odd, so (when n is odd) $n^2 + 2$ cannot be divisible by 4	A1	1.1b
When n is even, it is a multiple of 2, so "even $\times$ even" is a multiple of 4	dM1	2.1
Concludes that when n is even $n^2 + 2$ cannot be divisible by 4 because n^2 is divisible by 4…..AND STATES ……..trues for all n.	A1*	2.4
	(4)	

Example of proof via contradiction

Sets up the contradiction 'Assume that $n^2 + 2$ is divisible by 4 $\Rightarrow n^2 + 2 = 4k$ '	M1	2.1

$\Rightarrow n^2 = 4k - 2 = 2(2k-1)$ and concludes even Note that the M mark (for setting up the contradiction must have been awarded)	A1	1.1b
States that n^2 is even, then n is even and hence n^2 is a multiple of 4	dM1	2.1
Explains that if n^2 is a multiple of 4 then $n^2 + 2$ cannot be a multiple of 4 and hence divisible by 4 Hence there is a contradiction and concludes Hence true for all n.	A1*	2.4
	(4)	

A similar proof exists via contradiction where
A1: $n^2 = 2(2k-1) \Rightarrow n = \sqrt{2} \times \sqrt{2k-1}$
dM1: States that $2k-1$ is odd, so does not have a factor of 2, meaning that n is irrational

Silver Questions

Non-calculator

The total mark for this section is 28

Q1

Show that $\dfrac{3x+6}{x^2-3x-10} \div \dfrac{x+5}{x^3-25x}$ simplifies to ax where a is an integer.

(4)

(Total for Question 1 is 4 marks)

Q2

Express $\dfrac{4x}{x^2-9} - \dfrac{2}{x+3}$ as a single fraction in its simplest form.

(4)

(Total for Question 2 is 4 marks)

Q3

Express

$$\frac{3x+5}{x^2+x-12} - \frac{2}{x-3}$$

as a single fraction in its simplest form.

(4)

(Total for Question 3 is 4 marks)

Q4

Express in partial fractions

$$\frac{5x+3}{(2x+1)(x+1)^2}$$

(4)

(Total for Question 4 is 4 marks)

Q5

Express

$$\frac{3}{2x+3} - \frac{1}{2x-3} + \frac{6}{4x^2-9}$$

as a single fraction in its simplest form.

(4)

(Total for Question 5 is 4 marks)

Q6

Given that

$$\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 - 4}, \quad x \neq \pm 2$$

find the values of the constants a, b, c, d and e.

(4)

(Total for Question 6 is 4 marks)

Q7

Use algebra to prove that the square of any natural number is **either** a multiple of 3 **or** one more than a multiple of 3.

(4)

(Total for Question 4 is 10 marks)

End of Questions

Silver Mark Scheme

Q1

Question	Working	Answer	Notes
		$3x$	M1 Factorising numerator and denominator of first fraction $\frac{3(x+2)}{(x-5)(x+2)}$ $\left(=\frac{3}{(x-5)}\right)$ M1 Factorising denominator of second fraction $\frac{x+5}{x(x+5)(x-5)}$ $\left(=\frac{1}{x(x-5)}\right)$ M1 Multiplication by reciprocal $\frac{3(x+2)}{(x-5)(x+2)} \times \frac{x(x+5)(x-5)}{(x+5)}$ A1 Completing algebra to reach $3x$

Q2

Question Number	Scheme	Marks
	$x^2-9=(x+3)(x-3)$	B1
	$\frac{4x}{x^2-9}-\frac{2}{(x+3)}=\frac{4x-2(x-3)}{(x+3)(x-3)}$	M1
	$=\frac{2x+6}{(x+3)(x-3)}$	A1
	$=\frac{2\cancel{(x+3)}}{\cancel{(x+3)}(x-3)}$	
	$=\frac{2}{(x-3)}$	A1
		(4)

B1 $x^2 - 9 = (x+3)(x-3)$ This can occur anywhere.

M1 For combining the two fractions with a common denominator. The denominator must be correct and at least one numerator must have been adapted. Accept as separate fractions. Condone missing brackets.

For example accept $\dfrac{4x}{x^2-9} - \dfrac{2}{x+3} = \dfrac{4x(x+3) - 2(x^2-9)}{(x+3)(x^2-9)}$

accept separately $\dfrac{4x}{(x+3)(x-3)} - \dfrac{2}{(x+3)} = \dfrac{4x}{(x+3)(x-3)} - \dfrac{2x-3}{(x+3)(x-3)}$ condoning missing bracket

condone $\dfrac{4x}{x^2-9} - \dfrac{2}{x+3} = \dfrac{4x(x+3) - 2}{(x+3)(x^2-9)}$ ....as only one numerator has been adapted

A1 A correct intermediate form of $\dfrac{\text{simplified linear}}{\text{simplified quadratic}}$

Accept $\dfrac{2x+6}{(x+3)(x-3)}$, $\dfrac{2x+6}{x^2-9}$, and even $\dfrac{(2x+6)\,\cancel{(x+3)}}{(x^2-9)\,\cancel{(x+3)}}$,

A1 Further factorises and cancels (which may be implied) to reach the answer $\dfrac{2}{x-3}$

Do not penalise correct solutions that include incomplete lines Eg $\dfrac{4x - 2(x-3)}{(x+3)(x-3)} = \dfrac{4x - 2x + 6}{\ldots} = \dfrac{2x+6}{(x+3)(x-3)} = \dfrac{2}{x-3}$

This is not a "show that" question.

Note: Watch out for an answer of $\dfrac{2}{x+3}$ probably scored from $\dfrac{4x - 2(x-3)}{(x+3)(x-3)} = \dfrac{2x-6}{(x+3)(x-3)} = \dfrac{2(x-3)}{(x+3)(x-3)}$

This would score B1 M1 A0 A0

Q3

Question Number	Scheme	Marks
	(a) $x^2+x-12=(x+4)(x-3)$	B1
	Attempt as a single fraction $\dfrac{(3x+5)(x-3)-2(x^2+x-12)}{(x^2+x-12)(x-3)}$ or $\dfrac{3x+5-2(x+4)}{(x+4)(x-3)}$	M1
	$=\dfrac{x-3}{(x+4)(x-3)}$, $=\dfrac{1}{(x+4)}$ cao	A1, A1
		(4 marks)

Notes for Question

B1 For correctly factorising $x^2+x-12=(x+4)(x-3)$. It could appear anywhere in their solution

M1 For an attempt to combine two fractions. The denominator must be correct for 'their' fractions. The terms could be separate but one term must have been modified.
Condone invisible brackets.
Examples of work scoring this mark are;

$\dfrac{(3x+5)(x-3)}{(x^2+x-12)(x-3)}-\dfrac{2(x^2+x-12)}{(x^2+x-12)(x-3)}$ Two separate terms

$\dfrac{3x+5-2x+4}{(x+4)(x-3)}$ Single term, invisible bracket

$\dfrac{(3x+5)}{(x^2+x-12)(x-3)}-\dfrac{2(x^2+x-12)}{(x^2+x-12)(x-3)}$ Separate terms, only one numerator modified

A1 Correct un simplified answer $\dfrac{x-3}{(x+4)(x-3)}$

If $\dfrac{x^2-6x-9}{(x^2+x-12)(x-3)}$ scored M1 the fraction must be subsequently be reduced to a correct $\dfrac{x-3}{x^2+x-12}$ or $\dfrac{(x-3)(x-3)}{(x+4)(x-3)(x-3)}$ to score this mark.

A1 cao $\dfrac{1}{(x+4)}$

Do Not isw in this question.

The method of partial fractions is perfectly acceptable and can score full marks

$$\underbrace{\frac{3x+5}{(x+4)(x-3)}}_{B1}-\frac{2}{x-3}=\underbrace{\frac{1}{x+4}+\frac{2}{x-3}}_{M1A1}-\frac{2}{x-3}=\underbrace{\frac{1}{x+4}}_{A1}$$

Q4

Question Number	Scheme		Marks
	$\dfrac{5x+3}{(2x+1)(x+1)^2} \equiv \dfrac{A}{(2x+1)} + \dfrac{B}{(x+1)} + \dfrac{C}{(x+1)^2}$	At least one of "A" or "C" are correct.	B1
	$A = 2,\quad C = 2$	Breaks up their partial fraction correctly into three terms **and** both "A" $= 2$ and "C" $= 2$.	B1 cso
	$5x+3 \equiv A(x+1)^2 + B(2x+1)(x+1) + C(2x+1)$ $x=-1 \Rightarrow -2 = -C \Rightarrow C = 2$ $x = -\frac{1}{2} \Rightarrow -\frac{5}{2}+3 = \frac{1}{4}A \Rightarrow \frac{1}{2} = \frac{1}{4}A \Rightarrow A = 2$	Writes down ***a correct identity*** and attempts to find the value of either one "A" or "B" or "C".	M1
	Either $x^2: 0 = A + 2B$, constant: $3 = A + B + C$ $x: 5 = 2A + 3B + 2C$		
	leading to $B = -1$	Correct value for "B" which is found using a correct identity and follows from their partial fraction decomposition.	A1 cso
	So, $\dfrac{5x+3}{(2x+1)(x+1)^2} \equiv \dfrac{2}{(2x+1)} - \dfrac{1}{(x+1)} + \dfrac{2}{(x+1)^2}$		**[4]** **4**

Notes for Question

BE CAREFUL! Candidates will assign ***their own*** "A, B and C" for this question.

B1: At least one of "A" or "C" are correct.

B1: Breaks up their partial fraction correctly into three terms **and** both "A" $= 2$ and "C" $= 2$.

M1: Writes down ***a correct identity*** (although this can be implied) and attempts to find the value of either one of "A" or "B" or "C".
This can be achieved by ***either*** substituting values into their identity ***or*** comparing coefficients and solving the resulting equations simultaneously.

A1: Correct value for "B" which is found using a correct identity and follows from their partial fraction decomposition.
Note: If a candidate does not give partial fraction decomposition then:

- the 2nd B1 mark can follow from a correct identity.
- the final A1 mark can be awarded for a correct "B" if a candidate goes writes out their partial fractions at the end.

Note: The correct partial fraction from no working scores B1B1M1A1.

Note: A number of candidates will start this problem by writing out the correct identity and then attempt to find "A" or "B" or "C". Therefore the B1 marks can be awarded from this method.

Q5

Question Number	Scheme	Marks
	Factorise $4x^2-9=(2x-3)(2x+3)$	B1
	Use of common denominator $\frac{3}{2x+3}-\frac{1}{2x-3}+\frac{6}{4x^2-9}=\frac{3(2x-3)-1(2x+3)+6}{(2x+3)(2x-3)}$	M1
	$=\frac{4x-6}{(2x+3)(2x-3)}$	A1
	$=\frac{2\cancel{(2x-3)}}{(2x+3)\cancel{(2x-3)}}=\frac{2}{2x+3}$	A1 (4)
		4 marks
	Alternative where $4x^2-9$ is not factorised	
	$\frac{3}{2x+3}-\frac{1}{2x-3}+\frac{6}{4x^2-9}=\frac{3(2x-3)(4x^2-9)-1(2x+3)(4x^2-9)+6(2x+3)(2x-3)}{(2x+3)(2x-3)(4x^2-9)}$	M1
	$=\frac{2(2x-3)(4x^2-9)}{(2x+3)(2x-3)(4x^2-9)}$ or $\frac{(4x-6)(4x^2-9)}{(2x+3)(2x-3)(4x^2-9)}$ or $\frac{(2x-3)(8x^2-18)}{(2x+3)(2x-3)(4x^2-9)}$	B1
	$=\frac{(4x-6)\cancel{(4x^2-9)}}{(2x+3)(2x-3)\cancel{(4x^2-9)}}$ or $\frac{{}^{2}\cancel{(4x-6)}(4x^2-9)}{(2x+3)\cancel{(2x-3)}(4x^2-9)}$	A1
	$=\frac{2}{2x+3}$	A1

B1 For **factorising** $4x^2-9$ to $(2x-3)(2x+3)$ at any point. Note that this is not scored for combining the terms $(2x-3)(2x+3)$ and writing the product as $4x^2-9$

M1 Use of common denominator – combines three fractions to form one. The denominator must be correct for their fractions and at least one numerator must have been adapted. Condone missing brackets.

$\frac{16x^3-24x^2-36x+54}{(4x^2-9)^2}$ is a correct intermediate stage but needs to be factorised and cancelled before A1

Examples of incorrect fractions scoring this mark are: $\frac{3(2x-3)-2x+3+6}{(2x+3)(2x-3)}$ missing bracket

$\frac{3(4x^2-9)-4x^2-9+6(2x+3)(2x-3)}{(2x+3)(2x-3)(4x^2-9)}$ denominator correct and at least one numerator has been adapted.

A1 Correct simplified intermediate answer. It must be a CORRECT $\frac{\text{Linear}}{\text{Quadratic}}$ or $\frac{\text{Quadratic}}{\text{Cubic}}$

Accept versions of $\frac{4x-6}{(2x+3)(2x-3)}$ or $\frac{8x^2-18}{(2x+3)(4x^2-9)}$

A1 cao $=\frac{2}{2x+3}$

Allow recovery from invisible brackets for all 4 marks as the answer is not given.

Q6

Question Number	Scheme	Marks
By Division	$\begin{array}{r} 3x^2-2x+7 \\ x^2(+0x)-4\overline{)3x^4-2x^3-5x^2+(0x)-4} \\ \underline{3x^4+0x^3-12x^2} \\ -2x^3+7x^2+0x \\ \underline{-2x^3+0x^2\ +8x} \\ 7x^2-8x-4 \\ \underline{7x^2+0x-28} \\ -8x+24 \end{array}$	
	$a=3$	B1
	Long division as far as $\begin{array}{r} 3x^2-2x....... \\ x^2(+0x)-4\overline{)3x^4-2x^3-5x^2+(0x)-4} \\ \underline{3x^4+0x^3-12x^2} \\ -2x^3+................ \\ \underline{-2x^3+................} \end{array}$	M1
	Two of $b=-2\quad c=7\quad d=-8\quad e=24$	A1
	All four of $b=-2\quad c=7\quad d=-8\quad e=24$	A1
		(4 marks)

Notes for Question

B1 Stating $a=3$. This can also be scored by the coefficient of x^2 in $3x^2-2x+7$

M1 Using long division by x^2-4 and getting as far as the 'x' term. The coefficients need not be correct.
Award if you see the whole number part as $...x^2+...x$ following some working. You may also see this in a table/ grid.
Long division by $(x+2)$ will not score anything until $(x-2)$ has been divided into the new quotient. It is very unlikely to score full marks and the mark scheme can be applied.

A1 Achieving two of $b=-2$ $c=7$ $d=-8$ $e=24$.
The answers may be embedded within the division sum and can be implied.

A1 Achieving all of $b=-2$ $c=7$ $d=-8$ and $e=24$

Accept a correct long division for 3 out of the 4 marks scoring B1M1A1A0

Need to see a=…, b=…, or the values embedded in the rhs for all 4 marks

Q7

Question	Scheme	Marks	AOs
7	NB any natural number can be expressed in the form: $3k$, $3k + 1$, $3k + 2$ or equivalent e.g. $3k - 1$, $3k$, $3k + 1$		
	Attempts to square **any two** distinct cases of the above	M1	3.1a
	Achieves accurate results and makes a valid comment for **any two** of the possible three cases: E.g. $(3k)^2 = 9k^2 \left(= 3\times 3k^2\right)$ is a multiple of 3 $(3k+1)^2 = 9k^2 + 6k + 1 = 3\times\left(3k^2 + 2k\right) + 1$ is one more than a multiple of 3 $(3k+2)^2 = 9k^2 + 12k + 4 = 3\times\left(3k^2 + 4k + 1\right) + 1$ $\left(\text{or } (3k-1)^2 = 9k^2 - 6k + 1 = 3\times\left(3k^2 - 2k\right) + 1\right)$ is one more than a multiple of 3	A1 **M1 on EPEN**	1.1b
	Attempts to square in all 3 distinct cases. E.g. attempts to square $3k$, $3k + 1$, $3k + 2$ or e.g. $3k - 1$, $3k$, $3k + 1$	M1 **A1 on EPEN**	2.1
	Achieves accurate results for all three cases and gives a minimal conclusion (allow tick, QED etc.)	A1	2.4
		(4)	
		(4 marks)	

Notes:

M1: Makes the key step of attempting to write the natural numbers in any 2 of the 3 distinct forms or equivalent
expressions, as shown in the mark scheme, and attempts to square these expressions.

A1: Successfully shows for 2 cases that the squares are either a multiple of 3 or 1 more than a multiple
of 3 <u>using algebra</u>. This must be made explicit e.g. reaches $3\times\left(3k^2 + 2k\right) + 1$ and makes a statement that this is
one more than a multiple of 3 but also allow other rigorous arguments that reason why $9k^2 + 6k + 1$ is one more
than a multiple of 3 e.g. "$9k^2$ is a multiple of 3 and $6k$ is a multiple of 3 so $9k^2 + 6k + 1$ is one more than a multiple
of 3"

M1: Recognises that all natural numbers can be written in one of the 3 distinct forms or equivalent
expressions, as shown in the mark scheme, and attempts to square in all 3 cases.

A1: Successfully shows for all 3 cases that the squares are either a multiple of 3 or 1 more than a multiple of 3 using
algebra and makes a conclusion

Gold Questions

Non-calculator

The total mark for this section is 31

Q1

$$\frac{9x^2}{(x-1)^2(2x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(2x+1)}$$

Find the values of the constants A, B and C.

(4)

(Total for Question 1 is 4 marks)

Q2

(a) Simplify fully

$$\frac{2x^2+9x-5}{x^2+2x-15}$$

(3)

Given that

$$\ln(2x^2+9x-5) = 1 + \ln(x^2+2x-15), \quad x \neq -5,$$

(b) find x in terms of e.

(4)

(Total for Question 2 is 7 marks)

Q3

Express $\dfrac{9x^2+20x-10}{(x+2)(3x-1)}$ in partial fractions.

(4)

(Total for Question 3 is 4 marks)

Q4

Given that

$$\frac{2x^4 - 3x^2 + x + 1}{(x^2 - 1)} \equiv (ax^2 + bx + c) + \frac{dx + e}{(x^2 - 1)},$$

find the values of the constants a, b, c, d and e.

(4)

(Total for Question 4 is 4 marks)

Q5

Kayden claims that $3^x \geq 2^x$.

(i) Determine whether Kayden's claim is always true, sometimes true or never true, justifying your answer.

(2)

(ii) Prove that $\sqrt{3}$ is an irrational number.

(6)

(Total for Question 5 is 8 marks)

Q6

Prove by contradiction that there are no positive integers p and q such that

$$4p^2 - q^2 = 25$$

(4)

(Total for Question 6 is 4 marks)

End of Questions

Gold Mark Scheme

Q1

Question Number	Scheme		Marks	
	$9x^2 = A(x-1)(2x+1) + B(2x+1) + C(x-1)^2$		B1	
	$x \to 1$ $\quad 9 = 3B \Rightarrow B = 3$		M1	
	$x \to -\frac{1}{2}$ $\quad \frac{9}{4} = \left(-\frac{3}{2}\right)^2 C \Rightarrow C = 1$	Any two of A, B, C	A1	
	x^2 terms $\quad 9 = 2A + C \Rightarrow A = 4$	All three correct	A1	**(4)** **[4]**
	Alternatives for finding A.			
	x terms $\quad 0 = -A + 2B - 2C \Rightarrow A = 4$ Constant terms $\quad 0 = -A + B + C \Rightarrow A = 4$			

Q2

Question Number	Scheme	Marks
(a)	$\frac{(x+5)(2x-1)}{(x+5)(x-3)} = \frac{(2x-1)}{(x-3)}$	M1 B1 A1 aef (3)
(b)	$\ln\left(\frac{2x^2+9x-5}{x^2+2x-15}\right) = 1$	M1
	$\frac{2x^2+9x-5}{x^2+2x-15} = \mathrm{e}$	dM1
	$\frac{2x-1}{x-3} = \mathrm{e} \Rightarrow \quad 3\mathrm{e} - 1 = x(\mathrm{e} - 2)$	M1
	$\Rightarrow x = \frac{3\mathrm{e}-1}{\mathrm{e}-2}$	A1 aef cso
		(4) [7]
	(a) M1: An attempt to factorise the numerator. B1: Correct factorisation of denominator to give $(x+5)(x-3)$. Can be seen anywhere. (b) M1: Uses a correct law of logarithms to combine at least two terms. This usually is achieved by the subtraction law of logarithms to give $\ln\left(\frac{2x^2+9x-5}{x^2+2x-15}\right) = 1$. The product law of logarithms can be used to achieve $\ln(2x^2+9x-5) = \ln(\mathrm{e}(x^2+2x-15))$. The product and quotient law could also be used to achieve $\ln\left(\frac{2x^2+9x-5}{\mathrm{e}(x^2+2x-15)}\right) = 0$. dM1: Removing ln's correctly by the realisation that the anti-ln of 1 is e. Note that this mark is dependent on the previous method mark being awarded. M1: Collect x terms together and factorise. Note that this is not a dependent method mark. A1: $\frac{3\mathrm{e}-1}{\mathrm{e}-2}$ or $\frac{3\mathrm{e}^1-1}{\mathrm{e}^1-2}$ or $\frac{1-3\mathrm{e}}{2-\mathrm{e}}$. aef Note that the answer needs to be in terms of e. The decimal answer is 9.9610559… Note that the solution must be correct in order for you to award this final accuracy mark. **Note: See Appendix for an alternative method of long division.**	

Q3

Question Number	Scheme		Marks
	Method 1: Using one identity		
	$\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv A + \frac{B}{(x+2)} + \frac{C}{(3x-1)}$		
	$A = 3$	their constant term = 3	B1
	$9x^2+20x-10 \equiv A(x+2)(3x-1)+B(3x-1)+C(x+2)$	Forming a correct identity.	B1
	Either $x^2: \ 9 = 3A, \quad x: \ 20 = 5A+3B+C$ constant: $-10 = -2A - B + 2C$ or $x=-2 \Rightarrow 36-40-10=-7B \Rightarrow -14=-7B \Rightarrow B=2$	Attempts to find the value of either one of their B or their C from their identity.	M1
	$x=\frac{1}{3} \Rightarrow 1+\frac{20}{3}-10=\frac{7}{3}C \Rightarrow -\frac{7}{3}=\frac{7}{3}C \Rightarrow C=-1$	Correct values for their B and their C, which are found using a correct identity.	A1
			[4]
	Method 2: Long Division		
	$\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \frac{5x-4}{(x+2)(3x-1)}$	their constant term = 3	B1
	So, $\frac{5x-4}{(x+2)(3x-1)} \equiv \frac{B}{(x+2)} + \frac{C}{(3x-1)}$		
	$5x-4 \equiv B(3x-1)+C(x+2)$	Forming a correct identity.	B1
	Either $x: \ 5=3B+C$, constant: $-4=-B+2C$ or $x=-2 \Rightarrow -10-4=-7B \Rightarrow -14=-7B \Rightarrow B=2$	Attempts to find the value of either one of their B or their C from their identity.	M1
	$x=\frac{1}{3} \Rightarrow \frac{5}{3}-4=\frac{7}{3}C \Rightarrow -\frac{7}{3}=\frac{7}{3}C \Rightarrow C=-1$	Correct values for their B and their C, which are found using $5x-4 \equiv B(3x-1)+C(x+2)$	A1
	So, $\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \frac{2}{(x+2)} - \frac{1}{(3x-1)}$		[4]
			4

1[st] **B1:** Their constant term must be equal to 3 for this mark.
2[nd] **B1 (M1 on epen):** Forming a correct identity. This can be implied by later working.
M1 (A1 on epen): Attempts to find the value of either one of their B or their C from their identity. This can be achieved by *either* substituting values into their identity *or* comparing coefficients and solving the resulting equations simultaneously.
A1: Correct values for their B and their C, which are found using a correct identity.
Note : $\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv \frac{A}{(x+2)} + \frac{B}{(3x-1)}$, leading to $9x^2+20x-10 \equiv A(3x-1)+B(x+2)$, leading to $A=2$ and $B=-1$ will gain a maximum of B0B0M1A0

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Note: You can imply the 2nd B1 from either $\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv \dfrac{A(x+2)(3x-1)+B(3x-1)+C(x+2)}{(x+2)(3x-1)}$

or $\dfrac{5x-4}{(x+2)(3x-1)} \equiv \dfrac{B(3x-1)+C(x+2)}{(x+2)(3x-1)}$

Alternative Method 1: Initially dividing by (x + 2)

$$\frac{9x^2+20x-10}{"(x+2)"(3x-1)} \equiv \frac{9x+2}{(3x-1)} - \frac{14}{(x+2)(3x-1)}$$

$$\equiv 3 + \frac{5}{(3x-1)} - \frac{14}{(x+2)(3x-1)}$$

B1: their constant term = 3

So, $\dfrac{-14}{(x+2)(3x-1)} \equiv \dfrac{B}{(x+2)} + \dfrac{C}{(3x-1)}$

$-14 \equiv B(3x-1) + C(x+2)$

B1: Forming a correct identity.

$\Rightarrow B = 2, C = -6$

M1: Attempts to find either one of their B or their C from their identity.

So, $\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \dfrac{5}{(3x-1)} + \dfrac{2}{(x+2)} - \dfrac{6}{(3x-1)}$

and $\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \dfrac{2}{(x+2)} - \dfrac{1}{(3x-1)}$

A1: Correct answer in partial fractions.

Alternative Method 2: Initially dividing by (3x - 1)

$$\frac{9x^2+20x-10}{(x+2)"(3x-1)"} \equiv \frac{3x+\frac{23}{3}}{(x+2)} - \frac{\frac{7}{3}}{(x+2)(3x-1)}$$

$$\equiv 3 + \frac{\frac{5}{3}}{(x+2)} - \frac{\frac{7}{3}}{(x+2)(3x-1)}$$

B1: their constant term = 3

So, $\dfrac{-\frac{7}{3}}{(x+2)(3x-1)} \equiv \dfrac{B}{(x+2)} + \dfrac{C}{(3x-1)}$

$-\frac{7}{3} \equiv B(3x-1) + C(x+2)$

B1: Forming a correct identity.

$\Rightarrow B = \dfrac{1}{3}, C = -1$

M1: Attempts to find either one of their B or their C from their identity.

So, $\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \dfrac{\frac{5}{3}}{(x+2)} + \dfrac{\frac{1}{3}}{(x+2)} - \dfrac{1}{(3x-1)}$

and $\dfrac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \dfrac{2}{(x+2)} - \dfrac{1}{(3x-1)}$

A1: Correct answer in partial fractions.

Q4

Question Number	Scheme	Marks
	$\begin{array}{r\|l} & 2x^2 \quad\quad -1 \\ \hline x^2-1 & 2x^4 \quad -3x^2+x+1 \\ & 2x^4 \quad -2x^2 \\ \hline & \quad\quad -x^2+x+1 \\ & \quad\quad -x^2 \quad\;\; +1 \\ \hline & \quad\quad\quad\quad x \end{array}$	M1
	$a=2$ stated or implied	A1
	$c=-1$ stated or implied	A1
	$2x^2-1+\dfrac{x}{x^2-1}$ $a=2, b=0, c=-1, d=1, e=0$	
	$d=1$ and $b=0, e=0$ stated or implied	A1
		[4]

Q5

Question	Scheme	Marks	AOs
(i)	For an explanation or statement to show when the claim $3^x \ldots 2^x$ fails This could be e.g. • when $x = -1$, $\frac{1}{3} < \frac{1}{2}$ or $\frac{1}{3}$ is not greater than or equal to $\frac{1}{2}$ • when $x < 0$, $3^x < 2^x$ or 3^x is not greater than or equal to 2^x	M1	2.3
	followed by an explanation or statement to show when the claim $3^x \ldots 2^x$ is true. This could be e.g. • $x = 2$, $9 \ldots 4$ or 9 is greater than or equal to 4 • when $x \ldots 0$, $3^x \ldots 2^x$ and a correct conclusion. E.g. • so the claim $3^x \ldots 2^x$ is sometimes true	A1	2.4
		(2)	
(ii)	Assume that $\sqrt{3}$ is a rational number So $\sqrt{3} = \frac{p}{q}$, where p and q integers, $q \neq 0$, and the HCF of p and q is 1	M1	2.1
	$\Rightarrow p = \sqrt{3}q \Rightarrow p^2 = 3q^2$	M1	1.1b
	$\Rightarrow$ p^2 is divisible by 3 and so p is divisible by 3	A1	2.2a
	So $p = 3c$, where c is an integer From earlier, $p^2 = 3q^2 \Rightarrow (3c)^2 = 3q^2$	M1	2.1
	$\Rightarrow q^2 = 3c^2 \Rightarrow$ q^2 is divisible by 3 and so q is divisible by 3	A1	1.1b
	As both p and q are both divisible by 3 then the HCF of p and q is not 1 This contradiction implies that $\sqrt{3}$ is an irrational number	A1	2.4
		(6)	
			(8 marks)

Notes:	
(i)	
M1:	See scheme
A1:	See scheme
(ii)	
M1:	Uses a method of proof by contradiction by initially assuming that $\sqrt{3}$ is rational and expresses $\sqrt{3}$ in the form $\frac{p}{q}$, where p and q are correctly defined.
M1:	Writes $\sqrt{3} = \frac{p}{q}$ and rearranges to make p^2 the subject
A1:	Uses a logical argument to prove that p is divisible by 3
M1:	Uses the result that p is divisible by 3, (to construct the initial stage of proving that q is also divisible by 3), by substituting $p = 3c$ into their expression for p^2
A1:	Hence uses a correct argument, in the same way as before, to deduce that q is also divisible by 3
A1:	Completes the argument (as detailed on the scheme) that $\sqrt{3}$ is irrational. **Note:** All the previous 5 marks need to be scored in order to obtain the final A mark.

Q6

Question	Scheme	Marks	AOs
6	Sets up the contradiction and factorises: There are positive integers p and q such that $(2p+q)(2p-q)=25$	M1	2.1
	If true then $\begin{matrix}2p+q=25\\2p-q=1\end{matrix}$ or $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ **Award for deducing either of the above statements**	M1	2.2a
	Solutions are $p=6.5, q=12$ or $p=2.5, q=0$ Award for one of these	A1	1.1b
	This is a contradiction as there are no integer solutions hence there are no positive integers p and q such that $4p^2-q^2=25$	A1	2.1
		(4)	
	(4 marks)		

Notes:

M1: For the key step in setting up the contradiction and factorising

M1: For deducing that for p and q to be integers then either $\begin{matrix}2p+q=25\\2p-q=1\end{matrix}$ or $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ must be true.

Award for deducing either of the above statements.

You can ignore any reference to $\begin{matrix}2p+q=1\\2p-q=25\end{matrix}$ as this could not occur for positive p and q.

A1: For correctly solving one of the given statements,

For $\begin{matrix}2p+q=25\\2p-q=1\end{matrix}$ candidates only really need to proceed as far as $p=6.5$ to show the contradiction.

For $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ candidates only really need to find either p or q to show the contradiction.

Alt for $\begin{matrix}2p+q=5\\2p-q=5\end{matrix}$ candidates could state that $2p+q\neq 2p-q$ if p,q are positive integers.

A1: For a complete and rigorous argument with both possibilities and a correct conclusion.

Question	Scheme	Marks	AOs
6 Alt 1	Sets up the contradiction, attempts to make q^2 or $4p^2$ the subject and states that either $4p^2$ is even(*), or that q^2 (or q) is odd (**) Either There are positive integers p and q such that $4p^2 - q^2 = 25 \Rightarrow q^2 = 4p^2 - 25$ with * or ** Or There are positive integers p and q such that $4p^2 - q^2 = 25 \Rightarrow 4p^2 = q^2 + 25$ with * or **	M1	2.1
	Sets $q = 2n \pm 1$ and expands $(2n \pm 1)^2 = 4p^2 - 25$	M1	2.2a
	Proceeds to an expression such as $4p^2 = 4n^2 + 4n + 26 = 4(n^2 + n + 6) + 2$ $4p^2 = 4n^2 + 4n + 26 = 4(n^2 + n) + \frac{13}{2}$ $p^2 = n^2 + n + \frac{13}{2}$	A1	1.1b
	States This is a contradiction as $4p^2$ must be a multiple of 4 Or p^2 must be an integer And concludes there are no positive integers p and q such that $4p^2 - q^2 = 25$	A1	2.1
		(4)	

Alt 2

An approach using odd and even numbers is unlikely to score marks.

To make this consistent with the Alt method, score

M1: Set up the contradiction and start to consider one of the cases below where q is odd, $m \neq n$.

Solutions using the same variable will score no marks.

M1: Set up the contradiction and start to consider BOTH cases below where q is odd, $m \neq n$.

No requirement for evens

A1: Correct work and deduction for one of the two scenarios where q is odd

A1: Correct work and deductions for both scenarios where q is odd with a final conclusion

Options	Example of Calculation	Deduction
p (even) q (odd)	$4p^2 - q^2 = 4 \times (2m)^2 - (2n+1)^2 = 16m^2 - 4n^2 - 4n - 1$	One less than a multiple of 4 so cannot equal 25
p (odd) q (odd)	$4p^2 - q^2 = 4 \times (2m+1)^2 - (2n+1)^2 = 16m^2 + 16m - 4n^2 - 4n + 3$	Three more than a multiple of 4 so cannot equal 25

Topic 2: Functions and Graphs

Bronze, Silver, Gold and Platinum Worksheets for A Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel A Level Mathematics: Pure Mathematics Year 2' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high-level problem-solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Non-calculator

The total mark for this section is 32

Q1

The function f is defined by

$$f(x) = \frac{3x-7}{x-2} \qquad x \in \mathbb{R}, x \neq 2$$

(a) Find $f^{-1}(7)$.

(2)

(b) Show that $ff(x) = \frac{ax+b}{x-3}$ where a and b are integers to be found.

(3)

(Total for Question 1 is 5 marks)

Q2

$$g(x) = \frac{2x+5}{x-3} \qquad x \geqslant 5$$

(a) Find gg(5).

(2)

(b) State the range of g.

(1)

(c) Find $g^{-1}(x)$, stating its domain.

(3)

(Total for Question 2 is 6 marks)

Q3

The function f has domain $-2 \le x \le 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = f(x)$ is shown in Figure 1.

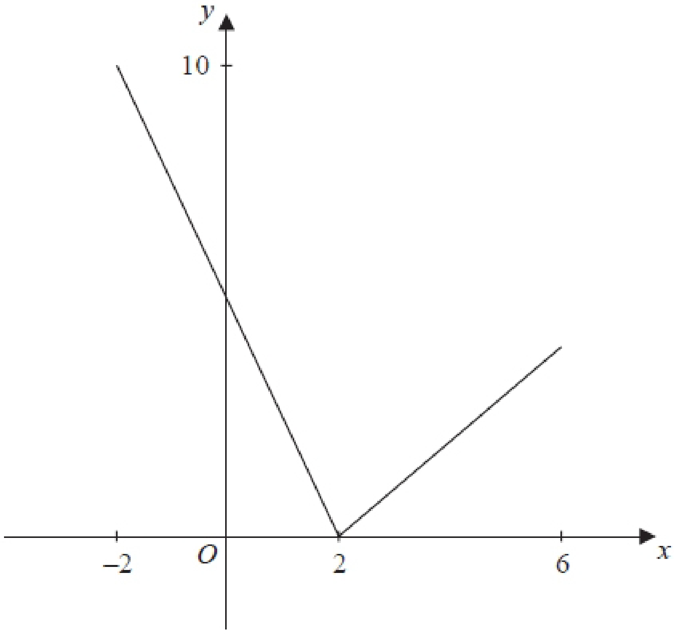


Figure 1

(a) Write down the range of f.

(1)

(b) Find ff(0).

(2)

The function g is defined by

$$g : x \rightarrow \frac{4 + 3x}{5 - x}, \qquad x \in \mathbb{R}, \qquad x \neq 5$$

(c) Find $g^{-1}(x)$.

(3)

(d) Solve the equation $gf(x) = 16$.

(5)

(Total for Question 3 is 11 marks)

Q4

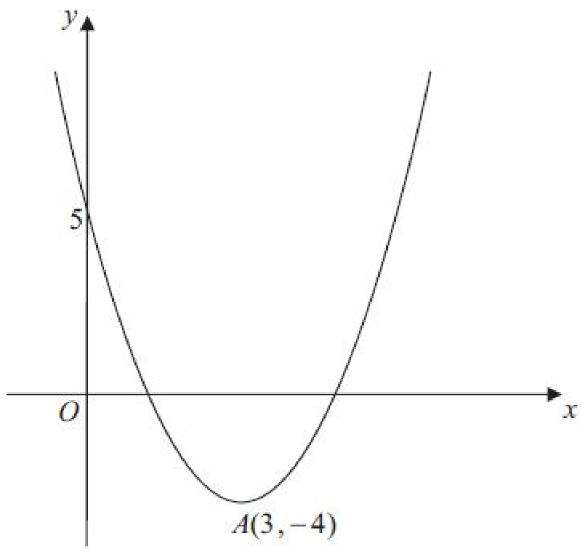


Figure 2

Figure 2 shows a sketch of the curve with the equation $y = \mathrm{f}(x), x \in \mathbb{R}$.

The curve has a turning point at $A(3, -4)$ and also passes through the point (0, 5).

(a) Write down the coordinates of the point to which A is transformed on the curve with equation

(i) $y = |\mathrm{f}(x)|$,

(ii) $y = 2\mathrm{f}\left(\frac{1}{2}x\right)$.

(4)

(b) Sketch the curve with equation

$$y = \mathrm{f}(|x|)$$

On your sketch show the coordinates of all turning points and the coordinates of the point at which the curve cuts the y-axis.

(3)

The curve with equation $y = \mathrm{f}(x)$ is a translation of the curve with equation $y = x^2$.

(c) Find $\mathrm{f}(x)$.

(2)

(d) Explain why the function f does not have an inverse.

(1)

(Total for Question 4 is 10 marks)

End of Questions

Bronze Mark Scheme

Q1

Question	Scheme	Marks	AOs
(a)	Either attempts $\frac{3x-7}{x-2}=7 \Rightarrow x=...$ Or attempts $f^{-1}(x)$ and substitutes in $x=7$	M1	3.1a
	$\frac{7}{4}$ oe	A1	1.1b
		(2)	
(b)	Attempts $ff(x)=\frac{3\times\left(\frac{3x-7}{x-2}\right)-7}{\left(\frac{3x-7}{x-2}\right)-2}=\frac{3\times(3x-7)-7(x-2)}{3x-7-2(x-2)}$	M1, dM1	1.1b 1.1b
	$=\frac{2x-7}{x-3}$	A1	2.1
		(3)	
			(5 marks)
Notes:			

(a)

M1: For either attempting to solve $\frac{3x-7}{x-2}=7$. Look for an attempt to multiply by the $(x-2)$ leading to a value for x.

Or score for substituting in $x=7$ in $f^{-1}(x)$. FYI $f^{-1}(x)=\frac{2x-7}{x-3}$

The method for finding $f^{-1}(x)$ should be sound, but you can condone slips.

A1: $\frac{7}{4}$

(b)

M1: For an attempt at fully substituting $\frac{3x-7}{x-2}$ into $f(x)$. Condone slips but the expression must have a correct form. E.g. $\frac{3\times\left(\frac{*-*}{*-*}\right)-a}{\left(\frac{*-*}{*-*}\right)-b}$ where a and b are positive constants.

dM1: Attempts to multiply **all** terms on the numerator and denominator by $(x-2)$ to create a fraction $\frac{P(x)}{Q(x)}$ where both $P(x)$ and $Q(x)$ are linear expressions. Condone $\frac{P(x)}{Q(x)}\times\frac{x-2}{x-2}$

A1: Reaches $\frac{2x-7}{x-3}$ via careful and accurate work. Implied by $a=2, b=-7$ following correct work.

Methods involving $\frac{3x-7}{x-2}\equiv a+\frac{b}{x-2}$ may be seen. The scheme can be applied in a similar way

FYI $\frac{3x-7}{x-2}\equiv 3-\frac{1}{x-2}$

Q2

Question	Scheme	Marks	AOs
	$g(x) = \dfrac{2x+5}{x-3},\ x \geq 5$		
(a) Way 1	$g(5) = \dfrac{2(5)+5}{5-3} = 7.5 \Rightarrow gg(5) = \dfrac{2("7.5")+5}{"7.5"-3}$	M1	1.1b
	$gg(5) = \dfrac{40}{9}\ \left(\text{or } 4\dfrac{4}{9} \text{ or } 4.\dot{4}\right)$	A1	1.1b
		(2)	
(a) Way 2	$gg(x) = \dfrac{2\left(\dfrac{2x+5}{x-3}\right)+5}{\left(\dfrac{2x+5}{x-3}\right)-3} \Rightarrow gg(5) = \dfrac{2\left(\dfrac{2(5)+5}{(5)-3}\right)+5}{\left(\dfrac{2(5)+5}{(5)-3}\right)-3}$	M1	1.1b
	$gg(5) = \dfrac{40}{9}\ \left(\text{or } 4\dfrac{4}{9} \text{ or } 4.\dot{4}\right)$	A1	1.1b
		(2)	
(b)	{Range:} $2 < y \leq \dfrac{15}{2}$	B1	1.1b
		(1)	
(c) Way 1	$y = \dfrac{2x+5}{x-3} \Rightarrow yx - 3y = 2x + 5 \Rightarrow yx - 2x = 3y + 5$	M1	1.1b
	$x(y-2) = 3y + 5 \Rightarrow x = \dfrac{3y+5}{y-2}\ \left\{\text{or } y = \dfrac{3x+5}{x-2}\right\}$	M1	2.1
	$g^{-1}(x) = \dfrac{3x+5}{x-2},\quad 2 < x \leq \dfrac{15}{2}$	A1ft	2.5
		(3)	
(c) Way 2	$y = \dfrac{2x-6+11}{x-3} \Rightarrow y = 2 + \dfrac{11}{x-3} \Rightarrow y - 2 = \dfrac{11}{x-3}$	M1	1.1b
	$x - 3 = \dfrac{11}{y-2} \Rightarrow x = \dfrac{11}{y-2} + 3\ \left\{\text{or } y = \dfrac{11}{x-2} + 3\right\}$	M1	2.1
	$g^{-1}(x) = \dfrac{11}{x-2} + 3,\quad 2 < x \leq \dfrac{15}{2}$	A1ft	2.5
		(3)	
		(6 marks)	

Notes for Question

(a)	
M1:	Full method of attempting $g(5)$ and substituting the result into g
Note:	**Way 2:** Attempts to substitute $x = 5$ into $\dfrac{2\left(\dfrac{2x+5}{x-3}\right)+5}{\left(\dfrac{2x+5}{x-3}\right)-3}$, o.e. Note that $gg(x) = \dfrac{9x-5}{14-x}$
A1:	Obtains $\dfrac{40}{9}$ or $4\dfrac{4}{9}$ or $4.\dot{4}$ or an exact equivalent
Note:	Give A0 for 4.4 or 4.444... without reference to $\dfrac{40}{9}$ or $4\dfrac{4}{9}$ or $4.\dot{4}$

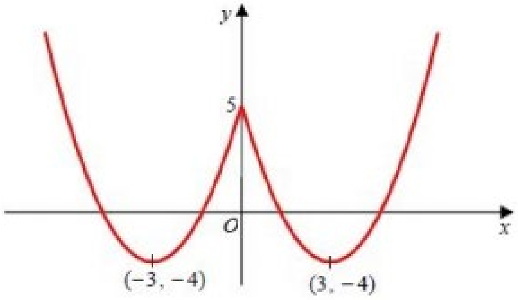
Notes for Question Continued	
(b)	
B1:	States $2 < y \le \frac{15}{2}$ Accept any of $2 < \text{g} \le \frac{15}{2}$, $2 < \text{g}(x) \le \frac{15}{2}$, $\left(2, \frac{15}{2}\right]$
Note:	Accept $\text{g}(x) > 2$ **and** $\text{g}(x) \le \frac{15}{2}$ o.e.
(c) Way 1	
M1:	Correct method of cross multiplication followed by an attempt to collect terms in x or terms in a swapped y
M1:	A complete method (i.e. as above and also factorising and dividing) to find the inverse
A1ft:	Uses correct notation to correctly define the inverse function g^{-1}, where the domain of g^{-1} stated correctly or correctly followed through (using correct notation) on the values shown in their range in part (b). Allow $\text{g}^{-1} : x \to$. Condone $\text{g}^{-1} = ...$ Do not accept $y = ...$
Note:	Correct notation is required when stating the domain of $\text{g}^{-1}(x)$. Allow $2 < x \le \frac{15}{2}$ or $\left(2, \frac{15}{2}\right]$ Do not allow any of e.g. $2 < \text{g} \le \frac{15}{2}$, $2 < \text{g}^{-1}(x) \le \frac{15}{2}$
Note:	Do not allow A1ft for following through their range in (b) to give a domain for g^{-1} as $x \in \mathbb{R}$
(c) Way 2	
M1:	Writes $y = \frac{2x+5}{x-3}$ in the form $y = 2 \pm \frac{k}{x-3}$, $k \neq 0$ and rearranges to isolate y and 2 on one side of their equation. **Note:** Allow the equivalent method with x swapped with y
M1:	A complete method to find the inverse
A1ft:	As in Way 1
Note:	If a candidate scores no marks in part (c), but • states the domain of g^{-1} correctly, **or** • states a domain of g^{-1} which is correctly followed through on the values shown in their range in part (b) then give special case (SC) M1 M0 A0

Q3

Question Number	Scheme	Marks
(a)	$0 \leqslant f(x) \leqslant 10$	B1
		(1)
(b)	$ff(0) = f(5), = 3$	B1,B1
		(2)
(c)	$y = \dfrac{4+3x}{5-x} \Rightarrow y(5-x) = 4+3x$	
	$\Rightarrow 5y - 4 = xy + 3x$	M1
	$\Rightarrow 5y-4 = x(y+3) \Rightarrow x = \dfrac{5y-4}{y+3}$	dM1
	$g^{-1}(x) = \dfrac{5x-4}{3+x}$	A1
		(3)
(d)	$gf(x) = 16 \Rightarrow f(x) = g^{-1}(16) = 4$ oe	M1A1
	$f(x) = 4 \Rightarrow x = 6$	B1
	$f(x) = 4 \Rightarrow 5 - 2.5x = 4 \Rightarrow x = 0.4$ oe	M1A1
		(5)
		(11 marks)
Alt 1 to (d)	$gf(x) = 16 \Rightarrow \dfrac{4+3(ax+b)}{5-(ax+b)} = 16$	M1
	$ax+b = x-2$ *or* $5-2.5x$	A1
	$\Rightarrow x = 6$	B1
	$\dfrac{4+3(5-2.5x)}{5-(5-2.5x)} = 16 \Rightarrow x = ...$	M1
	$\Rightarrow x = 0.4$ oe	A1 (5)

Notes for Question	
(a)	
B1	Correct range. Allow $0 \leqslant f(x) \leqslant 10$, $0 \leqslant f \leqslant 10$, $0 \leqslant y \leqslant 10$, $0 \leqslant \text{range} \leqslant 10$, $[0,10]$
	Allow $f(x) \geqslant 0$ and $f(x) \leqslant 10$ but not $f(x) \geqslant 0$ or $f(x) \leqslant 10$
	Do Not Allow $0 \leqslant x \leqslant 10$. The inequality must include BOTH ends
(b)	
B1	For correct one application of the function at x=0
	Possible ways to score this mark are f(0)=5, f(5) $0 \rightarrow 5 \rightarrow \ldots$
B1:	3 ('3' can score both marks as long as no incorrect working is seen.)
(c)	
M1	For an attempt to make x or a replaced y the subject of the formula. This can be scored for putting $y = g(x)$, multiplying across, expanding and collecting x terms on one side of the equation. Condone slips on the signs
dM1	Take out a common factor of x (or a replaced y) and divide, to make x subject of formula. Only allow **one sign error** for this mark
A1	Correct answer. No need to state the domain. Allow $g^{-1}(x) = \dfrac{5x-4}{3+x}$ $y = \dfrac{5x-4}{3+x}$
	Accept alternatives such as $y = \dfrac{4-5x}{-3-x}$ and $y = \dfrac{5-\frac{4}{x}}{1+\frac{3}{x}}$
(d)	
M1	Stating or implying that $f(x) = g^{-1}(16)$. For example accept $\dfrac{4+3f(x)}{5-f(x)} = 16 \Rightarrow f(x) = ..$
A1	Stating $f(x) = 4$ or implying that solutions are where $f(x) = 4$
B1	$x = 6$ and may be given if there is no working
M1	Full method to obtain other value from line $y = 5 - 2.5x$
	$5 - 2.5x = 4 \Rightarrow x = \ldots$.
	Alternatively this could be done by similar triangles. Look for $\dfrac{2}{5} = \dfrac{2-x}{4}(oe) \Rightarrow x = ..$
A1	0.4 or 2/5
Alt 1 to (d)	
M1	Writes $gf(x) = 16$ with a linear $f(x)$. The order of gf(x) must be correct
	Condone invisible brackets. Even accept if there is a modulus sign.
A1	Uses $f(x) = x - 2$ or $f(x) = 5 - 2.5x$ in the equation $gf(x) = 16$
B1	$x = 6$ and may be given if there is no working
M1	Attempt at solving $\dfrac{4+3(5-2.5x)}{5-(5-2.5x)} = 16 \Rightarrow x = \ldots$. The bracketing must be correct and there must be no more than one error in their calculation
A1	$x = 0.4, \dfrac{2}{5}$ or equivalent

Q4

Question Number	Scheme	Marks
(a) (i)	$(3, 4)$	B1 B1
(ii)	$(6, -8)$	B1 B1
		(4)
(b)		B1 B1 B1
		(3)
(c)	$\mathrm{f}(x) = (x-3)^2 - 4$ or $\mathrm{f}(x) = x^2 - 6x + 5$	M1A1
		(2)
(d)	Either: The function f is a many-one {mapping}. Or: The function f is not a one-one {mapping}.	B1
		(1) **[10]**
	(b) B1: Correct shape for $x \geqslant 0$, with the curve meeting the positive y-axis and the turning point is found below the x-axis. (providing candidate does not copy the whole of the original curve and adds nothing else to their sketch.). B1: Curve is symmetrical about the y-axis or correct shape of curve for $x < 0$. **Note:** The first two B1B1 can only be awarded if the curve has the correct shape, with a cusp on the positive y-axis and with both turning points located in the correct quadrants. Otherwise award B1B0. B1: Correct turning points of $(-3, -4)$ and $(3, -4)$. Also, $(\{0\}, 5)$ is marked where the graph cuts through the y-axis. Allow $(5, 0)$ rather than $(0, 5)$ if marked in the "correct" place on the y-axis. (c) M1: Either states $\mathrm{f}(x)$ in the form $(x \pm \alpha)^2 \pm \beta$; $\alpha, \beta \neq 0$ Or uses a complete method on $\mathrm{f}(x) = x^2 + ax + b$, with $\mathrm{f}(0) = 5$ and $\mathrm{f}(3) = -4$ to find both a and b. A1: Either $(x-3)^2 - 4$ or $x^2 - 6x + 5$ (d) B1: Or: The inverse is a one-many {mapping and not a function}. Or: Because $\mathrm{f}(0) = 5$ and also $\mathrm{f}(6) = 5$. Or: One y-coordinate has 2 corresponding x-coordinates {and therefore cannot have an inverse}.	

Silver Questions

Non-calculator

The total mark for this section is 31

Q1

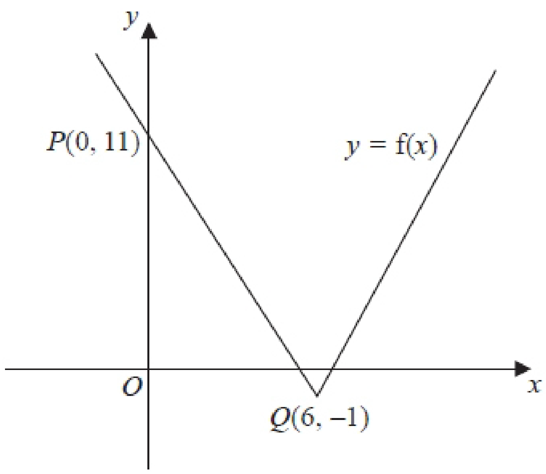


Figure 1

Figure 1 shows part of the graph with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point $Q(6, -1)$.

The graph crosses the y-axis at the point $P(0, 11)$.

Sketch, on separate diagrams, the graphs of

(a) $y = |\mathrm{f}(x)|$

(2)

(b) $y = 2\mathrm{f}(-x) + 3$

(3)

(Total for Question 1 is 5 marks)

Q2

The function f is defined by

$$f : x \mapsto 4 - \ln(x+2), \quad x \in \mathbb{R},\ x \geqslant -1$$

(a) Find $f^{-1}(x)$.

(3)

(b) Find the domain of $f^{-1}(x)$.

(1)

The function g is defined by

$$g : x \mapsto e^{x^2} - 2, \quad x \in \mathbb{R}$$

(c) Find $fg(x)$, giving your answer in its simplest form.

(3)

(d) Find the range of fg.

(1)

(Total for Question 2 is 8 marks)

Q3

The function f is defined by

$$f : x \mapsto |2x - 5|, \quad x \in \mathbb{R}$$

(a) Sketch the graph with equation $y = f(x)$, showing the coordinates of the points where the graph cuts or meets the axes.

(2)

(b) Solve $f(x) = 15 + x$.

(3)

The function g is defined by

$$g : x \mapsto x^2 - 4x + 1, \quad x \in \mathbb{R}, \quad 0 \leqslant x \leqslant 5$$

(c) Find fg(2).

(2)

(d) Find the range of g.

(3)

(Total for Question 3 is 10 marks)

Q4

Given that a and b are positive constants,

(a) on separate diagrams, sketch the graph with equation

(i) $y = |2x - a|$
(ii) $y = |2x - a| + b$

Show, on each sketch, the coordinates of each point at which the graph crosses or meets the axes.

(4)

Given that the equation

$$|2x - a| + b = \frac{3}{2}x + 8$$

has a solution at $x = 0$ and a solution at $x = c$,

(b) find c in terms of a.

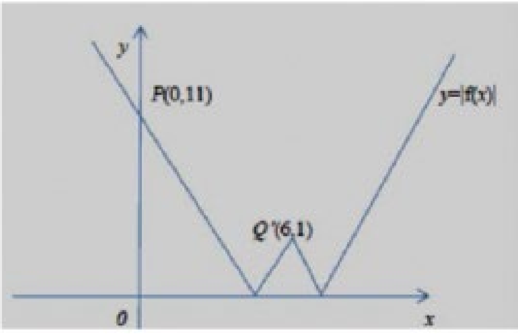
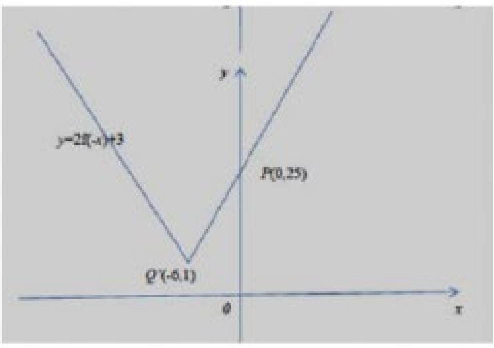
(4)

(Total for Question 4 is 8 marks)

End of Questions

Silver Mark Scheme

Q1

Question Number	Scheme		Marks
(a)		'W' Shape (0, 11) and (6, 1)	B1 B1 (2)
(b)		'V' shape (-6,1) (0,25)	B1 B1 B1 (3)

(a)

B1 A W shape in any position. The arms of the W do not need to be symmetrical but the two bottom points must appear to be at the same height. Do not accept rounded W's.
A correct sketch of $y = \mathrm{f}(|x|)$ would score this mark.

B1 A W shape in quadrants 1 and 2 sitting on the x axis with $P' = (0,11)$ **and** $Q' = (6,1)$. It is not necessary to see them labelled. Accept 11 being marked on the y axis for P'. Condone $P' = (11,0)$ marked on the correct axis, but $Q' = (1,6)$ is B0

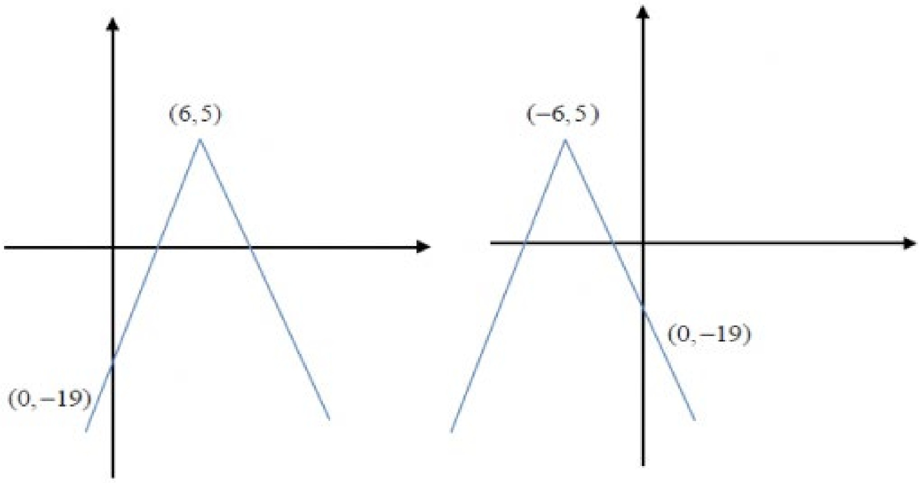
(b)

B1 Score for a V shape in any position on the grid. The arms of the V do not need to be symmetrical. Do not accept rounded or upside down V's for this mark.

B1 $Q' = (-6,1)$. It does not need to be labelled but it must correspond to the minimum point on the curve and be in the correct quadrant.

B1 $P' = (0,25)$. It does not need to be labelled but it must correspond to the y intercept and the line must cross the axis. Accept 25 marked on the correct axis. Condone $P' = (25,0)$ marked on the positive y axis.

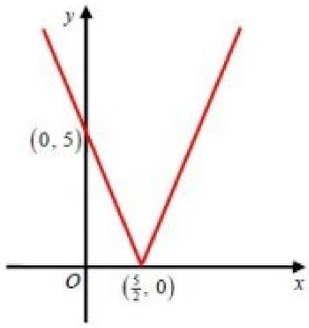
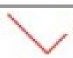
Special case: A candidate who mistakenly sketches $y = -2\mathrm{f}(x) + 3$ or $y = -2\mathrm{f}(-x) + 3$ will arrive at one of the following. They can be awarded SC B1B0B0



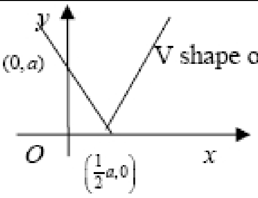
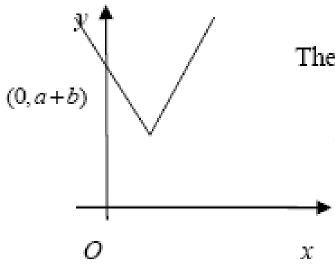
Q2

(a)	$y = 4 - \ln(x+2)$ $\ln(x+2) = 4 - y$ $x + 2 = e^{4-y}$ $x = e^{4-y} - 2$ $f^{-1}(x) = e^{4-x} - 2$ oe	M1 M1A1 (3)
(b)	$x \leq 4$	B1 (1)
(c)	$fg(x) = 4 - \ln(e^{x^2} - 2 + 2)$ $fg(x) = 4 - x^2$	M1 dM1A1 (3)
(d)	$fg(x) \leq 4$	B1ft (1)
		8 Marks

Q3

Question Number	Scheme	Marks
(a)		M1A1 (2)
(b)	$\underline{x = 20}$ $2x - 5 = -(15 + x)\ ; \Rightarrow \underline{x = -\frac{10}{3}}$	B1 M1;A1 oe. (3)
(c)	$\text{fg}(2) = \text{f}(-3) = \lvert 2(-3) - 5 \rvert;\ = \lvert -11 \rvert = 11$	M1;A1 (2)
(d)	$\text{g}(x) = x^2 - 4x + 1 = (x-2)^2 - 4 + 1 = (x-2)^2 - 3$. Hence $\text{g}_{\min} = -3$ Either $\text{g}_{\min} = -3$ or $\text{g}(x) \geqslant -3$ or $\text{g}(5) = 25 - 20 + 1 = 6$ $\underline{-3 \leqslant \text{g}(x) \leqslant 6}$ or $\underline{-3 \leqslant y \leqslant 6}$	M1 B1 A1 (3) **[10]**
	(a) M1: V or  or  graph with vertex on the x-axis. A1: $(\frac{5}{2}, \{0\})$ and $(\{0\}, 5)$ seen and the graph appears in both the first and second quadrants. (b) M1: Either $2x - 5 = -(15 + x)$ or $-(2x - 5) = 15 + x$ (c) M1: ***Full method*** of inserting $\text{g}(2)$ into $\text{f}(x) = \lvert 2x - 5 \rvert$ or for inserting $x = 2$ into $\lvert 2(x^2 - 4x + 1) - 5 \rvert$. There must be evidence of the modulus being applied. (d) M1: **Full method** to establish the minimum of g. Eg: $(x \pm \alpha)^2 + \beta$ leading to $\text{g}_{\min} = \beta$. Or for candidate to differentiate the quadratic, set the result equal to zero, find x and insert this value of x back into $\text{f}(x)$ in order to find the minimum. B1: For either finding the correct minimum value of g (can be implied by $\text{g}(x) \geqslant -3$ or $\text{g}(x) > -3$) or for stating that $\text{g}(5) = 6$. A1: $\underline{-3 \leqslant \text{g}(x) \leqslant 6}$ or $\underline{-3 \leqslant y \leqslant 6}$ or $\underline{-3 \leqslant \text{g} \leqslant 6}$. **Note that:** $-3 \leqslant x \leqslant 6$ is A0. **Note that:** $-3 \leqslant \text{f}(x) \leqslant 6$ is A0. **Note that**: $-3 \geqslant \text{g}(x) \geqslant 6$ is A0. **Note that:** $\text{g}(x) \geqslant -3$ or $\text{g}(x) > -3$ or $x \geqslant -3$ or $x > -3$ with no working gains M1B1A0. **Note that for the final Accuracy Mark**: If a candidate writes down $-3 < \text{g}(x) < 6$ or $-3 < y < 6$, then award M1B1A0. If, however, a candidate writes down $\text{g}(x) \geqslant -3$, $\text{g}(x) \leqslant 6$, then award A0. If a candidate writes down $\text{g}(x) \geqslant -3$ or $\text{g}(x) \leqslant 6$, then award A0.	

Q4

Question Number	Scheme	Marks
(a)(i)	 V shape on x - axis **or** coordinates $\left(\frac{1}{2}a, 0\right)$ **and** $(0, a)$	B1
	Correct shape, position and coordinates	B1
(ii)	 Their "V" shape translated up or $(0, a+b)$	B1ft
	Correct shape, position and $(0, a+b)$	B1
		(4)
(b)	States or uses $a+b=8$	B1
	Attempts to solve $\lvert 2x-a\rvert + b = \frac{3}{2}x+8$ in either x or with $x = c$	
	$2c-a+b=\frac{3}{2}c+8 \Rightarrow kc = \mathrm{f}(a,b)$	M1
	Combines $kc = \mathrm{f}(a,b)$ with $a+b=8 \quad \Rightarrow c = 4a$	dM1 A1
		(4)
		(8 marks)

(a)(i)

B1 V shape sitting anywhere on the x- axis **or** for $\left(\frac{1}{2}a, 0\right)$ and $(0, a)$ lying on the curve.

Condone non -symmetrical graphs and ones lying on just one side of the y -axis

B1 V shape sitting on the positive x-axis at $\left(\frac{1}{2}a, 0\right)$, cutting the y-axis at $(0, a)$ and lying in both quadrants 1 and 2

Accept $\frac{1}{2}a$ and a marked on the correct axis. Condone say $(a, 0)$ for $(0, a)$ as long as it is on the correct axis.

Condone a dotted line appearing on the diagram as many reflect $y = 2x-a$ to sketch $y = \lvert 2x-a\rvert$

If it is a solid line then it would not score the shape mark.

(a)(ii)

B1ft Follow through on (a)(i). Their graph translated up. Allow on U shapes and non symmetrical graphs.
Alternatively score for the $(0, a+b)$ lying on the curve

B1 V shape lying in quadrants 1 and 2 with the vertex in quadrant 1 cutting the y- axis at $(0, a+b)$
Ignore any coordinates given for the vertex.

(b)

B1 States or uses $a+b=8$ or exact equivalent. Condone use of capital letters throughout
It is not scored for just $|0-a|+b=8$

M1 This M is for an understanding of the modulus.
It is scored for an attempt at solving $(2x-a)+b=\frac{3}{2}x+8$ or $-(2x-a)+b=\frac{3}{2}x+8$ in either x or with x replaced by c. The signs of the $2x$ and the a must be different. $|2x-a|\neq 2x+a$

You may see $(2x-a)+b=\frac{3}{2}x+8 \Rightarrow kx=\text{f}(a,b)$

You may see $-2x+a+b=\frac{3}{2}x+8 \Rightarrow kx=\text{f}(a,b)$

You may see $(2x-a)+b=\frac{3}{2}x+8 \Rightarrow kx=\text{f}(a,b)$ being solved with b replaced with **their** $a+b=8$

You may see $-2c+a+b=\frac{3}{2}c+8 \Rightarrow kc=\text{f}(a,b)$ being solved with b replaced with **their** $a+b=8$

dM1 This dM mark is scored for combining $b=8-a$ with $(2x-a)+b=\frac{3}{2}x+8$ (or their $kx=\text{f}(a,b)$ resulting from that equation) resulting in a link between x and a **Both equations must have been correct initially.**
Alternatively for combining $b=8-a$ with their $2c-a+b=\frac{3}{2}c+8$ (or their $kc=\text{f}(a,b)$ resulting from that equation) resulting in a link between c and a

You may condone sign slips in finding the link between x (or c) and a
If you see an approach that involves making $|2x-a|$ the subject followed by squaring, and you feel that it deserves credit, please send to review. The solution proceeds as follows

Look for $|2x-a|=\frac{3}{2}x+8-b \Rightarrow |2x-a|=\frac{3}{2}x+a \Rightarrow (2x-a)^2=\left(\frac{3}{2}x+a\right)^2 \Rightarrow 7x\left(\frac{1}{4}x-a\right)=0$

A1 $c=4a$ ONLY

Special Case where they have the roots linked with the incorrect branch of the curve.

They have $x=0$ as the solution to $2x-a+b=\frac{3}{2}x+8 \Rightarrow -a+b=8$..................(1)

They have $x=c$ as the solution to $-2x+a+b=\frac{3}{2}x+8 \Rightarrow \frac{7}{2}x=a+b-8$............(2)

Solve (1) and (2) $\Rightarrow x=\frac{4}{7}a$

Hence $\Rightarrow c=\frac{4}{7}a$

This would score B0 M1 dM0 A0 anyway but should be awarded SC B0, M1 dM1, A0 for above work leading to either $x=\frac{4}{7}a$ or $c=\frac{4}{7}a$

Gold Questions

Non-calculator

The total mark for this section is 35

Q1

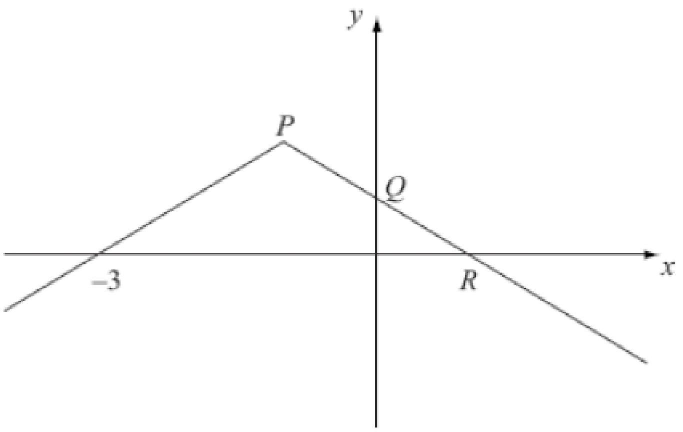


Figure 1

Figure 1 shows the graph of $y = f(x)$, $x \in \mathbb{R}$,

The graph consists of two line segments that meet at the point P.

The graph cuts the y-axis at the point Q and the x-axis at the points $(-3, 0)$ and R.

Sketch, on separate diagrams, the graphs of

(a) $y = |\,f(x)\,|$,

(2)

(b) $y = f(-x)$.

(2)

Given that $f(x) = 2 - |\,x + 1\,|$,

(c) find the coordinates of the points P, Q and R,

(3)

(d) solve $f(x) = \frac{1}{2}x$.

(5)

(Total for Question 1 is 12 marks)

Q2

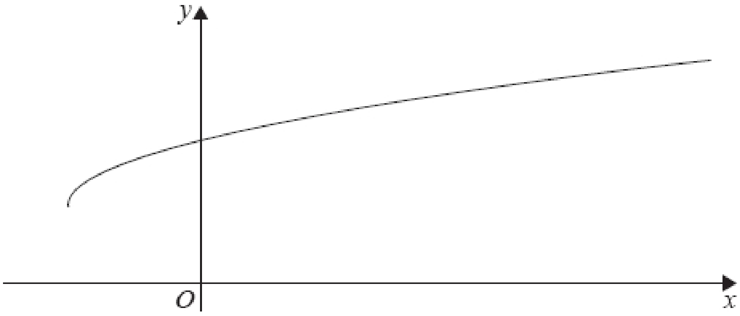


Figure 1

Figure 1 shows a sketch of part of the graph of $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = 3 + \sqrt{x+2}\,, \quad x \geq -2$$

(a) Find $\mathrm{g}^{-1}(x)$ and state its domain.

(3)

(b) Find the exact value of x for which

$$\mathrm{g}(x) = x$$

(4)

(c) Hence state the value of a for which

$$\mathrm{g}(a) = \mathrm{g}^{-1}(a)$$

(1)

(Total for Question 2 is 8 marks)

Q3

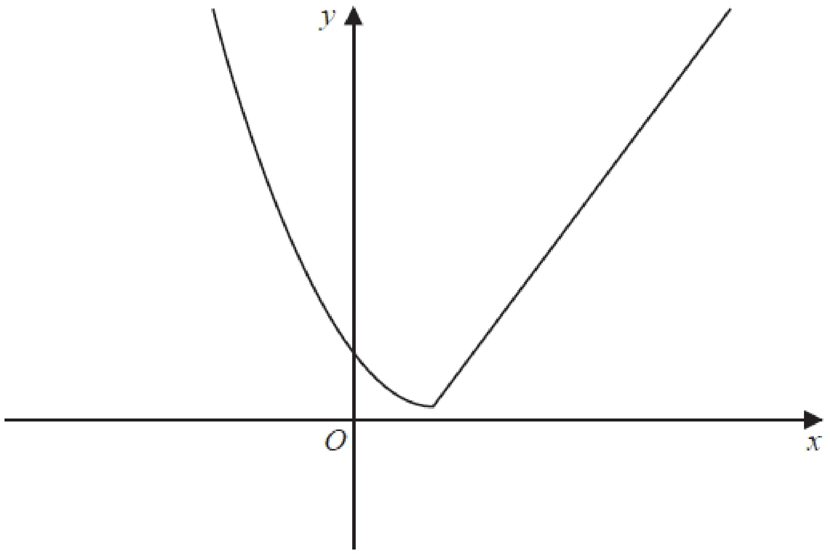


Figure 4

Figure 4 shows a sketch of the graph of $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = \begin{cases} (x-2)^2 + 1 & x \leqslant 2 \\ 4x - 7 & x > 2 \end{cases}$$

(a) Find the value of gg(0).

(2)

(b) Find all values of x for which

$$\mathrm{g}(x) > 28$$

(4)

The function h is defined by

$$\mathrm{h}(x) = (x-2)^2 + 1 \qquad x \leq 2$$

(c) Explain why h has an inverse but g does not.

(1)

(d) Solve the equation

$$\mathrm{h}^{-1}(x) = -\frac{1}{2}$$

(3)

(Total for Question 3 is 10 marks)

Q4

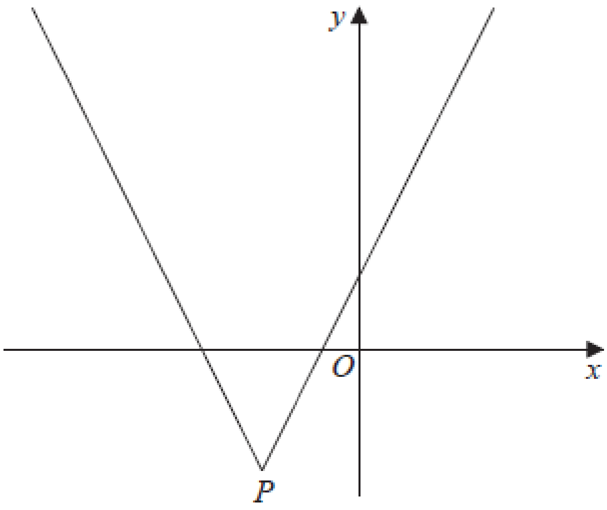


Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = 2\,|\,x + 4\,| - 5$$

The vertex of the graph, at point P, is $(-4, -5)$.

(*a*) Solve the equation

$$3x + 40 = 2\,|\,x + 4\,| - 5$$

(2)

A line l has equation $y = ax$, where a is a constant.

Given that l intersects $y = 2\,|\,x + 4\,| - 5$ at least once,

(b) Find the range of possible values of a, writing your answer in set notation.

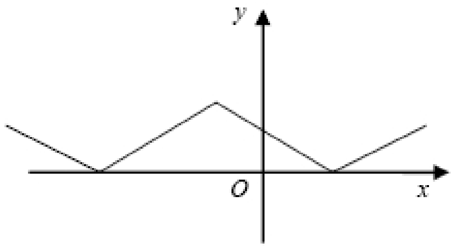
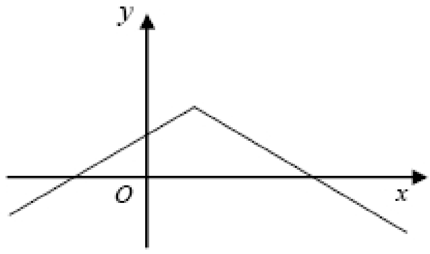
(3)

(Total for Question 4 is 5 marks)

End of Questions

Gold Mark Scheme

Q1

Question Number	Scheme	Marks
(a)	 shape	B1
	Vertices correctly placed	B1 (2)
(b)	 shape	B1
	Vertex and intersections with axes correctly placed	B1 (2)
(c)	$P:(-1,2)$	B1
	$Q:(0,1)$	B1
	$R:(1,0)$	B1 (3)
(d)	$x>-1;\quad 2-x-1=\frac{1}{2}x$	M1 A1
	Leading to $x=\frac{2}{3}$	A1
	$x<-1;\quad 2+x+1=\frac{1}{2}x$	M1
	Leading to $x=-6$	A1 (5)
		(12 marks)

Q2

Question Number	Scheme	Marks
(a)	$y=3+\sqrt{x+2} \Rightarrow y-3=\sqrt{x+2} \Rightarrow x=(y-3)^2-2$	M1 A1
	$\Rightarrow g^{-1}(x)=(x-3)^2-2, \quad \text{with } x \geqslant 3$	A1
		(3)
(b)	$g(x)=x \Rightarrow 3+\sqrt{x+2}=x$	
	$\Rightarrow x+2=(x-3)^2 \Rightarrow x^2-7x+7=0$	M1, A1
	$\Rightarrow x=\dfrac{7\pm\sqrt{21}}{2} \Rightarrow x=\dfrac{7+\sqrt{21}}{2}$ only	M1, A1
		(4)
(c)	$a=\dfrac{7+\sqrt{21}}{2}$	B1 ft
		(1)
		9 marks
(b) Alt	Solves $g^{-1}(x)=x \Rightarrow (x-3)^2-2=x$	
	$\Rightarrow x^2-7x+7=0$	M1, A1
	$\Rightarrow x=\dfrac{7\pm\sqrt{21}}{2} \Rightarrow x=\dfrac{7+\sqrt{21}}{2}$ only	dM1, A1
		(4)

Q3

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$g(0) = 5$	M1	This mark is given for a method to find g(0)
	$gg(0) = g(5) = 13$	A1	This mark is given for a correct value for gg(0)
(b)	$(x-2)^2 + 1 > 28$ $(x-2)^2 > 27$ $x - 2 > 3\sqrt{3}$	M1	This mark is given for a method to solve $g(x) > 28$ when $x \le 2$
	$x < 2 - 3\sqrt{3}$	A1	
	$4x - 7 > 28$ $4x > 35$ $x > \frac{35}{4}$	M1	This mark is given for a solving $g(x) > 28$ when $x > 2$
	$x < 2 - 3\sqrt{3}$ and $x > \frac{35}{4}$	A1	This mark is given for a correct range of values of x for which $g(x) > 28$ stated
(c)	h^{-1} exists since h is a one-to-one function; g^{-1} does not exists since g is a many-to-one function	B1	This mark is given for a valid explanation
(d)	$h^{-1}(x) = 2 - \sqrt{(x-1)}$	B1	This mark is given for finding an expression for $h^{-1}(x)$
	$2 \pm \sqrt{(x-1)} = -\frac{1}{2}$	M1	This mark is given for a method to rearrange to find a value for x
	$x = 7.25$	A1	This mark is given for a correct value of x
			(Total 10 marks)

Q4

Question	Scheme	Marks	AOs
(a)	$3x+40=-2(x+4)-5 \Rightarrow x=...$	M1	1.1b
	$x=-10.6$	A1	2.1
		(2)	
(b)	$a>2$	B1	2.2a
	$y=ax \Rightarrow -5=-4a \Rightarrow a=\frac{5}{4}$	M1	3.1a
	$\{a:a \leqslant 1.25\} \cup \{a:a>2\}$	A1	2.5
		(3)	
			(7 marks)

Notes:

(a)

M1: Attempts to solve $3x+40=-2(x+4)-5 \Rightarrow x=...$ Must reach a value for x.

You may see the attempt crossed out but you can still take this as an attempt to solve the required equation.

A1: $x=-10.6$ oe e.g. $-\frac{53}{5}$ only. If other values are given, e.g. $x=-37$ they must be rejected or the $-\frac{53}{5}$ clearly chosen as their answer. Ignore any attempts to find y.

Alternative by squaring:

$$3x+40=2|x+4|-5 \Rightarrow 3x+45=2|x+4| \Rightarrow 9x^2+270x+2025=4\left(x^2+8x+16\right)$$

$$\Rightarrow 5x^2+238x+1961=0 \Rightarrow x=-37,-\frac{53}{5}$$

M1 for isolating the $|x+4|$, squaring both sides and solving the resulting quadratic

A1 for selecting the $-\frac{53}{5}$

Correct answer with no working scores both marks.

(b)

B1: Deduces that $a>2$

M1: Attempts to find a value for a using their $P(-4,-5)$

Alternatively attempts to solve $ax=2(x+4)-5$ and $ax=2(x+4)-5$ to obtain a value for a.

A1: Correct range in acceptable set notation.

Examples:

$\{a:a \leqslant 1.25\} \cup \{a:a>2\}$

$\{a:a \leqslant 1.25\}, \{a:a>2\}$

$\{a:a \leqslant 1.25 \text{ or } a>2\}$

$\{a:a \leqslant 1.25,\ a>2\}$

$(-\infty,1.25] \cup (2,\infty)$

$(-\infty,1.25], (2,\infty)$

Platinum Questions

Non-calculator

The total mark for this section is 28

Q1

The functions f and g are defined by

$$\mathrm{f}(x) = 2\sqrt{1-\mathrm{e}^{-x}} \qquad x \in \mathbb{R},\ x \geq 0$$
$$\mathrm{g}(x) = \ln\left(4-x^2\right) \qquad x \in \mathbb{R},\ -2 < x < 2$$

(*a*) (i) Explain why fg cannot be formed as a composite function.

(ii) Explain why gf can be formed as a composite function.

(2)

(*b*) (i) Find gf (x), giving the answer in the form gf (x) = $a + bx$, where a and b are constants.

(ii) State the domain and range of gf.

(5)

(*c*) Sketch the graph of the function gf.

On your sketch, you should show the coordinates of any points where the graph meets or crosses the coordinate axes.

(2)

The circle C with centre (0, −ln 9) touches the line with equation y = gf (x) at precisely one point.

(*d*) Find an equation of the circle C.

(3)

(+S1)

(Total for Question 1 is 13 marks)

Q2

The function f is given by

$$f(x) = \sqrt{x+2} \quad \text{for} \quad x \in \mathbb{R},\ x \geqslant 0$$

(*a*) Find $f^{-1}(x)$ and state the domain of f^{-1}

(2)

The function g is given by

$$g(x)\ x^2 - 4x + 5 \quad \text{for} \quad x \in \mathbb{R}, x \geqslant 0$$

(*b*) Find the range of g.

(2)

(*c*) Solve the equation $fg(x) = x$.

(3)

(Total for Question 2 is 7 marks)

Q3

(a) On the same diagram, sketch

$$y = (x+1)(2-x) \quad \text{and} \quad y = -x^2 2|x|.$$

Mark clearly the coordinates of the points where these curves cross the coordinate axes.

(3)

(b) Find the x-coordinates of the points of intersection of these two curves.

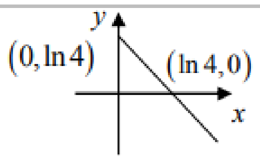
(5)

(Total for Question 3 is 8 marks)

End of Questions

Platinum Mark Scheme

Q1

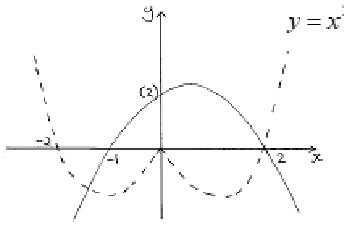
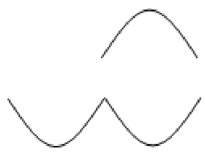
Question	Scheme		Marks	AOs
(a)	As the ranges of f and g are $0 \leqslant \mathrm{f}(x) < 2$ and $(-\infty <)\ \mathrm{g}(x) \leqslant \ln 4$			
	A function fg cannot be formed as the range of g does not lie in the domain of f.	A reason **or** example is acceptable (range for g not needed but if no example is given the range must be given and correct) e.g $\mathrm{g}\left(\frac{7}{4}\right) = \ln\left(\frac{15}{16}\right) < 0$ so range of g is not in domain of f.	B1	2
	A function gf can be formed as the range of f lies in the domain of g.	Correct range for f must have been found and reason given.	B1	2
			(2)	
(b)	$\mathrm{g}(\mathrm{f}(x)) = \ln\left(4 - \left(2\sqrt{1-\mathrm{e}^{-x}}\right)^2\right)$	Attempts the composite.	M1	1
	$= \ln\left(4 - \left(2\sqrt{1-\mathrm{e}^{-x}}\right)^2\right) = \ln\left(4 - 4\left(1-\mathrm{e}^{-x}\right)\right)$ $= \ln\left(4\mathrm{e}^{-x}\right) = \ln 4 + \ln\left(\mathrm{e}^{-x}\right)$	Correct composite with square evaluated, but need not be simplified.	A1	1
	$\mathrm{gf}(x) = \ln 4 - x$ or $2\ln 2 - x$	Correct form.	A1	1
	Domain is $x \in \mathbb{R}, x \geqslant 0$	Correct domain	B1	2
	Range is $(-\infty <)\ \mathrm{gf}(x) \leqslant \ln 4$	Correct range	B1	2
			(5)	
(c)		A line consistent with their gradient and intercept from (b).	M1	1
		Line starting at $(0, \ln 4)$ and passing through $(\ln 4, 0)$	A1	1
			(2)	

(d)	(If X is centre, P is $(0, \ln 4)$ and Q is point where circle touches line then triangle XPQ is isosceles right angled, so) $2r^2 = (\ln 4 - (-\ln 9))^2 \Rightarrow r^2 = ...$	A complete method to find r or r^2 where r is the radius – longer methods are possible here.	**M1 (S+)**	3
	$r^2 = \frac{1}{2}(\ln 36)^2 = 2(\ln 6)^2$ oe or $r = \sqrt{2}\ln 6$ etc	Correct r or r^2 award when first seen, need not be simplified.	**A1**	3
	So equation of C is $x^2 + (y + \ln 9)^2 = 2(\ln 6)^2$ oe	Correct equation, need not be simplified but do not isw if eg.	**A1**	3
		$(\ln 6)^2$ is incorrectly simplified to $\ln 36$		
			(3)	
	S1 mark: Award S1 for a clear and concise solution that scores 10+ marks and includes the S+ point – ie must be a well explained solution with all terminology and notation correct (though there may be variations on the notation used).		**S1**	2
			(12+1 marks)	
Notes:				
(d) S+: For succinct solution				

Q2

Question	Scheme	Marks	Notes
(a)	$f^{-1}(x) = x^2 - 2$	B1	
	Domain is $x \in \mathbb{R},\ x \geqslant \sqrt{2}$	B1 (2)	
(b)	$g(x) = (x-2)^2 + 1$ (or differentiation or equivalent)	M1	Suitable method to find min.
	So range is $g(x) \geqslant 1$	A1 (2)	
(c)	fg(x) = x: $\sqrt{x^2 - 4x + 7} = x$ <u>or</u> $g(x) = f^{-1}(x)$: $x^2 - 4x + 5 = $(a)	M1	Attempt suitable eq
	$x^2 - 4x + 7 = x^2$ <u>or</u> $x^2 - 4x + 5 = x^2 - 2$	A1	Simplify $x^2 + .. = x^2$
	$4x = 7$ so $x = \frac{7}{4}$	A1 (3)	
		[7]	

Q3

Question Number	Scheme	Marks	Notes
(a)	$y = x^2 - 2\|x\|$  	B1 B1	Don't insist on labels
	-2, -1, 2, (0,2)	B1 (3)	
	$y = (x+1)(2-x)$		
(b)	One intersection at $x = 2$	B1	
	Second at $(x+1)(2-x) = x(x+2)$	M1	Attempt correct equation Must be $x + 2$ on RHS
	$(0 =)\ 2x^2 + x - 2$	A1	Correct 3TQ
	$x = \dfrac{-1 \pm \sqrt{1+16}}{4}$, since root is in (-2, -1) $x = \dfrac{-1 - \sqrt{17}}{4}$	M1	Solving
		A1 cso (5)	Must choose -
		[8]	

Topic 3: Sequences & Series

Bronze, Silver, Gold and Platinum Worksheets for A Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel A Level Mathematics: Pure Mathematics Year 2' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high-level problem-solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Non-calculator

The total mark for this section is 29

Q1

A girl saves money over a period of 200 weeks. She saves 5p in Week 1, 7p in Week 2, 9p in Week 3, and so on until Week 200. Her weekly savings form an arithmetic sequence.

(a) Find the amount she saves in Week 200.

(3)

(b) Calculate her total savings over the complete 200 week period.

(3)

(Total for Question 1 is 6 marks)

Q2

A sequence of positive numbers is defined by

$$a_{n+1} = \sqrt{\left(a_n^{\ 2} + 3\right)}, \qquad n \geq 1,$$
$$a_1 = 2$$

(a) Find a_2 and a_3, leaving your answers in surd form.

(2)

(b) Show that $a_5 = 4$

(2)

(Total for Question 2 is 4 marks)

Q3

Each year, Andy pays into a savings scheme. In year one he pays in £600. His payments increase by £120 each year so that he pays £720 in year two, £840 in year three and so on, so that his payments form an arithmetic sequence.

(a) Find out how much Andy pays into the savings scheme in year ten.

(2)

Kim starts paying money into a different savings scheme at the same time as Andy. In year one she pays in £130. Her payments increase each year so that she pays £210 in year two, £290 in year three and so on, so that her payments form a different arithmetic sequence.

At the end of year N, Andy has paid, in total, twice as much money into his savings scheme as Kim has paid, in total, into her savings scheme.

(b) Find the value of N.

(5)

(Total for Question 3 is 7 marks)

Q4

The third term of a geometric sequence is 324 and the sixth term is 96.

(a) Show that the common ratio of the sequence is $\frac{2}{3}$.

(2)

(b) Find the first term of the sequence.

(2)

(c) Find the sum to infinity of the sequence.

(2)

(Total for Question 4 is 6 marks)

Q5

A sequence $x_1, x_2, x_3, \ldots$ is defined by

$$x_1 = 1,$$
$$x_{n+1} = ax_n - 3, \quad n \geq 1,$$

where a is a constant.

(a) Find an expression for x_2 in terms of a.

(1)

(b) Show that $x_3 = a^2 - 3a - 3$.

(2)

Given that $x_3 = 7$,

(c) find the possible values of a.

(3)

(Total for Question 5 is 6 marks)

End of Questions

Bronze Mark Scheme

Q1

Question number	Scheme	Marks
	(a) Identify $a = 5$ and $d = 2$ (May be implied)	B1
	$(u_{200} =) a + (200-1)d \quad (= 5 + (200 - 1) \times 2)$	M1
	$= \underline{403}$(p) or (£) $\underline{4.03}$	A1 (3)
	(b) $(S_{200} =) \frac{200}{2}[2a + (200-1)d]$ or $\frac{200}{2}(a + \text{"their 403"})$	M1
	$= \frac{200}{2}[2\times5 + (200-1)\times2]$ or $\frac{200}{2}(5 + \text{"their 403"})$	A1
	$= \underline{40\ 800}$ or $\underline{£408}$	A1 (3)
		6
(a)	B1 can be implied if the correct answer is obtained. If 403 is <u>not</u> obtained then the values of a and d must be clearly identified as $a = 5$ and $d = 2$. **This mark can be awarded at any point.**	
	M1 for attempt to use nth term formula with $n = 200$. Follow through their a and d. Must have use of $n = 200$ and one of a or d correct or correct follow through. Must be 199 not 200.	
	A1 for 403 or 4.03 (i.e. condone missing £ sign here). Condone £403 here.	
N.B.	$a = 3$, $d = 2$ is B0 and $a + 200d$ is M0 <u>BUT</u> $3 + 200\times2$ is B1M1 and A1 if it leads to 403. Answer only of 403 (or 4.03) scores 3/3.	
(b)	M1 for use of correct sum formula with $n = 200$. Follow through their a and d and their 403. Must have <u>some</u> use of $n = 200$, and some of a, d or l correct or correct follow through.	
	1st A1 for any correct expression (i.e. must have $a = 5$ and $d = 2$) but can f.t. their 403 still.	
	2nd A1 for 40800 or £408 (i.e. the £ sign is required before we accept 408 this time). 40800p is fine for A1 but £40800 is A0.	
ALT	<u>Listing</u>	
(a)	They might score B1 if a =5 and $d = 2$ are clearly identified. Then award M1A1 together for 403.	
(b)	$\sum_{r=1}^{200}(2r+3)$. Give M1 for $2\times\frac{200}{2}\times(201) + 3k$ (with $k > 1$), A1 for $k = 200$ and A1 for 40800.	

Q2

Question Number	Scheme	Marks
(a)	$a_2 = \left(\sqrt{4+3}\right) = \sqrt{7}$	B1
	$a_3 = \sqrt{\text{"their 7"}+3} = \sqrt{10}$	B1ft (2)
(b)	$a_4 = \sqrt{10+3}\left(=\sqrt{13}\right)$	M1
	$a_5 = \sqrt{13+3} = 4$ *	A1 cso (2)
		4
	Notes	
(a)	1st B1 for $\sqrt{7}$ only 2nd B1ft follow through their "7" in correct formula provided they have $\sqrt{n}$,where n is an integer.	
(b)	M1 for an attempt to find a_4. Should see $\sqrt{\text{"their"}(a_3)^2+3}$. Must see evidence for M1. $a_4 = \sqrt{13}$ provided this follows from their a_3 working or answer is sufficient A1cso for a correct solution (M1 explicit) must include the = 4. Ending at $\sqrt{16}$ only is A0 and ending with ± 4 is A0. Ignore any incorrect statements that are not used e.g. common difference = $\sqrt{3}$ Listing: A full list: $2\left(=\sqrt{4}\right)$, $\sqrt{7}$, $\sqrt{10}$, $\sqrt{13}$, $\sqrt{16}=4$ is fine for M1A1	
ALT	Formula: Some may state (or use) $a_n = \sqrt{3n+1}$ leading to $a_5 = \sqrt{3\times5+1} = 4$. This will get marks in (a) [if correct values are seen] and can score the M1 in (b) if $a_n = \sqrt{3n+1}$ or $a_4 = \sqrt{13}$ are seen.	
$\pm\sqrt{\ }$	If $\pm\sqrt{\ }$ appear any where ignore in part (a) and withhold the final A mark only	

Q3

Question Number	Scheme		Marks
(a)	$a+(n-1)d=600+9\times120$	This mark is for: $600+9\times120$ or $600+8\times120$	M1
	$=(£)1680$	1680 with or without the "£"	A1
	Answer only scores both marks		
	Listing M1: Lists ten terms starting £600, £720, £840, £960, … A1: Identifies the 10th term as $(£)1680$		
			(2)
(b)	**Allow the use of n instead of N throughout in (b)**		
	$d=80$ for Kim	Identifies or uses $d=80$ for Kim	B1
	$\frac{N}{2}\{2\times600+(N-1)\times120\}$ OR $\frac{N}{2}\{2\times130+(N-1)\times80\}$	Attempts a sum formula for Andy or Kim. A correct formula must be seen or implied with: a = 600, d = 120 for Andy or a = 130, d = 80 for Kim. If B0 was scored, allow M1 here if Kim's incorrect "d" is used.	M1
	$\frac{N}{2}\{2\times600+(N-1)\times120\}=2\times\frac{N}{2}\{2\times130+(N-1)\times80\}$ A **correct** equation in any form		A1
	$20N=360\Rightarrow N=...$	Proceeds to find a value for N. (Allow if it leads to $N<0$) **Dependent on the first method mark and must be an equation that uses Andy's and Kim's sum.**	dM1
	$(N=)18$	Ignore N/n = 0 and if a correct value of N is seen, isw any further reference to years etc.	A1
	See below for listing approach		
	If you see N = 18 with no working send to Review		
			(5)
			(7 marks)

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Andy	600	1320	2160	3120	4200	5400	6720	8160	9720	11400	13200	15120	17160	19320	21600	24000	26520	29160
Kim	130	340	630	1000	1450	1980	2590	3280	4050	4900	5830	6840	7930	9100	10350	11680	13090	14580
Kimx2	260	680	1260	2000	2900	3960	5180	6560	8100	9800	11660	13680	15860	18200	20700	23360	26180	29160

B1: States or uses $d=80$ for Kim
M1: Attempts to find the total savings for Andy or Kim – must see the correct pattern for Andy (600, 1320, 2160,…) or Kim (130, 340, 630,…) (or Kimx2)
A1: Correct totals for Andy and Kim (or Kimx2) at least as far as n = 18
M1: Identifies when Andy's total = 2xKim's total
A1: N = 18

Q4

Question Number	Scheme	Marks
(a)	$324r^3 = 96$ or $r^3 = \frac{96}{324}$ or $r^3 = \frac{8}{27}$	M1
	$r = \frac{2}{3}$ (*)	A1cso (2)
(b)	$a\left(\frac{2}{3}\right)^2 = 324$ or $a\left(\frac{2}{3}\right)^5 = 96$ $a = \dots,$ 729	M1, A1 (2)
(c)	$S_\infty = \frac{729}{1-\frac{2}{3}}$, $= 2187$	M1, A1 (2) [9]
(a)	M1 for forming an equation for r^3 based on 96 and 324 (e.g. $96r^3 = 324$ scores M1). The equation must involve multiplication/division rather than addition/subtraction. A1 Do not penalise solutions with working in decimals, providing these are correctly rounded or truncated to at least 2dp <u>and</u> the final answer 2/3 is seen. <u>Alternative</u>: (verification) M1 Using $r^3 = \frac{8}{27}$ and multiplying 324 by this (or multiplying by $r = \frac{2}{3}$ three times). A1 Obtaining 96 (cso). (A conclusion is not required). $324 \times \left(\frac{2}{3}\right)^3 = 96$ (no real evidence of calculation) is not quite enough and scores M1 A0.	
(b)	M1 for the use of a correct formula or for 'working back' by dividing by $\frac{2}{3}$ (or by their r) twice from 324 (or 5 times from 96). Exceptionally, allow M1 also for using $ar^3 = 324$ or $ar^6 = 96$ instead of $ar^2 = 324$ or $ar^5 = 96$, or for dividing by r three times from 324 (or 6 times from 96)… but no other exceptions are allowed.	
(c)	M1 for use of correct sum to infinity formula with their a. For this mark, if a value of r different from the given value is being used, M1 can still be allowed providing $\lvert r\rvert < 1$.	

Q5

Question Number	Scheme	Marks
(a)	$[x_2 =]\, a-3$	B1 (1)
(b)	$[x_3 =]\ ax_2 - 3$ or $a(a-3)-3$	B1
	$= a(a-3) - {}_5\ = a^2 - 3a - 3$ (*)	A1 cso (2)
(c)	$a^2 - 3a - 3 = 7$	
	$a^2 - 3a - 10 = 0$ or $a^2 - 3a = 10$	M1
	$(a-5)(a+2) = 0$	M1
	$a = 5$ or -2	A1 (3)
		(6 marks)

Silver Questions

Non-calculator

The total mark for this section is 35

Q1

A boy saves some money over a period of 60 weeks. He saves 10p in week 1, 15p in week 2, 20p in week 3 and so on until week 60. His weekly savings form an arithmetic sequence.

(a) Find how much he saves in week 15

(2)

(b) Calculate the total amount he saves over the 60 week period.

(3)

The boy's sister also saves some money each week over a period of m weeks. She saves 10p in week 1, 20p in week 2, 30p in week 3 and so on so that her weekly savings form an arithmetic sequence. She saves a total of £63 in the m weeks.

(c) Show that

$$m(m + 1) = 35 \times 36$$

(4)

(d) Hence write down the value of m.

(1)

(Total for Question 1 is 10 marks)

Q2

A sequence of numbers a_1, a_2, a_3, ...is defined by

$$a_{n+1} = \frac{k(a_n + 2)}{a_n} \qquad n \in \mathbb{N}$$

where k is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 2$

(a) show that

$$k^2 + k - 2 = 0$$

(3)

(b) For this sequence explain why $k \neq 1$

(1)

(c) Find the value of

$$\sum_{r=1}^{80} a_r$$

(3)

(Total for Question 2 is 7 marks)

Q3

An arithmetic sequence has first term a and common difference d. The sum of the first 10 terms of the sequence is 162.

(a) Show that $10a + 45d = 162$

(2)

Given also that the sixth term of the sequence is 17,

(b) write down a second equation in a and d,

(1)

(c) find the value of a and the value of d.

(4)

(Total for Question 3 is 7 marks)

Q4

The first term of an arithmetic series is a and the common difference is d.

The 18th term of the series is 25 and the 21st term of the series is $32\frac{1}{2}$.

(a) Use this information to write down two equations for a and d.

(2)

(b) Show that $a = -17.5$ and find the value of d.

(2)

The sum of the first n terms of the series is 2750.

(c) Show that n is given by

$$n^2 - 15n = 55 \times 40.$$

(4)

(d) Hence find the value of n.

(3)

(Total for Question 4 is 11 marks)

End of Questions

Silver Mark Scheme

Q1

Question Number	Scheme	Marks
	Boy's Sequence: 10, 15, 20, 25, ...	
(a)	$\{a = 10, d = 5 \Rightarrow T_{15} =\}\ a + 14d = 10 + 14(5);\ = 80$ or $0.1 + 14(0.05);\ = £0.80$	M1; A1 [2]
(b)	$\{S_{60} =\}\ \frac{60}{2}[2(10) + 59(5)]$	M1 A1
	$= 30(315) = 9450$ or £94.50	A1 [3]
	Boy's Sister's Sequence: 10, 20, 30, 40, ...	
(c)	$\{a = 10, d = 10 \Rightarrow S_m =\}\ \frac{m}{2}(2(10) + (m-1)(10))$ $\left(\text{or } \frac{m}{2} \times 10(m+1) \text{ or } 5m(m+1)\right)$	M1 A1
	63 or $6300 = \frac{m}{2}(2(10) + (m-1)(10))$	dM1
	$6300 = \frac{m}{2}(10)(m+1)$ or $12600 = 10m(m+1)$	
	$1260 = m(m+1)$	
	$35 \times 36 = m(m+1)$ (*)	A1 cso [4]
(d)	$\{m =\}\ 35$	B1 [1]
		10

Q2

Question	Scheme	Marks	AOs
(a)	Uses the sequence formula $a_{n+1} = \frac{k(a_n + 2)}{a_n}$ once with $a_1 = 2$	M1	1.1b
	$(a_1 = 2),\ a_2 = 2k,\ a_3 = k+1,\ a_4 = \frac{k(k+3)}{k+1}$ Finds four consecutive terms and sets a_4 equal to a_1 (oe)	M1	3.1a
	$\frac{k(k+3)}{k+1} = 2 \Rightarrow k^2 + 3k = 2k + 2 \Rightarrow k^2 + k - 2 = 0$ *	A1*	2.1
		(3)	
(b)	States that when $k = 1$, all terms are the same and concludes that the sequence does not have a period of order 3	B1	2.3
		(1)	
(c)	Deduces the repeating terms are $a_{1/4} = 2,\ a_{2/5} = -4,\ a_{3/6} = -1,$	B1	2.2a
	$\sum_{n=1}^{80} a_k = 26 \times (2 + -4 + -1) + 2 + -4$	M1	3.1a
	$= -80$	A1	1.1b
		(3)	
		(7 marks)	

Q3

Question Number	Scheme	Marks
(a)	$S_{10} = \frac{10}{2}[2a+9d]$ or $S_{10} = a+a+d+a+2d+a+3d+a+4d+a+5da+6d+a+7d+a+8d+a+9d$ $162 = 10a + 45d$ *	M1 A1cso (2)
(b)	$(u_n = a+(n-1)d \Rightarrow)17 = a+5d$	B1 (1)
	$10\times(b)$ gives $10a+50d = 170$ (a) is $10a + 45d = 162$	M1
	Subtract $5d = 8$ so $d = \underline{1.6}$ o.e.	A1
	Solving for a $a = 17 - 5d$	M1
	so $a = \underline{9}$	A1
		(4) 7

Q4

Question Number	Scheme	Marks
(a)	$a+17d = 25$ or equiv. (for 1st B1), $a+20d = 32.5$ or equiv. (for 2nd B1),	B1, B1 (2)
(b)	<u>Solving</u> (Subtract) $3d = 7.5$ so $d = \underline{2.5}$ $a = 32.5 - 20\times 2.5$ so $a = \underline{-17.5}$ (*)	M1 A1cso (2)
(c)	$2750 = \frac{n}{2}\left[-35+\frac{5}{2}(n-1)\right]$ { $4\times 2750 = n(5n-75)$ } $4\times 550 = n(n-15)$ $\underline{n^2 - 15n = 55\times 40}$ (*)	M1A1ft M1 A1cso (4)
(d)	$n^2 - 15n - 55\times 40 = 0$ or $n^2 - 15n - 2200 = 0$ $(n-55)(n+40) = 0$ $n = \ldots$ $\underline{n = 55}$ (ignore - 40)	M1 M1 A1 (3) [11]

Gold Questions

Non-calculator

The total mark for this section is 29

Q1

Lewis played a game of space invaders. He scored points for each spaceship that he captured.

Lewis scored 140 points for capturing his first spaceship.

He scored 160 points for capturing his second spaceship, 180 points for capturing his third spaceship, and so on.

The number of points scored for capturing each successive spaceship formed an arithmetic sequence.

(a) Find the number of points that Lewis scored for capturing his 20th spaceship.

(2)

(b) Find the total number of points Lewis scored for capturing his first 20 spaceships.

(3)

Sian played an adventure game. She scored points for each dragon that she captured. The number of points that Sian scored for capturing each successive dragon formed an arithmetic sequence.

Sian captured n dragons and the total number of points that she scored for capturing all n dragons was 8500.

Given that Sian scored 300 points for capturing her first dragon and then 700 points for capturing her nth dragon,

(c) find the value of n.

(3)

(Total for Question 1 is 8 marks)

Q2

In the year 2000 a shop sold 150 computers. Each year the shop sold 10 more computers than $\sqrt{\frac{1+4x}{1-x}} \approx 1+\frac{5}{2}x-\frac{5}{8}x^2$ the year before, so that the shop sold 160 computers in 2001, 170 computers in 2002, and so on forming an arithmetic sequence.

(a) Show that the shop sold 220 computers in 2007.

(2)

(b) Calculate the total number of computers the shop sold from 2000 to 2013 inclusive.

(3)

In the year 2000, the selling price of each computer was £900. The selling price fell by £20 each year, so that in 2001 the selling price was £880, in 2002 the selling price was £860, and so on forming an arithmetic sequence.

(c) In a particular year, the selling price of each computer in £s was equal to three times the number of computers the shop sold in that year. By forming and solving an equation, find the year in which this occurred. **(4)**

(Total for Question 2 is 9 marks)

Q3

A geometric series has common ratio r and first term a.

Given $r \neq 1$ and $a \neq 0$

(a) prove that

$$S_n = \frac{a(1-r^n)}{1-r}$$

(4)

Given also that S_{10} is four times S_5

(b) find the exact value of r.

(4)

(Total for Question 3 is 8 marks)

Q4

In a geometric series the common ratio is r and sum to n terms is S_n

Given that

$$S_\infty = \frac{8}{7} \times S_6$$

show that $r = \pm\frac{1}{\sqrt{k}}$, where k is an integer to be found.

(4)

(Total for Question 4 is 4 marks)

End of Questions

Gold Mark Scheme

Q1

Question Number	Scheme	Marks
(a)	Lewis; arithmetic series, $a = 140$, $d = 20$. $T_{20} = 140 + (20-1)(20)$; $= 520$ Or lists 20 terms to get to 520 OR $120 + (20)(20)$	M1; A1 [2]
(b)	Method 1 **Either:** Uses $\frac{1}{2}n(2a+(n-1)d)$ $\frac{20}{2}(2\times140+(20-1)(20))$ Method 2 **Or:** Uses $\frac{1}{2}n(a+l)$ $\frac{20}{2}(140+"520")$ ft 520 6600	M1 A1 A1 [3]
(c)	Sian; arithmetic series, $a = 300$, $l = 700$, $S_n = 8500$ **Either:** Attempt to use $8500 = \frac{n}{2}(a+l)$ $8500 = \frac{n}{2}(300+700)$ **Or:** May use both $8500 = \frac{1}{2}n(2a+(n-1)d)$ and $l = a+(n-1)d$ **and eliminate** d $8500 = \frac{n}{2}(600+400)$ $\Rightarrow n = 17$	M1 A1 A1 [3]
		8 marks
	Notes	
(a)	**M1:** Attempt to use formula for 20[th] **term** of Arithmetic series with first term **140** and d = **20. Normal formula rules** apply – see General principles at the start of the mark scheme re "Method Marks" **Or:** uses $120 + 20n$ with $n = 20$ **Or: Listing method :** Lists 140, 160, 180, 200, 220, 240, 260, 280, … 520. M1A1 if correct M0A0 if wrong. (So 2 marks or zero) **A1:** For 520	
(b)	**M1:** An attempt to apply $\frac{1}{2}n(2a+(n-1)d)$ **or** $\frac{1}{2}n(a+l)$ with their values for a, n, d and l **A1:** Uses $a = 140$, $d = 20$, $n = 20$ in their formula (two alternatives given above) but **ft on their value of** l from (a) if they use Method 2. **A1:** 6600 cao **Or: Listing method :** Lists 140, 160, 180, 200, 220, 240, 260, 280, … 520 and adds **6600** gets M1A1A1- any other answer gets M1 A0A0 provided there are 20 numbers, the first is 140 and the last is 520.	
(c) **First method**	**M1:** Attempt to use $S_n = \frac{n}{2}(a+l)$ with their values for a, and l and S =8500 **A1:** Uses formula with correct values **A1:** Finds exact value 17	
Alternative method	**M1:** If both formulae $8500 = \frac{1}{2}n(2a+(n-1)d)$ and $l = a+(n-1)d$ are used, then d **must be eliminated** before this mark is awarded by valid work. Should not be using $d = 400$. This would be **M0**. **A1:** Correct equation in n only then **A1** for 17 exactly **Trial and error methods:** Finds $d = 25$ and $n = 17$ and list from 300 to 700 with total checked – 3/3	

Q2

Question Number	Scheme	Marks
	(a) Use n^{th}term $= a + (n-1)d$ with $d = 10$; $a = 150$ and $n = 8$, or $a = 160$ and $n = 7$, or $a = 170$ and $n = 6$: $= 150+7\times10$ or $160 + 6\times10$ or $170 + 5\times10$ $= 220$* (Or gives clear list – see note)	M1 A1* (2)
Or	If answer 220 is assumed and 150 + (n – 1) 10 =220 or variation is solved for n= Then n = 8, so 2007 is the year (must conclude the year)	M1 A1* (2)
	(b) Use $S_n = \frac{n}{2}\{2a+(n-1)10\}$ | Or $S_n = \frac{n}{2}\{a+l\}$ and $l = a + (n-1)10$ $= 7(300+13\times10)$ or $7(150 + 280)$ $= 7\times430$ $= 3010$	M1 A1 A1 (3)
	(c) Cost in year n $= 900+(n-1)\times$-20 Sales in year $n = 150+(n-1)\times10$ Cost =3×Sales $\Rightarrow 900+(n-1)\times$-$20 = 3\times(150+(n-1)\times10)$ $900-20n+20 = 450+30n-30$ $500 = 50n$ $n = 10$ Year is 2009 As n is not defined they may work correctly from another base year to get the answer 2009 and their n may not equal 10. If doubtful – send to review.	M1 M1 M1 A1 (4)
		(9 marks)

Q3

Question	Scheme	Marks	AOs
(a)	$S_n = a + ar + ar^2 + \dots\dots + ar^{n-1}$	B1	1.2
	$rS_n = ar + ar^2 + ar^3 + \dots\dots + ar^n \Rightarrow S_n - rS_n = \dots$	M1	2.1
	$S_n - rS_n = a - ar^n$	A1	1.1b
	$S_n(1-r) = a(1-r^n) \Rightarrow S_n = \frac{a(1-r^n)}{(1-r)}$*	A1*	2.1
		(4)	
(b)	$\frac{a(1-r^{10})}{1-r} = 4\times\frac{a(1-r^5)}{1-r}$ or $4\times\frac{a(1-r^{10})}{1-r} = \frac{a(1-r^5)}{1-r}$ Equation in r^{10} and r^5 (and possibly $1-r$)	M1	3.1a
	$1-r^{10} = 4(1-r^5)$	A1	1.1b
	$r^{10} - 4r^5 + 3 = 0 \Rightarrow (r^5-1)(r^5-3) = 0 \Rightarrow r^5 = \dots$ or e.g. $1-r^{10} = 4(1-r^5) \Rightarrow (1-r^5)(1+r^5) = 4(1-r^5) \Rightarrow r^5 = \dots$	dM1	2.1
	$r = \sqrt[5]{3}$ oe only	A1	1.1b
		(4)	
			(8 marks)

Q4

Question	Scheme	Marks	AOs
	Attempts $S_\infty = \frac{8}{7} \times S_6 \Rightarrow \frac{a}{1-r} = \frac{8}{7} \times \frac{a(1-r^6)}{1-r}$	M1	2.1
	$\Rightarrow 1 = \frac{8}{7} \times (1-r^6)$	M1	2.1
	$\Rightarrow r^6 = \frac{1}{8} \Rightarrow r = ..$	M1	1.1b
	$\Rightarrow r = \pm\frac{1}{\sqrt{2}}$ (so $k = 2$)	A1	1.1b
			(4 marks)

Notes:

M1: Substitutes the correct formulae for S_∞ and S_6 into the given equation $S_\infty = \frac{8}{7} \times S_6$

M1: Proceeds to an equation just in r

M1: Solves using a correct method

A1: Proceeds to $r = \pm\frac{1}{\sqrt{2}}$ giving $k = 2$

Platinum Questions

Non-calculator

The total mark for this section is 26

Q1

The sum of the first p terms of an arithmetic series is q and the sum of the first q terms of the same arithmetic series is p, where p and q are positive integers and $p \neq q$.

Giving simplified answers in terms of p and q, find

(a) the common difference of the terms in this series,

(5)

(b) the first term of the series,

(3)

(c) the sum of the first $(p + q)$ terms of the series.

(3)

(Total for Question 1 is 11 marks)

Q2

(a) The sides of the triangle ABC have lengths $BC = a$, $AC = b$ and $AB = c$, where $a < b < c$. The sizes of the angles A, B and C form an arithmetic sequence.

(i) Show that the area of triangle ABC is $ac\frac{\sqrt{3}}{4}$.

(4)

Given that $a = 2$ and $\sin A = \frac{\sqrt{15}}{5}$, find

(ii) the value of b,

(2)

(iii) the value of c.

(4)

(b) The internal angles of an n-sided polygon form an arithmetic sequence with first term 143° and common difference 2°.

Given that all of the internal angles are less than 180°, find the value of n.

(5)

(Total for Question 2 is 15 marks)

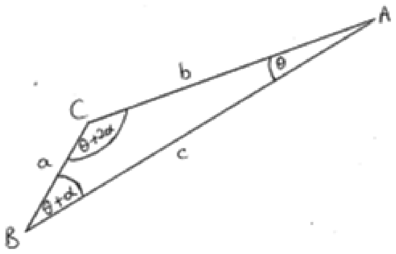
End of Questions

Platinum Mark Scheme

Q1.

Q.	Scheme	Marks	Notes
2(a)	$q=\frac{p}{2}(2a+(p-1)d)$ and $p=\frac{q}{2}(2a+(q-1)d)$	M1 A1	Attempt one sum formula Both correct expressions
	$2\left(\frac{q}{p}-\frac{p}{q}\right)=d(p-1-q+1)$	dM1 A1	Eliminate a. Dep on 1st M1 Must use 2 indep. eqns Correct elimination of a
	$d=\frac{2(q^2-p^2)}{pq(p-q)}$; $\underline{d=\frac{-2(p+q)}{pq}}$	A1 (5)	Correct simplified $d=$
(b)	$2a=\frac{2q}{p}+\frac{(p-1)2(q+p)}{pq}$; $a=\frac{q^2(q-1)-p^2(p-1)}{pq(q-p)}$	M1	Substitute for d in a correct sum formula i.e. eqn in a only
		dM1	Rearrange to $a=$. Dep M1
	$\frac{q^2+qp+p^2-p-q}{pq}$ or $\frac{q^2+(p-1)(q+p)}{pq}$ or $\frac{p^2+(q-1)(q+p)}{pq}$	A1 (3)	Correct single fraction with denom $=pq$
(c)	$S_{p+q}=\frac{p+q}{2}\left(\frac{2q}{p}+\frac{(p-1)2(q+p)}{pq}+\frac{-2(p+q)}{pq}(p+q-1)\right)$	M1	Attempt sum formula with $n=(p+q)$ and ft their a and d
	$=\frac{p+q}{2}\left[\frac{2(q^2+qp+p^2-p-q)}{pq}-\frac{2(p+q-1)(p+q)}{pq}\right]$	M1	Attempt to simplify- denominator $=pq$ or $2pq$
	$\frac{p+q}{pq}[-pq]=-[p+q]$	A1 (3) **[11]**	A1for $-(p+q)$ (S+ for concise simplification/factorising)

Q2.

Question Number	Scheme	Marks	Notes
Q5 (a) (i)			
	$\theta+(\theta+\alpha)+(\theta+2\alpha)=180$	M1	Equate $S_3=180$
	$3\theta+3\alpha = 180$		
	$\therefore \hat{B}=(\theta+\alpha) = 60°$	A1	Show $\hat{B}=60°$
	Area $=\frac{1}{2}ac\sin(\theta+\alpha)$	M1	Use of $\frac{1}{2}ac\sin B$
	$=\frac{1}{2}ac\frac{\sqrt{3}}{2} = \frac{ac\sqrt{3}}{4}$ (*)	A1 (4)	
(ii)	<u>Sine Rule</u> $\frac{b}{\sin(\theta+\alpha)}=\frac{a}{\sin A}$ <u>OR</u> $\frac{1}{2}bc\sin A=\frac{ac\sqrt{3}}{4}$	M1	Correct use of sine rule or $\frac{1}{2}bc\sin A$ and (a)
	$\therefore$ $b = 2\times\frac{5}{\sqrt{15}}\times\frac{\sqrt{3}}{2}=\sqrt{5}$	A1 (2)	
(iii)	<u>Cosine Rule</u> $b^2 = a^2+c^2-2ac\cos(\theta+\alpha)$	M1	Use of cos rule where all terms are known, except c.
	$5 = 4+c^2-2\times2\times c\ \frac{1}{2}$		
	$0 = c^2-2c-1$ OR $c^2-2\sqrt{2}+1=0$	M1	Sub & simplify -> 3TQ
	$c = \frac{2\pm\sqrt{4+4}}{2}$	M1	Solving
	$c = 1+\sqrt{2}$ OR $(3+2\sqrt{2})^{1/2}$	A1 (4)	
(b)	$S_n=\frac{n}{2}[2\times143+2(n-1)] = \{n(142+n)\}$	M1	For use of S_n needn't be simplified.
	Sum of internal angles $= 180(n-2)$	B1	
	$n(142+n)=180(n-2) \Rightarrow 0=n^2-38n+360$	A1	Correct 3TQ.
	$0=(n-19)^2-19^2+360$	M1	Attempt to solve relevant 3TQ
	$n-19=\pm1$ $(n=20$ or $18)$		
	Internal angles all <180 $u_{20}=143+19\times2>180$		] S+
	$u_{18}=143+17\times2<180$		
	$\therefore n=18$	A1 (5) [15]	

Pearson

Topic 4: Binomial Expansion

Bronze, Silver, Gold and Platinum Worksheets for

A Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel A Level Mathematics: Pure Mathematics Year 2' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Non-calculator

The total mark for this section is 25

Q1

Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(3 - 2x)^5$, giving each term in its simplest form.

(4)

(Total for Question 1 is 4 marks)

Q2

Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(1 + \frac{3x}{2}\right)^8$$

Giving each term in its simplest form.

(4)

(Total for Question 2 is 4 marks)

Q3

$$f(x) = \frac{1}{\sqrt{(4+x)}}, \qquad |x| < 4$$

Find the binomial expansion of f(x) in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

(6)

(Total for Question 3 is 6 marks)

Q4

$$f(x) = \frac{1}{\sqrt{(9+4x^2)}}, \qquad |x| < \frac{3}{2}$$

Find the first three non-zero terms of the binomial expansion of f(x) in ascending powers of x. Give each coefficient as a simplified fraction.

(6)

(Total for Question 4 is 6 marks)

Q5

(a) Use the binomial expansion, in ascending powers of x, to show that

$$\sqrt{(4-x)} = 2 - \frac{1}{4}x + kx^2 + \ldots$$

Where k is a rational constant to be found.

(4)

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

(b) State, giving a reason, if the expansion is valid for this value of x.

(1)

(Total for Question 5 is 5 marks)

End of Questions

Bronze Mark Scheme

Q1

Question Number	Scheme	Marks
	$(3-2x)^5 = 243$, $\quad \ldots\ldots + 5\times(3)^4(-2x) = -810x \quad \ldots\ldots$ $+\frac{5\times 4}{2}(3)^3(-2x)^2 = \qquad +1080x^2$	B1, B1 M1 A1 (4) **[4]**
Notes	First term must be 243 for **B1**, writing just 3^5 is B0 (Mark their final answers except in second line of special cases below). Term must be simplified to $-810x$ for **B1** The x is required for this mark. The **method** mark **(M1)** is **generous** and is awarded for an attempt at Binomial to get the third term. There must be an x^2 (or no x- i.e. not wrong power) and attempt at Binomial Coefficient and at dealing with powers of 3 and 2. The power of 3 should not be one, but the power of 2 may be one (regarded as bracketing slip). So allow $\binom{5}{2}$ or $\binom{5}{3}$ or ${}^5C_2$ or ${}^5C_3$ or even $\left(\frac{5}{2}\right)$ or $\left(\frac{5}{3}\right)$ or use of '10' (maybe from Pascal's triangle) May see ${}^5C_2(3)^3(-2x)^2$ or ${}^5C_2(3)^3(-2x^2)$ or ${}^5C_2(3)^5(-\frac{2}{3}x^2)$ or $10(3)^3(2x)^2$ which would each score the **M1** **A1** is c.a.o and needs $1080x^2$ (if $1080x^2$ is written with no working this is awarded both marks i.e. **M1 A1.**)	
Special cases	$243+810x+1080x^2$ is **B1B0M1A1** (condone no negative signs) Follows correct answer with $27-90x+120x^2$ can isw here (sp case)– full marks for correct answer Misreads *ascending* and gives $-32x^5+240x^4-720x^3$ is marked as **B1B0M1A0** special case and must be completely correct. *(If any slips could get B0B0M1A0)* Ignores 3 and expands $(1\pm 2x)^5$ is **0/4** 243, -810x, 1080x^2 is full marks but 243, -810, 1080 is **B1,B0,M1,A0** NB Alternative method $3^5(1-\frac{2}{3}x)^5 = 3^5 - 5\times 3^5\times(\frac{2}{3}x)+\binom{5}{3}3^5(-\frac{2}{3}x)^2+\ldots$ is **B0B0M1A0** – answers must be simplified to 243 $-810x$ $+1080x^2$ for full marks (awarded as before) Special case $3(1-\frac{2}{3}x)^5 = 3-5\times 3\times(\frac{2}{3}x)+\binom{5}{3}3(-\frac{2}{3}x)^2+\ldots$ is **B0, B0, M1, A0** Or $\quad 3(1-2x)^5$ is **B0B0M0A0**	

Q2

Question Number	Scheme		Marks
	$\left(1+\frac{3x}{2}\right)^8$		
	$1+12x$	Both terms correct as printed (allow $12x^1$ but not 1^8)	B1
	$\ldots+\frac{8(7)}{2!}\left(\frac{3x}{2}\right)^2+\frac{8(7)(6)}{3!}\left(\frac{3x}{2}\right)^3+\ldots$ $\ldots+{}^8C_2\left(\frac{3x}{2}\right)^2+{}^8C_3\left(\frac{3x}{2}\right)^3+\ldots$	$\left(\frac{8(7)}{2!}\times\ldots\times x^2\right)$ or $\left(\frac{8(7)(6)}{3!}\times\ldots\times x^3\right)$ or $\left({}^8C_2\times\ldots\times x^2\right)$ or $\left({}^8C_3\times\ldots\times x^3\right)$ M1: For <u>either</u> the x^2 term <u>or</u> the x^3 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 2 and/or 3 or signs) may be wrong or missing.	M1
	<u>Special Case:</u> Allow this M1 <u>only</u> for an attempt at a descending expansion provided the equivalent conditions are met for any term <u>other than the first</u> e.g. $\ldots+8\left(\frac{3x}{2}\right)^7(1)+\frac{8(7)}{2!}\left(\frac{3x}{2}\right)^6(1)^2+\ldots$ $\ldots+{}^8C_1\left(\frac{3x}{2}\right)^7+{}^8C_2\left(\frac{3x}{2}\right)^6+\ldots$		
	$\ldots+63x^2+189x^3+\ldots$	A1: **Either** $63x^2$ **or** $189x^3$ A1: **Both** $63x^2$ **and** $189x^3$	A1A1
	Terms may be listed but must be positive		
			[4]
			Total 4
	Note it is common not to square the 2 in the denominator of $\left(\frac{3x}{2}\right)$ and this gives $1+12x+126x^2+756x^3$. This could score B1M1A0A0.		
	Note $\ldots+{}^8C_2\left(1^6+\frac{3x}{2}\right)^2+{}^8C_3\left(1^5+\frac{3x}{2}\right)^3+\ldots$ would score M0 unless a correct method was implied by later work		

Q3

Question Number	Scheme	Marks
Q	$f(x)=\frac{1}{\sqrt{(4+x)}}=(4+x)^{-\frac{1}{2}}$	M1
	$=(4)^{-\frac{1}{2}}(1+\ ...\ )^{...}$ $\qquad \frac{1}{2}(1+\ ...\ )^{...}$ or $\frac{1}{2\sqrt{(1+...)}}$	B1
	$= \ ...\left(1+(-\frac{1}{2})\left(\frac{x}{4}\right)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2}\left(\frac{x}{4}\right)^2+\frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}\left(\frac{x}{4}\right)^3+\ ...\right)$	M1 A1ft
	ft their $\left(\frac{x}{4}\right)$	
	$=\frac{1}{2}-\frac{1}{16}x,+\frac{3}{256}x^2-\frac{5}{2048}x^3+\ ...$	A1, A1 (6)
		[6]
	Alternative	
	$f(x)=\frac{1}{\sqrt{(4+x)}}=(4+x)^{-\frac{1}{2}}$	M1
	$=\underline{4^{-\frac{1}{2}}}+(-\frac{1}{2})4^{-\frac{3}{2}}x+\frac{(-\frac{1}{2})(-\frac{3}{2})}{1.2}4^{-\frac{5}{2}}x^2+\frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{1.2.3}4^{-\frac{7}{2}}x^3+\ ...$	<u>B1</u> M1 A1
	$=\frac{1}{2}-\frac{1}{16}x,+\frac{3}{256}x^2-\frac{5}{2048}x^3+\ ...$	A1, A1 (6)

Q4

Question Number	Scheme		Marks	
	$\mathrm{f}(x)=(\ \dots\ +\ \dots)^{-\frac{1}{2}}$		M1	
	$=9^{-\frac{1}{2}}(\ \dots\ +\ \dots)^{\dots}$	$3^{-1},\ \frac{1}{3}$ or $\frac{1}{9^{\frac{1}{2}}}$	B1	
	$(1+kx^2)^n = 1+nkx^2+\dots$	n not a natural number, $k\neq 1$	M1	
	$(1+kx^2)^{-\frac{1}{2}} = \dots + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(kx^2)^2$	ft their $k\neq 1$	A1 ft	
	$\left(1+\frac{4}{9}x^2\right)^{-\frac{1}{2}}=1-\frac{2}{9}x^2+\frac{2}{27}x^4$		A1	
	$\mathrm{f}(x)=\frac{1}{3}-\frac{2}{27}x^2+\frac{2}{81}x^4$		A1	**(6)**
				[6]

Q5

Question	Scheme	Marks	AOs
(a)	$\sqrt{(4-x)}=2\left(1-\frac{1}{4}x\right)^{\frac{1}{2}}$	M1	2.1
	$\left(1-\frac{1}{4}x\right)^{\frac{1}{2}}=1+\frac{1}{2}\left(-\frac{1}{4}x\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{1}{4}x\right)^2+\ldots$	M1	1.1b
	$\sqrt{(4-x)}=2\left(1-\frac{1}{8}x-\frac{1}{128}x^2+..\right)$	A1	1.1b
	$\sqrt{(4-x)}=2-\frac{1}{4}x-\frac{1}{64}x^2+\ldots$ and $k=-\frac{1}{64}$	A1	1.1b
		(4)	
(b)	The expansion is valid for $\lvert x\rvert<4$, so $x=1$ can be used	B1	2.4
		(1)	
		(5 marks)	

Notes:

(a)

M1: Takes out a factor of 4 and writes $\sqrt{(4-x)}=2(1\pm\ldots)^{\frac{1}{2}}$

M1: For an attempt at the binomial expansion with $n=\frac{1}{2}$

Eg. $(1+ax)^{\frac{1}{2}}=1+\frac{1}{2}(ax)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(ax)^2+\ldots$

A1: Correct expression inside the bracket $1-\frac{1}{8}x-\frac{1}{128}x^2+$ which may be left unsimplified

A1: $\sqrt{(4-x)}=2-\frac{1}{4}x-\frac{1}{64}x^2+\ldots$ and $k=-\frac{1}{64}$

(b)

B1: The expansion is valid for $\lvert x\rvert<4$, so $x=1$ can be used

Silver Questions

Non-calculator

The total mark for this section is 35

Q1

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(3 + bx)^5$$

where b is a non-zero constant. Give each term in its simplest form.

(4)

Given that, in this expansion, the coefficient of x^2 is twice the coefficient of x,

(b) find the value of b.

(2)

(Total for Question 1 is 6 marks)

Q2

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 9x)^4$$

Giving each term in its simplest form.

(4)

$$f(x) = (1 + kx)(2 - 9x)^4, \text{ where } k \text{ is a constant}$$

The expansion, in ascending powers of x, of $f(x)$ up to and including the term in x^2 is

$$A - 232x + Bx^2$$

Where A and B are constants.

(b) Write down the value of A.

(1)

(c) Find the value of k.

(2)

(d) Hence find the value of B.

(2)

(Total for Question 2 is 9 marks)

Q3

$$\frac{2x^2+5x-10}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}$$

(a) Find the values of the constants A, B and C.

(4)

(b) Hence, or otherwise, expand $\frac{2x^2+5x-10}{(x-1)(x+2)}$ in ascending powers of x, as far as the term in x^2. Give each coefficient as a simplified fraction.

(7)

(Total for Question 3 is 11 marks)

Q4

(a) Use the binomial expansion to show that

$$\sqrt{\left(\frac{1+x}{1-x}\right)} \approx 1 + x + \frac{1}{2}x^2, \quad |x| < 1$$

(6)

(b) Substitute $x = \frac{1}{26}$ into

$$\sqrt{\left(\frac{1+x}{1-x}\right)} = 1 + x + \frac{1}{2}x^2$$

To obtain an approximation to $\sqrt{3}$. Give your answer in the form $\frac{a}{b}$ where a and b are integers.

(3)

(Total for Question 4 is 9 marks)

End of Questions

Silver Mark Scheme

Q1

Question Number	Scheme		Marks
(a)	$\{(3+bx)^5\} = (3)^5 + {}^5C_1(3)^4(bx) + {}^5C_2(3)^3(bx)^2 + \ldots$ $= 243 + 405bx + 270b^2x^2 + \ldots$	243 as a constant term seen. $405bx$ $({}^5C_1 \times \ldots \times x)$ or $({}^5C_2 \times \ldots \times x^2)$ $270b^2x^2$ or $270(bx)^2$	B1 B1 M1 A1 [4]
(b)	$\{2(\text{coeff } x) = \text{coeff } x^2\} \Rightarrow 2(405b) = 270b^2$ So, $\left\{b = \frac{810}{270} \Rightarrow\right\} b = 3$	Establishes an equation from their coefficients. Condone 2 on the wrong side of the equation. $b = 3$ (Ignore $b = 0$, if seen.)	M1 A1 [2] 6

(a)	The terms can be "listed" rather than added. Ignore any extra terms. 1st B1: A constant term of 243 seen. Just writing $(3)^5$ is B0. 2nd B1: Term must be simplified to $405bx$ for B1. The x is required for this mark. Note $405 + bx$ is B0. M1: For either the x term or the x^2 term. Requires correct binomial coefficient in any form with the correct power of x, but the other part of the coefficient (perhaps including powers of 3 and/or b) may be wrong or missing. Allow binomial coefficients such as $\binom{5}{2}, \left(\frac{5}{2}\right), \binom{5}{1}, \left(\frac{5}{1}\right), {}^5C_2, {}^5C_1$. A1: For either $270b^2x^2$ or $270(bx)^2$. (If $270bx^2$ follows $270(bx)^2$, isw and allow A1.) **Alternative:** Note that a factor of 3^5 can be taken out first: $3^5\left(1+\frac{bx}{3}\right)^5$, but the mark scheme still applies. **Ignore subsequent working (isw):** Isw if necessary after correct working: e.g. $243 + 405bx + 270b^2x^2 + \ldots$ leading to $9 + 15bx + 10b^2x^2 + \ldots$ scores B1B1M1A1 isw. Also note that full marks could also be available in part (b), here. Special Case: Candidate writing down the first three terms in ***descending*** powers of x usually get $(bx)^5 + {}^5C_4(3)^1(bx)^4 + {}^5C_3(3)^2(bx)^3 + \ldots = b^5x^5 + 15b^4x^4 + 90b^3x^3 + \ldots$ So award SC: B0B0M1A0 for either $({}^5C_4 \times \ldots \times x^4)$ or $({}^5C_3 \times \ldots \times x^3)$
(b)	M1 for equating 2 times their coefficient of x to the coefficient of x^2 to get an equation in b, or equating their coefficient of x to 2 times that of x^2, to get an equation in b. Allow this M mark even if the equation is trivial, providing their coefficients from part (a) have been used, eg: $2(405b) = 270b$, but beware $b = 3$ from this, which is A0. An equation in b alone is required: e.g. $2(405b)x = 270b^2x^2 \Rightarrow b = 3$ or similar will be **Special Case** SC: M1A0 (as equation in coefficients only is not seen here). e.g. $2(405b)x = 270b^2x^2 \Rightarrow 2(405b) = 270b^2 \Rightarrow b = 3$ will get M1A1 (as coefficients rather than terms have now been considered). **Note:** Answer of 3 from no working scores M1A0. **Note:** The mistake $k\left(1+\frac{bx}{3}\right)^5, k \neq 243$ would give a maximum of 3 marks: B0B0M1A0, M1A1 **Note:** For $270bx^2$ in part (a), followed by $2(405b) = 270b^2 \Rightarrow b = 3$, in part (b), allow recovery M1A1.

Q2

Question Number	Scheme		Marks
	(a) $(2-9x)^4 = 2^4 + {}^4C_1 2^3(-9x) + {}^4C_2 2^2(-9x)^2$, (b) $f(x) = (1+kx)(2-9x)^4 = A - 232x + Bx^2$		
(a) Way 1	First term of 16 in their final series		B1
	At least one of $({}^4C_1 \times ... \times x)$ or $({}^4C_2 \times ... \times x^2)$		M1
	$= (16) - 288x + 1944x^2$	At least one of $-288x$ or $+1944x^2$	A1
		Both $-288x$ and $+1944x^2$	A1
			[4]
(a)	$(2-9x)^4 = (4-36x+81x^2)(4-36x+81x^2)$		
		First term of 16 in their final series	B1
Way 2	$= 16 - 144x + 324x^2 - 144x + 1296x^2 + 324x^2$	Attempts to multiply a 3 term quadratic by the same 3 term quadratic to achieve either 2 terms in x or at least 2 terms in x^2.	M1
	$= (16) - 288x + 1944x^2$	At least one of $-288x$ or $+1944x^2$	A1
		Both $-288x$ and $+1944x^2$	A1
			[4]
(a) Way 3	$\{(2-9x)^4 =\}\ 2^4\left(1 - \frac{9}{2}x\right)^4$	First term of 16 in final series	B1
	$= 2^4\left(1 + \underline{4\left(-\frac{9}{2}x\right) + \frac{4(3)}{2}\left(-\frac{9}{2}x\right)^2} + ...\right)$	At least one of $(4 \times ... \times x)$ or $\left(\frac{4(3)}{2} \times ... \times x^2\right)$	M1
	$= (16) - 288x + 1944x^2$	At least one of $-288x$ or $+1944x^2$	A1
		Both $-288x$ and $+1944x^2$	A1
			[4]
	Parts (b), (c) and (d) may be marked together		
(b)	$A = "16"$	Follow through their value from (a)	B1ft
			[1]
(c)	$\{(1+kx)(2-9x)^4\} = (1+kx)(16 - 288x + \{1944x^2 + ...\})$	*May be seen in part (b) or (d) and can be implied by work in parts (c) or (d).*	M1
	x terms: $-288x + 16kx = -232x$		
	giving, $16k = 56 \Rightarrow \underline{k = \frac{7}{2}}$	$\underline{k = \frac{7}{2}}$	A1
			[2]
(d)	x^2 terms: $1944x^2 - 288kx^2$		
	So, $B = 1944 - 288\left(\frac{7}{2}\right);\ = 1944 - 1008 = 936$	See notes	M1
		936	A1
			[2]
			9

		Question Notes
(a) Ways 1 and 3	B1 cao	16
	M1	Correct binomial coefficient associated with correct power of x i.e $\left({}^4C_1 \times ... \times x\right)$ or $\left({}^4C_2 \times ... \times x^2\right)$ They may have 4 and 6 or 4 and $\frac{4(3)}{2}$ or even $\binom{4}{1}$ and $\binom{4}{2}$ as their coefficients. Allow missing signs and brackets for the M marks.
	1st A1	At least one of $-288x$ or $+1944x^2$ (allow +- 288x)
	2nd A1	Both $-288x$ and $+1944x^2$ (May list terms separated by commas) Also full marks for correct answer with no working here. Again allow +- 288x
	Note	If the candidate then divides their final correct answer through by 8 or any other common factor then isw and mark correct series when first seen. So (a) B1M1A1A1 .It is likely that this approach will be followed by (b) B0, (c) M1A0, (d) M1A0 if they continue with their new series e.g. $2-36x+283x^2+...$ (Do not ft the value 2 as a mark was awarded for 16)
Way 2b	Special Case	**Slight Variation on the solution given in the scheme** $(2-9x)^4 = (2-9x)(2-9x)(4-36x+81x^2)$ $= (2-9x)(8-108x+486x^2+...)$ $= 16 - 216x + 972x^2 - 72x + 972x^2$ — First term of 16: **B1**; Multiplies out to give either 2 terms in x or 2 terms in x^2: **M1** $= (16) - 288x + 1944x^2 + ...$ — At least one of $-288x$ or $+1944x^2$: **A1**; Both $-288x$ and $+1944x^2$: **A1**
(b)	B1ft	**Parts (b), (c) and (d) may be marked together.** Must **identify** $A = 16$ or $A =$ *their* constant term found in part (a). Or may write just 16 if this is clearly their answer to part (b). If they expand their series and have 16 as first term of a series it is not sufficient for this mark.
(c)	M1	Candidate shows intention to multiply $(1+kx)$ by part of their series from (a) e.g. Just $(1+kx)(16-288x+...)$ or $(1+kx)(16-288x+1944x^2+...)$ are fine for M1.
	Note	This mark can also be implied by candidate multiplying out to find **two terms** (or coefficients) in x. i.e. f.t. their $-288x+16kx$ N.B. $-288kx = -232x$ with no evidence of brackets is M0 – allow copying slips, or use of factored series, as this is a method mark
	A1	$k=\frac{7}{2}$ o.e. so 3.5 is acceptable
(d)	M1	Multiplies out their $(1+kx)(16-288x+1944x^2+...)$ to give **exactly** two terms (or coefficients) in x^2 and attempts to find B using **these two terms** and a numerical value of k.
	A1	936
	Note	Award A0 for $B = 936x^2$ But allow A1 for $B = 936x^2$ **followed by** $B = 936$ and treat this as a correction Correct answers in parts (c) and (d) with no method shown may be awarded full credit.

Q3

Question Number	Scheme	Marks
	(a) $A = 2$	B1
	$2x^2 + 5x - 10 = A(x-1)(x+2) + B(x+2) + C(x-1)$	
	$x \to 1$ $\quad -3 = 3B \Rightarrow B = -1$	M1 A1
	$x \to -2$ $\quad -12 = -3C \Rightarrow C = 4$	A1 **(4)**
	(b) $\dfrac{2x^2+5x-10}{(x-1)(x+2)} = 2 + (1-x)^{-1} + 2\left(1+\dfrac{x}{2}\right)^{-1}$	M1
	$(1-x)^{-1} = 1 + x + x^2 + \ldots$	B1
	$\left(1+\dfrac{x}{2}\right)^{-1} = 1 - \dfrac{x}{2} + \dfrac{x^2}{4} + \ldots$	B1
	$\dfrac{2x^2+5x-10}{(x-1)(x+2)} = (2+1+2) + (1-1)x + \left(1+\dfrac{1}{2}\right)x^2 + \ldots$	M1
	$= 5 + \ldots$ $\quad$ ft their $A - B + \frac{1}{2}C$	A1 ft
	$= \ldots + \dfrac{3}{2}x^2 + \ldots$ $\quad$ $0x$ stated or implied	A1 A1 **(7)**
		[11]

Q4

Question Number	Scheme		Marks
(a)	$\left\{\sqrt{\left(\dfrac{1+x}{1-x}\right)}\right\} = (1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$	$(1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$	B1
	$= \left(1+\left(\frac{1}{2}\right)x+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2+...\right)\times\left(1+\left(-\frac{1}{2}\right)(-x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2+...\right)$	See notes	M1 A1 A1
	$= \left(1+\frac{1}{2}x-\frac{1}{8}x^2+...\right)\times\left(1+\frac{1}{2}x+\frac{3}{8}x^2+...\right)$		
	$= 1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{1}{2}x+\frac{1}{4}x^2-\frac{1}{8}x^2+...$	See notes	M1
	$= 1+x+\frac{1}{2}x^2$	*Answer is given in the question.*	A1 *
			[6]
(b)	$\sqrt{\left(\dfrac{1+(\frac{1}{26})}{1-(\frac{1}{26})}\right)} = 1+\left(\frac{1}{26}\right)+\frac{1}{2}\left(\frac{1}{26}\right)^2$		M1
	ie: $\dfrac{3\sqrt{3}}{5} = \dfrac{1405}{1352}$		B1
	so, $\sqrt{3} = \dfrac{7025}{4056}$	$\dfrac{7025}{4056}$	A1 cao
			[3] 9

Notes for Question

(a)

B1: $(1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$ or $\sqrt{(1+x)}(1-x)^{-\frac{1}{2}}$ seen or implied. (Also allow $\left((1+x)(1-x)^{-1}\right)^{\frac{1}{2}}$).

M1: Expands $(1+x)^{\frac{1}{2}}$ to give any 2 out of 3 terms simplified or un-simplified,

Eg: $1+\frac{1}{2}x$ or $+\left(\frac{1}{2}\right)x+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2$ or $1+ .... +\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2$

or expands $(1-x)^{-\frac{1}{2}}$ to give any 2 out of 3 terms simplified or un-simplified,

Eg: $1+\left(-\frac{1}{2}\right)(-x)$ or $+\left(-\frac{1}{2}\right)(-x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2$ or $1+ .... +\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2$

Also allow: $1+ .... +\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(x)^2$ for M1.

A1: At least one binomial expansion correct (either un-simplified or simplified). (ignore x^3 and x^4 terms)
A1: Two binomial expansions are correct (either un-simplified or simplified). (ignore x^3 and x^4 terms)
Note: Candidates can give decimal equivalents when expanding out their binomial expansions.
M1: Multiplies out to give 1, exactly two terms in x and exactly three terms in x^2.
A1: Candidate achieves the result on the exam paper. Make sure that their working is sound.

Special Case: Award SC FINAL M1A1 for *a correct* $\left(1+\frac{1}{2}x-\frac{1}{8}x^2+...\right)\times\left(1+\frac{1}{2}x+\frac{3}{8}x^2+...\right)$ multiplied out with no errors to give either $1+x+\frac{3}{8}x^2+\frac{1}{4}x^2-\frac{1}{8}x^2$ or $1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{1}{2}x+\frac{1}{8}x^2$ or $1+\frac{1}{2}x+\frac{1}{4}x^2+\frac{1}{2}x+\frac{1}{4}x^2$ or $1+\frac{1}{2}x+\frac{5}{8}x^2+\frac{1}{2}x-\frac{1}{8}x^2$ leading to the correct answer of $1+x+\frac{1}{2}x^2$.

Notes for Question Continued		
(a) ctd	**Note:** If a candidate writes down either $(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + ...$ **or** $(1-x)^{-\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + ...$ with no working then you can award 1st M1, 1st A1. **Note:** If a candidate writes down both correct binomial expansions with no working, then you can award 1st M1, 1st A1, 2nd A1.	
(b)	**M1:** Substitutes $x = \frac{1}{26}$ into **both** sides of $\sqrt{\left(\frac{1+x}{1-x}\right)}$ and $1 + x + \frac{1}{2}x^2$ **B1:** For sight of $\sqrt{\frac{27}{25}}$ (or better) and $\frac{1405}{1352}$ or equivalent fraction Eg: $\frac{3\sqrt{3}}{5}$ and $\frac{1405}{1352}$ or $0.6\sqrt{3}$ and $\frac{1405}{1352}$ or $\frac{3\sqrt{3}}{5}$ and $1\frac{53}{1352}$ or $\sqrt{3}$ and $\frac{5}{3}\left(\frac{1405}{1352}\right)$ are fine for B1. **A1:** $\frac{7025}{4056}$ **or any equivalent fraction**, eg: $\frac{14050}{8112}$ or $\frac{182650}{105456}$ etc. **Special Case:** Award SC: M1B1A0 for $\sqrt{3} \approx 1.732001972..$ or truncated 1.732001 or awrt 1.732002. **Note that** $\frac{7025}{4056} = 1.732001972...$ and $\sqrt{3} = 1.732050808...$	
Aliter (a) Way 2	$\left\{\sqrt{\left(\frac{1+x}{1-x}\right)} = \sqrt{\frac{(1+x)(1-x)}{(1+x)(1-x)}} = \sqrt{\frac{(1-x^2)}{(1-x)^2}} = \right\} = (1-x^2)^{\frac{1}{2}}(1-x)^{-1}$ $(1-x^2)^{\frac{1}{2}}(1-x)^{-1}$	B1
	$= \left(1 + \left(\frac{1}{2}\right)(-x^2) + ...\right) \times \left(1 + (-1)(-x) + \frac{(-1)(-2)}{2!}(-x)^2 + ...\right)$ See notes	M1A1A1
	$= \left(1 - \frac{1}{2}x^2 + ...\right) \times (1 + x + x^2 + ...)$	
	$= 1 + x + x^2 - \frac{1}{2}x^2$ See notes	M1
	$= 1 + x + \frac{1}{2}x^2$ *Answer is given in the question.*	A1 *
		[6]
Aliter (a) Way 2	**B1:** $(1-x^2)^{\frac{1}{2}}(1-x)^{-1}$ seen or implied. **M1:** Expands $(1-x^2)^{\frac{1}{2}}$ to give both terms simplified or un-simplified, $1 + \left(\frac{1}{2}\right)(-x^2)$ or expands $(1-x)^{-1}$ to give any 2 out of 3 terms simplified or un-simplified, Eg: $1 + (-1)(-x)$ or $... + (-1)(-x) + \frac{(-1)(-2)}{2!}(-x)^2$ or $1 + ... + \frac{(-1)(-2)}{2!}(-x)^2$ **A1:** At least one binomial expansion correct (either un-simplified or simplified). (ignore x^3 and x^4 terms) **A1:** Two binomial expansions are correct (either un-simplified or simplified). (ignore x^3 and x^4 terms) **M1:** Multiplies out to give 1, exactly one term in x and exactly two terms in x^2. **A1:** Candidate achieves the result on the exam paper. Make sure that their working is sound.	

Notes for Question Continued		
Aliter (a) Way 3	$\left\{\sqrt{\left(\frac{1+x}{1-x}\right)} = \sqrt{\frac{(1+x)(1+x)}{(1-x)(1+x)}} = \right\} = (1+x)(1-x^2)^{-\frac{1}{2}}$ $(1+x)(1-x^2)^{-\frac{1}{2}}$	B1
	$= (1+x)\left(1+\frac{1}{2}x^2+...\right)$ Must follow on from above.	M1A1A1
	$= 1+x+\frac{1}{2}x^2$	dM1A1
	Note: The final M1 mark is dependent on the previous method mark for Way 3.	
Aliter (a) Way 4	**Assuming the result on the Question Paper. (You need to be convinced that a candidate is applying this method before you apply the Mark Scheme for Way 4).**	
	$\left\{\sqrt{\left(\frac{1+x}{1-x}\right)} = \frac{\sqrt{(1+x)}}{\sqrt{(1-x)}} = 1+x+\frac{1}{2}x^2\right\} \Rightarrow (1+x)^{\frac{1}{2}} = \left(1+x+\frac{1}{2}x^2\right)(1-x)^{\frac{1}{2}}$	B1
	$(1+x)^{\frac{1}{2}} = 1+\left(\frac{1}{2}\right)x+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2+... \left\{= 1+\frac{1}{2}x-\frac{1}{8}x^2+...\right\},$ $(1-x)^{\frac{1}{2}} = 1+\left(\frac{1}{2}\right)(-x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(-x)^2+... \left\{= 1-\frac{1}{2}x-\frac{1}{8}x^2+...\right\}$	M1A1A1
	$\text{RHS} = \left(1+x+\frac{1}{2}x^2\right)(1-x)^{\frac{1}{2}} = \left(1+x+\frac{1}{2}x^2\right)\left(1-\frac{1}{2}x-\frac{1}{8}x^2+...\right)$	
	$= 1-\frac{1}{2}x-\frac{1}{8}x^2+x-\frac{1}{2}x^2+\frac{1}{2}x^2$ See notes	M1
	$= 1+\frac{1}{2}x-\frac{1}{8}x^2$	
	$\text{So, LHS} = 1+\frac{1}{2}x-\frac{1}{8}x^2 = \text{RHS}$	A1 *
		[6]

B1: $(1+x)^{\frac{1}{2}} = \left(1+x+\frac{1}{2}x^2\right)(1-x)^{\frac{1}{2}}$ seen or implied.

M1: For Way 4, this M1 mark is dependent on the first B1 mark.

Expands $(1+x)^{\frac{1}{2}}$ to give any 2 out of 3 terms simplified or un-simplified,

Eg: $1+\frac{1}{2}x$ or $+\left(\frac{1}{2}\right)x+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2$ or $1+ .... +\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2$

or expands $(1-x)^{\frac{1}{2}}$ to give any 2 out of 3 terms simplified or un-simplified,

Eg: $1+\left(\frac{1}{2}\right)(-x)$ or $+\left(\frac{1}{2}\right)(-x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(-x)^2$ or $1+ .... +\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(-x)^2$

A1: At least one binomial expansion correct (either un-simplified or simplified). (ignore x^3 and x^4 terms)

A1: Two binomial expansions are correct (either un-simplified or simplified). (ignore x^3 and x^4 terms)

M1: For Way 4, this M1 mark is dependent on the first B1 mark.
Multiplies out RHS to give 1, exactly two terms in x and exactly three terms in x^2.

A1: Candidate achieves the result on the exam paper. Candidate needs to have correctly processed both the LHS and RHS of $(1+x)^{\frac{1}{2}} = \left(1+x+\frac{1}{2}x^2\right)(1-x)^{\frac{1}{2}}$.

Gold Questions

Non-calculator

The total mark for this section is 35

Q1

$$f(x) = \frac{27x^2 + 32x + 16}{(3x+2)^2(1-x)}, \quad |x| < \frac{2}{3}$$

Given that f (x) can be expressed in the form

$$f(x) = \frac{A}{(3x+2)} + \frac{B}{(3x+2)^2} + \frac{C}{(1-x)},$$

(a) Find the values of B and C and show that $A = 0$.

(4)

(b) Hence, or otherwise, find the series expansion of f (x), in ascending powers of x, up to and including the term in x^2.
Simplify each term.

(6)

(Total for Question 1 is 10 marks)

Q2

(a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$\frac{1}{\sqrt{4-x}}$$

Giving each coefficient in its simplest form.

(4)

The expansion can be used to find an approximation to $\sqrt{2}$. Possible values of x that could be substituted into this expansion are:

- $x = -14$ because $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{18}} = \frac{\sqrt{2}}{6}$
- $x = 2$ because $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
- $x = -\frac{1}{2}$ because $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{\frac{9}{2}}} = \frac{\sqrt{2}}{3}$

(b) Without evaluating your expansion,

(i) state, giving a reason, which of the three values of x should not be used

(1)

(ii) state, giving a reason, which of the three values of x would lead to the most accurate approximation to $\sqrt{2}$.

(1)

(Total for Question 2 is 6 marks)

Q3

$$f(x) = \frac{6}{\sqrt{(9-4x)}}, \qquad |x| < \frac{9}{4}.$$

(*a*) Find the binomial expansion of f(x) in ascending powers of x, up to and including the term in x^3. Give each coefficient in its simplest form.

(6)

Use your answer to part (*a*) to find the binomial expansion in ascending powers of x, up to and including the term in x^3, of

(*b*) $g(x) = \frac{6}{\sqrt{(9+4x)}}, \qquad |x| < \frac{9}{4},$

(1)

(*c*) $h(x) = \frac{6}{\sqrt{(9-8x)}}, \qquad |x| < \frac{9}{8}.$

(2)

(Total for Question 3 is 9 marks)

Q4

(a) Use the binomial theorem to expand

$$(2-3x)^{-2}, \qquad |x| < \frac{2}{3},$$

In ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

(5)

$$f(x) = \frac{a+bx}{(2-3x)^2}, \qquad |x| < \frac{2}{3}, \qquad \text{where } a \text{ and } b \text{ are constants.}$$

In the binomial expansion of f (x), in ascending powers of x, the coefficient of x is 0 and the coefficient of x^2 is $\frac{9}{16}$. Find

(b) The value of a and the value of b.

(5)

(Total for Question 4 is 10 marks)

End of Questions

Pearson

Gold Mark Scheme

Q1

Question Number	Scheme		Marks
(a)	$27x^2+32x+16 \equiv A(3x+2)(1-x)+B(1-x)+C(3x+2)^2$	Forming this identity	M1
	$x=-\frac{2}{3}$, $12-\frac{64}{3}+16=\left(\frac{5}{3}\right)B \Rightarrow \frac{20}{3}=\left(\frac{5}{3}\right)B \Rightarrow B=4$ $x=1$, $27+32+16=25C \Rightarrow 75=25C \Rightarrow C=3$	Substitutes either $x=-\frac{2}{3}$ or $x=1$ into their identity or equates 3 terms or substitutes in values to write down three simultaneous equations.	M1
		Both $B=4$ and $C=3$ **(Note the A1 is dependent on both method marks in this part.)**	A1
	Equate x^2: $27=-3A+9C \Rightarrow 27=-3A+27 \Rightarrow 0=-3A$ $\Rightarrow A=0$ $x=0$, $16=2A+B+4C$ $\Rightarrow 16=2A+4+12 \Rightarrow 0=2A \Rightarrow A=0$	Compares coefficients or substitutes in a third x-value or uses simultaneous equations to show $A=0$.	B1
			(4)
(b)	$f(x)=\dfrac{4}{(3x+2)^2}+\dfrac{3}{(1-x)}$		
	$=4(3x+2)^{-2}+3(1-x)^{-1}$	Moving powers to top on any one of the two expressions	M1
	$=4\left[2\left(1+\frac{3}{2}x\right)^{-2}\right]+3(1-x)^{-1}$		
	$=1\left(1+\frac{3}{2}x\right)^{-2}+3(1-x)^{-1}$		
	$=1\left\{\underline{1+(-2)\left(\frac{3x}{2}\right);+\frac{(-2)(-3)}{2!}\left(\frac{3x}{2}\right)^2+\ldots}\right\}$	Either $1\pm(-2)\left(\frac{3x}{2}\right)$ or $1\pm(-1)(-x)$ from either first or second expansions respectively	dM1;
	$+3\left\{\underline{1+(-1)(-x);+\frac{(-1)(-2)}{2!}(-x)^2+\ldots}\right\}$	Ignoring 1 and 3, any one correct $\{\ldots\ldots\ldots\}$ expansion.	A1
		Both $\{\ldots\ldots\ldots\}$ correct.	A1
	$=\left\{1-3x+\frac{27}{4}x^2+\ldots\right\}+3\left\{1+x+x^2+\ldots\right\}$		
	$=4+0x;+\frac{39}{4}x^2$	$4+(0x)$; $\frac{39}{4}x^2$	A1; A1
			(6)
			10

Q2

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\dfrac{1}{\sqrt{4-x}} = (4-x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}}\left(1-\dfrac{x}{4}\right)^{-\frac{1}{2}}$	M1	This mark is given for rearranging $\dfrac{1}{\sqrt{4-x}}$ to attempt a binomial expansion
	$\left(1-\dfrac{x}{4}\right)^{-\frac{1}{2}} =$ $1+\left(-\dfrac{1}{2}\right)\left(-\dfrac{x}{4}\right)+\dfrac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\dfrac{x}{4}\right)$	M1	This mark is given for an attempt at a binomial expansion
		A1	This mark is given for a fully correct binomial expansion
	$\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{2}+\dfrac{1}{16}x+\dfrac{3}{256}x^2$	A1	This mark is given for a fully correct expansion with the first three terms
(b)(i)	$x = -14$, since the expansion is only valid for $\lvert x\rvert < 4$	B1	This mark is given for the correct value chosen with a correct reason
(b)(ii)	$x = -\dfrac{1}{2}$, since the smaller value will give the more accurate approximation	B1	This mark is given for the correct value chosen with a correct reason

Q3

(a)	$f(x) = \dots(\dots - \dots x)^{-\frac{1}{2}}$	M1
	$= 6 \times 9^{-\frac{1}{2}}(\dots)$ $\quad \frac{6}{9^{\frac{1}{2}}}, \frac{6}{3}$, 2 or equivalent	B1
	$= \dots\left(1 + (-\tfrac{1}{2})(kx); + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(kx)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}(kx)^3 + \dots\right)$	M1 A1ft
	$= 2\left(1 + \frac{2}{9}x + \dots\right)$ $\quad$ or $2 + \frac{4}{9}x$	A1
	$= 2 + \frac{4}{9}x + \frac{4}{27}x^2 + \frac{40}{729}x^3 + \dots$	A1
		(6)
(b)	$g(x) = 2 - \frac{4}{9}x + \frac{4}{27}x^2 - \frac{40}{729}x^3 + \dots$	B1ft
		(1)
(c)	$h(x) = 2 + \frac{4}{9}(2x) + \frac{4}{27}(2x)^2 + \frac{40}{729}(2x)^3 + \dots$	M1 A1
	$\left(= 2 + \frac{8}{9}x + \frac{16}{27}x^2 + \frac{320}{729}x^3 + \dots\right)$	
		(2)
		(9 marks)

Q4

Question Number	Scheme	Marks
(a)	$(2-3x)^{-2}=2^{-2}\left(1-\frac{3}{2}x\right)^{-2}$	B1
	$\left(1-\frac{3}{2}x\right)^{-2}=1+(-2)\left(-\frac{3}{2}x\right)+\frac{-2.-3}{1.2}\left(-\frac{3}{2}x\right)^{2}+\frac{-2.-3.-4}{1.2.3}\left(-\frac{3}{2}x\right)^{3}+\ \dots$	M1 A1
	$=1+3x+\frac{27}{4}x^{2}+\frac{27}{2}x^{3}+\ \dots$	
	$(2-3x)^{-2}=\frac{1}{4}+\frac{3}{4}x+\frac{27}{16}x^{2}+\frac{27}{8}x^{3}+\ \dots$	M1 A1 (5)
(b)	$f(x)=(a+bx)\left(\frac{1}{4}+\frac{3}{4}x+\frac{27}{16}x^{2}+\frac{27}{8}x^{3}+\ \dots\right)$	
	Coefficient of x; $\frac{3a}{4}+\frac{b}{4}=0$ $(3a+b=0)$	M1
	Coefficient of x^2; $\frac{27a}{16}+\frac{3b}{4}=\frac{9}{16}$ $(9a+4b=3)$ A1 either correct	M1 A1
	Leading to $a=-1,\ b=3$	M1 A1 (5)

Platinum Questions

Non-calculator

The total mark for this section is 25

Q1

(*a*) (i) Write down the binomial series expansion of

$$\left(1+\frac{2}{n}\right)^n \qquad n \in \square,\ n > 2$$

in powers of $\left(\frac{2}{n}\right)$ up to and including the term in $\left(\frac{2}{n}\right)^3$.

(ii) Hence prove that, form $n \in \square$, n ≥ 3

$$\left(1+\frac{2}{n}\right)^n \geq \frac{19}{3}-\frac{6}{n}$$

(3)

(*b*) Use the binomial series expansion of $\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$ to show that $\sqrt{3} < \frac{7}{4}$

(4)

$$\mathrm{f}(x) = \left(1+\frac{2}{x}\right)^x - 3^{\frac{x}{6}} \qquad x \in \square,\ x > 0$$

Given that the function f (x) is continuous and that $\sqrt[6]{3} > \frac{6}{5}$

(*c*) prove that f (x) = 0 has a root in the interval [9, 10]

(5)

(+S1)

(Total for Question 1 is 13 marks)

Q2

(a) Find the binomial series expansion for $(4+y)^{\frac{1}{2}}$ in ascending powers of y up to and including the term in y^3. Simplify the coefficient of each term.

(3)

(b) Hence show that the binomial series expansion for $\left(4+5x+x^2\right)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^3 is

$$2+\frac{5x}{4}-\frac{9x^2}{64}+\frac{45x^3}{512}$$

(3)

(c) Show that the binomial series expansion of $\left(4+5x+x^2\right)^{\frac{1}{2}}$ will converge for $-\frac{1}{2}\leq x\leq\frac{1}{2}$.

(6)

(Total for Question 2 is 12 marks)

End of Questions

Platinum Mark Scheme

Q1

Question	Scheme	Marks	AOs
1(a)(i)	$\left(1+\frac{2}{n}\right)^n = 1+n\left(\frac{2}{n}\right)+\frac{n(n-1)}{2}\left(\frac{2}{n}\right)^2+\frac{n(n-1)(n-2)}{6}\left(\frac{2}{n}\right)^3+\ldots$ No need to simplify but allow if simplified forms are given instead.	**B1**	1
(ii)	$\left(1+\frac{2}{n}\right)^n = 1+2+\frac{2(n-1)}{n}+\frac{4(n^2-3n+2)}{3n^2}+\ldots$(non-negative terms) $=1+2+2-\frac{2}{n}+\frac{4}{3}-\frac{4}{n}+\left(\frac{8}{3n^2}+\right)\ldots$(non-negative terms) Simplifies terms and cancels common factors (+… may be missing).	**M1**	2
	$\therefore\left(1+\frac{2}{n}\right)^n \geqslant \frac{19}{3}-\frac{6}{n}$ (as all other terms are non-negative.) * (Accept all other terms are positive, but S- if no reason is given.)	**A1*** **(S-)**	2
		(3)	
(b)	Expands to e.g. $\left(1-\frac{x}{4}\right)^{\frac{1}{2}} = 1+\frac{1}{2}\left(-\frac{x}{4}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}\left(-\frac{x}{4}\right)^2+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{6}\left(-\frac{x}{4}\right)^3+\ldots$ Enough terms to deduce pattern of signs should be given.	**M1** **(S+)**	1
	$\left(1-\frac{x}{4}\right)^{\frac{1}{2}} < 1-\frac{x}{8}$ since we can see all remaining terms are negative. Deduces inequality noting all remaining terms are negative. (Allow if $\leqslant$ used as equality can be rejected as $\sqrt{3}$ is not rational.)	**B1**	3
	(Series converges for $\lvert x\rvert < 4$ so) substituting $x=1$ into the equation gives $\left(\frac{3}{4}\right)^{\frac{1}{2}} < 1-\frac{1}{8}=\frac{7}{8}$ (M0 if attempt with an x with $\lvert x\rvert$ >4)	**(S+)** **M1**	2

	Hence $\frac{\sqrt{3}}{2} < \frac{7}{8} \Rightarrow \sqrt{3} < \frac{7}{4}$ * Simplifies and rearranges, no incorrect working seen. Reason for inequality must have been given.	**A1***	2
		(4)	
(c)	$f(9) = \left(1+\frac{2}{9}\right)^9 - 3^{9/6} \geqslant \frac{19}{3} - \frac{6}{9} - 3\sqrt{3}$ using the result of (a)(ii)	**M1**	3
	$\geqslant \frac{17}{3} - 3\times\frac{7}{4} = 5\frac{2}{3} - 5\frac{1}{4} = \frac{2}{3} - \frac{1}{4} > 0$ using the result of (b)	**A1**	3
	$f(10) = \left(1+\frac{2}{10}\right)^{10} - 3^{\frac{10}{6}} = \left(\frac{6}{5}\right)^{10} - \left(\sqrt[6]{3}\right)^{10}$	**M1**	1
	But $\left(\sqrt[6]{3}\right)^{10} > \left(\frac{6}{5}\right)^{10}$ (as $g(x) = x^{10}$ is an increasing function), so $f(10) < 0$ follows. (Correct reason must be given.)	**A1 (S+)**	3
	So $f(x)$ changes sign on [9,10], and as it is a continuous function, thus there is a root in the interval [9,10] Must have scored both M's.	**A1 (S-)**	2
		(5)	
	S1 mark: Award S1 for a clear and concise solution that scores 10+ marks without the S- or scores 9+ and includes an S+ point but no S-.	**S1**	2
		(1)	
			(12 +1 marks)

Notes:

(a) S+ for noting other terms are non-negative rather than positive, or considers cases $n \geqslant 3$ and $n = 1, 2$ separately.
(b) S+ for reasoning for why terms after the second are all negative.
S+ for considering the domain of convergence to give valid expression.
(c) S+ for mentioning increasing function.
S- if continuity not mentioned.

Q2

Question	Scheme	Marks	Notes
(a)	$(4+y)^{\frac{1}{2}}=2\left(1+\frac{y}{4}\right)^{\frac{1}{2}}$	M1	Correct prep or dealing with 4
	$=[2]\left(1+\frac{y}{8}+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}\left(\frac{y}{4}\right)^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\left(\frac{y}{4}\right)^3\ldots\right)$	M1	Clear use of bin for 3rd or 4th terms. Condone missing 2
	$=2+\frac{y}{4}-\frac{y^2}{64}+\frac{y^3}{512}$	A1	Allow o.e. for coefficients
		(3)	
ALT	$(4+y)^{\frac{1}{2}}=4^{\frac{1}{2}}+\frac{1}{2}4^{-\frac{1}{2}}y+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}4^{-\frac{3}{2}}y^2+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}4^{-\frac{5}{2}}y^3$		1st M1 for the powers of 4
(b)	Let $y=5x+x^2$ so $2+\frac{5x}{4}+\frac{x^2}{4}-\frac{\left(25x^2+10x^3+[x^4]\right)}{64}+\frac{\left(125x^3\ldots\right)}{512}$	M1	Some attempt to sub. for y Ignore higher order terms.
		M1	Clearly attempt x^2
	$=2+\frac{5x}{4}-\frac{9x^2}{64}+\frac{45x^3}{512}$ (*)	A1cso	
		(3)	
ALT	$\left(4+5x+x^2\right)^{\frac{1}{2}}=(4+x)^{\frac{1}{2}}(1+x)^{\frac{1}{2}}=(a)\times\left(1+\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{16}\right)$		For 1st M1
(c)	$\left(x^2+5x+4\right)^{\frac{1}{2}}=(4+x)^{\frac{1}{2}}(1+x)^{\frac{1}{2}}$ | $\left\|5x+x^2\right\|<4$ so $5x+x^2=4$	M1	Attempt factors or a correct equ
	| $x=\frac{-5\pm\sqrt{41}}{2}$	A1	Correct fact. Or cvs
	$-4<x<4$ and $-1<x<1$ | $5x+x^2=-4 \Rightarrow (x+4)(x+1)$	M1	Solves 2nd eqn or 1 correct interval
	| $x=-1$ or -4	A1	A1 All correct
	So $-1<x<1$ | So $-1<x<\frac{\sqrt{41}-5}{2}$ or …	M1	M1 for a suitable combined region
	So series is convergent for $-\frac{1}{2}\leqslant x\leqslant\frac{1}{2}$	A1 (6)	A1 cso

Topic 5: Radians

Bronze, Silver and Gold Worksheets for A Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 20 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel A Level Mathematics: Pure Mathematics Year 2' textbook.

Please note the questions in this topic are calculator questions.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculator

The total mark for this section is 25

Q1

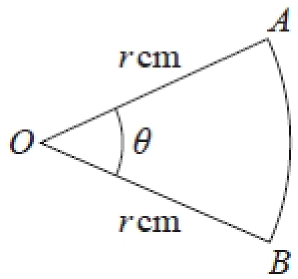


Figure 1

Figure 1 shows a sector AOB of a circle with centre O and radius r cm.

The angle AOB is θ radians.
The area of the sector AOB is 11 cm^2.

Given that the perimeter of the sector is 4 times the length of the arc AB, find the exact value of r.

(4)

(Total for Question 1 is 4 marks)

Q2

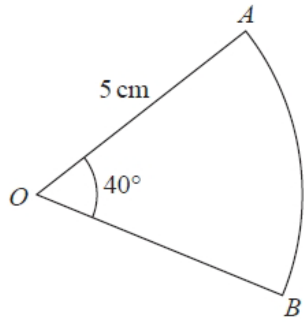


Figure 1

Figure 1 shows a sector *AOB* of a circle with centre *O*, radius 5 cm and angle *AOB* = 40°

The attempt of a student to find the area of the sector is shown below.

$$\text{Area of sector} = \frac{1}{2}r^2\theta$$
$$= \frac{1}{2} \times 5^2 \times 40$$
$$= 500 \text{ cm}^2$$

(a) Explain the error made by this student.

(1)

(b) Write out a correct solution.

(2)

(Total for Question 2 is 3 marks)

Q3

(a) Sketch, for $0 \leq x \leq 2\pi$, the graph of $y = \sin\left(x + \frac{\pi}{6}\right)$.

(2)

(b) Write down the exact coordinates of the points where the graph meets the coordinate axes.

(3)

(c) Solve, for $0 \leq x \leq 2\pi$, the equation

$$\sin\left(x + \frac{\pi}{6}\right) = 0.65,$$

giving your answers in radians to 2 decimal places.

(5)

(Total for Question 3 is 10 marks)

Q4

(i) Solve, for $0 \leq \theta < 180°$, the equation

$$\frac{\sin 2\theta}{(4\sin 2\theta - 1)} = 1$$

giving your answers to 1 decimal place.

(3)

(ii) Solve, for $0 \leq x < 2\pi$, the equation

$$5\sin^2 x - 2\cos x - 5 = 0$$

giving your answers to 2 decimal places.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

(Total for Question 4 is 8 marks)

End of Questions

Bronze Mark Scheme

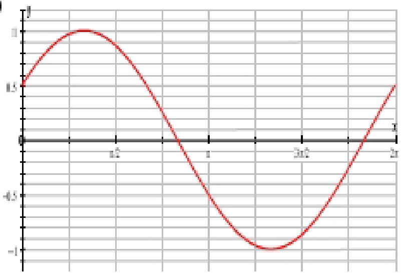
Q1

Question	Scheme	Marks	AOs
	States or uses $\frac{1}{2}r^2\theta = 11$	B1	1.1b
	States or uses $2r + r\theta = 4r\theta$	B1	1.1b
	Attempts to solve, full method $r = ...$	M1	3.1a
	$r = \sqrt{33}$	A1	1.1b
			[4]
			(4 marks)

Q2

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	The formula is only valid when the angle *AOB* is given in radians	B1	This mark is given for a correct explanation
(b)	$\frac{40}{360} \times \pi \times 5^2$	M1	This mark is given for a correct method to find the area of the sector
	$\frac{25\pi}{9}$ cm^2	A1	This mark is given for a correct value for the area of the sector
			(Total 3 marks)

Q3

Question number	Scheme	Marks
	(a)  Sine wave (anywhere) with at least 2 turning points.	M1
	Starting on positive y-axis, going up to a max., then min. below x-axis, no further turning points in range, finishing above x-axis at $x = 2\pi$ or 360°. There must be <u>some</u> indication of scale on the y-axis… (e.g. 1, −1 or 0.5) Ignore parts of graph outside 0 to 2π. n.b. Give credit if necessary for what is seen on an initial sketch (before any transformation has been performed).	A1 (2)
	(b) $\left(0, \frac{1}{2}\right)$, $\left(\frac{5\pi}{6}, 0\right)$, $\left(\frac{11\pi}{6}, 0\right)$ (Ignore any extra solutions) (<u>Not</u> 150°, 330°)	B1, B1, B1 (3)
	$\left(\pi - \frac{\pi}{6}\right)$ and $\left(2\pi - \frac{\pi}{6}\right)$ are insufficient, but if <u>both</u> are seen allow B1 B0.	
	(c) awrt 0.71 radians (0.70758…), or awrt 40.5° (40.5416…) (α)	B1
	$(\pi - \alpha)$ (2.43…) or $(180 - \alpha)$ <u>if α is in degrees</u>. $\left[\underline{\text{NOT}}\ \pi - \left(\alpha - \frac{\pi}{6}\right)\right]$	M1
	Subtract $\frac{\pi}{6}$ from α (or from $(\pi - \alpha)$)… or subtract 30 <u>if α is in degrees</u>	M1
	0.18 (or 0.06π), 1.91 (or 0.61π) Allow awrt (The 1st A mark is dependent on just the 2nd M mark)	A1, A1 (5)
		10
	(b) The zeros are not required, i.e. allow 0.5, etc. (and also allow coordinates the wrong way round). These marks are also awarded if the exact intercept values are seen in part (a), but if values in (b) and (a) are contradictory, (b) takes precedence. (c) B1: If the required value of α is <u>not seen</u>, this mark can be given by implication if a final answer rounding to 0.18 or 0.19 (or a final answer rounding to 1.91 or 1.90) is achieved. (Also see premature approx. note*). <u>Special case:</u> $\sin\left(x + \frac{\pi}{6}\right) = 0.65 \Rightarrow \sin x + \sin\frac{\pi}{6} = 0.65 \Rightarrow \sin x = 0.15$ $x = \arcsin 0.15 = 0.15056\ldots$ and $x = \pi - 0.15056 = 2.99$ (B0 M1 M0 A0 A0) (This special case mark is also available for degrees… 180 – 8.62…) Extra solutions outside 0 to 2π : Ignore. Extra solutions between 0 and 2π : Loses the final A mark. *<u>Premature approximation</u> in part (c): e.g. α = 41°, 180 – 41 = 139, 41 – 30 = 11 and 139 – 30 = 109 Changing to radians: 0.19 and 1.90 This would score B1 (required value of α not seen, but there is a final answer 0.19 (or 1.90)), M1 M1 A0 A0.	

Q4

Question Number	Scheme		Marks
(i)	$\dfrac{\sin 2\theta}{(4\sin 2\theta - 1)} = 1;\ \ 0 \leqslant \theta < 180^{\circ}$		
	$\sin 2\theta = \dfrac{1}{3}$	$\sin 2\theta = k$ where $-1 < k < 1$ **Must be 2θ and not θ.**	M1
	$\{2\theta = \{19.4712..., 160.5288...\}\}$		
	$\theta = \{9.7356..., 80.2644...\}$	A1: Either awrt 9.7 or awrt 80.3 A1: Both awrt 9.7 and awrt 80.3	A1 A1
	Do not penalise poor accuracy more than once e.g. 9.8 and 80.2 from correct work could score M1A1A0		
	If <u>both</u> answers are correct in radians award A1A0 otherwise A0A0 Correct answers are 0.2 and 1.4		
	Extra solutions in range in an otherwise fully correct solution deduct the last A1		
			[3]
(ii)	$5\sin^2 x - 2\cos x - 5 = 0,\ \ 0 \le x < 2\pi.$		
	$5(1-\cos^2 x) - 2\cos x - 5 = 0$	Applies $\sin^2 x = 1 - \cos^2 x$	M1
	$5\cos^2 x + 2\cos x = 0$ $\cos x(5\cos x + 2) = 0$ $\Rightarrow \cos x = ...$	Cancelling out $\cos x$ or a valid attempt at solving the quadratic in cosx **and** giving cosx = … **Dependent on the previous method mark.**	dM1
	awrt 1.98 or awrt 4.3(0)	Degrees: 113.58, 246.42	A1
	Both 1.98 and 4.3(0)	or their α and their $2\pi - \alpha$, where $\alpha \neq \dfrac{\pi}{2}$. If working in degrees allow 360 – their α	A1ft
	awrt 1.57 or $\dfrac{\pi}{2}$ **and** 4.71 or $\dfrac{3\pi}{2}$ **or** 90° and 270°	**These answers only but ignore other answers <u>outside</u> the range**	B1
			[5]
	NB: $x = \text{awrt}\left\{1.98,\ 4.3(0),\ 1.57 \text{ or } \dfrac{\pi}{2},\ 4.71 \text{ or } \dfrac{3\pi}{2}\right\}$		**8**
	Answers in degrees: 113.58, 246.42, 90, 270 Could score M1M1A0A1ftB1 (4/5)		

Silver Questions

20 Marks

Calculator

The total mark for this section is 20

Q1

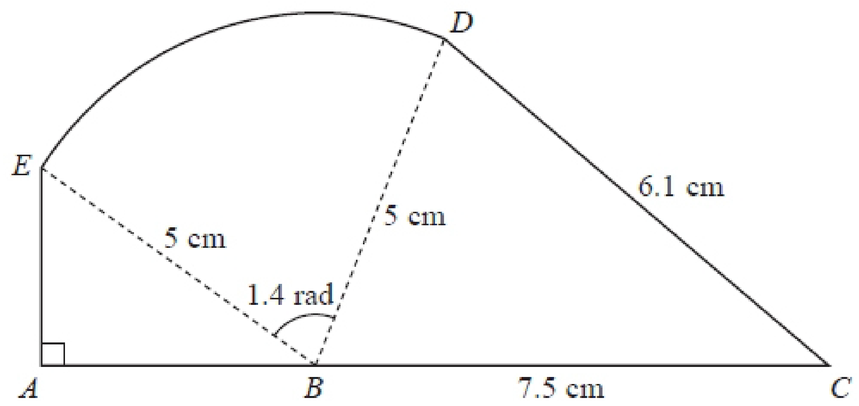


Figure 2

The shape *ABCDEA*, as shown in Figure 2, consists of a right-angled triangle *EAB* and a triangle *DBC* joined to a sector *BDE* of a circle with radius 5 cm and centre *B*.

The points *A*, *B* and *C* lie on a straight line with *BC* = 7.5 cm.

Angle *EAB* = $\frac{\pi}{2}$ radians, angle *EBD* = 1.4 radians and *CD* = 6.1 cm.

(a) Find, in cm^2, the area of the sector *BDE*.

(2)

(b) Find the size of the angle *DBC*, giving your answer in radians to 3 decimal places.

(2)

(c) Find, in cm^2, the area of the shape *ABCDEA*, giving your answer to 3 significant figures.

(5)

(Total for Question 1 is 9 marks)

Q2

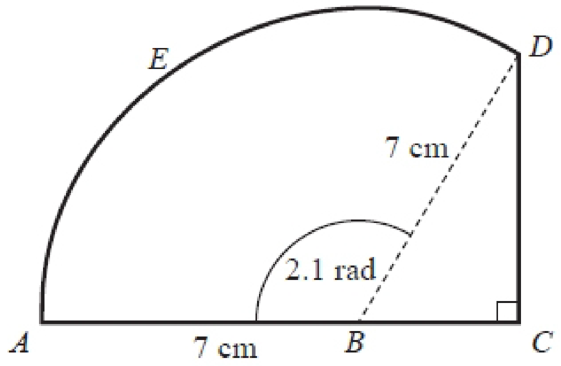


Figure 2

Figure 2 shows the shape *ABCDEA* which consists of a right-angled triangle *BCD* joined to a sector *ABDEA* of a circle with radius 7 cm and centre *B*.

A, B and *C* lie on a straight line with *AB* = 7 cm.

Given that the size of angle *ABD* is exactly 2.1 radians,

(a) find, in cm, the length of the arc *DEA*,

(2)

(b) find, in cm, the perimeter of the shape *ABCDEA*, giving your answer to 1 decimal place.

(4)

(Total for Question 2 is 6 marks)

Q3

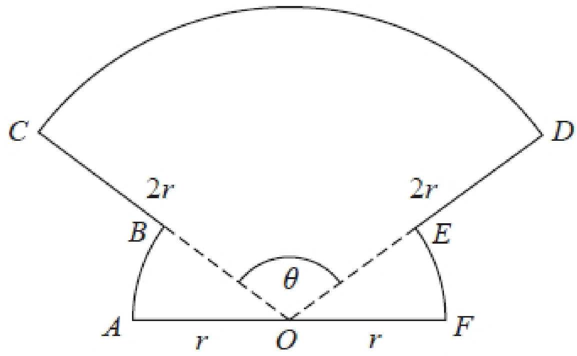


Figure 1

The shape *OABCDEFO* shown in Figure 1 is a design for a logo.

In the design

- *OAB* is a sector of a circle centre *O* and radius *r*
- sector *OFE* is congruent to sector *OAB*
- *ODC* is a sector of a circle centre *O* and radius 2*r*
- *AOF* is a straight line

Given that the size of angle *COD* is θ radians,

(a) write down, in terms of θ, the size of angle *AOB*

(1)

(b) Show that the area of the logo is

$$\frac{1}{2}r^2(3\theta + \pi)$$

(2)

(c) Find the perimeter of the logo, giving your answer in simplest form in terms of r, θ and π.

(2)

(Total for Question 3 is 5 marks)

End of Questions

Silver Mark Scheme

Q1

Question Number	Scheme		Marks
(a)	Area $BDE = \frac{1}{2}(5)^2(1.4)$	M1: Use of the correct formula or method for the area of the sector	M1A1
	$= 17.5\ (\text{cm}^2)$	A1: 17.5 oe	
			[2]
(b)	**Parts (b) and (c) can be marked together**		
	$6.1^2 = 5^2 + 7.5^2 - (2 \times 5 \times 7.5 \cos DBC)$ or $\cos DBC = \frac{5^2 + 7.5^2 - 6.1^2}{2 \times 5 \times 7.5}$ (or equivalent)		M1
	M1: A correct statement involving the angle DBC		
	Angle $DBC = 0.943201...$	awrt 0.943	A1
	Note that work for (b) may be seen on the diagram or in part (c)		
			[2]
(c)	**Note that candidates may work in degrees in (c)** (Angle $DBC = 54.04....$degrees)		
	Area $CBD = \frac{1}{2}5(7.5)\sin(0.943)$		
	Angle $EBA = \pi - 1.4 - "0.943"$ **(Maybe seen on the diagram)**	Area $CBD = \frac{1}{2}5(7.5)\sin(\text{their } 0.943)$ or awrt 15.2. (Note area of $CBD = 15.177...$) A correct method for the area of triangle CBD which can be implied by awrt 15.2	M1
	$\pi - 1.4 - "\text{their } 0.943"$ A value for angle EBA of awrt 0.8 (from 0.7985926536... or 0.7983916536...) or value for angle EBA of $(1.74159... - \text{their angle } DBC)$ would imply this mark.		M1
	$AB = 5\cos(\pi - 1.4 - "0.943")$ or $AE = 5\sin(\pi - 1.4 - "0.943")$		
		$AB = 5\cos(\pi - 1.4 - \text{their } 0.943)$ $AB = 5\cos(0.79859...) = 3.488577938...$ Allow M1 for $AB =$ awrt 3.49 Or $AE = 5\sin(\pi - 1.4 - \text{their } 0.943)$ $AE = 5\sin(0.79859...) = 3.581874365688...$ Allow M1 for $AE =$ awrt 3.58 **It must be clear that $\pi - 1.4 - "0.943"$ is being used for angle EBA.** **Note that some candidates use the sin rule here but it must be used correctly – do not allow mixing of degrees and radians.**	M1
	Area $EAB = \frac{1}{2}5\cos(\pi - 1.4 - "0.943") \times 5\sin(\pi - 1.4 - "0.943")$		
	This is dependent on the previous M1 and there must be no other errors in finding the area of triangle EAB		dM1
	Allow M1 for area $EAB =$ awrt 6.2		
	Area $ABCDE = 15.17... + 17.5 + 6.24... = 38.92...$		
		awrt 38.9	A1cso
			[5]
	Note that a sign error in (b) can give the obtuse angle (2.198....) and could lead to the correct answer in (c) – this would lose the final mark in (c)		Total 9

Q2

Question Number	Scheme		Marks
(a)	Length $DEA = 7(2.1) = 14.7$	M1: 7×2.1 only A1: 14.7	M1A1
			[2]
(b)	Angle $CBD = \pi - 2.1$	**May be seen on the diagram** (allow awrt 1.0 and allow 180 – 120). Could score for sight of **Angle CBD = awrt 60 degrees.**	M1
	Both $7\cos(\pi - 2.1)$ **and** $7\sin(\pi - 2.1)$ or **Both** $7\cos(\pi - 2.1)$ **and** $\sqrt{7^2 - (7\cos(\pi - 2.1))^2}$ or **Both** $7\sin(\pi - 2.1)$ **and** $\sqrt{7^2 - (7\sin(\pi - 2.1))^2}$ **Or equivalents to these**	A correct attempt to find BC and BD. You can ignore how the candidate assigns BC and CD. $7\cos(\pi - 2.1)$ can be implied by awrt 3.5 and $7\sin(\pi - 2.1)$ can be implied by awrt 6. Note if the sin rule is used, do not allow mixing of degrees and radians unless their answer implies a correct interpretation. **Dependent on the previous method mark.**	dM1
	Note that 2.1 radians is 120 degrees (to 3sf) which if used gives angle CBD as 60 degrees. If used this gives a correct perimeter of 31.3 and could score full marks.		
	$P = 7\cos(\pi - 2.1) + 7\sin(\pi - 2.1) + 7 + 14.7$	their BC + their CD + 7 + their DEA **Dependent on <u>both</u> previous method marks**	ddM1
	$= 31.2764...$	Awrt 31.3	A1
			[4]
			Total 6

Q3

Question	Scheme	Marks	AOs
(a)	Angle $AOB = \dfrac{\pi - \theta}{2}$	B1	2.2a
		(1)	
(b)	Area $= 2 \times \dfrac{1}{2} r^2 \left(\dfrac{\pi - \theta}{2}\right) + \dfrac{1}{2}(2r)^2 \theta$	M1	2.1
	$= \dfrac{1}{2} r^2 \pi - \dfrac{1}{2} r^2 \theta + 2r^2 \theta = \dfrac{3}{2} r^2 \theta + \dfrac{1}{2} r^2 \pi = \dfrac{1}{2} r^2 (3\theta + \pi)$ *	A1*	1.1b
		(2)	
(c)	Perimeter $= 4r + 2r\left(\dfrac{\pi - \theta}{2}\right) + 2r\theta$	M1	3.1a
	$= 4r + r\pi + r\theta$ or e.g. $r(4 + \pi + \theta)$	A1	1.1b
		(2)	
			(5 marks)

Gold Questions

Calculator

The total mark for this section is 30

Q1

A circle C has centre M (6, 4) and radius 3.

(a) Write down the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = r^2.$$

(2)

Figure 3

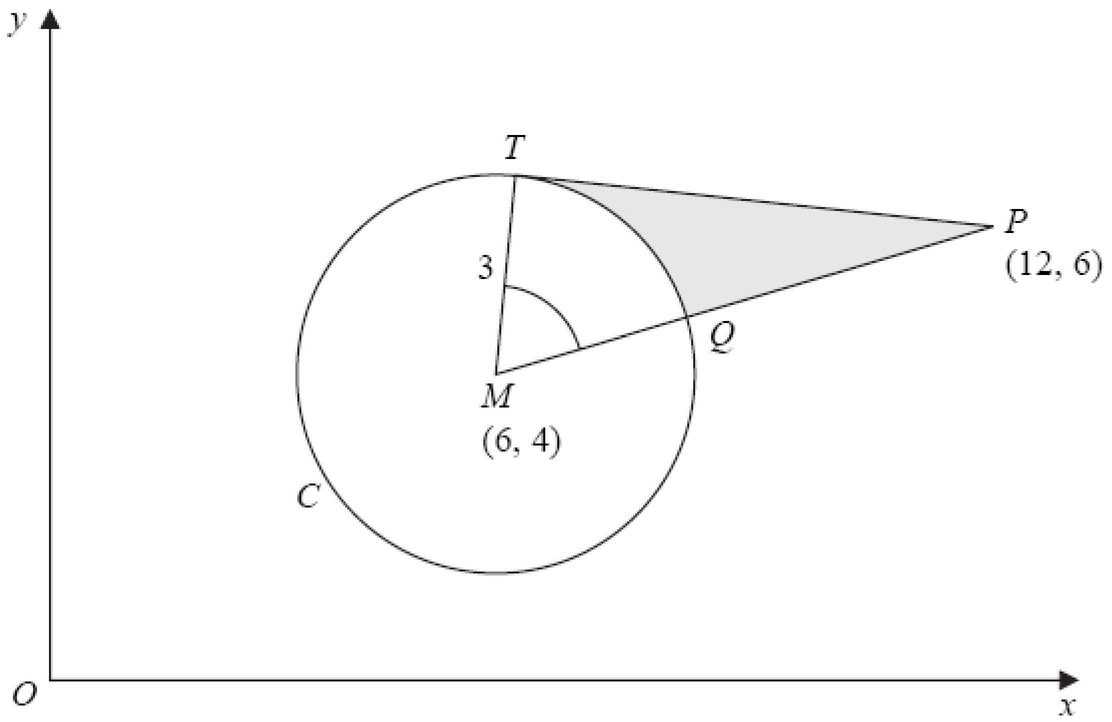


Figure 3 shows the circle C. The point T lies on the circle and the tangent at T passes through the point P (12, 6). The line MP cuts the circle at Q.

(b) Show that the angle TMQ is 1.0766 radians to 4 decimal places.

(4)

The shaded region TPQ is bounded by the straight lines TP, QP and the arc TQ, as shown in Figure 3.

(c) Find the area of the shaded region TPQ. Give your answer to 3 decimal places.

(5)

(Total for Question 1 is 11 marks)

Q2

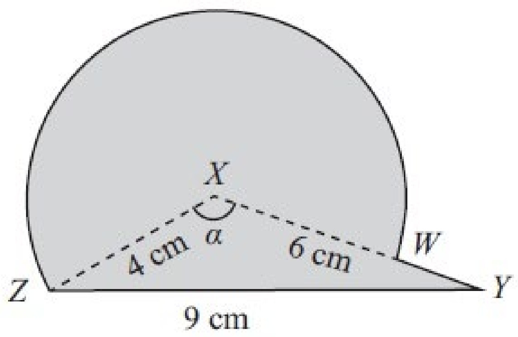


Figure 1

The triangle XYZ in Figure 1 has $XY = 6$ cm, $YZ = 9$ cm, $ZX = 4$ cm and angle $ZXY = a$. The point W lies on the line XY.

The circular arc ZW, in Figure 1 is a major arc of the circle with centre X and radius 4 cm.

(a) Show that, to 3 significant figures, $a = 2.22$ radians.

(2)

(b) Find the area, in cm^2, of the major sector $XZWX$.

(3)

The region enclosed by the major arc ZW of the circle and the lines WY and YZ is shown shaded in Figure 1.

Calculate

(c) the area of this shaded region,

(3)

(d) the perimeter $ZWYX$ of this shaded region.

(4)

(Total for Question 2 is 12 marks)

Q3

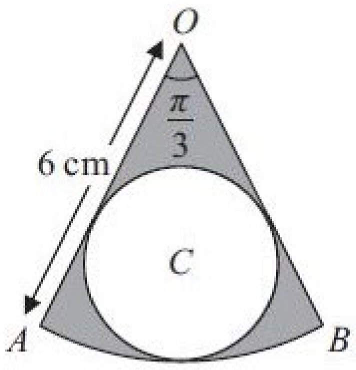


Figure 1

The shape shown in Figure 1 is a pattern for a pendant. It consists of a sector *OAB* of a circle centre *O*, of radius 6 cm, and angle $AOB = \frac{\pi}{3}$. The circle *C*, inside the sector, touches the two straight edges, *OA* and *OB*, and the arc *AB* as shown.

Find

(a) the area of the sector *OAB*,

(2)

(b) the radius of the circle *C*.

(3)

The region outside the circle *C* and inside the sector *OAB* is shown shaded in Figure 1.

(c) Find the area of the shaded region.

(2)

(Total for Question 3 is 7 marks)

End of Questions

Gold Mark Scheme

Q1

Question Number	Scheme	Marks
(a)	$(x-6)^2 + (y-4)^2 =$; 3^2	B1; B1 (2)
(b)	Complete method for MP: $= \sqrt{(12-6)^2 + (6-4)^2}$	M1
	$= \sqrt{40}$ or awrt 6.325	A1
	[These first two marks can be scored if seen as part of solution for (c)]	
	Complete method for $\cos\theta$, $\sin\theta$ or $\tan\theta$	M1
	e.g. $\cos\theta = \frac{MT}{MP} = \frac{3}{candidate's\sqrt{40}}$ (= 0.4743) (θ = 61.6835°)	
	[If TP = 6 is used, then M0]	
	$\theta = 1.0766$ rad **AG**	A1 (4)
(c)	Complete method for area TMP; e.g. $= \frac{1}{2} \times 3 \times \sqrt{40} \sin\theta$	M1
	$= \frac{3}{2}\sqrt{31}$ (= 8.3516..) allow awrt 8.35	A1
	Area (sector) $MTQ = 0.5 \times 3^2 \times 1.0766$ (= 4.8446…)	M1
	Area TPQ = candidate's (8.3516.. − 4.8446..)	M1
	= 3.507 awrt [Note: 3.51 is A0]	A1 (5) **[11]**

Q2

Question Number	Scheme		Marks
(a)	$9^2 = 4^2 + 6^2 - 2 \times 4 \times 6\cos\alpha \Rightarrow \cos\alpha = ...$	Correct use of cosine rule leading to a value for cos α	M1
	$\cos\alpha = \frac{4^2+6^2-9^2}{2\times4\times6}\left(= -\frac{29}{48} = -0.604..\right)$		
	$\alpha = 2.22$ *	Cso (2.22 must be seen here)	A1
	(NB $\alpha = 2.219516005$)		**(2)**
(a) Way 2	$XY^2 = 4^2 + 6^2 - 2\times4\times6\cos 2.22 \Rightarrow XY^2 = ..$	Correct use of cosine rule leading to a value for XY^2	M1
	$XY^2 = 81.01....$		
	$XY = 9.00....$		A1
			(2)
(b)	$2\pi - 2.22(= 4.06366......)$	$2\pi - 2.22$ or awrt 4.06	B1
	$\frac{1}{2}\times4^2\times"4.06"$	Correct method for major sector area.	M1
	32.5	Awrt 32.5	A1
			(3)
(b) Way2	**Circle – Minor sector**		
	$\pi\times4^2$	Correct expression for circle area	B1
	$\pi\times4^2 - \frac{1}{2}\times4^2\times2.22 = 32.5$	Correct method for circle - minor sector area	M1
	$= 32.5$	Awrt 32.5	A1
			(3)
(c)	Area of triangle = $\frac{1}{2}\times4\times6\times\sin 2.22(= 9.56)$	Correct expression for the area of triangle XYZ	B1
	So area required = "9.56" + "32.5"	Their Triangle XYZ + (part (b) answer or correct attempt at major sector)	M1
	= 42.1 cm^2 or 42.0 cm^2	Awrt 42.1 or 42.0 (Or just 42)	A1
			(3)
(d)	$\text{Arc length} = 4\times4.06(= 16.24)$ Or $8\pi - 4\times2.22$	M1: $4\times \textit{their}(2\pi - 2.22)$ Or circumference – minor arc A1: Correct ft expression	M1A1ft
	Perimeter = $ZY + WY +$ Arc Length	9 + 2 + Any Arc	M1
	Perimeter = 27.2 or 27.3	Awrt 27.2 or awrt 27.3	A1
			(4)
			[12]

Q3

Question Number	Scheme		Marks
(a)	$\frac{1}{2}r^2\theta = \frac{1}{2}(6)^2\left(\frac{\pi}{3}\right) = 6\pi$ or 18.85 or awrt 18.8 $(\text{cm})^2$	Using $\frac{1}{2}r^2\theta$ (See notes)	M1
		6π or 18.85 or awrt 18.8	A1
			[2]
(b)	$\sin\left(\frac{\pi}{6}\right) = \frac{r}{6-r}$	$\sin\left(\frac{\pi}{6}\right)$ or $\sin 30^\circ = \frac{r}{6-r}$	M1
	$\frac{1}{2} = \frac{r}{6-r}$	Replaces sin by numeric value	dM1
	$6 - r = 2r \Rightarrow r = 2$	$r = 2$	A1 cso
			[3]
(c)	Area $= 6\pi - \pi(2)^2 = 2\pi$ or awrt 6.3 $(\text{cm})^2$	their area of sector $- \pi r^2$	M1
		2π or awrt 6.3	A1 cao
			[2]
			7

(a)	M1: Needs θ in radians for this formula. Candidate could convert to degrees and use the degrees formula. A1: Does not need units. Answer should be either 6π or 18.85 or awrt 18.8 Correct answer with no working is M1A1. This M1A1 can only be awarded in part (a).
(b)	M1: Also allow $\cos\left(\frac{\pi}{3}\right)$ or $\cos 60^\circ = \frac{r}{6-r}$. 1st M1: Needs correct trigonometry method. Candidates could state $\sin\left(\frac{\pi}{6}\right) = \frac{r}{x}$ and $x + r = 6$ or equivalent in their working to gain this method mark. dM1: Replaces sin by numerical value. $0.009... = \frac{r}{6-r}$ from working "incorrectly" in degrees is fine here for dM1. A1: For $r = 2$ from correct solution only. **Alternative:** 1st M1 for $\frac{r}{OC} = \sin 30$ or $\frac{r}{OC} = \cos 60$. 2nd M1 for $OC = 2r$ and then A1 for $r = 2$. **Note** seeing $OC = 2r$ is M1M1. **Special Case:** If a candidate states an answer of $r = 2$ (must be in part (b)) as a guess or from an incorrect method then award SC: M0M0B1. Such a candidate could then go on to score M1A1 in part (c).
(c)	M1: For "their area of sector – their area of circle", where $r > 0$ is ft from their answer to part (b). Allow the method mark if "their area of sector" < "their area of circle". The candidate must show somewhere in their working that they are subtracting the correct way round, even if their answer is negative. Some candidates in part (c) will either use their value of r from part (b) or even introduce a value of r in part (c). You can apply the scheme to award either M0A0 or M1A0 or M1A1 to these candidates. **Note:** Candidates can get M1 by writing "their part (a) answer $- \pi r^2$", where the radius of the circle is not substituted. A1: cao – accept exact answer or awrt 6.3 Correct answer only with no working in (c) gets M1A1 **Beware: The answer in (c) is the same as the arc length of the pendant**

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Topic 6: Trigonometric Functions

Bronze, Silver, Gold and Platinum Worksheets for A Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel A Level Mathematics: Pure Mathematics Year 2' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high-level problem-solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

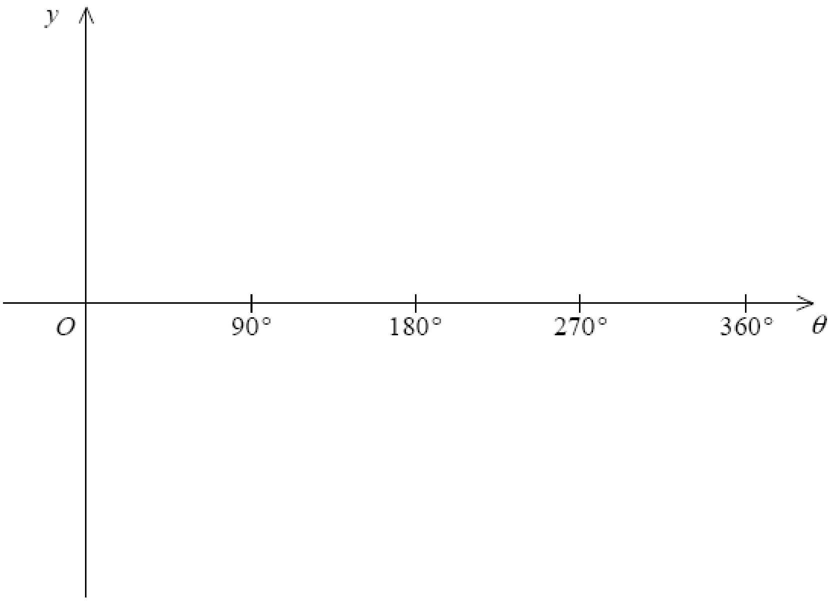
Bronze Questions

Non-calculator

The total mark for this section is 27

Q1.

On the axes below, sketch the graph of $y = 2 \text{ cosec } 2\theta$ for $0° < \theta < 360°$.



(2)

(Total for Question 1 is 2 marks)

Q2

Solve

$$\sec^2 \theta = 4$$

giving your answers in terms of π.

(3)

(Total for Question 2 is 3 marks)

Q3

(i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta.$$

(4)

(ii) Hence, or otherwise, for $0 < \theta < 60^{\circ}$, solve

$$8 \sin^3 \theta - 6 \sin \theta + 1 = 0.$$

(5)

(Total for Question 3 is 9 marks)

Q4

Given that

$$\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} \equiv 2\cot 2\theta \qquad \theta \neq (90n)^{\circ},\ n \in \mathbb{Z}$$

Solve, for $90^{\circ} < \theta < 180^{\circ}$, the equation

$$\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} = 2\,.$$

(3)

(Total for Question 4 is 3 marks)

Q5

Solve, for $0 \leq \theta \leq 180^{\circ}$,

$$2\cot^2 3\theta = 3 \operatorname{cosec} 3\theta$$

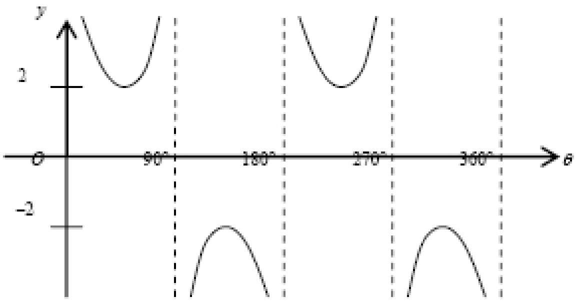
(10)

(Total for Question 5 is 10 marks)

End of Questions

Bronze Mark Scheme

Q1

	Shape (May be translated but need to see 4"sections") T.P.s at $y = \pm 2$, asymptotic at correct x-values (dotted lines not required)	B1 B1 dep. (2)

Q2

	(c)	$\sec^2\theta = 4 \Rightarrow \sec\theta = \pm 2 \Rightarrow \cos\theta = \pm\frac{1}{2}$ $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$	M1 A1,A1 (3) **(9 marks)**

c) M1 For $\sec^2\theta = 4$ leading to a solution of $\cos\theta$ by taking the root and inverting in either order .
Similarly accept $\tan^2\theta = 3, \sin^2\theta = \frac{3}{4}$ leading to solutions of $\tan\theta, \sin\theta$. Also accept $\cos 2\theta = -\frac{1}{2}$

A1 Obtains one correct answer usually $\theta = \frac{\pi}{3}$ Do not accept decimal answers or degrees

A1 Obtains both correct answers. $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$ Do not award if there are extra solutions inside the range.
Ignore solutions outside the range.

Q3

Question Number	Scheme		Marks	
(i)	$\sin 3\theta = \sin(2\theta + \theta)$			
	$= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$			
	$= 2\sin\theta\cos\theta.\cos\theta + (1 - 2\sin^2\theta)\sin\theta$		M1 A1	
	$= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$		M1	
	$= 3\sin\theta - 4\sin^3\theta$ ∗	cso	A1	(4)
(ii)	$8\sin^3\theta - 6\sin\theta + 1 = 0$			
	$-2\sin 3\theta + 1 = 0$		M1 A1	
	$\sin 3\theta = \frac{1}{2}$		M1	
	$3\theta = 30^\circ, 150^\circ$			
	$\theta = 10^\circ, 50^\circ$		A1 A1	(5)

Q4

Working or answer an examiner might expect to see	**Mark**	**Notes**
$\tan 2\theta = 1$	M1	This mark is given for deducing the value of $\tan 2\theta$
$180^\circ + 45^\circ$	M1	This mark is given for finding the solution in the third quadrant for arctan1
$\theta = 112.5^\circ$	A1	This mark is given for finding a correct value for θ

Q5

Question No	Scheme	Marks
	Uses the identity $cot^2(3\theta) = cosec^2(3\theta) - 1$ *in*	M1
	$2cot^2 3\theta = 3cosec\theta$	
	$2cosec^2 3\theta - 3cosec3\theta - 2 = 0$	A1
	$(2cosec3\theta + 1)(cosec3\theta + 2) = 0$	dM1
	$cosec3\theta = 2$	A1
	$\theta = \frac{invsin\left(\frac{1}{2}\right)}{3}, \frac{30°}{3} = 10°$	ddM1, A1
	$\theta = \frac{180° - invsin\left(\frac{1}{2}\right)}{3} = 50°$	ddM1,A1
	$\theta = \frac{360° + invsin(\frac{1}{2})}{3} = 130°$	ddM1
	All 4 correct values $10°, 50°, 130°, 170°$	A1 (10 marks)

Silver Questions

Non-calculator

The total mark for this section is 27

Q1

(a) Use the identity $\cos^2\theta + \sin^2\theta = 1$ to prove that $\tan^2\theta = \sec^2\theta - 1$.

(2)

(b) Solve, for $0 \le \theta < 360°$, the equation

$$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2$$

(6)

(Total for Question 1 is 8 marks)

Q2

Solve

$$\operatorname{cosec}^2 2x - \cot 2x = 1$$

for $0 \le x \le 180^\circ$.

(7)

(Total for Question 2 is 7 marks)

Q3

Solve, for $0 \le \theta < 2\pi$, the equation

$$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta$$

You must show all of your working. Give your answers in terms of π.

(6)

(Total for Question 2 is 6 marks)

Q4

Given that

$$2\cot 2x + \tan x \equiv \cot x \qquad x \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

Hence, solve, for $-\pi \leq x < \pi$,

$$4\cot 2x + 2\tan x = \operatorname{cosec}^2 x$$

Give your answers in exact form.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

(Total for Question 4 is 6 marks)

End of Questions

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Silver Mark Scheme

Q1

Question Number	Scheme		Marks
Q (a)	$\cos^2\theta + \sin^2\theta = 1 \quad (\div \cos^2\theta)$		
	$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$	Dividing $\cos^2\theta + \sin^2\theta = 1$ by $\cos^2\theta$ to give underlined equation.	M1
	$1 + \tan^2\theta = \sec^2\theta$		
	$\tan^2\theta = \sec^2\theta - 1$ (as required) **AG**	Complete proof. No errors seen.	A1 cso
			(2)
(b)	$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2$, (eqn *) $\quad 0 \le \theta < 360^\circ$		
	$2(\sec^2\theta - 1) + 4\sec\theta + \sec^2\theta = 2$	Substituting $\tan^2\theta = \sec^2\theta - 1$ into eqn * to get a quadratic in $\sec\theta$ only	M1
	$2\sec^2\theta - 2 + 4\sec\theta + \sec^2\theta = 2$		
	$3\sec^2\theta + 4\sec\theta - 4 = 0$	Forming a three term "one sided" quadratic expression in $\sec\theta$.	M1
	$(\sec\theta + 2)(3\sec\theta - 2) = 0$	Attempt to factorise or solve a quadratic.	M1
	$\sec\theta = -2$ or $\sec\theta = \frac{2}{3}$		
	$\frac{1}{\cos\theta} = -2$ or $\frac{1}{\cos\theta} = \frac{2}{3}$		
	$\cos\theta = -\frac{1}{2}$; or $\cos\theta = \frac{3}{2}$	$\cos\theta = -\frac{1}{2}$	A1;
	$\alpha = 120^\circ$ or $\alpha =$ no solutions		
	$\theta_1 = 120^\circ$	120°	A1
	$\theta_2 = 240^\circ$	240° or $\theta_2 = 360^\circ - \theta_1$ when solving using $\cos\theta = \ldots$	B1 $\sqrt{}$
	$\theta = \{120^\circ, 240^\circ\}$	Note the final A1 mark has been changed to a B1 mark.	(6)
			[8]

Q2

Question Number	Scheme		Marks
	$\text{cosec}^2 2x - \cot 2x = 1, \quad (\text{eqn } *) \quad 0 \le x \le 180°$		
	Using $\text{cosec}^2 2x = 1 + \cot^2 2x$ gives $1 + \cot^2 2x - \cot 2x = 1$	Writing down or using $\text{cosec}^2 2x = \pm 1 \pm \cot^2 2x$ or $\text{cosec}^2\theta = \pm 1 \pm \cot^2\theta$.	M1
	$\underline{\cot^2 2x - \cot 2x = 0}$ or $\cot^2 2x = \cot 2x$	For either $\underline{\cot^2 2x - \cot 2x}\ \{= 0\}$ or $\cot^2 2x = \cot 2x$	A1
	$\cot 2x(\cot 2x - 1) = 0$ or $\cot 2x = 1$	Attempt to factorise or solve a quadratic (See rules for factorising quadratics) or cancelling out $\cot 2x$ from both sides.	dM1
	$\cot 2x = 0$ or $\cot 2x = 1$	Both $\cot 2x = 0$ and $\cot 2x = 1$.	A1
	$\cot 2x = 0 \Rightarrow (\tan 2x \to \infty) \Rightarrow 2x = 90, 270$ $\Rightarrow x = 45, 135$ $\cot 2x = 1 \Rightarrow \tan 2x = 1 \Rightarrow 2x = 45, 225$ $\Rightarrow x = 22.5, 112.5$	Candidate attempts to divide at least one of their principal angles by 2. This will be usually implied by seeing $x = 22.5$ resulting from $\cot 2x = 1$.	ddM1
	Overall, $x = \{22.5, 45, 112.5, 135\}$	***Both*** $x = 22.5$ and $x = 112.5$ ***Both*** $x = 45$ and $x = 135$	A1 B1
			[7]

If there are any EXTRA solutions inside the range $0 \le x \le 180°$ and the candidate would otherwise score FULL MARKS then withhold the final accuracy mark (the sixth mark in this question).
Also ignore EXTRA solutions outside the range $0 \le x \le 180°$.

Q3

Question Number	Scheme	Marks
(ii)	$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta \Rightarrow 3\sec^2\theta + 3\sec\theta = 2(\sec^2\theta - 1)$	M1
	$\sec^2\theta + 3\sec\theta + 2 = 0$	
	$(\sec\theta + 2)(\sec\theta + 1) = 0$	M1
	$\sec\theta = -2, -1$	A1
	$\cos\theta = -0.5, -1$	M1
	$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \pi$	A1A1
		(6)
		(9 marks)
ALT (ii)	$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta \Rightarrow 3\times\frac{1}{\cos^2\theta} + 3\times\frac{1}{\cos\theta} = 2\times\frac{\sin^2\theta}{\cos^2\theta}$	
	$3 + 3\cos\theta = 2\sin^2\theta$	
	$3 + 3\cos\theta = 2(1 - \cos^2\theta)$	M1
	$2\cos^2\theta + 3\cos\theta + 1 = 0$	
	$(2\cos\theta + 1)(\cos\theta + 1) = 0 \Rightarrow \cos\theta = -0.5, -1$	M1A1
	$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \pi$	M1,A1,A1
		(6)

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Notes for Question

(ii)

M1 Replaces $\tan^2\theta$ by $\pm\sec^2\theta \pm 1$ to produce an equation in just $\sec\theta$

M1 Award for a forming a 3TQ=0 in $\sec\theta$ and applying a correct method for factorising, or using the formula, or completing the square to find two answers to $\sec\theta$

If they replace $\sec\theta = \frac{1}{\cos\theta}$ it is for forming a 3TQ in $\cos\theta$ and applying a correct method for finding two answers to $\cos\theta$

A1 Correct answers to $\sec\theta = -2, -1$ or $\cos\theta = -\frac{1}{2}, -1$

M1 Award for using the identity $\sec\theta = \frac{1}{\cos\theta}$ and proceeding to find at least one value for θ.

If the 3TQ was in cosine then it is for finding at least one value of θ.

A1 Two correct values of θ. All method marks must have been scored.

Accept two of $120°, 180°, 240°$ or two of $\frac{2\pi}{3}, \frac{4\pi}{3}, \pi$ or two of awrt 2dp 2.09, 3.14, 4.19

A1 All three answers correct. They must be given in terms of π as stated in the question.

Accept $0.\dot{6}\pi, 1.\dot{3}\pi, \pi$

Withhold this mark if further values in the range are given. All method marks must have been scored.

Ignore any answers outside the range.

Alt (ii)

M1 Award for replacing $\sec^2\theta$ with $\frac{1}{\cos^2\theta}$, $\sec\theta$ with $\frac{1}{\cos\theta}$, $\tan^2\theta$ with $\frac{\sin^2\theta}{\cos^2\theta}$ multiplying through by $\cos^2\theta$ (seen in at least 2 terms) and replacing $\sin^2\theta$ with $\pm 1 \pm \cos^2\theta$ to produce an equation in just $\cos\theta$

M1 Award for a forming a 3TQ=0 in $\cos\theta$ and applying a correct method for factorising, or using the formula, or completing the square to find two answers to $\cos\theta$

A1 $\cos\theta = -\frac{1}{2}, -1$

M1 Proceeding to finding at least one value of θ from an equation in $\cos\theta$.

A1 Two correct values of θ. All method marks must have been scored

Accept two of $120°, 180°, 240°$ or two of $\frac{2\pi}{3}, \frac{4\pi}{3}, \pi$ or two of awrt 2dp 2.09, 3.14, 4.19

A1 All three answers correct. They must be given in terms of π as stated in the question.

Accept $0.\dot{6}\pi, 1.\dot{3}\pi, \pi$

All method marks must have been scored. Withhold this mark if further values in the range are given.

Ignore any answers outside the range

Q4

Question Number	Scheme	Marks
	$4\cot 2x + 2\tan x = cosec^2 x \Rightarrow 2\cot x = cosec^2 x$	
	$\Rightarrow 2\cot x = 1 + cot^2 x$	M1
	$\Rightarrow 0 = cot^2 x - 2\cot x + 1$	A1
	$\Rightarrow 0 = (\cot x - 1)^2$	M1
	$\Rightarrow \cot x = 1$	
	$\Rightarrow \tan x = 1$	M1
	$\Rightarrow x = -\frac{3\pi}{4}, \frac{\pi}{4}$	A2

Gold Questions

Non-calculator

The total mark for this section is 29

Q1

Given that

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2\operatorname{cosec}2\theta, \qquad \theta \neq 90n°.$$

Solve, for $0° < \theta < 360°$, the equation

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 1$$

giving your answers in exact form.

(6)

(Total for Question 1 is 6 marks)

Q2

Given that $\sin 2x = 2\sin x\cos x$, find, for $0 < x < \pi$, all the solutions of the equation

$$\operatorname{cosec} x - 4\cos x = 0$$

(5)

(Total for Question 2 is 5 marks)

Q3

Given that

$$\operatorname{cosec} 2x + \cot 2x = \cot x, \quad x \neq 90n°, \quad n \in \mathbb{R}.$$

Hence, or otherwise, solve, for $0 \leq \theta < 180°$,

$$\operatorname{cosec}(4\theta + 10°) + \cot(4\theta + 10°) = \sqrt{3}$$

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

(Total for Question 3 is 5 marks)

Q4

In this question you must show all stages of your working.

(a) Show that

$$\operatorname{cosec}\theta - \sin\theta \equiv \cos\theta\cot\theta \qquad \theta \neq (180n)° \qquad n \in \mathbb{Z}$$

(3)

(b) Hence, or otherwise, solve for $0 < x < 180°$

$$\operatorname{cosec} x - \sin x = \cos x \cot(3x - 50°)$$

(5)

(Total for Question 4 is 8 marks)

Q5

(a) For $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, sketch the graph of $y = \mathrm{g}(x)$ where

$$\mathrm{g}(x) = \arcsin x \qquad -1 \leq x \leq 1$$

(2)

(b) Find the exact value of x for which

$$3\mathrm{g}(x + 1) + \pi = 0$$

(3)

(Total for Question 5 is 5 marks)

End of Questions

Gold Mark Scheme

Q1

Question Number	Scheme	Marks
	$2cosec\ 2\theta = 1$	M1
	$\sin 2\theta = \frac{1}{2}$	A1
	$2\theta = 30°, 150°, 390°, 510°$	M2
	$\theta = 15°, 75°, 195°, 255°$	A2

Q2

Question Number	Scheme		Marks
(b)	$cosec\ x - 4\cos x = 0, \quad 0 < x < \pi$		
	$\frac{1}{\sin x} - 4\cos x = 0$	Using $\mathrm{cosec}\, x = \frac{1}{\sin x}$	M1
	$\frac{1}{\sin x} = 4\cos x$		
	$1 = 4\sin x\cos x$		
	$1 = 2(2\sin x\cos x)$		
	$1 = 2\sin 2x$		
	$\sin 2x = \frac{1}{2}$	$\sin 2x = k$, where $-1 < k < 1$ and $k \neq 0$	M1
		$\sin 2x = \frac{1}{2}$	A1
	Radians $2x = \frac{\pi}{6}, \frac{5\pi}{6}$		
	Degrees $2x = 30°, 150°$		
	Radians $x = \frac{\pi}{12}, \frac{5\pi}{12}$		A1
	Degrees $x = 15°, 75°$		A1 cao
		Solutions for the final two A marks must be given in x only. If there are any EXTRA solutions inside the range $0 < x < \pi$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 < x < \pi$.	(5)
			[6]

Q3

(b)	$\text{cosec}(4\theta+10°)+\cot(4\theta+10°)=\sqrt{3}$		
	$\cot(2\theta \pm ..°)=\sqrt{3}$	M1	
	$2\theta \pm ...=30° \Rightarrow \theta=12.5°$	dM1, A1	
	$2\theta \pm ...=180+PV° \Rightarrow \theta=..°$	dM1	
	$\theta=102.5°$	A1	
			(5)

M1 Attempt to use the solution to part (a) with $2x=4\theta+10 \Rightarrow$ to write or imply $\cot(2\theta \pm ...°)=\sqrt{3}$

Watch for attempts which start $\cot\alpha=\sqrt{3}$. The method mark here is not scored until the α has been replaced by $2\theta \pm ...°$

Accept a solution from $\cot(2x \pm ...°)=\sqrt{3}$ where θ has been replaced by another variable.

dM1 Proceeds from the previous method and uses $\tan..=\frac{1}{\cot..}$ and $\arctan\left(\frac{1}{\sqrt{3}}\right)=30°$ to solve $2\theta \pm ...°=30° \Rightarrow \theta=..$

A1 $\theta=12.5°$ or exact equivalent. Condone answers such as $x=12.5°$

dM1 This mark is for the correct method to find a second solution to θ. It is dependent upon the first M only.
Accept $2\theta \pm ...=180+PV° \Rightarrow \theta=..°$

A1 $\theta=102.5°$ or exact equivalent. Condone answers such as $x=102.5°$
Ignore any solutions outside the range. This mark is withheld for any extra solutions within the range.
If radians appear they could just lose the answer marks. So for example

$2\theta \pm ...=\frac{\pi}{6}(0.524) \Rightarrow \theta=..$is M1dM1A0 followed by

$2\theta \pm ...=\pi+'\frac{\pi}{6}' \Rightarrow \theta=..$ dM1A0

Special case 1: For candidates in (b) who solve $\cot(4\theta \pm ...°)=\sqrt{3}$ the mark scheme is severe, so we are awarding a special case solution, scoring 00011.

$\cot(4\theta+\beta°)=\sqrt{3} \Rightarrow 4\theta+\beta=30° \Rightarrow \theta=..$ is M0M0A0 where $\beta=5°$ or $10°$

$\Rightarrow 4\theta+\beta=210° \Rightarrow \theta=..$ can score M1A1 Special case.

If $\beta=5°, \theta=51.25$ If $\beta=10°, \theta=50$

Special case 2: Just answers in (b) **with no working** scores 1 1 0 0 0 for 12.5 and 102.5

BUT $\cot(2\theta \pm 5°)=\sqrt{3} \Rightarrow \theta=12.5°, 102.5°$ scores all available marks.

Q4

Question	Scheme	Marks	AOs
(a)	States or uses $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$	B1	1.2
	$\operatorname{cosec}\theta - \sin\theta = \dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1-\sin^2\theta}{\sin\theta}$	M1	2.1
	$= \dfrac{\cos^2\theta}{\sin\theta} = \cos\theta \times \dfrac{\cos\theta}{\sin\theta} = \cos\theta\cot\theta$ *	A1*	2.1
		(3)	
(b)	$\operatorname{cosec} x - \sin x = \cos x \cot(3x - 50°)$ $\Rightarrow \cos x \cot x = \cos x \cot(3x - 50°)$		
	$\cot x = \cot(3x - 50°) \Rightarrow x = 3x - 50°$	M1	3.1a
	$x = 25°$	A1	1.1b
	Also $\cot x = \cot(3x - 50°) \Rightarrow x + 180° = 3x - 50°$	M1	2.1
	$x = 115°$	A1	1.1b
	Deduces $x = 90°$	B1	2.2a
		(5)	
			(8 marks)
Notes:			

(a) Condone a full proof in x (or other variable) instead of θ's here

B1: States or uses $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ Do not accept $\operatorname{cosec}\theta = \dfrac{1}{\sin}$ with the θ missing

M1: For the key step in forming a single fraction/common denominator

E.g. $\operatorname{cosec}\theta - \sin\theta = \dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1-\sin^2\theta}{\sin\theta}$. Allow if written separately $\dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1}{\sin\theta} - \dfrac{\sin^2\theta}{\sin\theta}$

Condone missing variables for this M mark

A1*: Shows careful work with all necessary steps shown leading to given answer. See scheme for necessary steps. There should not be any notational or bracketing errors.

(b) Condone θ's instead of x's here

M1: Uses part (a), cancels or factorises out the $\cos x$ term, to establish that one solution is found when $x = 3x - 50°$.

You may see solutions where $\cot A - \cot B = 0 \Rightarrow \cot(A-B) = 0$ or $\tan A - \tan B = 0 \Rightarrow \tan(A-B) = 0$.

As long as they don't state $\cot A - \cot B = \cot(A-B)$ or $\tan A - \tan B = \tan(A-B)$ this is acceptable

A1: $x = 25°$

M1: For the key step in realising that $\cot x$ has a period of $180°$ and a second solution can be found by solving $x + 180° = 3x - 50°$. The sight of $x = 115°$ can imply this mark provided the step $x = 3x - 50°$ has been seen. Using reciprocal functions it is for realising that $\tan x$ has a period of $180°$

A1: $x = 115°$ Withhold this mark if there are additional values in the range (0,180) but ignore values outside.

B1: Deduces that a solution can be found from $\cos x = 0 \Rightarrow x = 90°$. Ignore additional values here.

....

Solutions with limited working. The question demands that candidates show all stages of working.

SC: $\cos x \cot x = \cos x \cot(3x - 50°) \Rightarrow \cot x = \cot(3x - 50°) \Rightarrow x = 25°, 115°$

They have shown some working so can score B1, B1 marked on epen as 11000

Alt 1- Right hand side to left hand side

Question	Scheme	Marks	AOs
(a)	States or uses $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$	B1	1.2
	$\cos\theta\cot\theta = \dfrac{\cos^2\theta}{\sin\theta} = \dfrac{1-\sin^2\theta}{\sin\theta}$	M1	2.1
	$= \dfrac{1}{\sin\theta} - \sin\theta = \operatorname{cosec}\theta - \sin\theta$ *	A1*	2.1
		(3)	

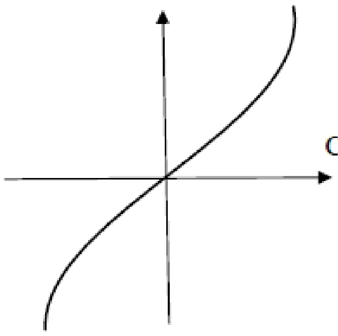
Alt 2- Works on both sides

Question	Scheme	Marks	AOs
(a)	States or uses $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ or $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$	B1	1.2
	$LHS = \dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1-\sin^2\theta}{\sin\theta} = \dfrac{\cos^2\theta}{\sin\theta}$ $RHS = \cos\theta\cot\theta = \dfrac{\cos^2\theta}{\sin\theta}$	M1	2.1
	States a conclusion E.g. "HENCE TRUE", "QED" or $\operatorname{cosec}\theta - \sin\theta \equiv \cos\theta\cot\theta$ o.e. (condone $=$ for $\equiv$)	A1*	2.1
		(3)	

Alt (b)

Question	Scheme	Marks	AOs
	$\cot x = \cot(3x-50°) \Rightarrow \dfrac{\cos x}{\sin x} = \dfrac{\cos(3x-50°)}{\sin(3x-50°)}$ $\sin(3x-50°)\cos x - \cos(3x-50°)\sin x = 0$ $\sin((3x-50°)-x) = 0$ $2x - 50° = 0$	M1	3.1a
	$x = 25°$	A1	1.1b
	Also $2x - 50° = 180°$	M1	2.1
	$x = 115°$	A1	1.1b
	Deduces $\cos x = 0 \Rightarrow x = 90°$	B1	2.2a
		(5)	

Q5

Question	Scheme	Marks
(a)	 Correct position or curvature	M1
	Correct position and curvature	A1
		(2)
(b)	$3\arcsin(x+1)+\pi=0 \Rightarrow \arcsin(x+1)=-\frac{\pi}{3}$	M1
	$\Rightarrow (x+1)=\sin\left(-\frac{\pi}{3}\right)$	
	$\Rightarrow x=-1-\frac{\sqrt{3}}{2}$	dM1A1
		(3)
		(5 marks)

(a) Ignore any scales that appear on the axes

M1 Accept for the method mark
Either one of the two sections with correct curvature passing through (0,0),
Or both sections condoning dubious curvature passing through (0,0) (but do not accept any negative gradients)
Or a curve with a different range or an "extended range"
See the next page for a useful guide for clarification of this mark.

A1 A curve only in quadrants one and three passing through the point (0,0) with a gradient that is always positive. The gradient should appear to be approx ∞ at each end. If you are unsure use review
If range and domain are given then ignore.

(b)

M1 Substitutes $g(x+1)=\arcsin(x+1)$ in $3g(x+1)+\pi=0$ and attempts to make $\arcsin(x+1)$ the subject

Accept $\arcsin(x+1)=\pm\frac{\pi}{3}$ or even $g(x+1)=\pm\frac{\pi}{3}$. Condone $\frac{\pi}{3}$ in decimal form awrt1.047

dM1 Proceeds by evaluating $\sin\left(\pm\frac{\pi}{3}\right)$ and making x the subject.

Accept for this mark $\Rightarrow x=\pm\frac{\sqrt{3}}{2}\pm1$. Accept decimal such as -1.866

Do not allow this mark if the candidate works in mixed modes (radians and degrees)
You may condone invisible brackets for both M's as long as the candidate is working correctly with the function

A1 $-1-\frac{\sqrt{3}}{2}$ oe with no other solutions. Remember to isw after a correct answer

Be careful with single fractions. $-\frac{2-\sqrt{3}}{2}$ and $\frac{-2+\sqrt{3}}{2}$ are incorrect but $-\frac{2+\sqrt{3}}{2}$ is correct

Note: It is possible for a candidate to change $\frac{\pi}{3}$ to $60°$ and work in degrees for all marks

Platinum Questions

Non-calculator

The total mark for this section is 28

Q1

Solve, for $0 \leq \theta \leq 180°$,

$$\tan(\theta + 35°) = \cot(\theta - 53°).$$

(4)

(Total for Question 1 is 4 marks)

Q2

The angle θ, $0 < \theta < \frac{\pi}{2}$, satisfies

$$\tan\theta \tan 2\theta = \sum_{r=0}^{\infty} 2\cos^r 2\theta.$$

(*a*) Show that $\tan\theta = 3^p$, where p is a rational number to be found.

(8)

(*b*) Hence show that $\frac{\pi}{6} < \theta < \frac{\pi}{4}$.

(2)

(Total for Question 2 is 10 marks)

Q3

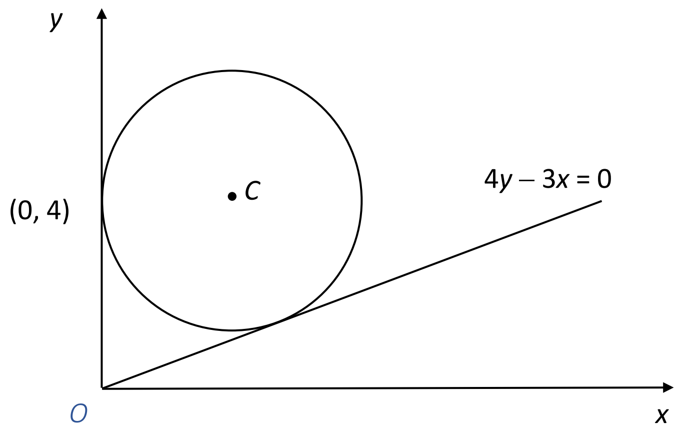


Figure 1

The circle, with centre C and radius r, touches the y-axis at (0, 4) and also touches the line with equation $4y - 3x = 0$, as shown in Figure 1.

(*a*) (i) Find the value of r.

(ii) Show that arctan $\left(\frac{3}{4}\right)$ + 2 acrtan $\left(\frac{1}{2}\right) = \frac{1}{2}\pi$.

(8)

The line with equation $4x + 3y = q$, $q > 12$, is a tangent to the circle.

(*b*) Find the value of q.

(4)

(Total for Question 3 is 12 marks)

End of Questions

Platinum Mark Scheme

Q1

	$\dfrac{\sin(\theta+35)}{\cos(\theta+35)}=\dfrac{\cos(\theta-53)}{\sin(\theta-53)}$	M1	Use of correct defns for tan and cot
	$0=\cos(\theta-53)\cos(\theta+35)-\sin(\theta+35)\sin(\theta-53)$ $0=\cos(2\theta-53+35)$	M1	Use of cos(A+B) rule to reach single trig function
	$2\theta-18=90, 270$ so $\underline{\boldsymbol{x=54, 144}}$	A1A1 **(4)**	A1 for 54 and A1 for 144
	Use of tan $(A\pm B)$ doesn't score until $\tan2\theta=\tan(90--18)$		
ALT	$\tan(\theta+35)=\tan[90-(\theta-53)]$ $\theta+35=90-(\theta-53)$ or $\theta+35=90-(\theta-53)+180$	M1 M1	Use of $\cot x=\pm\tan(90\pm x)$ either

Q2

	RHS = GP $a=2$, $r=\cos2\theta$ $S_\infty=\dfrac{2}{1-\cos2\theta}$	M1,A1	Identify GP and attempt sum to ∞ for M1
	$\cos2\theta=1-2\sin^2\theta\Rightarrow(\text{RHS})=\text{cosec}^2\theta$ (Allow $\frac{k}{\sin^2\theta}$)	M1	Use $\cos2\theta$ to simplify
	$\tan2\theta=\dfrac{2\tan\theta}{1-\tan^2\theta}\Rightarrow(\text{LHS})=\dfrac{2\tan^2\theta}{1-\tan^2\theta}$	M1	Use of $\tan2\theta$ on LHS
	Equating: $\dfrac{2\tan^2\theta}{1-\tan^2\theta}=1+\cot^2\theta=\dfrac{1+\tan^2\theta}{\tan^2\theta}$	M1	Equate LHS=RHS and use formula to get eqn in $\tan\theta$ or single trig func.
	so $3\tan^4\theta-1=0$	A1	Correct eqn (either line)
	$\tan^4\theta=\frac{1}{3}\Rightarrow\tan\theta=\left(\frac{1}{3}\right)^{\frac{1}{4}}$	dM1	Solve their equn leading to $\tan\theta=$…Dep on 4th M
	$\tan\theta=3^{-\frac{1}{4}}$ or $p=-\dfrac{1}{4}$	A1 (8)	
(b)	$1>3^{-\frac{1}{4}}>3^{-\frac{1}{2}}\Rightarrow\tan\frac{\pi}{4}>\tan\theta>\tan\frac{\pi}{6}$	M1	
	$\Rightarrow\frac{\pi}{4}>\theta>\frac{\pi}{6}$	A1 (2) **[10]**	cso

Q3

4. (*a*)	Centre $(r, 4)$, $\tan\theta = \frac{3}{4}$ or $OA = 4$ seen or implied anywhere	B1 B1
(i)	Method α Finding cords of A	
	$[A = (4k, 3k) \Rightarrow OA = 5k = 4$ or $(4\cos\theta, 4\sin\theta)$ with attempt at $\theta]$	M1
	$A = (\frac{16}{5}, \frac{12}{5})$ can be written down	A1
	Complete method for r: $\frac{4 - y_A}{r - x_A} = -\frac{4}{3}$ or $(r - x_A)^2 + (4 - y_A)^2 = r^2$	M1
	$r = 2$	A1
(ii)	$\theta + 2\alpha = \frac{1}{2}\pi$	M1
	$\arctan\left(\frac{3}{4}\right) + 2\arctan\left(\frac{1}{2}\right) = \frac{\pi}{2}$ AG (no errors, convincing)	A1 (8) cso
(b) Method α:	Tangent is perp. to given line; intercept = $(4 + r)\operatorname{cosec}\theta$	M1 A1
	Complete method for q; e.g. $0 + 3(\text{intercept}) = q$	M1
	$\Rightarrow q = 30$	A1 (4)
Method β	(Other variations)	
	Finding pt. on $4x + 3y = q$	M1 A1
	e.g where circle meets it $\left(\frac{18}{5}, \frac{26}{5}\right)$	
	where $4y = 3x$ meets it $(\frac{4q}{25}, \frac{3q}{25})$ $\left(\frac{24}{5}, \frac{18}{5}\right)$	
	Complete method to find q, $q = 30$	M1 A1
		(12 marks)

Topic 7: Trigonometry and Modelling

Bronze, Silver, Gold and Platinum Worksheets for A Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel A Level Mathematics: Pure Mathematics Year 2' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Non-calculator

The total mark for this section is 32

Q1

Prove that

$$1 - \cos 2\theta \equiv \tan\theta \sin 2\theta, \quad \theta \neq \frac{(2n+1)\pi}{2}, \quad n \in \mathbb{Z}$$

(3)

(Total for Question 1 is 3 marks)

Q2

Prove that

$$\sin 2x - \tan x \equiv \tan x \cos 2x, \quad x \neq (2n+1)90^\circ, \quad n \in \mathbb{Z}$$

(4)

(Total for Question 2 is 4 marks)

Q3

Given that

$$\tan\theta^\circ = p, \text{ where } p \text{ is a constant, } p \neq \pm 1$$

use standard trigonometric identities, to find in terms of p,

(a) $\tan 2\theta^\circ$

(2)

(b) $\cos\theta^\circ$

(2)

(c) $\cot(\theta - 45)^\circ$

(2)

Write each answer in its simplest form.

(Total for Question 3 is 6 marks)

Q4

(a) Use the double angle formulae and the identity

$$\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$$

to obtain an expression for cos 3x in terms of powers of cos x only.

(4)

(b) (i) Prove that

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} \equiv 2\sec x, \qquad x \neq (2n+1)\frac{\pi}{2}.$$

(4)

(ii) Hence find, for $0 < x < 2\pi$, all the solutions of

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 4.$$

(3)

(Total for Question 4 is 11 marks)

Q5

(a) Show that

$$\text{cosec}\,\theta - \sin\theta \equiv \cos\theta \cot\theta \qquad \theta \neq (180n)° \qquad n \in \mathbb{Z}$$

(3)

(b) Hence, or otherwise, solve for $0 < x < 180°$

$$\text{cosec}\, x - \sin x = \cos x \cot (3x - 50°)$$

(5)

(Total for Question 5 is 8 marks)

End of Questions

Bronze Mark Scheme

Q1

Question	Scheme	Marks	AOs
	$1-\cos 2\theta \equiv \tan\theta \sin 2\theta,\ \theta \neq \frac{(2n+1)\pi}{2},\ n \in \mathbb{Z}$		
(a) Way 1	$\tan\theta\sin 2\theta = \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta)$	M1	1.1b
	$= \left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)(2\sin\theta\cancel{\cos\theta}) = 2\sin^2\theta = 1-\cos 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
(a) Way 2	$1-\cos 2\theta = 1-(1-2\sin^2\theta) = 2\sin^2\theta$	M1	1.1b
	$= \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta) = \tan\theta\sin 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	

Notes for Question	
(a)	Way 1
M1:	Applies $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to $\tan\theta\sin 2\theta$
M1:	Cancels as scheme (may be implied) and attempts to use $\cos 2\theta = 1-2\sin^2\theta$
A1*:	For a correct proof showing all steps of the argument
(a) Way 2	
M1:	For using $\cos 2\theta = 1-2\sin^2\theta$
Note:	If the form $\cos 2\theta = \cos^2\theta - \sin^2\theta$ or $\cos 2\theta = 2\cos^2\theta - 1$ is used, the mark cannot be awarded until $\cos^2\theta$ has been replaced by $1-\sin^2\theta$
M1:	Attempts to write their $2\sin^2\theta$ in terms of $\tan\theta$ and $\sin 2\theta$ using $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and
	$\sin 2\theta = 2\sin\theta\cos\theta$ within the given expression
A1*:	For a correct proof showing all steps of the argument
Note:	If a proof meets in the middle; e.g. they show LHS $= 2\sin^2\theta$ and RHS $= 2\sin^2\theta$; then some indication must be given that the proof is complete. E.g. $1-\cos 2\theta \equiv \tan\theta\sin 2\theta$, QED, box

Q2

Question Number	Scheme	Marks
(a)	$\sin 2x - \tan x = 2\sin x\cos x - \tan x$	M1
	$= \dfrac{2\sin x\cos^2 x}{\cos x} - \dfrac{\sin x}{\cos x}$	M1
	$= \dfrac{\sin x}{\cos x} \times (2\cos^2 x - 1)$	
	$= \tan x \cos 2x$	dM1 A1*
		(4)

(a)

M1 Uses a correct double angle identity involving $\sin 2x$ Accept $\sin(x+x) = \sin x\cos x + \cos x\sin x$

M1 Uses $\tan x = \dfrac{\sin x}{\cos x}$ with $\sin 2x = 2\sin x\cos x$ and attempts to combine the two terms using a common denominator. This can be awarded on two separate terms with a common denominator.
Alternatively uses $\sin x = \tan x\cos x$ and attempts to combine two terms using factorisation of $\tan x$

dM1 Both M's must have been scored. Uses a correct double angle identity involving $\cos 2x$.

A1* A fully correct solution with no errors or omissions. All notation must be correct and variables must be consistent
Withhold this mark if for instance they write $\tan x = \dfrac{\sin}{\cos}$

If the candidate $\times \cos x$ on line 1 and/or $\div \sin x$ they cannot score any more than one mark unless they are working with both sides of the equation or it is fully explained.

(a) Alt 1	$\tan x\cos 2x = \tan x\left(2\cos^2 x - 1\right)$	M1
	$= 2\tan x\cos^2 x - \tan x$	
	$= 2\dfrac{\sin x}{\cos x}\cos^2 x - \tan x$	M1
	$= 2\sin x\cos x - \tan x$	
	$= \sin 2x - \tan x$	dM1 A1
		(4)

a) Alt 1 Starting from the rhs

M1 Uses a correct double angle identity for $\cos 2x$. Accept any correct version including $\cos(x+x) = \cos x\cos x - \sin x\sin x$

M1 Uses $\tan x = \dfrac{\sin x}{\cos x}$ with $\cos 2x = 2\cos^2 x - 1$ and attempts to multiply out the bracket

dM1 Both M's must have been scored. It is for using $2\sin x\cos x = \sin 2x$

A1* A fully correct solution with no errors or omissions. All notation must be correct and variables must be consistent.
See Main scheme for how to deal with candidates who $\div \tan x$

(a) Alt 2	$\sin 2x - \tan x \equiv \tan x \cos 2x$	
	$2\sin x\cos x - \tan x \equiv \tan x(2\cos^2 x - 1)$	M1
	$2\sin x\cos x - \cancel{\tan x} \equiv 2\tan x\cos^2 x - \cancel{\tan x}$	
	$2\sin x\cos x \equiv 2\dfrac{\sin x}{\cos x}\cos^2 x$	M1
	$2\sin x\cos x \equiv 2\sin x\cos x$	dM1
	+statement that it must be true	A1*

a) Alt 2 Candidates who use both sides

M1 Uses a correct double angle identity involving $\sin 2x$ **or** $\cos 2x$. Can be scored from either side
Accept $\sin(x+x) = \sin x\cos x + \cos x\sin x$ or $\cos(x+x) = \cos x\cos x - \sin x\sin x$

M1 Uses $\tan x = \dfrac{\sin x}{\cos x}$ with $\cos 2x = 2\cos^2 x - 1$ and cancels the $\tan x$ term from both sides

dM1 Uses a correct double angle identity involving $\sin 2x$ Both previous M's must have been scored

A1* A fully correct solution with no errors or omissions AND statement "hence true", "a tick", "QED". W[5]
All notation must be correct and variables must be consistent

....

It is possible to solve part (b) without using the given identity. There are various ways of doing this, one of which is shown below.

$\sin 2x - \tan x = 3\tan x\sin x \Rightarrow \quad 2\sin x\cos x - \dfrac{\sin x}{\cos x} = 3\dfrac{\sin x}{\cos x}\sin x$

$2\sin x\cos^2 x - \sin x = 3\sin^2 x$ — M1 Equation in $\sin x$ and $\cos x$

$2\sin x(1-\sin^2 x) - \sin x = 3\sin^2 x$ — M1 Equation in $\sin x$ only

$(2\sin^2 x + 3\sin x - 1)\sin x = 0$

$x = ..$ — M1 Solving equation to find at least one x

Two of $x = 16.3°, 163.7°, 0, 180°$ — A1

All four of $x = 16.3°, 163.7°, 0, 180°$ and no extras A1

Q3

Question Number	Scheme	Marks
(a)	$\tan 2\theta° = \dfrac{2\tan\theta°}{1-\tan^2\theta°} = \dfrac{2p}{1-p^2}$ Final answer	M1A1 **(2)**
(b)	$\cos\theta° = \dfrac{1}{\sec\theta°} = \dfrac{1}{\sqrt{1+\tan^2\theta°}} = \dfrac{1}{\sqrt{1+p^2}}$ Final answer	M1A1 **(2)**
(c)	$\cot(\theta-45)° = \dfrac{1}{\tan(\theta-45)°} = \dfrac{1+\tan\theta°\tan 45°}{\tan\theta° - \tan 45°} = \dfrac{1+p}{p-1}$ Final answer	M1A1 **(2)** **(6 marks)**

(a)

M1 Attempt to use the double angle formula for tangent followed by the substitution $\tan\theta = p$.

For example accept $\tan 2\theta^\circ = \frac{2\tan\theta^\circ}{1\pm\tan^2\theta^\circ} = \frac{2p}{1\pm p^2}$

Condone unconventional notation such as $\tan 2\theta^\circ = \frac{2\tan\theta^\circ}{1\pm\tan\theta^{2\circ}}$ followed by an attempt to substitute $\tan\theta = p$ for the M mark. Recovery from this notation is allowed for the A1.

Alternatively use $\tan(A+B) = \frac{\tan A + \tan B}{1\pm\tan A\tan B}$ with an attempt at substituting $\tan A = \tan B = p$. The unsimplified answer $\frac{p+p}{1-p\times p}$ is evidence

It is possible to use $\tan 2\theta^\circ = \frac{\sin 2\theta^\circ}{\cos 2\theta^\circ} = \frac{2\sin\theta^\circ\cos\theta^\circ}{2\cos^2\theta^\circ - 1} = \frac{2\times\frac{p}{\sqrt{1\pm p^2}}\times\frac{1}{\sqrt{1\pm p^2}}}{2\times\frac{1}{1\pm p^2}-1}$ but it is unlikely to succeed.

A1 Correct **simplified** answer of $\tan 2\theta^\circ = \frac{2p}{1-p^2}$ or $\frac{2p}{(1-p)(1+p)}$.

Do not allow if they "simplify" to $\frac{2}{1-p}$

Allow the correct answer for both marks as long as no incorrect working is seen.

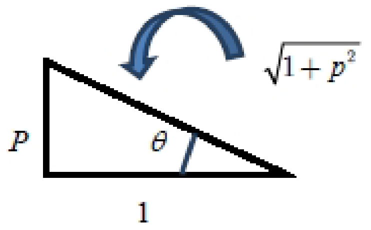
(b)

M1 Attempt to use **both** $\cos\theta = \frac{1}{\sec\theta}$ **and** $1+\tan^2\theta = \sec^2\theta$ with $\tan\theta = p$ in an attempt to obtain an expression for $\cos\theta$ in terms of p. Condone a slip in the sign of the second identity.

Evidence would be $\cos^2\theta = \frac{1}{\pm 1 \pm p^2}$

Alternatively use a triangle method, attempt Pythagoras' theorem and use $\cos\theta = \frac{adj}{hyp}$

The attempt to use Pythagoras must attempt to use the squares of the lengths.



A1 $\cos\theta^\circ = \frac{1}{\sqrt{1+p^2}}$ Accept versions such as $\cos\theta^\circ = \sqrt{\frac{1}{1+p^2}}$, $\cos\theta^\circ = \pm\frac{1}{\sqrt{1+p^2}}$

Withhold this mark if the candidate goes on to write $\cos\theta^\circ = \frac{1}{1+p}$

(c)

M1 Use the correct identity $\cot(\theta - 45) = \frac{1}{\tan(\theta - 45)}$ and an attempt to use the $\tan(A-B)$ formula with $A=\theta$, $B=45$ and $\tan\theta = p$.

For example accept an unsimplified answer such as $\frac{1}{\frac{\tan\theta \pm \tan 45}{1 \pm \tan\theta\tan 45}} = \frac{1}{\frac{p \pm \tan 45}{1 \pm p\tan 45}}$

It is possible to use $\cot(\theta - 45) = \frac{\cos(\theta - 45)}{\sin(\theta - 45)}$ and an attempt to use the formulae for $\sin(A-B)$ and $\cos(A-B)$ with $A=\theta$, $B=45$. $\sin\theta = \frac{p}{\sqrt{1 \pm p^2}}$ and $\cos\theta = \frac{1}{\sqrt{1 \pm p^2}}$

Sight of an expression $\frac{\frac{1}{\sqrt{1 \pm p^2}}\cos 45 \pm \frac{p}{\sqrt{1 \pm p^2}}\sin 45}{\frac{p}{\sqrt{1 \pm p^2}}\cos 45 \pm \frac{1}{\sqrt{1 \pm p^2}}\sin 45}$ is evidence.

A1 Uses $\tan 45 = 1$ or $\sin 45 = \cos 45 = \frac{\sqrt{2}}{2}$ *oe* and simplifies answer.

Accept $-\frac{1+p}{1-p}$ or $1 + \frac{2}{p-1}$

Note that there is no isw in any parts of this question.

Q4

Question Number	Scheme	Marks
	(a) $\cos(2x+x) = \cos 2x\cos x - \sin 2x\sin x$	M1
	$= (2\cos^2 x - 1)\cos x - (2\sin x\cos x)\sin x$	M1
	$= (2\cos^2 x - 1)\cos x - 2(1-\cos^2 x)\cos x$ any correct expression	A1
	$= 4\cos^3 x - 3\cos x$	A1 (4)
	(b)(i) $\dfrac{\cos x}{1+\sin x} + \dfrac{1+\sin x}{\cos x} = \dfrac{\cos^2 x + (1+\sin x)^2}{(1+\sin x)\cos x}$	M1
	$= \dfrac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{(1+\sin x)\cos x}$	A1
	$= \dfrac{2(1+\sin x)}{(1+\sin x)\cos x}$	M1
	$= \dfrac{2}{\cos x} = 2\sec x$ * cso	A1 (4)
	(c) $\sec x = 2$ *or* $\cos x = \dfrac{1}{2}$	M1
	$x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$ accept awrt 1.05, 5.24	A1, A1 (3)
		[11]

Q5

Question	Scheme	Marks	AOs
(a)	States or uses $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$	B1	1.2
	$\operatorname{cosec}\theta - \sin\theta = \dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1-\sin^2\theta}{\sin\theta}$	M1	2.1
	$= \dfrac{\cos^2\theta}{\sin\theta} = \cos\theta \times \dfrac{\cos\theta}{\sin\theta} = \cos\theta\cot\theta$ *	A1*	2.1
		(3)	
(b)	$\operatorname{cosec} x - \sin x = \cos x \cot(3x - 50°)$ $\Rightarrow \cos x \cot x = \cos x \cot(3x - 50°)$		
	$\cot x = \cot(3x - 50°) \Rightarrow x = 3x - 50°$	M1	3.1a
	$x = 25°$	A1	1.1b
	Also $\cot x = \cot(3x - 50°) \Rightarrow x + 180° = 3x - 50°$	M1	2.1
	$x = 115°$	A1	1.1b
	Deduces $x = 90°$	B1	2.2a
		(5)	
			(8 marks)

Notes:

(a) Condone a full proof in x (or other variable) instead of θ's here

B1: States or uses $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ Do not accept $\operatorname{cosec}\theta = \dfrac{1}{\sin}$ with the θ missing

M1: For the key step in forming a single fraction/common denominator

E.g. $\operatorname{cosec}\theta - \sin\theta = \dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1-\sin^2\theta}{\sin\theta}$. Allow if written separately $\dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1}{\sin\theta} - \dfrac{\sin^2\theta}{\sin\theta}$

Condone missing variables for this M mark

A1*: Shows careful work with all necessary steps shown leading to given answer. See scheme for necessary steps. There should not be any notational or bracketing errors.

(b) Condone θ's instead of x's here

M1: Uses part (a), cancels or factorises out the $\cos x$ term, to establish that one solution is found when $x = 3x - 50°$.

You may see solutions where $\cot A - \cot B = 0 \Rightarrow \cot(A-B) = 0$ or $\tan A - \tan B = 0 \Rightarrow \tan(A-B) = 0$.

As long as they don't state $\cot A - \cot B = \cot(A-B)$ or $\tan A - \tan B = \tan(A-B)$ this is acceptable

A1: $x = 25°$

M1: For the key step in realising that $\cot x$ has a period of $180°$ and a second solution can be found by solving $x + 180° = 3x - 50°$. The sight of $x = 115°$ can imply this mark provided the step $x = 3x - 50°$ has been seen. Using reciprocal functions it is for realising that $\tan x$ has a period of $180°$

A1: $x = 115°$ Withhold this mark if there are additional values in the range (0,180) but ignore values outside.

B1: Deduces that a solution can be found from $\cos x = 0 \Rightarrow x = 90°$. Ignore additional values here.

....

Solutions with limited working. The question demands that candidates show all stages of working.

SC: $\cos x \cot x = \cos x \cot(3x - 50°) \Rightarrow \cot x = \cot(3x - 50°) \Rightarrow x = 25°, 115°$

They have shown some working so can score B1, B1 marked on epen as 11000

Alt 1- Right hand side to left hand side

Question	Scheme	Marks	AOs
(a)	States or uses $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$	B1	1.2
	$\cos\theta\cot\theta = \dfrac{\cos^2\theta}{\sin\theta} = \dfrac{1-\sin^2\theta}{\sin\theta}$	M1	2.1
	$= \dfrac{1}{\sin\theta} - \sin\theta = \text{cosec}\,\theta - \sin\theta$ *	A1*	2.1
		(3)	

Alt 2- Works on both sides

Question	Scheme	Marks	AOs
(a)	States or uses $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ or $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$	B1	1.2
	$LHS = \dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1-\sin^2\theta}{\sin\theta} = \dfrac{\cos^2\theta}{\sin\theta}$ $RHS = \cos\theta\cot\theta = \dfrac{\cos^2\theta}{\sin\theta}$	M1	2.1
	States a conclusion E.g. "HENCE TRUE", "QED" or $\text{cosec}\,\theta - \sin\theta \equiv \cos\theta\cot\theta$ o.e. (condone = for ≡)	A1*	2.1
		(3)	

Alt (b)

Question	Scheme	Marks	AOs
	$\cot x = \cot(3x-50°) \Rightarrow \dfrac{\cos x}{\sin x} = \dfrac{\cos(3x-50°)}{\sin(3x-50°)}$ $\sin(3x-50°)\cos x - \cos(3x-50°)\sin x = 0$ $\sin((3x-50°)-x) = 0$ $2x-50° = 0$	M1	3.1a
	$x = 25°$	A1	1.1b
	Also $2x-50° = 180°$	M1	2.1
	$x = 115°$	A1	1.1b
	Deduces $\cos x = 0 \Rightarrow x = 90°$	B1	2.2a
		(5)	

Silver Questions

Non-calculator

The total mark for this section is 34

Q1

(i) Use an appropriate double angle formula to show that

$$\text{cosec}2x = \lambda \text{ cosec } x \sec x,$$

and state the value of the constant λ.

(3)

(ii) Solve, for $0 \leq \theta < 2\pi$, the equation

$$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta$$

You must show all your working. Give your answers in terms of π.

(6)

(Total for Question 1 is 9 marks)

Q2

(a) Starting from the formulae for $\sin(A + B)$ and $\cos(A + B)$, prove that

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

(4)

(b) Deduce that

$$\tan\left(\theta + \frac{\pi}{6}\right) = \frac{1 + \sqrt{3}\tan\theta}{\sqrt{3} - \tan\theta}$$

(3)

(c) Hence, or otherwise, solve, for $0 \leq \theta \leq \pi$,

$$1 + \sqrt{3}\tan\theta = (\sqrt{3} - \tan\theta)\tan(\pi - \theta)$$

Give your answers as multiples of π.

(6)

(Total for Question 2 is 13 marks)

Q3

(a) Prove that

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \tan\theta, \quad \theta \neq 90n°, \; n \in \mathbb{Z}$$

(4)

(b) Hence, or otherwise,

(i) show that $\tan 15° = 2 - \sqrt{3}$,

(3)

(ii) solve, for $0 < x < 360°$,

$$\operatorname{cosec} 4x - \cot 4x = 1$$

(5)

(Total for Question 3 is 12 marks)

End of Questions

Pearson

Silver Mark Scheme

Q1

Question Number	Scheme	Marks
(i)	$\text{cosec}2x = \dfrac{1}{\sin 2x}$	M1
	$= \dfrac{1}{2\sin x \cos x}$	M1
	$= \dfrac{1}{2}\text{cosec}x\sec x \Rightarrow \quad \lambda = \dfrac{1}{2}$	A1
		(3)
(ii)	$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta \Rightarrow 3\sec^2\theta + 3\sec\theta = 2(\sec^2\theta - 1)$	M1
	$\sec^2\theta + 3\sec\theta + 2 = 0$	
	$(\sec\theta + 2)(\sec\theta + 1) = 0$	M1
	$\sec\theta = -2, -1$	A1
	$\cos\theta = -0.5, -1$	M1
	$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$	A1A1
		(6)
		(9 marks)
ALT (ii)	$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta \Rightarrow 3\times\dfrac{1}{\cos^2\theta} + 3\times\dfrac{1}{\cos\theta} = 2\times\dfrac{\sin^2\theta}{\cos^2\theta}$	
	$3 + 3\cos\theta = 2\sin^2\theta$	
	$3 + 3\cos\theta = 2(1-\cos^2\theta)$	M1
	$2\cos^2\theta + 3\cos\theta + 1 = 0$	
	$(2\cos\theta + 1)(\cos\theta + 1) = 0 \Rightarrow \cos\theta = -0.5, -1$	M1A1
	$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$	M1,A1,A1
		(6)
		(9 marks)

Notes for Question

(i)

M1 Uses the identity $\text{cosec}2x = \frac{1}{\sin 2x}$

M1 Uses the correct identity for $\sin 2x = 2\sin x\cos x$ in their expression.
Accept $\sin 2x = \sin x\cos x + \cos x\sin x$

A1 $\lambda = \frac{1}{2}$ following correct working

(ii)

M1 Replaces $\tan^2\theta$ by $\pm\sec^2\theta \pm 1$ to produce an equation in just $\sec\theta$

M1 Award for a forming a 3TQ=0 in $\sec\theta$ and applying a correct method for factorising, or using the formula, or completing the square to find two answers to $\sec\theta$

If they replace $\sec\theta = \frac{1}{\cos\theta}$ it is for forming a 3TQ in $\cos\theta$ and applying a correct method for finding two answers to $\cos\theta$

A1 Correct answers to $\sec\theta = -2, -1$ or $\cos\theta = -\frac{1}{2}, -1$

M1 Award for using the identity $\sec\theta = \frac{1}{\cos\theta}$ and proceeding to find at least one value for θ.

If the 3TQ was in cosine then it is for finding at least one value of θ.

A1 Two correct values of θ. All method marks must have been scored.

Accept two of $120°, 180°, 240°$ or two of $\frac{2\pi}{3}, \frac{4\pi}{3}, \pi$ or two of awrt 2dp 2.09, 3.14, 4.19

A1 All three answers correct. They must be given in terms of π as stated in the question.

Accept $0.\dot{6}\pi, 1.\dot{3}\pi, \pi$

Withhold this mark if further values in the range are given. All method marks must have been scored.
Ignore any answers outside the range.

Alt (ii)

M1 Award for replacing $\sec^2\theta$ with $\frac{1}{\cos^2\theta}$, $\sec\theta$ with $\frac{1}{\cos\theta}$, $\tan^2\theta$ with $\frac{\sin^2\theta}{\cos^2\theta}$ multiplying through by $\cos^2\theta$ (seen in at least 2 terms) and replacing $\sin^2\theta$ with $\pm 1 \pm \cos^2\theta$ to produce an equation in just $\cos\theta$

M1 Award for a forming a 3TQ=0 in $\cos\theta$ and applying a correct method for factorising, or using the formula, or completing the square to find two answers to $\cos\theta$

A1 $\cos\theta = -\frac{1}{2}, -1$

M1 Proceeding to finding at least one value of θ from an equation in $\cos\theta$.

A1 Two correct values of θ. All method marks must have been scored

Accept two of $120°, 180°, 240°$ or two of $\frac{2\pi}{3}, \frac{4\pi}{3}, \pi$ or two of awrt 2dp 2.09, 3.14, 4.19

A1 All three answers correct. They must be given in terms of π as stated in the question.

Notes for Question Continued

Accept $0.\dot{6}\pi, 1.\dot{3}\pi, \pi$

All method marks must have been scored. Withhold this mark if further values in the range are given.
Ignore any answers outside the range

Q2

Question No	Scheme	Marks
	(a) $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{sinAcosB + cosAsinB}{cosAcosB - sinAsinB}$	M1A1
	$= \frac{\frac{sinA}{cosA}+\frac{sinB}{cosB}}{1-\frac{sinAsinB}{cosAcosB}}$ $(\div\ cosAcosB)$	M1
	$= \frac{tanA + tanB}{1 - tanAtanB}$	A1 *
		(4)
	(b) $\tan\left(\theta+\frac{\pi}{6}\right) = \frac{tan\theta + tan\frac{\pi}{6}}{1-tan\theta tan\frac{\pi}{6}}$	M1
	$= \frac{tan\theta+\frac{1}{\sqrt{3}}}{1-tan\theta\frac{1}{\sqrt{3}}}$	M1
	$= \frac{\sqrt{3}tan\theta+1}{\sqrt{3}-tan\theta}$	A1 *
		(3)
	(c) $\tan\left(\theta+\frac{\pi}{6}\right) = \tan(\pi-\theta)$.	M1
	$\left(\theta+\frac{\pi}{6}\right) = (\pi-\theta)$	dM1
	$\theta = \frac{5}{12}\pi$	ddM1 A1
	$\tan\left(\theta+\frac{\pi}{6}\right) = \tan(2\pi-\theta)$	dddM1
	$\theta = \frac{11}{12}\pi$	A1
		(6)
		(13 MARKS)

Q3

Question Number	Scheme		Marks
(a)	$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \frac{1-\cos 2\theta}{\sin 2\theta}$		M1
	$= \frac{2\sin^2\theta}{2\sin\theta\cos\theta}$		M1A1
	$= \frac{\sin\theta}{\cos\theta} = \tan\theta$	cso	A1* (4)
(b)(i)	$\tan 15° = \frac{1}{\sin 30°} - \frac{\cos 30°}{\sin 30°}$		M1
	$\tan 15° = \frac{1}{\frac{1}{2}} - \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2-\sqrt{3}$	cso	dM1 A1* (3)
(b)(ii)	$\tan 2x = 1$		M1
	$2x = 45°$		A1
	$2x = 45° + 180°$		M1
	$x = 22.5°, 112.5°, 202.5°, 292.5°$		A1(any two) A1 (5)
			12 Marks
	Alt for (b)(i) $\tan 15° = \tan(60° - 45°)$ *or* $\tan(45° - 30°)$		
	$\tan 15° = \frac{\tan 60 - \tan 45}{1 + \tan 60 \tan 45}$ *or* $\frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$		M1
	$\tan 15° = \frac{\sqrt{3}-1}{1+\sqrt{3}}$ *or* $\frac{1-\frac{\sqrt{3}}{3}}{1+\frac{\sqrt{3}}{3}}$		M1
	Rationalises to produce $\tan 15° = 2-\sqrt{3}$		A1*

Gold Questions

Non-calculator

The total mark for this section is 33

Q1

Show that

$$\operatorname{cosec} 2x + \cot 2x = \cot x, \quad x \neq 90n°, \quad n \in \mathbb{Z}.$$

(5)

(Total for Question 1 is 5 marks)

Q2

Prove

$$\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} \equiv 2\cot 2\theta \qquad \theta \neq (90n)°,\ n \in \mathbb{Z}$$

(4)

(Total for Question 2 is 4 marks)

Q3

(i) Solve, for $\theta \leq \theta < \pi$, the equation

$$\sin 3\theta - \sqrt{3}\cos 3\theta = 0$$

giving your answers in terms of π.

(3)

(ii) Given that

$$4\sin^2 x + \cos x = 4 - k, \qquad 0 \leqslant k \leqslant 3$$

find $\cos x$ in terms of k.

(3)

(Total for Question 3 is 6 marks)

Q4

Prove that

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A}, \qquad A \neq \frac{(2n+1)\pi}{4}, \; n \in \mathbb{Z}$$

(5)

(Total for Question 4 is 5 marks)

Q5

(a) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

(4)

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8\sin^3\theta - 6\sin\theta + 1 = 0.$$

Give your answers in terms of π.

(5)

(b) Using $\sin(\theta - \alpha) = \sin\theta\cos\alpha - \cos\theta\sin\alpha$, or otherwise, show that

$$\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2}).$$

(4)

(Total for Question 5 is 13 marks)

End of Questions

Gold Mark Scheme

Q1

Question Number	Scheme	Marks
(a)	$\text{cosec}2x+\cot 2x=\frac{1}{\sin 2x}+\frac{\cos 2x}{\sin 2x}$	M1
	$=\frac{1+\cos 2x}{\sin 2x}$	M1
	$=\frac{1+2\cos^2 x-1}{2\sin x\cos x}$	
	$=\frac{2\cos^2 x}{2\sin x\cos x}$	M1 A1
	$=\frac{\cos x}{\sin x}=\cot x$	A1*
		(5)

(a)

M1 Writing $\text{cosec}2x=\frac{1}{\sin 2x}$ **and** $\cot 2x=\frac{\cos 2x}{\sin 2x}$ *or* $\frac{1}{\tan 2x}$

M1 Writing the lhs as a single fraction $\frac{a+b}{c}$. The denominator must be correct for their terms.

M1 Uses the appropriate double angle formulae/trig identities to produce a fraction in a form containing no addition or subtraction signs. A form $\frac{p\times q}{s\times t}$ or similar

A1 A correct intermediate line. Accept $\frac{2\cos^2 x}{2\sin x\cos x}$ or $\frac{2\sin x\cos x}{2\sin x\cos x\tan x}$ or similar

This cannot be scored if errors have been made

A1* Completes the proof by cancelling and using either $\frac{\cos x}{\sin x}=\cot x$ or $\frac{1}{\tan x}=\cot x$

The cancelling could be implied by seeing $\frac{2}{2}\frac{\cos x}{\sin x}\frac{\cos x}{\cos x}=\cot x$

The proof cannot rely on expressions like $\cot=\frac{\cos}{\sin}$ (with missing x's) for the final A1

Question Number	Scheme	Marks
(a)Alt 1	$\text{cosec}2x+\cot 2x=\frac{1}{\sin 2x}+\frac{1}{\tan 2x}$	1^{ST} M1
	$=\frac{1}{2\sin x\cos x}+\frac{1-\tan^2 x}{2\tan x}$	
	$=\frac{\tan x+(1-\tan^2 x)\sin x\cos x}{2\sin x\cos x\tan x}$ or $=\frac{2\tan x+2(1-\tan^2 x)\sin x\cos x}{4\sin x\cos x\tan x}$	2^{nd} M1
	$=\frac{\tan x+\sin x\cos x-\tan^2 x\sin x\cos x}{2\sin x\cos x\tan x}$	
	$=\frac{\tan x+\sin x\cos x-\tan x\sin^2 x}{2\sin x\cos x\tan x}$	
	$=\frac{\tan x(1-\sin^2 x)+\sin x\cos x}{2\sin x\cos x\tan x}$	
	$=\frac{\tan x\cos^2 x+\sin x\cos x}{2\sin x\cos x\tan x}$	
	$=\frac{\sin x\cos x+\sin x\cos x}{2\sin x\cos x\tan x}$	
	$=\frac{2\sin x\cos x}{2\sin x\cos x\tan x}$ oe	3^{rd} M1A1
	$=\frac{1}{\tan x}=\cot x$	A1* (5)
(a)Alt 2	**Example of how main scheme could work in a roundabout route**	
	$\text{cosec}2x+\cot 2x=\cot x\Leftrightarrow\frac{1}{\sin 2x}+\frac{1}{\tan 2x}=\frac{1}{\tan x}$	1^{st} M1
	$\Leftrightarrow \tan 2x\tan x+\sin 2x\tan x=\sin 2x\tan 2x$	2^{nd} M1
	$\Leftrightarrow\frac{2\tan x}{1-\tan^2 x}\times\tan x+2\sin x\cos x\times\frac{\sin x}{\cos x}=2\sin x\cos x\times\frac{2\tan x}{1-\tan^2 x}$	
	$\Leftrightarrow\frac{2\tan^2 x}{1-\tan^2 x}+2\sin^2 x=\frac{4\sin^2 x}{1-\tan^2 x}$	
	$\times(1-\tan^2 x)\Leftrightarrow 2\tan^2 x+2\sin^2 x(1-\tan^2 x)=4\sin^2 x$	
	$\Leftrightarrow 2\tan^2 x-2\sin^2 x\tan^2 x=2\sin^2 x$	
	$\Leftrightarrow 2\tan^2 x(1-\sin^2 x)=2\sin^2 x$	3^{rd} M1
	$\div 2\tan^2 x\Leftrightarrow 1-\sin^2 x=\cos^2 x$	A1
	As this is true, initial statement is true	A1*
		(5)

Q2

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{\cos 3\theta}{\sin\theta}+\frac{\sin 3\theta}{\cos\theta}=\frac{\cos 3\theta\cos\theta+\sin 3\theta}{\sin\theta\cos\theta}$	M1	This mark is given for a method to form a single fraction
	$=\frac{\cos(3\theta-\theta)}{\sin\theta\cos\theta}$	M1	This mark is given for a method to use a compound angle formula on the numerator
	$=\frac{\cos 2\theta}{\frac{1}{2}\sin 2\theta}$	M1	This mark is given for a method to use a compound angle formula on the denominator
	$=2\cot 2\theta$	A1	This mark is given for a fully correct proof to show the answer required

Q3

Question Number	Scheme		Marks
(i)	Way 1: Divides by $\cos 3\theta$ to give $\tan 3\theta=\sqrt{3}$ so $(3\theta)=\frac{\pi}{3}$	Or Way 2: Squares both sides, uses $\cos^2 3\theta+\sin^2 3\theta=1$, obtains $\cos 3\theta=\pm\frac{1}{2}$ or $\sin 3\theta=\pm\frac{\sqrt{3}}{2}$ so $(3\theta)=\frac{\pi}{3}$	M1
	Adds π or 2π to previous value of angle (to give $\frac{4\pi}{3}$ or $\frac{7\pi}{3}$)		M1
	So $\theta=\frac{\pi}{9},\frac{4\pi}{9},\frac{7\pi}{9}$ (all three, no extra in range)		A1 (3)
(ii)(a)	$4(1-\cos^2 x)+\cos x=4-k$	Applies $\sin^2 x=1-\cos^2 x$	M1
	Attempts to solve $4\cos^2 x-\cos x-k=0$, to give $\cos x=$		dM1
	$\cos x=\frac{1\pm\sqrt{1+16k}}{8}$ or $\cos x=\frac{1}{8}\pm\sqrt{\frac{1}{64}+\frac{k}{4}}$ or other correct equivalent		A1 (3)

Notes
(i) **M1**: Obtains $\frac{\pi}{3}$. Allow $x=\frac{\pi}{3}$ or even $\theta=\frac{\pi}{3}$. Need not see working here. May be implied by $\theta=\frac{\pi}{9}$ in final answer (allow $(3\theta)=1.05$ or $\theta=0.349$ as decimals or $(3\theta)=60$ or $\theta=20$ as degrees for this mark) Do not allow $\tan 3\theta=-\sqrt{3}$ nor $\tan 3\theta=\pm\frac{1}{\sqrt{3}}$ **M1**: Adding π or 2π to a previous value however obtained. It is not dependent on the previous mark. (May be implied by final answer of $\theta=\frac{4\pi}{9}$ **or** $\frac{7\pi}{9}$). This mark may also be given for answers as decimals [4.19 or 7.33], or degrees (240 or 420). **A1**: Need all three correct answers in terms of π and **no extras in range**. **Three correct answers implies M1M1A1** NB : $\theta=20°$, $80°$, $140°$ earns M1M1A0 and 0.349, 1.40 and 2.44 earns M1M1A0 (ii) (a) **M1:** Applies $\sin^2 x=1-\cos^2 x$ (allow even if brackets are missing e.g. $4\times 1-\cos^2 x$). This must be awarded in (ii) (a) for an expression with k not after $k=3$ is substituted. **dM1**: Uses formula or completion of square to obtain $\cos x=$ expression in k (Factorisation attempt is M0) **A1:** cao - award for their final simplified expression

Q4

Question Number	Scheme	Marks
(a)	$\sec 2A+\tan 2A=\frac{1}{\cos 2A}+\frac{\sin 2A}{\cos 2A}$	**B1**
	$=\frac{1+\sin 2A}{\cos 2A}$	M1
	$=\frac{1+2\sin A\cos A}{\cos^2 A-\sin^2 A}$	M1
	$=\frac{\cos^2 A+\sin^2 A+2\sin A\cos A}{\cos^2 A-\sin^2 A}$	
	$=\frac{(\cos A+\sin A)(\cos A+\sin A)}{(\cos A+\sin A)(\cos A-\sin A)}$	M1
	$=\frac{\cos A+\sin A}{\cos A-\sin A}$	A1*
		(5)

(a)

B1 A correct identity for $\sec 2A = \frac{1}{\cos 2A}$ **OR** $\tan 2A = \frac{\sin 2A}{\cos 2A}$.

It need not be in the proof and it could be implied by the sight of $\sec 2A = \frac{1}{\cos^2 A - \sin^2 A}$

M1 For setting their expression as a single fraction. The denominator must be correct for their fractions and at least two terms on the numerator.

This is usually scored for $\frac{1+\cos 2A \tan 2A}{\cos 2A}$ or $\frac{1+\sin 2A}{\cos 2A}$

M1 For getting an expression in just $\sin A$ and $\cos A$ by using the double angle identities

$\sin 2A = 2\sin A\cos A$ and $\cos 2A = \cos^2 A - \sin^2 A$, $2\cos^2 A - 1$ or $1 - 2\sin^2 A$.

Alternatively for getting an expression in just $\sin A$ and $\cos A$ by using the double angle identities $\sin 2A = 2\sin A\cos A$ and $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$ with $\tan A = \frac{\sin A}{\cos A}$.

For example $= \frac{1}{\cos^2 A - \sin^2 A} + \frac{2\sin A/\cos A}{1 - \sin^2 A/\cos^2 A}$ is B1M0M1 so far

M1 In the main scheme it is for replacing 1 by $\cos^2 A + \sin^2 A$ and factorising both numerator and denominator

A1* Cancelling to produce given answer with no errors.

Allow a consistent use of another variable such as θ, but mixing up variables will lose the A1*.

Question Number	Scheme	Marks
Alt I **From RHS**	$\frac{\cos A+\sin A}{\cos A-\sin A}=\frac{\cos A+\sin A}{\cos A-\sin A}\times\frac{\cos A+\sin A}{\cos A+\sin A}$ $=\frac{\cos^2 A+\sin^2 A+2\sin A\cos A}{\cos^2 A-\sin^2 A}$ $=\frac{1+\sin 2A}{\cos 2A}$ $=\frac{1}{\cos 2A}+\frac{\sin 2A}{\cos 2A}$ $=\sec 2A+\tan 2A$	(Pythagoras) **M1** (Double Angle) **M1** (Single Fraction) **M1** **B1**(Identity), **A1***
Alt II **Both sides**	Assume true $\sec 2A+\tan 2A=\frac{\cos A+\sin A}{\cos A-\sin A}$ $\frac{1}{\cos 2A}+\frac{\sin 2A}{\cos 2A}=\frac{\cos A+\sin A}{\cos A-\sin A}$ $\frac{1+\sin 2A}{\cos 2A}=\frac{\cos A+\sin A}{\cos A-\sin A}$ $\frac{1+2\sin A\cos A}{\cos^2 A-\sin^2 A}=\frac{\cos A+\sin A}{\cos A-\sin A}$ $\times(\cos A-\sin A)\Rightarrow\frac{1+2\sin A\cos A}{\cos A+\sin A}=\cos A+\sin A$ $1+2\sin A\cos A=\cos^2 A+2\sin A\cos A+\sin^2 A=1+2\sin A\cos A$ **True**	B1 (identity) M1 (single fraction) M1(double angles) M1(Pythagoras)A1*
Alt 111 **Very difficult**	$\sec 2A+\tan 2A=\frac{1}{\cos 2A}+\tan 2A$ $=\frac{1}{\cos 2A}+\frac{2\tan A}{1-\tan^2 A}$ $=\frac{1-\tan^2 A+2\tan A\cos 2A}{\cos 2A(1-\tan^2 A)}$ $=\frac{1-\tan^2 A+2\tan A(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)(1-\tan^2 A)}$ $=\frac{1-\frac{\sin^2 A}{\cos^2 A}+2\frac{\sin A}{\cos A}(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)\left(1-\frac{\sin^2 A}{\cos^2 A}\right)}$ $\times\cos^2 A=\frac{\cos^2 A-\sin^2 A+2\sin A\cos A(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)(\cos^2 A-\sin^2 A)}$ $=\frac{(\cos^2 A-\sin^2 A)(1+2\sin A\cos A)}{(\cos^2 A-\sin^2 A)(\cos^2 A-\sin^2 A)}$ Final two marks as in main scheme	(Identity) **B1** (Single fraction) **M1** (Double Angle and in just sin and cos) **M1** **M1A1***

Q5

Question Number	Scheme		Marks	
(a)(i)	$\sin 3\theta = \sin(2\theta + \theta)$			
	$= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$			
	$= 2\sin\theta\cos\theta.\cos\theta + (1 - 2\sin^2\theta)\sin\theta$		M1 A1	
	$= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$		M1	
	$= 3\sin\theta - 4\sin^3\theta$ ✱	cso	A1	(4)
(ii)	$8\sin^3\theta - 6\sin\theta + 1 = 0$			
	$-2\sin 3\theta + 1 = 0$		M1 A1	
	$\sin 3\theta = \frac{1}{2}$		M1	
	$3\theta = \frac{\pi}{6}, \frac{5\pi}{6}$			
	$\theta = \frac{\pi}{18}, \frac{5\pi}{18}$		A1 A1	(5)
(b)	$\sin 15° = \sin(60° - 45°) = \sin 60° \cos 45° - \cos 60° \sin 45°$		M1	
	$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$		M1 A1	
	$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ ✱	cso	A1	(4)
				[13]
	Alternatives to (b)			
	① $\sin 15° = \sin(45° - 30°) = \sin 45° \cos 30° - \cos 45° \sin 30°$		M1	
	$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$		M1 A1	
	$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ ✱	cso	A1	(4)
	② Using $\cos 2\theta = 1 - 2\sin^2\theta$, $\cos 30° = 1 - 2\sin^2 15°$			
	$2\sin^2 15° = 1 - \cos 30° = 1 - \frac{\sqrt{3}}{2}$			
	$\sin^2 15° = \frac{2 - \sqrt{3}}{4}$		M1 A1	
	$\left(\frac{1}{4}(\sqrt{6} - \sqrt{2})\right)^2 = \frac{1}{16}(6 + 2 - 2\sqrt{12}) = \frac{2 - \sqrt{3}}{4}$		M1	
	Hence $\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ ✱	cso	A1	(4)

Platinum Questions

Non-calculator

The total mark for this section is 33

Q1

(*a*) Prove the identity

$$(\sin x + \cos y)\cos(x - y) \equiv (1 + \sin(x - y))(\cos x + \sin y)$$

(5)

(*b*) Hence, or otherwise, show that

$$\frac{\sin 5\theta + \cos 3\theta}{\cos 5\theta + \sin 3\theta} \equiv \frac{1 + \tan\theta}{1 - \tan\theta}$$

(6)

(*c*) Given that $k > 1$, show that the equation $\dfrac{\sin 5\theta + \cos 3\theta}{\cos 5\theta + \sin 3\theta} = k$ has a unique solution in the interval $0 < \theta < \dfrac{\pi}{4}$

(4)

(+S2)

(Total for Question 1 is 17 marks)

Q2

Solve, for $0 \leqslant x \leqslant 360°$,

$$\sin 47° \cos^3 x + \cos 47° \sin x \cos^2 x = \frac{1}{2}\cos^2 x.$$

(7)

(Total for Question 2 is 7 marks)

Q3

Solve for $0 < x < 360°$

$$\cot 2x - \tan 78° = \frac{(\sec x)(\sec 78°)}{2}$$

where x is not an integer multiple of 90°.

(9)

(Total for Question 3 is 9 marks)

End of Questions

Platinum Mark Scheme

Q1

Question	Scheme	Marks	Notes
4. (a)	LHS $\equiv S_xC_xC_y + S_x^2S_y + C_xC_y^2 + S_xS_yC_y$	M1	Applies $\cos(x-y)$ formula and expands the brackets.
	$\equiv S_xC_xC_y + (1 - C_x^2)\,S_y + C_x(1 - S_y^2) + S_xS_yC_y$	M1	Replaces $\cos^2x$ by $1 - \sin^2x$ and $\sin^2x$ by $1 - \cos^2x$ respectively
	$\equiv S_y + C_x + C_x(S_xC_y - C_xS_y) + S_y(S_xC_y - C_xS_y)$	M1	Expands, rearranges and factorises appropriately
	$\equiv S_y + C_x + (S_y + C_x)(S_xC_y - C_xS_y)$	M1	Factors out the $(S_y + C_x)$
	$\equiv (S_y + C_x)(1 + \sin(x-y)) \equiv$ RHS	A1	Applies $\sin(x-y)$ formula and completes to RHS (no conclusion needed this way)
		(5)	(S+ for a direct approach)
	Alt 1 $\cos(x-y) = \cos x\cos y + \sin x\sin y$ and $\sin(x-y) = \sin x\cos y - \cos x\sin y$	M1	Use of both expansions (NB may be awarded for use of one and complete expansion of one side as per main)
	$S_xC_xC_y + S_x^2S_y + C_xC_y^2 + S_xS_yC_y$ $\equiv C_x + S_y + S_xC_xC_y - C_x^2S_y + S_xS_yC_y - C_xS_y^2$	M1	Full expansion both sides (may be seen separately)
	$\Leftrightarrow S_x^2S_y + C_xC_y^2 \equiv C_x + S_y - C_x^2S_y - C_xS_y^2$	M1	Cancelling terms
	$\Leftrightarrow S_x^2S_y + C_xC_y^2 \equiv C_x\left(1 - S_y^2\right) + S_y\left(1 - C_x^2\right)$	M1	Use of relevant trig. identities;
	$\equiv C_xC_y^2 + S_x^2S_y$ Hence the result is true	A1 (S+)	all correct, with concluding statement.
		(5)	
	Alt 2 $\sin x\cos(x-y) + \cos y\cos(x-y) \equiv$ $\cos x + \sin y + \cos x\sin(x-y) + \sin y\sin(x-y)$	M1	Multiplying out both sides. May be done as separate statements
	$\Leftrightarrow \left[\sin x\cos(x-y) - \cos x\sin(x-y)\right]$ $+ \left[\cos y\cos(x-y) - \sin y\sin(x-y)\right] \equiv$ $\cos x + \sin y$	M1 M1	(Equates sides and) attempts to rearrange to useful form Collecting into useable groupings
	Then L $\equiv \sin\left(x - (x-y)\right) + \cos\left(y + (x-y)\right)$	M1	Use of $\sin(A - B)$ and $\cos(A + B)$ formulae
	$\equiv \sin(y) + \cos(x) \equiv$ R Hence the identity is true	A1 (S+)	All correctly shown and conclusion
		(5)	

(b)	Re-arranging and setting $x = 5\theta, y = 3\theta$ $\Rightarrow \frac{\sin 5\theta + \cos 3\theta}{\cos 5\theta + \sin 3\theta} \equiv \frac{1 + \sin 2\theta}{\cos 2\theta}$	M1	Use of **(a)**'s result
	$= \frac{1 + 2sc}{c^2 - s^2}$	M1 A1	Use of double-angle formulae
	$= \frac{(c+s)^2}{(c-s)(c+s)} = \frac{c+s}{c-s}$ *	M1	Factorisation & cancelling
	$= \frac{c/c + s/c}{c/c - s/c} = \frac{1+t}{1-t}$ **	M1 A1	Converting to tans **Given Answer** all correctly obtained
		(6)	
(c)	$k = \frac{1+t}{1-t} \Rightarrow k - kt = 1 + t \Rightarrow k - 1 = t(k+1)$ $\Rightarrow \tan\theta = \frac{k-1}{k+1}$ or $\theta = \arctan\frac{k-1}{k+1}$	M1 A1	Rearranging to tan θ = …
	Explain 1-1-ness of mapping $k \to \theta$	B1	e.g by graph of $y = \frac{x-1}{x+1}$ or its gradient > 0 always
	$0 < k - 1 < k + 1$ so $0 < \frac{k-1}{k+1} < 1$ $\Rightarrow 0 < \tan\theta < 1 \Rightarrow 0 < \theta < \frac{1}{4}\pi$	B1	For convincing reasoning that each k gives a θ in the required interval
		(4)	
S2	**S1**: Award S1 for a clear solution that EITHER Scores 9+ in (a) and (b) with one part fully correct and concise OR that scores 12+ in total and includes an S+ point but may be laboured in places. **S2**: Award S2 for a clear **and concise** solution throughout that scores at least 12 marks and includes an S+ point.	(2)	
	S+ opportunities: for a direct proof or correct use of ⇔ notation through proof in (a) for noting that we require $c + s \neq 0$ (i.e. $\tan\theta \neq -1$ or $k \neq 0$) at * and/or that $t = \tan\theta \neq 1$ (i.e. $k \neq 1$) at ** for completely clear handling of the trig. identities throughout for any innovative ways used throughout the question.		
			Total 15 + 2 marks

Q2

2.	$\cos^2 x(\sin 47\cos x + \cos 47\sin x) = \frac{1}{2}\cos^2 x$	M1	Factor or cancel $\cos^2 x$
	$\cos^2 x\sin(47 + x) = \frac{1}{2}\cos^2 x$	M1	Use of $\sin(A + B)$
	$[\cos^2 x(2\sin(47 + x) - 1) = 0]$ so $\cos x = 0$ and $x = \mathbf{90,\ 270}$	M1A1	A1 for both
	$\sin(47 + x) = \frac{1}{2}$; so $47 + x = 30,\ 150,\ 390$	M1	M1 for any one for 47+x
	$x = \mathbf{103,\ 343}$	A1,A1	Deduct only from last 2 A marks
	Ignore solutions out of range, extras in range deduct A1 each upto A2	[7]	

Q3

3.	$\text{LHS} = \frac{\cos 2x}{\sin 2x} - \frac{\sin 78}{\cos 78}$	M1	Cot and tan to sin and cos
	$\cos(2x + 78) = \frac{1}{2}(\sin 2x\cos 78\sec x\sec 78)$	M1	Use of $\cos(A + B)$
	$[\cos(2x + 78) =]\ \frac{1}{2}[2\sin x\cos x\cos 78\sec 78\sec x]$	M1	Use of $\sin 2x$ and some cancelling
	$\cos(2x + 78) = \sin x$ or $\cos(2x + 78) = \cos(90 - x)$	A1	
	$2x + 78 = 90 - x$	M1	Non-trig eqn in x Allow $90 \pm x$
	$\underline{\mathbf{x = 4}}$	A1	
	$2x + 78 = 270 + x$ $\underline{\mathbf{x = 192}}$		Award A1 for each of these 3 solutions found. Extras inside the range -1 i.e allow upto 4 answers. If more than 4 then deduct 1 for each in range
	$2x + 78 = 450 - x$ $\underline{\mathbf{x = 124}}$	A3/2/1	
	$2x + 78 = 810 - x$ $\underline{\mathbf{x = 244}}$		
		[9]	

Topic 8: Parametric Equations

(Includes Parametric Differentiation and Integration)

Bronze, Silver, Gold and Platinum Worksheets for

A Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel A Level Mathematics: Pure Mathematics Year 2' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Non-calculator

The total mark for this section is 27

Q1

A curve C has parametric equations

$$x = 2t - 1, \quad y = 4t - 7 + \frac{3}{t}, \quad t \neq 0$$

Show that the Cartesian equation of the curve C can be written in the form

$$y = \frac{2x^2 + ax + b}{x + 1}, \quad x \neq -1$$

Where a and b are integers to be found.

(3)

(Total for Question 1 is 3 marks)

Q2

The curve C has parametric equations

$$x = 3t - 4, \ y = 5 - \frac{6}{t}, \quad t > 0$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.

(2)

The point P lies on C where $t = \frac{1}{2}$.

(b) Find the equation of the tangent to C at the point P. Give your answer in the form $y = px + q$, where p and q are integers to be determined.

(3)

(c) Show that the Cartesian equation for C can be written in the form

$$y = \frac{ax + b}{x + 4}, \quad x > -4$$

Where a and b are integers to be determined.

(3)

(Total for Question 2 is 8 marks)

Q3

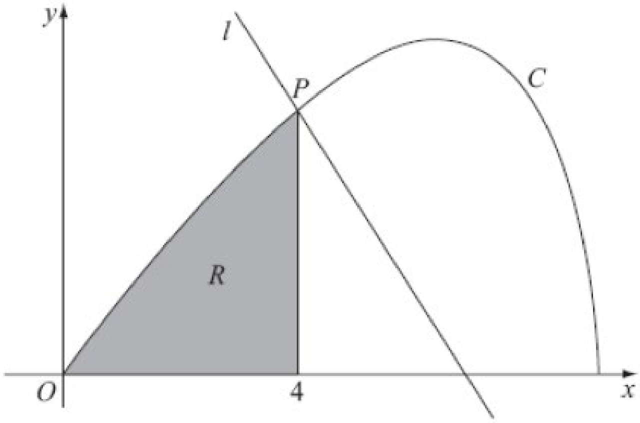


Figure 3

Figure 3 shows the curve C with parametric equations

$$x = 8\cos t, \qquad y = 4\sin 2t, \qquad 0 \leq t \leq \frac{\pi}{2}.$$

The point P lies on C and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of t at the point P.

(2)

The line l is a normal to C at P.

(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$.

(6)

The finite region R is enclosed by the curve C, the x-axis and the line $x = 4$, as shown shaded in Figure 3.

(c) Show that the area of R is given by the integral $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64 \sin^2 t \cos t \, dt$.

(4)

(d) Use this integral to find the area of R, giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(4)

(Total for Question 3 is 16 marks)

End of Questions

Bronze Mark Scheme

Q1

Question	Scheme	Marks	AOs
	Attempts to substitute $=\dfrac{x+1}{2}$ into $y \Rightarrow y = 4\left(\dfrac{x+1}{2}\right)-7 + \dfrac{6}{(x+1)}$	M1	2.1
	Attempts to write as a single fraction $y = \dfrac{(2x-5)(x+1)+6}{(x+1)}$	M1	2.1
	$y = \dfrac{2x^2-3x+1}{x+1} \quad a=-3, b=1$	A1	1.1b
		(3 marks)	

Notes:

M1: Score for an attempt at substituting $t = \dfrac{x+1}{2}$ or equivalent into $y = 4t - 7 + \dfrac{3}{t}$

M1: Award this for an attempt at a single fraction with a correct common denominator.

Their $4\left(\dfrac{x+1}{2}\right)-7$ term may be simplified first

A1: Correct answer only $y = \dfrac{2x^2-3x+1}{x+1} \quad a=-3, b=1$

Q2

Question Number	Scheme	Notes	Marks
	$x = 3t - 4,\ y = 5 - \frac{6}{t},\quad t > 0$		
(a)	$\frac{dx}{dt} = 3,\ \frac{dy}{dt} = 6t^{-2}$		
	$\frac{dy}{dx} = \frac{6t^{-2}}{3}\ \left\{= \frac{6}{3t^2} = 2t^{-2} = \frac{2}{t^2}\right\}$	their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ to give $\frac{dy}{dx}$ in terms of t **or** their $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$ to give $\frac{dy}{dx}$ in terms of t	M1
		$\frac{6t^{-2}}{3}$, simplified or un-simplified, in terms of t. **See note.**	A1 isw
	Award **Special Case 1st M1** if **both** $\frac{dx}{dt}$ **and** $\frac{dy}{dt}$ are stated **correctly** and **explicitly**.		**[2]**
	Note: You can recover the work for part (a) in part (b).		
(a) **Way 2**	$y = 5 - \frac{18}{x+4} \Rightarrow \frac{dy}{dx} = \frac{18}{(x+4)^2} = \frac{18}{(3t)^2}$	Writes $\frac{dy}{dx}$ in the form $\frac{\pm\lambda}{(x+4)^2}$, ***and*** writes $\frac{dy}{dx}$ as a function of t.	M1
		Correct un-simplified or simplified answer, in terms of t. **See note.**	A1 isw
			[2]
(b)	$\left\{t = \frac{1}{2} \Rightarrow\right\} P\left(-\frac{5}{2}, -7\right)$	$x = -\frac{5}{2},\ y = -7$ **or** $P\left(-\frac{5}{2}, -7\right)$ seen or implied.	B1
	$\frac{dy}{dx} = \frac{2}{\left(\frac{1}{2}\right)^2}$ **and either** • $y - "-7" = "8"\left(x - "-\frac{5}{2}"\right)$ • $"-7" = ("8")("-\frac{5}{2}") + c$ So, $y = (\text{their } m_T)x + "c"$	**Some** attempt to substitute $t = 0.5$ into their $\frac{dy}{dx}$ which contains t in order to find m_T **and either** applies $y - (\text{their } y_p) = (\text{their } m_T)(x - \text{their } x_p)$ **or** finds c from $(\text{their } y_p) = (\text{their } m_T)(\text{their } x_p) + c$ and uses their numerical c in $y = (\text{their } m_T)x + c$	M1
	T: $y = 8x + 13$	$y = 8x + 13$ or $y = 13 + 8x$	A1 cso
	Note: their x_P, their y_P and their m_T must be numerical values in order to award M1		**[3]**
(c) **Way 1**	$\left\{t = \frac{x+4}{3} \Rightarrow\right\} y = 5 - \frac{6}{\left(\frac{x+4}{3}\right)}$	An attempt to eliminate t. **See notes.**	M1
		Achieves a correct equation in x and y only	A1 o.e.
	$\Rightarrow y = 5 - \frac{18}{x+4} \Rightarrow y = \frac{5(x+4) - 18}{x+4}$		
	So, $y = \frac{5x+2}{x+4},\ \{x > -4\}$	$y = \frac{5x+2}{x+4}$ (or implied equation)	A1 cso
			[3]
(c) **Way 2**	$\left\{t = \frac{6}{5-y} \Rightarrow\right\} x = \frac{18}{5-y} - 4$	An attempt to eliminate t. **See notes.**	M1
		Achieves a correct equation in x and y only	A1 o.e.
	$\Rightarrow (x+4)(5-y) = 18 \Rightarrow 5x - xy + 20 - 4y = 18$		
	$\{\Rightarrow 5x + 2 = y(x+4)\}$ So, $y = \frac{5x+2}{x+4},\ \{x > -4\}$	$y = \frac{5x+2}{x+4}$ (or implied equation)	A1 cso
			[3]
	Note: Some or all of the work for part (c) can be recovered in part (a) or part (b)		**8**

Question Number	Scheme	Notes	Marks
(c) **Way 3**	$y = \dfrac{3at - 4a + b}{3t - 4 + 4} = \dfrac{3at}{3t} - \dfrac{4a - b}{3t} = a - \dfrac{4a-b}{3t} \Rightarrow a = 5$	A full method leading to the value of a being found	M1
		$y = a - \dfrac{4a-b}{3t}$ and $a = 5$	A1
	$\dfrac{4a-b}{3} = 6 \Rightarrow b = 4(5) - 6(3) = 2$	**Both** $a = 5$ **and** $b = 2$	A1
			[3]

Question Notes

(a)	**Note**	Condone $\dfrac{dy}{dx} = \dfrac{\left(\frac{6}{t^2}\right)}{3}$ for A1
	Note	You can ignore subsequent working following on from a correct expression for $\dfrac{dy}{dx}$ in terms of t.
(b)	**Note**	Using a changed gradient (i.e. applying $\dfrac{-1}{\text{their } \frac{dy}{dx}}$ or $\dfrac{1}{\text{their } \frac{dy}{dx}}$ or $-\left(\text{their } \frac{dy}{dx}\right)$) is M0.
	Note	**Final A1:** A correct solution is required from a correct $\dfrac{dy}{dx}$.
	Note	**Final A1:** You can ignore subsequent working following on from a correct solution.
(c)	**Note**	1st **M1:** A full attempt to eliminate t is defined as either • rearranging one of the parametric equations to make t the subject and substituting for t in the other parametric equation (only the RHS of the equation required for M mark) • rearranging both parametric equations to make t the subject and putting the results equal to each other.
	Note	Award M1A1 for $\dfrac{6}{5-y} = \dfrac{x+4}{3}$ or equivalent.

Q3

Question Number	Scheme	Marks
(a)	At $P(4, 2\sqrt{3})$ either $\underline{4 = 8\cos t}$ or $\underline{2\sqrt{3} = 4\sin 2t}$	M1
	$\Rightarrow$ only solution is $\underline{t = \frac{\pi}{3}}$ where $0 \leqslant t \leqslant \frac{\pi}{2}$	A1
(b)	$x = 8\cos t, \quad y = 4\sin 2t$	
	$\frac{dx}{dt} = -8\sin t, \quad \frac{dy}{dt} = 8\cos 2t$	M1 A1
	At P, $\frac{dy}{dx} = \underline{\frac{8\cos\left(\frac{2\pi}{3}\right)}{-8\sin\left(\frac{\pi}{3}\right)}}$	M1
	$\left\{ = \frac{8\left(-\frac{1}{2}\right)}{(-8)\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58 \right\}$	
	Hence $m(N) = -\sqrt{3}$ or $\frac{-1}{\frac{1}{\sqrt{3}}}$	M1
	N: $y - 2\sqrt{3} = -\sqrt{3}(x - 4)$	M1
	N: $\underline{y = -\sqrt{3}x + 6\sqrt{3}}$ (*)	A1 cso (6)
(c)	$A = \int_0^4 y\, dx = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 4\sin 2t.(-8\sin t)\, dt$	M1 A1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32\sin 2t.\sin t\, dt = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32(2\sin t\cos t).\sin t\, dt$	M1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -64.\sin^2 t\cos t\, dt$	
	$A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64.\sin^2 t\cos t\, dt$ (*)	A1 (4)
(d)	$A = 64\left[\frac{\sin^3 t}{3}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ or $A = 64\left[\frac{u^3}{3}\right]_{\frac{\sqrt{3}}{2}}^{1}$	M1 A1
	$A = 64\left[\frac{1}{3} - \left(\frac{1}{3}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}\right)\right]$	M1
	$A = 64\left(\frac{1}{3} - \frac{1}{8}\sqrt{3}\right) = \underline{\frac{64}{3} - 8\sqrt{3}}$	A1 (4)
		(16 marks)

Silver Questions

Non-calculator

The total mark for this section is 28

Q1

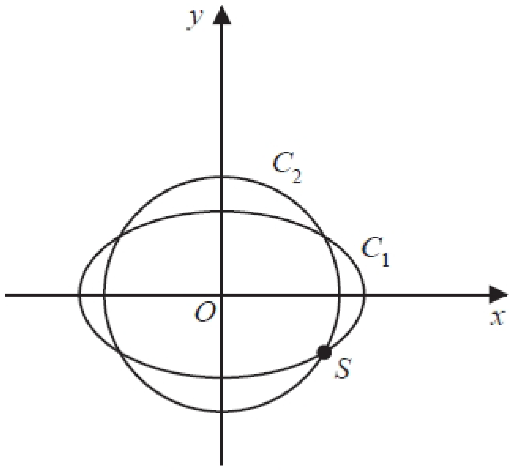


Figure 2

The curve C_1 with parametric equations

$$x = 10\cos t, \quad y = 4\sqrt{2}\sin t, \quad 0 \leqslant t < 2\pi$$

Meets the circle C_2 with equation

$$x^2 + y^2 = 66$$

At four distinct points as shown in Figure 2.

Given that one of these points, S, lies in the 4th quadrant, find the Cartesian coordinates of S.

(6)

(Total for Question 1 is 6 marks)

Q2

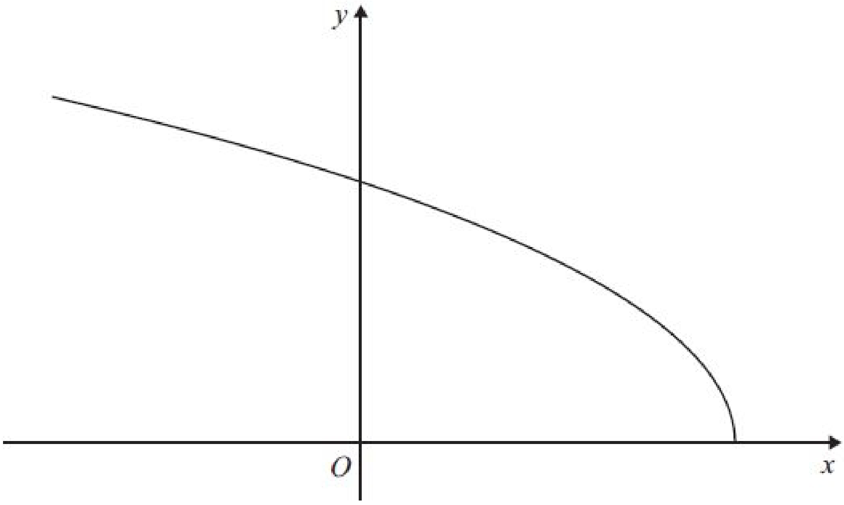


Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = 2\cos 2t, \qquad y = 6\sin t, \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

(a) Find the gradient of the curve at the point where $t = \frac{\pi}{3}$.

(4)

(b) Find a Cartesian equation of the curve in the form

$$y = \mathrm{f}(x), \quad -k \leqslant x \leqslant k,$$

Stating the value of the constant k.

(4)

(c) Write down the range of $\mathrm{f}(x)$.

(2)

(Total for Question 2 is 10 marks)

Q3

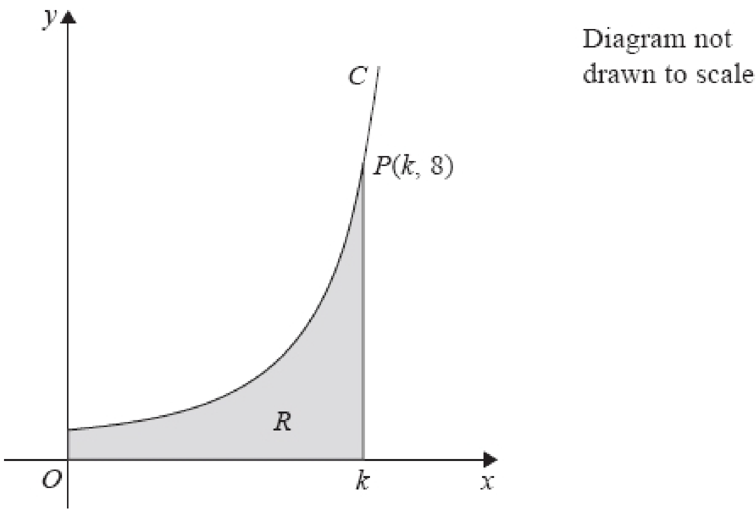


Figure 4

Figure 4 shows a sketch of part of the curve C with parametric equations

$$x = 3\theta \sin\theta, \quad y = \sec^3\theta, \quad 0 \leq \theta \leq \frac{\pi}{2}$$

The point $P(k, 8)$ lies on C, where k is a constant.

(a) Find the exact value of k.

(2)

The finite region R, shown shaded in Figure 4, is bounded by the curve C, the y-axis, the x-axis and the line with equation $x = k$.

(b) Show that the area of R can be expressed in the form

$$\lambda \int_{\alpha}^{\beta} \left(\theta \sec^2\theta + \tan\theta \sec^2\theta\right) \mathrm{d}\theta$$

Where λ, α and β are constants to be determined.

(4)

(c) Hence use integration to find the exact value of the area of R.

(6)

(Total for Question 3 is 12 marks)

End of Questions

<u>Silver Mark Scheme</u>

Q1

Part	Working or answer an examiner might expect to see	Mark	Notes
	$(10 \cos t)^2 + (4\sqrt{2} \sin t)^2 = 66$	M1	This mark is given for combining the two equations to show where the curve and circle meet
	$100 (\cos t)^2 + 32(1 - \cos t)^2 = 66$	M1	This mark is given for forming an equation in cos t only
	$68 \cos^2 t = 34$	A1	This mark is given for simplifying to find an equation in terms of cos t
	$\cos t = \pm\frac{1}{\sqrt{2}} \quad \Rightarrow \; t = \frac{\pi}{4}$	M1	This mark is given for finding a value for t
	$x = 10 \times \frac{1}{\sqrt{2}}$ $y = 4\sqrt{2} \times -\sin \frac{\pi}{4} = 4\sqrt{2} \times -\frac{1}{\sqrt{2}}$	M1	This mark is given for a method to substitute back into the original equations to find value for x and y
	$S = (5\sqrt{2}, -4)$	A1	This mark is given for the correct coordinates of S
			(Total 6 marks)

Q2

Question Number	Scheme	Marks
Q (a)	$\frac{dx}{dt} = -4\sin 2t$, $\frac{dy}{dt} = 6\cos t$	B1, B1
	$\frac{dy}{dx} = -\frac{6\cos t}{4\sin 2t}$ $\left(= -\frac{3}{4\sin t}\right)$	M1
	At $t = \frac{\pi}{3}$, $m = -\frac{3}{4\times\frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{2}$ accept equivalents, awrt -0.87	A1 (4)
(b)	Use of $\cos 2t = 1 - 2\sin^2 t$	M1
	$\cos 2t = \frac{x}{2}$, $\sin t = \frac{y}{6}$	
	$\frac{x}{2} = 1 - 2\left(\frac{y}{6}\right)^2$	M1
	Leading to $y = \sqrt{(18-9x)}$ $\left(= 3\sqrt{(2-x)}\right)$ cao	A1
	$-2 \le x \le 2$ $k = 2$	B1 (4)
(c)	$0 \le f(x) \le 6$ either $0 \le f(x)$ or $f(x) \le 6$	B1
	Fully correct. Accept $0 \le y \le 6$, $[0, 6]$	B1 (2)
		[10]
	Alternatives to (a) where the parameter is eliminated	
	① $y = (18-9x)^{\frac{1}{2}}$	
	$\frac{dy}{dx} = \frac{1}{2}(18-9x)^{-\frac{1}{2}} \times (-9)$	B1
	At $t = \frac{\pi}{3}$, $x = \cos\frac{2\pi}{3} = -1$	B1
	$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{\sqrt{(27)}} \times -9 = -\frac{\sqrt{3}}{2}$	M1 A1 (4)
	② $y^2 = 18 - 9x$	
	$2y\frac{dy}{dx} = -9$	B1
	At $t = \frac{\pi}{3}$, $y = 6\sin\frac{\pi}{3} = 3\sqrt{3}$	B1
	$\frac{dy}{dx} = -\frac{9}{2\times 3\sqrt{3}} = -\frac{\sqrt{3}}{2}$	M1 A1 (4)

Q3

Question Number	Scheme	Notes	Marks
	$x = 3\theta\sin\theta,\ y = \sec^3\theta,\ 0 \le \theta < \frac{\pi}{2}$		
(a)	$\{\text{When } y = 8,\}\ 8 = \sec^3\theta \Rightarrow \cos^3\theta = \frac{1}{8} \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$ $k \text{ (or } x) = 3\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{3}\right)$	Sets $y = 8$ to find θ **and** attempts to substitute their θ into $x = 3\theta\sin\theta$	M1
	so $k \text{ (or } x) = \frac{\sqrt{3}\pi}{2}$	$\frac{\sqrt{3}\pi}{2}$ or $\frac{3\pi}{2\sqrt{3}}$	A1
	Note: Obtaining two value for k without accepting the correct value is final A0		[2]
(b)	$\frac{dx}{d\theta} = 3\sin\theta + 3\theta\cos\theta$	$3\theta\sin\theta \to 3\sin\theta + 3\theta\cos\theta$ Can be implied by later working	B1
	$\left\{\int y\frac{dx}{d\theta}\{d\theta\}\right\} = \int(\sec^3\theta)(3\sin\theta + 3\theta\cos\theta)\{d\theta\}$	Applies $(\pm K\sec^3\theta)\left(\text{their } \frac{dx}{d\theta}\right)$ Ignore integral sign and $d\theta$; $K \ne 0$	M1
	$= 3\int \theta\sec^2\theta + \tan\theta\sec^2\theta\, d\theta$	Achieves the correct result no errors in their working, e.g. bracketing or manipulation errors. **Must have** integral sign and $d\theta$ in their final answer.	A1 *
	$x = 0$ and $x = k \Rightarrow \underline{\alpha = 0}$ and $\underline{\beta = \frac{\pi}{3}}$	$\alpha = 0$ and $\beta = \frac{\pi}{3}$ or evidence of $0 \to 0$ and $k \to \frac{\pi}{3}$	B1
	Note: The work for the final B1 mark must be seen in part (b) only.		[4]

(c) Way 1	$\left\{\int \theta\sec^2\theta d\theta\right\} = \theta\tan\theta - \int\tan\theta\{d\theta\}$	$\theta\sec^2\theta \to A\theta g(\theta) - B\int g(\theta), A > 0, B > 0$, where $g(\theta)$is a trigonometric function in θ **and** $g(\theta) =$ their $\int\sec^2\theta d\theta$. [Note: $g(\theta) \ne \sec^2\theta$]	M1
		dependent on the previous M mark **Either** $\lambda\theta\sec^2\theta \to A\theta\tan\theta - B\int\tan\theta, A > 0, B > 0$ **or** $\theta\sec^2\theta \to \theta\tan\theta - \int\tan\theta$	dM1
	$= \theta\tan\theta - \ln(\sec\theta)$ **or** $= \theta\tan\theta + \ln(\cos\theta)$	$\theta\sec^2\theta \to \theta\tan\theta - \ln(\sec\theta)$ **or** $\theta\tan\theta + \ln(\cos\theta)$ **or** $\lambda\theta\sec^2\theta \to \lambda\theta\tan\theta - \lambda\ln(\sec\theta)$ or $\lambda\theta\tan\theta + \lambda\ln(\cos\theta)$	A1
	Note: Condone $\theta\sec^2\theta \to \theta\tan\theta - \ln(\sec x)$ or $\theta\tan\theta + \ln(\cos x)$ for A1		
	$\left\{\int\tan\theta\sec^2\theta d\theta\right\}$	$\tan\theta\sec^2\theta$ or $\lambda\tan\theta\sec^2\theta \to \pm C\tan^2\theta$ or $\pm C\sec^2\theta$ or $\pm Cu^{-2}$, where $u = \cos\theta$	M1
	$= \frac{1}{2}\tan^2\theta$ or $\frac{1}{2}\sec^2\theta$ or $\frac{1}{2u^2}$ where $u = \cos\theta$ or $\frac{1}{2}u^2$ where $u = \tan\theta$	$\tan\theta\sec^2\theta \to \frac{1}{2}\tan^2\theta$ or $\frac{1}{2}\sec^2\theta$ or $\frac{1}{2\cos^2\theta}$ or $\tan^2\theta - \frac{1}{2}\sec^2\theta$ or $0.5u^{-2}$, where $u = \cos\theta$ or $0.5u^2$, where $u = \tan\theta$ or $\lambda\tan\theta\sec^2\theta \to \frac{\lambda}{2}\tan^2\theta$ or $\frac{\lambda}{2}\sec^2\theta$ or $\frac{\lambda}{2\cos^2\theta}$ or $0.5\lambda u^{-2}$, where $u = \cos\theta$ or $0.5\lambda u^2$, where $u = \tan\theta$	A1
	$\{\text{Area}(R)\} = \left[3\theta\tan\theta - 3\ln(\sec\theta) + \frac{3}{2}\tan^2\theta\right]_0^{\frac{\pi}{3}}$ **or** $\left[3\theta\tan\theta - 3\ln(\sec\theta) + \frac{3}{2}\sec^2\theta\right]_0^{\frac{\pi}{3}}$		
	$= \left(3\left(\frac{\pi}{3}\right)\sqrt{3} - 3\ln 2 + \frac{3}{2}(3)\right) - (0)$ **or** $\left(3\left(\frac{\pi}{3}\right)\sqrt{3} - 3\ln 2 + \frac{3}{2}(4)\right) - \left(\frac{3}{2}\right)$		
	$= \frac{9}{2} + \sqrt{3}\pi - 3\ln 2$ **or** $\frac{9}{2} + \sqrt{3}\pi + 3\ln\left(\frac{1}{2}\right)$ **or** $\frac{9}{2} + \sqrt{3}\pi - \ln 8$ **or** $\ln\left(\frac{1}{8}e^{\frac{9}{2}+\sqrt{3}\pi}\right)$		A1 o.e.
			[6]
			12

Question Number	Scheme	Notes	Marks
(c) **Way 2**	**Way 2 for the first 5 marks:** Applying integration by parts on $\int(\theta+\tan\theta)\sec^2\theta\,d\theta$		
	$\int(\theta\sec^2\theta+\tan\theta\sec^2\theta)d\theta=\int(\theta+\tan\theta)\sec^2\theta\,d\theta,$	$\left\{\begin{array}{l} u=\theta+\tan\theta \Rightarrow \frac{du}{d\theta}=1+\sec^2\theta \\ \frac{dv}{d\theta}=\sec^2\theta \Rightarrow v=\tan\theta=g(\theta)\end{array}\right\}$	
	$h(\theta)$ and $g(\theta)$ are trigonometric functions in θ **and** $g(\theta)=$ their $\int\sec^2\theta\,d\theta$. [**Note:** $g(\theta)\neq\sec^2\theta$]		
	$=(\theta+\tan\theta)\tan\theta-\int(1+\sec^2\theta)\tan\theta\{d\theta\}$	$A(\theta+\tan\theta)g(\theta)-B\int(1+h(\theta))g(\theta),\ A>0,\ B>0$	M1
		dependent on the previous M mark **Either** $\lambda\left[(\theta+\tan\theta)\sec^2\theta\right]\rightarrow$ $A(\theta+\tan\theta)\tan\theta-B\int(1+h(\theta))\tan\theta,\ A\neq0,\ B>0$ **or** $(\theta+\tan\theta)\tan\theta-\int(1+h(\theta))\tan\theta$	dM1
	$=(\theta+\tan\theta)\tan\theta-\int(\tan\theta+\tan\theta\sec^2\theta)\{d\theta\}$		
	$=(\theta+\tan\theta)\tan\theta-\ln(\sec\theta)-\int\tan\theta\sec^2\theta\{d\theta\}$	$(\theta+\tan\theta)\tan\theta-\ln(\sec\theta)$ o.e. or $\lambda\left[(\theta+\tan\theta)\tan\theta-\ln(\sec\theta)\right]$ o.e.	A1
	$=(\theta+\tan\theta)\tan\theta-\ln(\sec\theta)-\frac{1}{2}\tan^2\theta$ or $=(\theta+\tan\theta)\tan\theta-\ln(\sec\theta)-\frac{1}{2}\sec^2\theta$ etc.	$\tan\theta\sec^2\theta\rightarrow\pm C\tan^2\theta$ or $\pm C\sec^2\theta$	M1
		$(\theta+\tan\theta)\tan\theta-\frac{1}{2}\tan^2\theta$ or $(\theta+\tan\theta)\tan\theta-\frac{1}{2}\sec^2\theta$	A1
	Note	Allow the first two marks in part (c) for $\theta\tan\theta-\int\tan\theta$ embedded in their working	
	Note	Allow the first three marks in part (c) for $\theta\tan\theta-\ln(\sec\theta)$ embedded in their working	
	Note	Allow 3rd M1 2nd A1 marks for either $\tan^2\theta-\frac{1}{2}\tan^2\theta$ or $\tan^2\theta-\frac{1}{2}\sec^2\theta$ embedded in their working	

	Question Notes	
(a)	**Note**	Allow M1 for an answer of $k=$ awrt 2.72 without reference to $\frac{\sqrt{3}\pi}{2}$ or $\frac{3\pi}{2\sqrt{3}}$
	Note	Allow M1 for an answer of $k=3\left(\arccos(\tfrac{1}{2})\right)\sin\left(\arccos(\tfrac{1}{2})\right)$ without reference to $\frac{\sqrt{3}\pi}{2}$ or $\frac{3\pi}{2\sqrt{3}}$
	Note	E.g. allow M1 for $\theta=60°$, leading to $k=3(60)\sin(60)$ or $k=90\sqrt{3}$

		Question Notes Continued
(b)	**Note**	To gain A1, $\mathrm{d}\theta$ does not need to appear until they obtain $3\int(\theta\sec^2\theta+\tan\theta\sec^2\theta)\mathrm{d}\theta$
	Note	For M1, their $\frac{\mathrm{d}x}{\mathrm{d}\theta}$, where their $\frac{\mathrm{d}x}{\mathrm{d}\theta}\neq 3\theta\sin\theta$, needs to be a trigonometric function in θ
	Note	Writing $\int(\sec^3\theta)(3\sin\theta+3\theta\cos\theta)=3\int(\theta\sec^2\theta+\tan\theta\sec^2\theta)\mathrm{d}\theta$ is sufficient for B1M1A1
	Note	Writing $\frac{\mathrm{d}x}{\mathrm{d}\theta}=3\sin\theta+3\theta\cos\theta$ followed by writing $\int y\frac{\mathrm{d}x}{\mathrm{d}\theta}\mathrm{d}\theta=3\int(\theta\sec^2\theta+\tan\theta\sec^2\theta)\mathrm{d}\theta$ is sufficient for B1M1A1
	Note	The final A mark would be lost for $\int\frac{1}{\cos^3\theta}3\sin\theta+3\theta\cos\theta=3\int(\theta\sec^2\theta+\tan\theta\sec^2\theta)\mathrm{d}\theta$ [lack of brackets in this particular case].
	Note	Give 2nd B0 for $\alpha=0$ and $\beta=60°$, without reference to $\beta=\frac{\pi}{3}$
(c)	**Note**	A decimal answer of 7.861956551... (without a correct **exact** answer) is A0.
	Note	First three marks are for integrating $\theta\sec^2\theta$ with respect to θ
	Note	Fourth and fifth marks are for integrating $\tan\theta\sec^2\theta$ with respect to θ
	Note	Candidates are not penalised for writing $\ln\lvert\sec\theta\rvert$ as either $\ln(\sec\theta)$ or $\ln\sec\theta$
	Note	$\theta\sec^2\theta\to\theta\tan\theta+\ln(\sec\theta)$ **WITH NO INTERMEDIATE WORKING** is M0M0A0
	Note	$\theta\sec^2\theta\to\theta\tan\theta-\ln(\cos\theta)$ **WITH NO INTERMEDIATE WORKING** is M0M0A0
	Note	$\theta\sec^2\theta\to\theta\tan\theta-\ln(\sec\theta)$ **WITH NO INTERMEDIATE WORKING** is M1M1A1
	Note	$\theta\sec^2\theta\to\theta\tan\theta+\ln(\cos\theta)$ **WITH NO INTERMEDIATE WORKING** is M1M1A1
	Note	Writing a correct $uv-\int v\frac{\mathrm{d}u}{\mathrm{d}x}$ with $u=\theta$, $\frac{\mathrm{d}v}{\mathrm{d}\theta}=\tan\theta$, $\frac{\mathrm{d}u}{\mathrm{d}\theta}=1$ and $v=$ their $g(\theta)$ and making one error in the direct application of this formula is 1st M1 only.

(c)	Alternative method for finding $\int\tan\theta\sec^2\theta\,\mathrm{d}\theta$		
	$\left\{\begin{array}{ll} u=\tan\theta & \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}\theta}=\sec^2\theta \\ \frac{\mathrm{d}v}{\mathrm{d}\theta}=\sec^2\theta & \Rightarrow v=\tan\theta\end{array}\right\}$		
	$\int\tan\theta\sec^2\theta\,\mathrm{d}\theta=\tan^2\theta-\int\tan\theta\sec^2\theta\,\mathrm{d}\theta$ $\Rightarrow 2\int\tan\theta\sec^2\theta\,\mathrm{d}\theta=\tan^2\theta$		
	$\int\tan\theta\sec^2\theta\,\mathrm{d}\theta=\frac{1}{2}\tan^2\theta$	$\tan\theta\sec^2\theta$ or $\to\pm C\tan^2\theta$	M1
		$\tan\theta\sec^2\theta\to\frac{1}{2}\tan^2\theta$	A1
	or $\left\{\begin{array}{ll} u=\sec\theta & \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}\theta}=\sec\theta\tan\theta \\ \frac{\mathrm{d}v}{\mathrm{d}\theta}=\sec\theta\tan\theta & \Rightarrow v=\sec\theta\end{array}\right\}$		
	$\Rightarrow \int\tan\theta\sec^2\theta\,\mathrm{d}\theta=\sec^2\theta-\int\sec^2\theta\tan\theta\,\mathrm{d}\theta$ $\Rightarrow 2\int\tan\theta\sec^2\theta\,\mathrm{d}\theta=\sec^2\theta$		
	$\int\tan\theta\sec^2\theta\,\mathrm{d}\theta=\frac{1}{2}\sec^2\theta$	$\tan\theta\sec^2\theta$ or $\to\pm C\sec^2\theta$	M1
		$\tan\theta\sec^2\theta\to\frac{1}{2}\sec^2\theta$	A1

Gold Questions

Non-calculator

The total mark for this section is 30

Q1

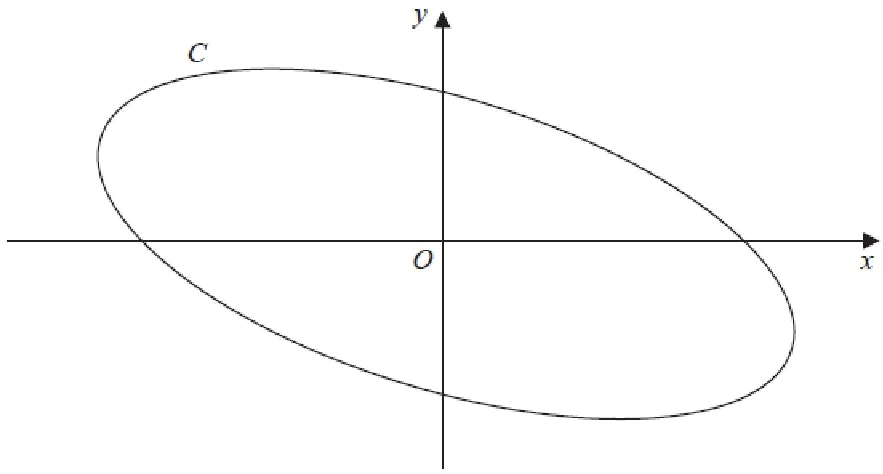


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t, \quad 0 \leq t \leq 2\pi$$

(a) Show that

$$x + y = \sqrt{3}\cos t$$

(3)

(b) Show that a Cartesian equation of C is

$$(x + y)^2 + ay^2 = b$$

Where a and b are integers to be determined.

(2)

(Total for Question 1 is 5 marks)

Q2

A curve C has parametric equations

$$x = 3 + 2\sin t, \qquad y = 4 + 2\cos 2t, \qquad 0 \leq t < 2\pi$$

(a) Show that all points on C satisfy $y = 6 - (x - 3)^2$

(2)

(b) (i) Sketch the curve C.

(ii) Explain briefly why C does not include all points of $y = 6 - (x - 3)^2, \quad x \in \mathbb{R}$.

(3)

The line with equation $x + y = k$, where k is a constant, intersects C at two distinct points.

(c) State the range of values of k, writing your answer in set notation.

(5)

(Total for Question 2 is 10 marks)

Q3

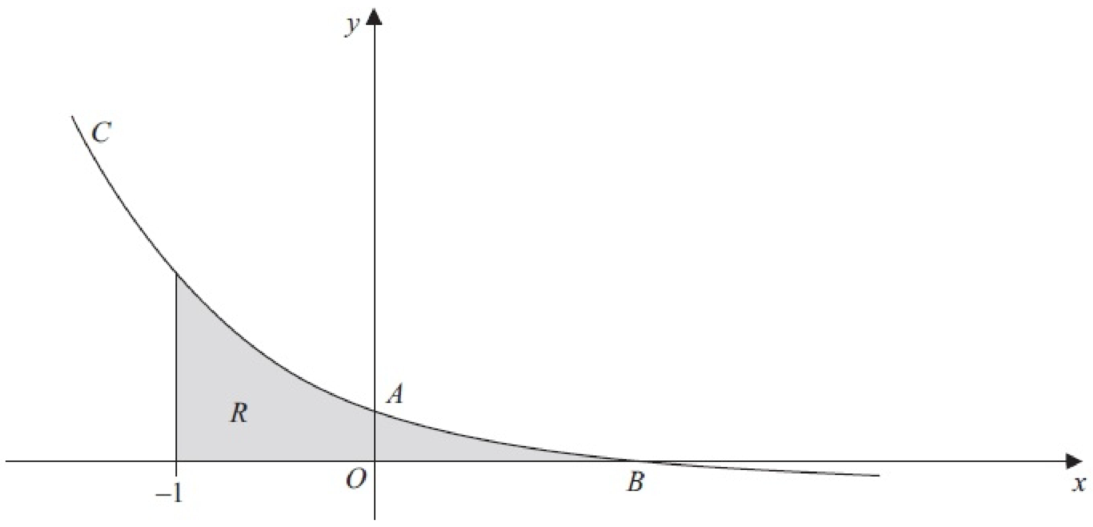


Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$$

The curve crosses the y-axis at the point A and crosses the x-axis at the point B.

(a) Show that A has coordinates (0, 3).

(2)

(b) Find the x coordinate of the point B.

(2)

(c) Find an equation of the normal to C at the point A.

(5)

The region R, as shown shaded in Figure 2, is bounded by the curve C, the line $x = -1$ and the x-axis.

(d) Use integration to find the exact area of R.

(6)

(Total for Question 3 is 15 marks)

End of Questions

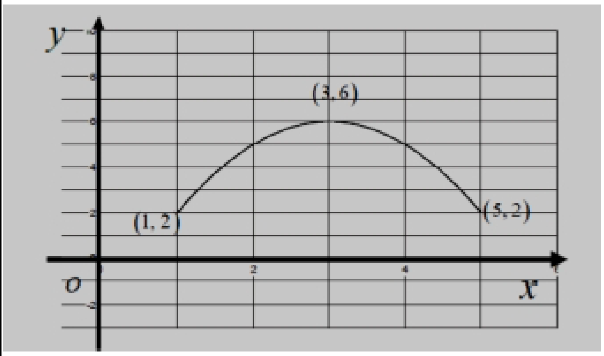
Gold Mark Scheme

Q1

Question Number	Scheme		Marks
	$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t$		
(a)	**Main Scheme**		
	$x = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right)$	$\cos\left(t + \frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$	M1 oe
	So, $\{x + y\} = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right) + 2\sin t$	Adds their expanded x (which is in terms of t) to $2\sin t$	dM1
	$= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) + 2\sin t$		
	$= 2\sqrt{3}\cos t$ *	Correct proof	A1 *
			[3]
(a)	**Alternative Method 1**		
	$x = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right)$	$\cos\left(t + \frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$	M1 oe
	$= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) = 2\sqrt{3}\cos t - 2\sin t$		
	So, $x = 2\sqrt{3}\cos t - y$	Forms an equation in x, y and t.	dM1
	$x + y = 2\sqrt{3}\cos t$ *	Correct proof	A1 *
			[3]
(b)	**Main Scheme**		
	$\left(\frac{x + y}{2\sqrt{3}}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.	M1
	$\Rightarrow \frac{(x + y)^2}{12} + \frac{y^2}{4} = 1$		
	$\Rightarrow (x + y)^2 + 3y^2 = 12$	$(x + y)^2 + 3y^2 = 12$ $\{a = 3, b = 12\}$	A1
			[2]
(b)	**Alternative Method 1**		
	$(x + y)^2 = 12\cos^2 t = 12(1 - \sin^2 t) = 12 - 12\sin^2 t$		
	So, $(x + y)^2 = 12 - 3y^2$	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.	M1
	$\Rightarrow (x + y)^2 + 3y^2 = 12$	$(x + y)^2 + 3y^2 = 12$	A1
			[2]
(b)	**Alternative Method 2**		
	$(x + y)^2 = 12\cos^2 t$		
	As $12\cos^2 t + 12\sin^2 t = 12$		
	then $(x + y)^2 + 3y^2 = 12$		M1, A1
			[2]
			5

		Question Notes
(a)	M1	$\cos\left(t+\frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$ or $\cos\left(t+\frac{\pi}{6}\right) \to \left(\frac{\sqrt{3}}{2}\right)\cos t \pm \left(\frac{1}{2}\right)\sin t$
	Note	**If a candidate states** $\cos(A+B) = \cos A\cos B \pm \sin A\sin B$, but there is an error ***in its application*** then give M1. **<u>Awarding the dM1 mark which is dependent on the first method mark</u>**
Main	dM1 **Note**	Adds their expanded x (which is in terms of t) to $2\sin t$ Writing $x + y = ...$ is not needed in the **Main Scheme** method.
Alt 1	dM1	Forms an equation in x, y and t.
	A1*	Evidence of $\cos\left(\frac{\pi}{6}\right)$ and $\sin\left(\frac{\pi}{6}\right)$ evaluated and the proof is correct with no errors.
	Note	$\{x+y\} = 4\cos\left(t+\frac{\pi}{6}\right) + 2\sin t$, by itself is M0M0A0.
(b)	M1 A1	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's. leading $(x+y)^2 + 3y^2 = 12$
	SC	Award **Special Case B1B0** for a candidate who writes down **either** • $(x+y)^2 + 3y^2 = 12$ from no working • $a = 3, b = 12$, but **<u>does not provide a correct proof</u>**.
	Note **Note**	Alternative method 2 is fine for M1 A1 Writing $(x+y)^2 = 12\cos^2 t$ followed by $12\cos^2 t + a(4\sin^2 t) = b \Rightarrow a = 3, b = 12$ is SC: B1B0
	Note	Writing $(x+y)^2 = 12\cos^2 t$ followed by $12\cos^2 t + a(4\sin^2 t) = b$ • states $a = 3, b = 12$ • and refers to either $\cos^2 t + \sin^2 t = 1$ or $12\cos^2 t + 12\sin^2 t = 12$ • and there is no incorrect working would get M1A1

Q2

Question	Scheme	Marks	AOs
(a)	Attempts to use $\cos 2t = 1 - 2\sin^2 t \Rightarrow \frac{y-4}{2} = 1 - 2\left(\frac{x-3}{2}\right)^2$	M1	2.1
	$\Rightarrow y - 4 = 2 - 4 \times \frac{(x-3)^2}{4} \Rightarrow y = 6 - (x-3)^2$ *	A1*	1.1b
		(2)	
(b)	 $\cap$ shaped parabola	M1	1.1b
	Fully correct with 'ends' at (1,2) & (5,2)	A1	1.1b
	Suitable reason : Eg states as $x = 3 + 2\sin t$, $1 \leqslant x \leqslant 5$	B1	2.4
		(3)	

(c)	Either finds the lower value for $k = 7$ or deduces that $k < \frac{37}{4}$	B1	2.2a
	Finds where $x + y = k$ meets $y = 6 - (x-3)^2$ $\Rightarrow k - x = 6 - (x-3)^2$ and proceeds to 3TQ in x or y	M1	3.1a
	Correct 3TQ in x $\quad x^2 - 7x + (k+3) = 0$ Or y $\quad y^2 + (7-2k)y + (k^2 - 6k + 3) = 0$	A1	1.1b
	Uses $b^2 - 4ac = 0 \Rightarrow \quad 49 - 4 \times 1 \times (k+3) = 0 \Rightarrow k = \left(\frac{37}{4}\right)$ or $(7-2k)^2 - 4 \times 1 \times (k^2 - 6k + 3) = 0 \Rightarrow k = \left(\frac{37}{4}\right)$	M1	2.1
	Range of values for $k = \left\{k : 7 \leqslant k < \frac{37}{4}\right\}$	A1	2.5
		(5)	
			(10 marks)

(a)

M1: Uses $\cos 2t = 1-2\sin^2 t$ in an attempt to eliminate t

A1*: Proceeds to $y = 6-(x-3)^2$ without any errors

Allow a proof where they start with $y = 6-(x-3)^2$ and substitute the parametric coordinates. M1 is scored for a correct $\cos 2t = 1-2\sin^2 t$ but A1 is only scored when both sides are seen to be the same AND a comment is made, hence proven, or similar .

(b)

M1: For sketching a $\cap$ parabola with a maximum in quadrant one. It does not need to be symmetrical

A1: For sketching a $\cap$ parabola with a maximum in quadrant one and with end coordinates of $(1,2)$ and $(5,2)$

B1: Any suitable explanation as to why C does not include all points of $y = 6-(x-3)^2, \quad x \in \mathbb{R}$

This should include a reference to **the limits on sin or cos** with a **link to a restriction on** x **or** y.

For example

'As $-1 \le \sin t \le 1$ then $1 \leqslant x \leqslant 5$' Condone in words '$x$ lies between 1 and 5' and strict inequalities

'As $\sin t \le 1$ then $x \leqslant 5$' Condone in words 'x is less than 5'

'As $-1 \le \cos(2t) \le 1$ then $2 \leqslant y \leqslant 6$' Condone in words '$y$ lies between 2 and 6'

Withhold if the statement is incorrect Eg "because the domain is $2 \leqslant x \leqslant 5$"

Do not allow a statement on the top limit of y as this is the same for both curves

(c)

B1: Deduces either

- the correct that the lower value of $k = 7$ This can be found by substituting into $(5,2)$

  $x + y = k \Rightarrow k = 7$ or substituting $x = 5$ into $x^2 - 7x + (k+3) = 0 \Rightarrow 25 - 35 + k + 3 = 0$
  $\Rightarrow k = 7$

- or deduces that $k < \dfrac{37}{4}$ This may be awarded from later work

M1: For an attempt at the upper value for k.

Finds where $x + y = k$ meets $y = 6-(x-3)^2$ once by using an appropriate method.

Eg. Sets $k - x = 6-(x-3)^2$ and proceeds to a 3TQ

A1: Correct 3TQ $x^2 - 7x + (k+3) = 0$ The $= 0$ may be implied by subsequent work

M1: Uses the "discriminant" condition. Accept use of $b^2 = 4ac$ oe or $b^2 ... 4ac$ where ... is any inequality leading to a critical value for k. Eg. one root $\Rightarrow 49 - 4\times1\times(k+3) = 0 \Rightarrow k = \dfrac{37}{4}$

A1: Range of values for $k = \left\{k : 7 \leqslant k < \dfrac{37}{4}\right\}$ Accept $k \in \left[7, \dfrac{37}{4}\right)$ or exact equivalent

ALT	As above	B1	2.2a
	Finds where $x + y = k$ meets $y = 6-(x-3)^2$ once by using an appropriate method. Eg. Sets gradient of $y = 6-(x-3)^2$ equal to -1	M1	3.1a
	$-2x + 6 = -1 \Rightarrow x = 3.5$	A1	1.1b
	Finds point of intersection and uses this to find upper value of k. $y = 6-(3.5-3)^2 = 5.75$ Hence using $k = 3.5 + 5.75 = 9.25$	M1	2.1
	Range of values for $k = \{k : 7 \leqslant k < 9.25\}$	A1	2.5

Q3

Question Number	Scheme		Marks
	Working parametrically:		
	$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$ or $y = e^{t\ln 2} - 1$		
(a)	$\{x = 0 \Rightarrow\}\ 0 = 1 - \frac{1}{2}t \Rightarrow t = 2$	Applies $x = 0$ to obtain a value for t.	M1
	When $t = 2$, $y = 2^2 - 1 = 3$	Correct value for y.	A1
			[2]
(b)	$\{y = 0 \Rightarrow\}\ 0 = 2^t - 1 \Rightarrow t = 0$	Applies $y = 0$ to obtain a value for t. (Must be seen in part (b)).	M1
	When $t = 0$, $x = 1 - \frac{1}{2}(0) = 1$	$x = 1$	A1
			[2]
(c)	$\frac{dx}{dt} = -\frac{1}{2}$ and either $\frac{dy}{dt} = 2^t \ln 2$ or $\frac{dy}{dt} = e^{t\ln 2}\ln 2$		B1
	$\frac{dy}{dx} = \frac{2^t \ln 2}{-\frac{1}{2}}$	Attempts their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$.	M1
	At A, $t = "2"$, so $m(\mathbf{T}) = -8\ln 2 \Rightarrow m(\mathbf{N}) = \frac{1}{8\ln 2}$	Applies $t = "2"$ and $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$	M1
	$y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or equivalent.	See notes.	M1 A1 oe cso
			[5]
(d)	$\text{Area}(R) = \int (2^t - 1)\left(-\frac{1}{2}\right)dt$	Complete substitution for both y and dx	M1
	$x = -1 \rightarrow t = 4$ and $x = 1 \rightarrow t = 0$		B1
	$= \left\{-\frac{1}{2}\right\}\left(\frac{2^t}{\ln 2} - t\right)$	Either $2^t \rightarrow \frac{2^t}{\ln 2}$ or $(2^t - 1) \rightarrow \frac{(2^t)}{\pm\alpha(\ln 2)} - t$ or $(2^t - 1) \rightarrow \pm\alpha(\ln 2)(2^t) - t$	M1*
		$(2^t - 1) \rightarrow \frac{2^t}{\ln 2} - t$	A1
	$\left\{-\frac{1}{2}\left[\frac{2^t}{\ln 2} - t\right]_4^0\right\} = -\frac{1}{2}\left(\left(\frac{1}{\ln 2}\right) - \left(\frac{16}{\ln 2} - 4\right)\right)$	**Depends on the previous method mark.** Substitutes their changed limits in t and subtracts either way round.	dM1*
	$= \frac{15}{2\ln 2} - 2$	$\frac{15}{2\ln 2} - 2$ or equivalent.	A1
			[6]
			15

Platinum Questions

Non-calculator

The total mark for this section is 13

Q1

The curve C has parametric equations

$$x = \cos^2 t,$$

$$y = \cos t \sin t,$$

where $0 \leq t < \pi$.

(a) Show that C is a circle and find its centre and its radius.

(5)

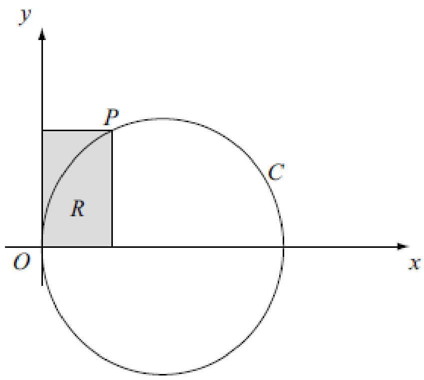


Figure 1

Figure 1 shows a sketch of C. The point P, with coordinates $(\cos^2 \alpha, \cos \alpha \sin \alpha)$, $0 < \alpha < \frac{\pi}{2}$, lies on C. The rectangle R has one side on the x-axis, one side on the y-axis and OP as a diagonal, where O is the origin.

(b) Show that the area of R is $\sin \alpha \cos^3 \alpha$.

(1)

(c) Find the maximum area of R, as α varies.

(7)

(Total for Question 1 is 13 marks)

End of Questions

Platinum Mark Scheme

Q1

Question	Scheme	Marks	Notes
.. (a)	$2y = 2\sin t\cos t = \sin 2t$	M1	Use of $\sin 2t$
	$2x = 2\cos^2 t \Rightarrow 2x - 1 = 2\cos^2 t - 1 = \cos 2t$	M1	Use of $\cos 2t$
	$(2x-1)^2 + (2y)^2 = 1$	M1	Successfully eliminating t and eqn. for circle
	$\left(x - \frac{1}{2}\right)^2 + y^2 = \left(\frac{1}{2}\right)^2$ so centre $\left(\frac{1}{2}, 0\right)$, $r = \frac{1}{2}$	A1A1 (5)	A1 for centre A1 for radius
(b)	Area of $R = \cos^2\alpha \times \sin\alpha\cos\alpha = \cos^3\alpha\sin\alpha$	B1(1)	Some evidence of xy leading to given result
(c)	$\frac{dA}{d\alpha} = \cos\alpha\cos^3\alpha - 3\cos^2\alpha\sin^2\alpha$	M1A1	M1 for use of product rule
	$\frac{dA}{d\alpha} = 0 \Rightarrow \cos^2\alpha\left(\cos^2\alpha - 3\sin^2\alpha\right) = 0$	M1	M1 for setting derivative =0 and attempting to solve
	$\cos^2\alpha = 0 \Rightarrow \left[\alpha = \frac{\pi}{2}\right]$ or $\tan^2\alpha = \frac{1}{3} \Rightarrow \alpha = \frac{\pi}{6}$ (or $30°$)	A1 A1	A1 for "trig" = ..A1 for α = .. Can ignore $\alpha = \frac{\pi}{2}$ but consider for S+
	$A'' = 2\sin\alpha\cos\alpha(3 - 8\cos^2\alpha)$ and show <0 for $\alpha = \frac{\pi}{6}$ or argument based on $\alpha = \frac{\pi}{2}$ gives min so this is max	M1	Some check that this value of α gives a max
	Maximum area is $\frac{3\sqrt{3}}{16}$ (o.e.)	B1 (7) **(13)**	Single fraction with rational denom

Topic 9: Differentiation

Bronze, Silver, Gold and Platinum Worksheets for A Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel A Level Mathematics: Pure Mathematics Year 2' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high-level problem-solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Non-calculator

The total mark for this section is 35

Q1

The curve C has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find

(i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 2$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

(Total for Question 1 is 7 marks)

Q2

Given $y = x(2x + 1)^4$, show that

$$\frac{dy}{dx} = (2x + 1)^n (Ax + B)$$

where n, A and B are constants to be found.

(4)

(Total for Question 2 is 4 marks)

Q3

A curve C has the equation $y^2 - 3y = x^3 + 8$.

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(4)

(b) Hence find the gradient of C at the point where $y = 3$.

(3)

(Total for Question 3 is 7 marks)

Q4

Find the gradient of the curve with equation

$$\ln y = 2x \ln x, \quad x > 0,\ y > 0$$

at the point on the curve where $x = 2$. Give your answer as an exact value.

(7)

(Total for Question 4 is 7 marks)

Q5

(i) Find, using calculus, the x coordinate of the turning point of the curve with equation

$$y = e^{3x} \cos 4x, \quad \frac{\pi}{4} \leqslant x < \frac{\pi}{2}$$

Write your answer in the form

$$x = m \arctan(b)$$

Where m and b are constants to be determined.

(5)

(ii) Given that $x = \sin^2 2y,\ 0 < y < \frac{\pi}{4}$, find $\frac{dy}{dx}$ as a function of y.

Write your answer in the form

$$\frac{dy}{dx} = p \operatorname{cosec}(qy), \qquad 0 < y < \frac{\pi}{4}$$

where p and q are constants to be determined.

(5)

(Total for Question 5 is 10 marks)

End of Questions

Bronze Mark Scheme

Q1

Question	Scheme	Marks	AOs
(a)	(i) $\frac{dy}{dx} = 12x^3 - 24x^2$	M1 A1	1.1b 1.1b
	(ii) $\frac{d^2y}{dx^2} = 36x^2 - 48x$	A1ft	1.1b
		(3)	
(b)	Substitutes $x = 2$ into their $\frac{dy}{dx} = 12\times 2^3 - 24\times 2^2$	M1	1.1b
	Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point"	A1	2.1
		(2)	
(c)	Substitutes $x = 2$ into their $\frac{d^2y}{dx^2} = 36\times 2^2 - 48\times 2$	M1	1.1b
	$\frac{d^2y}{dx^2} = 48 > 0$ and states "hence the stationary point is a minimum"	A1ft	2.2a
		(2)	
		(7 marks)	

Notes:

(a)(i)

M1: Differentiates to a cubic form

A1: $\frac{dy}{dx} = 12x^3 - 24x^2$

(a)(ii)

A1ft: Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx} = 36x^2 - 48x$

(b)

M1: Substitutes $x = 2$ into their $\frac{dy}{dx}$

A1: Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point" All aspects of the proof must be correct

(c)

M1: Substitutes $x = 2$ into their $\frac{d^2y}{dx^2}$

Alternatively calculates the gradient of C either side of $x = 2$

A1ft: For a correct calculation, a valid reason and a correct conclusion.

Follow through on an incorrect $\frac{d^2y}{dx^2}$

Q2

Question	Scheme	Marks	AOs
	Attempts the product and chain rule on $y = x(2x+1)^4$	M1	2.1
	$\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$	A1	1.1b
	Takes out a common factor $\frac{dy}{dx} = (2x+1)^3\{(2x+1)+8x\}$	M1	1.1b
	$\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n=3, A=10, B=1$	A1	1.1b
		(4 marks)	

Notes:

M1: Applies the product rule to reach $\frac{dy}{dx} = (2x+1)^4 + Bx(2x+1)^3$

A1: $\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$

M1: Takes out a common factor of $(2x+1)^3$

A1: The form of this answer is given. Look for $\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n=3, A=10, B=1$

Q3

Question Number	Scheme		Marks
(a)	C: $y^2 - 3y = x^3 + 8$		
	$\left\{\frac{\cancel{dx}}{\cancel{dx}} \times\right\}$ $2y\frac{dy}{dx} - 3\frac{dy}{dx} = 3x^2$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $\pm 3\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$.)	M1
		Correct equation.	A1
	$(2y-3)\frac{dy}{dx} = 3x^2$	A correct (condoning sign error) attempt to combine or factorise their '$2y\frac{dy}{dx} - 3\frac{dy}{dx}$'. Can be implied.	M1
	$\frac{dy}{dx} = \frac{3x^2}{2y-3}$	$\frac{3x^2}{2y-3}$	A1 oe
			(4)
(b)	$y = 3 \Rightarrow 9 - 3(3) = x^3 + 8$	Substitutes $y = 3$ into C.	M1
	$x^3 = -8 \Rightarrow \underline{x = -2}$	Only $\underline{x = -2}$	A1
	$(-2, 3) \Rightarrow \frac{dy}{dx} = \frac{3(4)}{6-3} \Rightarrow \frac{dy}{dx} = 4$	$\frac{dy}{dx} = 4$ from correct working. Also can be ft using their 'x' value and $y = 3$ in the correct part (a) of $\frac{dy}{dx} = \frac{3x^2}{2y-3}$	A1 $\sqrt{}$
			(3)
	1(b) final A1 $\sqrt{}$. Note if the candidate inserts their x value and $y = 3$ into $\frac{dy}{dx} = \frac{3x^2}{2y-3}$, then an answer of $\frac{dy}{dx} = $ their x^2, *may* indicate a correct follow through.		
			[7]

Q4

Question Number	Scheme	Marks
	$\frac{1}{y}\frac{dy}{dx} = \dots$	B1
	$\dots \quad = 2\ln x + 2x\left(\frac{1}{x}\right)$	M1 A1
	At $x = 2$, $\quad \ln y = 2(2)\ln 2$	M1
	leading to $\quad y = 16 \qquad$ Accept $y = e^{4\ln 2}$	A1
	At $(2, 16)$ $\quad \frac{1}{16}\frac{dy}{dx} = 2\ln 2 + 2$	M1
	$\frac{dy}{dx} = 16(2 + 2\ln 2)$	A1 (7)
		[7]
	Alternative	
	$y = e^{2x\ln x}$	B1
	$\frac{d}{dx}(2x\ln x) = 2\ln x + 2x\left(\frac{1}{x}\right)$	M1 A1
	$\frac{dy}{dx} = \left(2\ln x + 2x\left(\frac{1}{x}\right)\right)e^{2x\ln x}$	M1 A1
	At $x = 2$, $\quad \frac{dy}{dx} = (2\ln 2 + 2)e^{4\ln 2}$	M1
	$= 16(2 + 2\ln 2)$	A1 (7)

Q5

Question	Scheme	Marks
(i)	$y = e^{3x}\cos 4x \Rightarrow \left(\frac{dy}{dx}\right) = \cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x$	M1A1
	Sets $\cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x = 0 \Rightarrow 3\cos 4x - 4\sin 4x = 0$	M1
	$\Rightarrow x = \frac{1}{4}\arctan\frac{3}{4}$	M1 A1
		(5)
(ii)	$x = \sin^2 2y \Rightarrow \frac{dx}{dy} = 2\sin 2y \times 2\cos 2y$	M1A1
	Uses $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\frac{dx}{dy} = 2\sin 4y \Rightarrow \frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\operatorname{cosec} 4y$	M1A1
		(5) (10 marks)
(ii) Alt I	$x = \sin^2 2y \Rightarrow x = \frac{1}{2} - \frac{1}{2}\cos 4y$	2nd M1
	$\frac{dx}{dy} = 2\sin 4y$	1st M1 A1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\operatorname{cosec} 4y$	M1A1
		(5)
(ii) Alt II	$x^{\frac{1}{2}} = \sin 2y \Rightarrow \frac{1}{2}x^{-\frac{1}{2}} = 2\cos 2y\frac{dy}{dx}$	M1A1
	Uses $x^{\frac{1}{2}} = \sin 2y$ AND $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\operatorname{cosec} 4y$	M1A1
		(5)
(ii) Alt III	$x^{\frac{1}{2}} = \sin 2y \Rightarrow 2y = \text{invsin}\, x^{\frac{1}{2}} \Rightarrow 2\frac{dy}{dx} = \frac{1}{\sqrt{1-x}} \times \frac{1}{2}x^{-\frac{1}{2}}$	M1A1
	Uses $x^{\frac{1}{2}} = \sin 2y$, $\sqrt{1-x} = \cos 2y$ and $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\operatorname{cosec} 4y$	M1A1
		(5)

(i)

M1 Uses the product rule $uv' + vu'$ to achieve $\left(\frac{dy}{dx}\right) = Ae^{3x}\cos 4x \pm Be^{3x}\sin 4x \quad A, B \neq 0$

The product rule if stated must be correct

A1 Correct (unsimplified) $\frac{dy}{dx} = \cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x$

M1 Sets/implies their $\frac{dy}{dx}=0$ factorises/cancels)by e^{3x} to form a trig equation in just $\sin 4x$ and $\cos 4x$

M1 Uses the identity $\frac{\sin 4x}{\cos 4x}\equiv \tan 4x$, moves from $\tan 4x = C, C\neq 0$ using correct order of operations to $x=...$ Accept $x=$ awrt 0.16 (radians) $x=$ awrt 9.22 (degrees) for this mark.

If a candidate elects to pursue a more difficult method using $R\cos(\theta+\alpha)$, for example, the minimum expectation will be that they get (1) the identity correct, and (2) the values of R and α correct to 2dp. So for the correct equation you would only accept $5\cos(4x+\text{awrt } 0.93)$ or $5\sin(4x-\text{awrt } 0.64)$ before using the correct order of operations to $x=...$

Similarly candidates who square $3\cos 4x-4\sin 4x=0$ then use a Pythagorean identity should proceed from either $\sin 4x=\frac{3}{5}$ or $\cos 4x=\frac{4}{5}$ before using the correct order of operations …

A1 $\Rightarrow x=$ awrt 0.9463.

Ignore any answers outside the domain. Withhold mark for additional answers inside the domain

(ii)

M1 Uses chain rule (or product rule) to achieve $\pm P\sin 2y\cos 2y$ as a derivative.

There is no need for lhs to be seen/ correct

If the product rule is used look for $\cancel{\frac{dy}{dx}}=\pm A\sin 2y\cos 2y\pm B\sin 2y\cos 2y$,

A1 Both lhs and rhs correct (unsimplified). $\frac{dx}{dy}=2\sin 2y\times 2\cos 2y=(4\sin 2y\cos 2y)$ or

$1=2\sin 2y\times 2\cos 2y\frac{dy}{dx}$

M1 Uses $\sin 4y=2\sin 2y\cos 2y$ in their expression.

You may just see a statement such as $4\sin 2y\cos 2y=2\sin 4y$ which is fine.

Candidates who write $\cancel{\frac{dy}{dx}}=A\sin 2x\cos 2x$ can score this for $\cancel{\frac{dy}{dx}}=\frac{A}{2}\sin 4x$

M1 Uses $\frac{dy}{dx}=\frac{1}{\frac{dx}{dy}}$ **for their expression in** y. Concentrate on the trig identity rather than the coefficient in awarding this. Eg $\frac{dx}{dy}=2\sin 4y\Rightarrow\frac{dy}{dx}=2\operatorname{cosec}4y$ is condoned for the M1

If $\frac{dx}{dy}=a+b$ do not allow $\frac{dy}{dx}=\frac{1}{a}+\frac{1}{b}$

A1 $\frac{dy}{dx}=\frac{1}{2}\operatorname{cosec}4y$ If a candidate then proceeds to write down incorrect values of p and q then do not withhold the mark.

NB: See the three alternatives which may be less common but mark in exactly the same way. If you are uncertain as how to mark these please consult your team leader.

In **Alt I** the second M is for writing $x=\sin^2 2y\Rightarrow x=\pm\frac{1}{2}\pm\frac{1}{2}\cos 4y$ from $\cos 4y=\pm 1\pm 2\sin^2 2y$

In **Alt II** the first M is for writing $x^{\frac{1}{2}}=\sin 2y$ and differentiating both sides to $Px^{-\frac{1}{2}}=Q\cos 2y\cancel{\frac{dy}{dx}}$ oe

In **Alt 111** the first M is for writing $2y=\text{invsin}\left(x^{0.5}\right)$ oe and differentiating to $M\cancel{\frac{dy}{dx}}=N\frac{1}{\sqrt{1-\left(x^{0.5}\right)^2}}\times x^{-0.5}$

Silver Questions

Non-calculator

The total mark for this section is 36

Q1

Given that

$$y = 3x^2 + 6x^{\frac{1}{3}} + \frac{2x^3 - 7}{3\sqrt{x}}, \quad x > 0$$

Find $\frac{dy}{dx}$. Give each term in your answer in its simplified form.

(6)

(Total for Question 1 is 6 marks)

Q2

The curve C has the equation $2x + 3y^2 + 3x^2 y = 4x^2$.

The point P on the curve has coordinates $(-1, 1)$.

(a) Find the gradient of the curve at P.

(5)

(b) Hence find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(3)

(Total for Question 2 is 8 marks)

Q3

Given that θ is measured in radians, prove, from first principles, that the derivative of $\sin\theta$ is $\cos\theta$

You may assume the formula for $\sin(A \pm B)$ and that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$.

(5)

(Total for Question 3 is 5 marks)

Q4

$$y = \frac{5x^2 + 10x}{(x+1)^2} \qquad x \neq -1$$

(a) Show that $\frac{dy}{dx} = \frac{A}{(x+1)^n}$ where A and n are constants to be found.

(4)

(b) Hence deduce the range of values for x for which $\frac{dy}{dx} < 0$.

(1)

(Total for Question 4 is 5 marks)

Q5

(i) Given that

$$x = \sec^2 2y, \qquad 0 < y < \frac{\pi}{4}$$

show that

$$\frac{dy}{dx} = \frac{1}{4x\sqrt{(x-1)}}$$

(4)

(ii) Given that

$$y = (x^2 + x3)\ln 2x$$

find the exact value of $\frac{dy}{dx}$ at $x = \frac{e}{2}$, giving your answer in its simplest form.

(5)

(iii) Given that

$$f(x) = \frac{3\cos x}{(x+1)^{\frac{1}{3}}}, \qquad x \neq -1$$

show that

$$f'(x) = \frac{g(x)}{(x+1)^{\frac{4}{3}}}, \qquad x \neq -1$$

where g(x) is an expression to be found.

(3)

(Total for Question 5 is 12 marks)

End of Questions

Silver Mark Scheme

Q1

Question Number	Scheme	Notes	Marks
	$y = 3x^2 + 6x^{\frac{1}{3}} + \dfrac{2x^3 - 7}{3\sqrt{x}}$		
	$\dfrac{2x^3-7}{3\sqrt{x}} = \dfrac{2x^3}{3\sqrt{x}} - \dfrac{7}{3\sqrt{x}} = \dfrac{2}{3}x^{\frac{5}{2}} - \dfrac{7}{3}x^{-\frac{1}{2}}$	Attempts to split the fraction into 2 terms and obtains either $\alpha x^{\frac{5}{2}}$ or $\beta x^{-\frac{1}{2}}$. This may be implied by a correct power of x in their differentiation of one of these terms. But beware of $\beta x^{-\frac{1}{2}}$ coming from $\dfrac{2x^3-7}{3\sqrt{x}} = 2x^3 - 7 + 3x^{-\frac{1}{2}}$	M1
	$x^n \to x^{n-1}$	Differentiates by reducing power by one for any of their powers of x	M1
	$\left(\dfrac{dy}{dx} =\right)\ 6x + 2x^{-\frac{2}{3}} + \dfrac{5}{3}x^{\frac{3}{2}} + \dfrac{7}{6}x^{-\frac{3}{2}}$	A1: $6x$. Do not accept $6x^1$. Depends on second M mark only. Award when first seen and isw. A1: $2x^{-\frac{2}{3}}$. Must be simplified so do not accept e.g. $\dfrac{2}{1}x^{-\frac{2}{3}}$ but allow $\dfrac{2}{\sqrt[3]{x^2}}$. Depends on second M mark only. Award when first seen and isw. A1: $\dfrac{5}{3}x^{\frac{3}{2}}$. Must be simplified but allow e.g. $1\dfrac{2}{3}x^{1.5}$ or e.g. $\dfrac{5}{3}\sqrt{x^3}$. Award when first seen and isw. A1: $\dfrac{7}{6}x^{-\frac{3}{2}}$. Must be simplified but allow e.g. $1\dfrac{1}{6}x^{-1\frac{1}{2}}$ or e.g. $\dfrac{7}{6\sqrt{x^3}}$. Award when first seen and isw.	A1A1A1A1
	In an otherwise <u>fully correct solution</u>, penalise the presence of + c by deducting the final A1		
			[6]
	Use of Quotient Rule: First M1 and final A1A1 (Other marks as above)		
	$\dfrac{d\left(\dfrac{2x^3-7}{3\sqrt{x}}\right)}{dx} = \dfrac{3\sqrt{x}\left(6x^2\right) - \left(2x^3-7\right)\frac{3}{2}x^{-\frac{1}{2}}}{\left(3\sqrt{x}\right)^2}$	Uses **<u>correct</u>** quotient rule	M1
	$= \dfrac{10x^{\frac{5}{2}} + 7x^{-\frac{1}{2}}}{6x}$	A1: Correct first term of numerator and correct denominator A1: All correct as simplified as shown	A1A1
	So $\dfrac{dy}{dx} = 6x + 2x^{-\frac{2}{3}} + \dfrac{10x^{\frac{5}{2}} + 7x^{-\frac{1}{2}}}{6x}$ scores full marks		
			6 marks

Q2

Question Number	Scheme	Marks
(a)	$\left\{\cancel{\frac{dy}{dx}}\times\right\}\quad \underline{2+6y\frac{dy}{dx}}+\underline{\underline{\left(6xy+3x^2\frac{dy}{dx}\right)}}=\underline{8x}$	M1 A1 B1
	$\left\{\frac{dy}{dx}=\frac{8x-2-6xy}{6y+3x^2}\right\}$ *not necessarily required.*	
	At $P(-1,1)$, $m(\mathbf{T})=\frac{dy}{dx}=\frac{8(-1)-2-6(-1)(1)}{6(1)+3(-1)^2}=-\frac{4}{9}$	dM1 A1 cso
		[5]
(b)	So, $m(\mathrm{N})=\frac{-1}{-\frac{4}{9}}\left\{=\frac{9}{4}\right\}$	M1
	N: $y-1=\frac{9}{4}(x+1)$	M1
	N: $9x-4y+13=0$	A1
		[3] 8
(a)	**M1**: Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $3x^2\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx}=\right)$).	
	A1: $(2x+3y^2)\to\left(\underline{2+6y\frac{dy}{dx}}\right)$ and $(4x^2\to\underline{8x})$. **Note:** If an extra "sixth" term appears then award A0.	
	B1: $6xy+3x^2\frac{dy}{dx}$.	
	dM1: Substituting $x=-1$ and $y=1$ into an equation involving $\frac{dy}{dx}$. Allow this mark if either the numerator or denominator of $\frac{dy}{dx}=\frac{8x-2-6xy}{6y+3x^2}$ is substituted into or evaluated correctly.	
	If it is clear, however, that the candidate is intending to substitute $x=1$ and $y=-1$, then award M0.	
	Candidates who substitute $x=1$ and $y=-1$, will usually achieve $m(\mathrm{T})=-4$	
	Note that this mark is dependent on the previous method mark being awarded.	
	A1: For $-\frac{4}{9}$ or $-\frac{8}{18}$ or $-0.\dot{4}$ or awrt -0.44	
	If the candidate's solution is not completely correct, then do not give this mark.	
(b)	**M1**: Applies $m(\mathrm{N})=-\frac{1}{\text{their } m(\mathrm{T})}$.	
	M1: Uses $y-1=(m_N)(x--1)$ or finds c using $x=-1$ and $y=1$ and uses $y=(m_N)x+"c"$,	
	Where $m_N=-\frac{1}{\text{their } m(\mathrm{T})}$ or $m_N=\frac{1}{\text{their } m(\mathrm{T})}$ or $m_N=-\text{their } m(\mathrm{T})$.	
	A1: $9x-4y+13=0$ or $-9x+4y-13=0$ or $4y-9x-13=0$ or $18x-8y+26=0$ etc.	
	Must be "$=0$". So do not allow $9x+13=4y$ etc.	
	Note: $m_N=-\left(\frac{6y+3x^2}{8x-2-6xy}\right)$ is M0M0 unless a numerical value is then found for m_N.	
	Alternative method for part (a): Differentiating with respect to y	
	$\left\{\cancel{\frac{dx}{dx}}\times\right\}\quad \underline{2\frac{dx}{dy}+6y}+\underline{\underline{\left(6xy\frac{dx}{dy}+3x^2\right)}}=\underline{8x\frac{dx}{dy}}$	
	M1: Differentiates implicitly to include either $2\frac{dx}{dy}$ or $6xy\frac{dx}{dy}$ or $\pm kx\frac{dx}{dy}$. (Ignore $\left(\frac{dx}{dy}=\right)$).	
	A1: $(2x+3y^2)\to\left(\underline{2\frac{dx}{dy}+6y}\right)$ and $\left(4x^2\to\underline{8x\frac{dx}{dy}}\right)$. **Note:** If an extra "sixth" term appears then award A0.	
	B1: $6xy+3x^2\frac{dy}{dx}$.	
	dM1: Substituting $x=-1$ and $y=1$ into an equation involving $\frac{dx}{dy}$ or $\frac{dy}{dx}$. Allow this mark if either the numerator or denominator of $\frac{dx}{dy}=\frac{6y+3x^2}{8x-2-6xy}$ is substituted into or evaluated correctly.	
	If it is clear, however, that the candidate is intending to substitute $x=1$ and $y=-1$, then award M0.	
	Candidates who substitute $x=1$ and $y=-1$, will usually achieve $m(\mathrm{T})=-4$	
	Note that this mark is dependent on the previous method mark being awarded.	
	A1: For $-\frac{4}{9}$ or $-\frac{8}{18}$ or $-0.\dot{4}$ or awrt -0.44	
	If the candidate's solution is not completely correct, then do not give this mark.	

Q3

Question	Scheme	Marks	AOs
	Use of $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}$	B1	2.1
	Uses the compound angle identity for $\sin(A+B)$ with $A=\theta,\ B=h$ $\Rightarrow \sin(\theta+h)=\sin\theta\cos h+\cos\theta\sin h$	M1	1.1b
	Achieves $\dfrac{\sin(\theta+h)-\sin\theta}{h}=\dfrac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}$	A1	1.1b
	$=\dfrac{\sin h}{h}\cos\theta+\left(\dfrac{\cos h-1}{h}\right)\sin\theta$	M1	2.1
	Uses $h\to 0,\ \dfrac{\sin h}{h}\to 1$ and $\dfrac{\cos h-1}{h}\to 0$ Hence the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and the gradient of the chord $\to$ gradient of the curve $\Rightarrow \dfrac{dy}{d\theta}=\cos\theta$ *	A1*	2.5
			(5 marks)

Notes:

B1: States or implies that the gradient of the chord is $\dfrac{\sin(\theta+h)-\sin\theta}{h}$ or similar such as $\dfrac{\sin(\theta+\delta\theta)-\sin\theta}{\theta+\delta\theta-\theta}$ for a small h or $\delta\theta$

M1: Uses the compound angle identity for $\sin(A+B)$ with $A=\theta,\ B=h$ or $\delta\theta$

A1: Obtains $\dfrac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}$ or equivalent

M1: Writes their expression in terms of $\dfrac{\sin h}{h}$ and $\dfrac{\cos h-1}{h}$

A1*: Uses correct language to explain that $\dfrac{dy}{d\theta}=\cos\theta$

For this method they should use all of the given statements $h\to 0,\ \dfrac{\sin h}{h}\to 1,$ $\dfrac{\cos h-1}{h}\to 0$ meaning that the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$

and therefore the gradient of the chord $\to$ gradient of the curve $\Rightarrow \dfrac{dy}{d\theta}=\cos\theta$

Question	Scheme	Marks	AOs
alt	Use of $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}$	B1	2.1
	Sets $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\dfrac{\sin\left(\theta+\frac{h}{2}+\frac{h}{2}\right)-\sin\left(\theta+\frac{h}{2}-\frac{h}{2}\right)}{h}$ and uses the compound angle identity for sin(*A* + *B*) and sin(*A* – *B*) with $A=\theta+\frac{h}{2},\quad B=\frac{h}{2}$	M1	1.1b
	Achieves $\dfrac{\sin(\theta+h)-\sin\theta}{h}=$ $\dfrac{\left[\sin\left(\theta+\frac{h}{2}\right)\cos\left(\frac{h}{2}\right)+\cos\left(\theta+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)\right]-\left[\sin\left(\theta+\frac{h}{2}\right)\cos\left(\frac{h}{2}\right)-\cos\left(\theta+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)\right]}{h}$	A1	1.1b
	$=\dfrac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\times\cos\left(\theta+\frac{h}{2}\right)$	M1	2.1
	Uses $h\to 0,\ \frac{h}{2}\to 0$ hence $\dfrac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\to 1$ and $\cos\left(\theta+\frac{h}{2}\right)\to\cos\theta$ Therefore the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and the gradient of the chord $\to$ gradient of the curve $\Rightarrow\dfrac{dy}{d\theta}=\cos\theta$ *	A1*	2.5
		(5 marks)	

Additional notes:
A1*: Uses correct language to explain that $\dfrac{dy}{d\theta}=\cos\theta$. For this method they should use the (adapted) given statement $h\to 0,\ \frac{h}{2}\to 0$ hence $\dfrac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\to 1$ with $\cos\left(\theta+\frac{h}{2}\right)\to\cos\theta$ meaning that the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and therefore the gradient of the chord $\to$ gradient of the curve $\Rightarrow\dfrac{dy}{d\theta}=\cos\theta$

Q4

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\dfrac{dy}{dx} = \dfrac{(x+1)^2 \times (10x+10) - (5x^2+10x) \times 2(x+1)}{(x+1)^4}$	M1	This mark is given for an attempt to differentiate the expression for y
		A1	This mark is given for correctly differentiating the expression for y
	$\dfrac{dy}{dx} = \dfrac{(x+1) \times (10x+10) - (5x^2+10x) \times 2}{(x+1)^3}$	M1	This mark sis given for cancelling the expression through by $(x + 1)$
	$\dfrac{dy}{dx} = \dfrac{10}{(x+1)^3}$	A1	This mark is given for a fully correct expression for $\dfrac{dy}{dx}$
(b)	If $A > 0$ and $n = 1, 3$ then $x < -1$	B1	This mark is given for deducing that $\dfrac{dy}{dx} < 0 \Rightarrow x < -1$.

Q5

Question Number	Scheme	Marks
(i)	$\dfrac{dx}{dy} = 4\sec^2 2y \tan 2y$	B1
	Use $\dfrac{dy}{dx} = \dfrac{1}{\frac{dx}{dy}}$	M1
	Uses $\tan^2 2y = \sec^2 2y - 1$ and $\sec 2y = \sqrt{x}$ to get $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in terms of just x	M1
	$\dfrac{dy}{dx} = \dfrac{1}{4x(x-1)^{\frac{1}{2}}}$ (conclusion stated with no errors previously)	A1*
		(4)
(ii)	$\dfrac{dy}{dx} = (x^2 + x^3) \times \dfrac{2}{2x} + (2x + 3x^2) \ln 2x$	M1 A1 A1
	When $x = \dfrac{e}{2}$, $\dfrac{dy}{dx} = 3(\frac{e}{2}) + 4(\frac{e}{2})^2 = 3(\frac{e}{2}) + e^2$	dM1 A1
		(5)
(iii)	$f'(x) = \dfrac{(x+1)^{\frac{1}{3}}(-3\sin x) - 3\cos x(\frac{1}{3}(x+1)^{-\frac{2}{3}})}{(x+1)^{\frac{2}{3}}}$	M1 A1
	$f'(x) = \dfrac{-3(x+1)(\sin x) - \cos x}{(x+1)^{\frac{4}{3}}}$	A1
		(3)
		12 marks

(i)

B1 $\frac{dx}{dy} = 4\sec^2 2y \tan 2y$ or equivalent such as $\frac{dx}{dy} = 4\frac{\sin 2y \cos 2y}{\cos^4 2y}$

Accept $\frac{dx}{dy} = 2\sec 2y \tan 2y \times \sec 2y + 2\sec 2y \tan 2y \times \sec 2y$, $1 = 4\sec^2 2y \tan 2y \frac{dy}{dx}$

M1 Uses $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ to get an expression for $\frac{dy}{dx}$ in terms of y.

It may be scored following the award of the next M1 if $\frac{dx}{dy}$ has been written in terms of x.

Follow through on their expression but condone errors on the coefficient.

For example $\frac{dx}{dy} = 2\sec^2 2y \tan 2y \Rightarrow \frac{dy}{dx} = \frac{1}{2\sec^2 2y \tan 2y}$ is OK as is $\frac{dy}{dx} = \frac{2}{\sec^2 2y \tan 2y}$

Do not accept y' s going to x' s. So for example $\frac{dx}{dy} = 2\sec^2 2y \tan 2y \Rightarrow \frac{dy}{dx} = \frac{1}{2\sec^2 2x \tan 2x}$ is M0

M1 Uses $\tan^2 2y = \sec^2 2y - 1$ **and** $x = \sec^2 2y$ to get their $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in terms of just x

$\frac{dx}{dy} = 2\sec^2 2y \tan 2y \Rightarrow \frac{dx}{dy} = 2x\sqrt{(\sec^2 2y - 1)} = 2x\sqrt{x-1}$ is incorrect but scores M1

$\frac{dx}{dy} = 2\sec 2y \tan 2y \Rightarrow \frac{dx}{dy} = 2\sec 2y\sqrt{(\sec^2 2y - 1)} = 2\sqrt{x}\sqrt{x-1}$ is incorrect but scores M1

The stating and use $1 + \tan^2 x = \sec^2 x$ is unlikely to score this mark.

Accept $1 + \tan^2 2y = \sec^2 2y \Rightarrow 1 + \tan^2 2y = x \Rightarrow \tan 2y = \sqrt{x-1}$. So $\frac{dy}{dx} = \frac{1}{4\sec^2 2y \tan 2y} = \frac{1}{4x\sqrt{x-1}}$

Condone examples where the candidate adapts something to get the given answer

Eg. $\frac{dy}{dx} = \frac{1}{4\sec^2 2y \tan^2 2y} = \frac{1}{4\sec^2 2y\left(\sec^2 2y - 1\right)} = \frac{1}{4x\sqrt{(x-1)}}$

A1* Completely correct solution. This is a 'show that' question and it is a requirement that all elements are seen.

(ii)

M1 Uses the product rule to differentiate $(x^2 + x^3)\ln 2x$. If the rule is stated it must be correct. It may be implied by their $u = ..., u' = ..., v = ..., v' = ..$ followed by $vu' + uv'$. If the rule is neither stated nor implied only accept expressions of the form $\ln 2x \times (ax + bx^2) + (x^2 + x^3) \times \frac{C}{x}$

It is acceptable to multiply out the expression to get $x^2\ln 2x + x^3\ln 2x$ but the product rule must be applied to both terms

A1 One term correct (unsimplified). Either $(x^2+x^3)\times\frac{2}{2x}$ or $(2x+3x^2)\ln 2x$

If they have multiplied out before differentiating the equivalent would be two of the four terms correct.

A1 A completely correct (unsimplified) expression $\frac{dy}{dx}=(x^2+x^3)\times\frac{2}{2x}+(2x+3x^2)\ln 2x$

dM1 Fully substitutes $x=\frac{e}{2}$ (dependent on previous M mark) into their expression for $\frac{dy}{dx}=....$ Implied by awrt 11.5

A1 $\frac{dy}{dx}=3(\frac{e}{2})+e^2$ Accept equivalent simplified forms such as $\frac{dy}{dx}=1.5e+e^2$, $\frac{dy}{dx}=e(1.5+e)$, $\frac{dy}{dx}=\frac{e(2e+3)}{2}$

(iii)

M1 Uses quotient rule with $u=3\cos x$, $v=(x+1)^{\frac{1}{3}}$, $u'=\pm A\sin x$ and $v'=B(x+1)^{-\frac{2}{3}}$.

If the rule is quoted it must be correct. It may be implied by their $u=3\cos x$, $v=(x+1)^{\frac{1}{3}}$, $u'=\pm A\sin x$ and $v'=B(x+1)^{-\frac{2}{3}}$ followed by $\frac{vu'-uv'}{v^2}$

Additionally this could be scored by using the product rule with $u=3\cos x$, $v=(x+1)^{-\frac{1}{3}}$ $u'=\pm A\sin x$ and $v'=B(x+1)^{-\frac{4}{3}}$. If the rule is quoted it must be correct. It may be implied by their $u=3\cos x$, $v=(x+1)^{-\frac{1}{3}}$ $u'=\pm A\sin x$ and $v'=B(x+1)^{-\frac{4}{3}}$ followed by $vu'+uv'$

If it is not quoted nor implied only accept either of the two expressions

1) Using quotient form $\frac{(x+1)^{\frac{1}{3}}\times\pm A\sin x-3\cos x\times B(x+1)^{-\frac{2}{3}}}{\left((x+1)^{\frac{1}{3}}\right)^2}$ or $\frac{(x+1)^{\frac{1}{3}}\times\pm A\sin x-3\cos x\times B(x+1)^{-\frac{2}{3}}}{(x+1)^{\frac{1}{9}}}$

2) Using product form $(x+1)^{-\frac{1}{3}}\times\pm A\sin x+3\cos x\times B(x+1)^{-\frac{4}{3}}$

A1 A correct gradient. Accept $f'(x)=\frac{(x+1)^{\frac{1}{3}}(-3\sin x)-3\cos x(\frac{1}{3}(x+1)^{-\frac{2}{3}})}{\left((x+1)^{\frac{1}{3}}\right)^2}$

or $f'(x)=(x+1)^{-\frac{1}{3}}\times-3\sin x+3\cos x\times-\frac{1}{3}(x+1)^{-\frac{4}{3}}$

A1 $f'(x)=\frac{-3(x+1)(\sin x)-\cos x}{(x+1)^{\frac{4}{3}}}$ oe. or a statement that $g(x)=-3(x+1)(\sin x)-\cos x$ oe.

Gold Questions

Non-calculator

The total mark for this section is 35

Q1

(i) Differentiate with respect to x

(a) $x^2 \cos 3x$

(3)

(b) $\dfrac{\ln\left(x^2+1\right)}{x^2+1}$

(4)

(i) A curve C has the equation

$$y = \sqrt{(4x+1)}, \quad x > -\tfrac{1}{4}, \ y > 0$$

The point P on the curve has x-coordinate 2. Find an equation of the tangent to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

(Total for Question 1 is 13 marks)

Q2

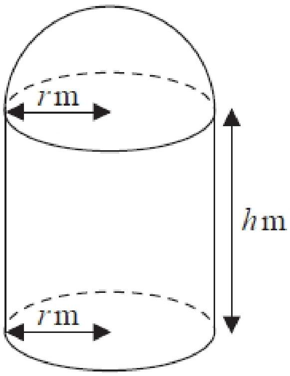


Figure 9

[A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$]

A manufacturer produces a storage tank.

The tank is modelled in the shape of a hollow circular cylinder closed at one end with a hemispherical shell at the other end as shown in Figure 9.

The walls of the tank are assumed to have negligible thickness.

The cylinder has radius r metres and height h metres and the hemisphere has radius r metres.

The volume of the tank is 6 m^3.

(a) Show that, according to the model, the surface area of the tank, in m^2, is given by

$$\frac{12}{r} + \frac{5}{3}\pi r^2$$

(4)

The manufacturer needs to minimise the surface area of the tank.

(b) Use calculus to find the radius of the tank for which the surface area is a minimum.

(4)

(Total for Question 2 is 8 marks)

Q3

A curve C has equation

$$2^x + y^2 = 2xy$$

Find the exact value of $\frac{dy}{dx}$ at the point on C with coordinates (3, 2).

(7)

(Total for Question 3 is 7 marks)

Q4

The curve C, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{dy}{dx}$ at the origin.

(2)

(b) (i) Use the small angle approximation for sin $2y$ to find an equation linking x and y for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i).

(2)

(c) Show that, for all points (x, y) lying on C,

$$\frac{dy}{dx} = \frac{1}{a\sqrt{b - x^2}}$$

where a and b are constants to be found.

(3)

(Total for Question 4 is 7 marks)

End of Questions

Gold Mark Scheme

Q1

Question Number	Scheme		Marks
Q (i)(a)	$y = x^2\cos 3x$		
	Apply product rule: $\begin{Bmatrix} u = x^2 & v = \cos 3x \\ \frac{du}{dx} = 2x & \frac{dv}{dx} = -3\sin 3x \end{Bmatrix}$		
	$\frac{dy}{dx} = 2x\cos 3x - 3x^2\sin 3x$	Applies $vu' + uv'$ correctly for their u, u', v, v' AND gives an expression of the form $\alpha x\cos 3x \pm \beta x^2 \sin 3x$	M1
		Any one term correct	A1
		Both terms correct and no further simplification to terms in $\cos\alpha x^2$ or $\sin\beta x^3$.	A1
			(3)
(b)	$y = \frac{\ln(x^2+1)}{x^2+1}$		
	$u = \ln(x^2+1) \Rightarrow \frac{du}{dx} = \frac{2x}{x^2+1}$	$\ln(x^2+1) \rightarrow \frac{\text{something}}{x^2+1}$	M1
		$\ln(x^2+1) \rightarrow \frac{2x}{x^2+1}$	A1
	Apply quotient rule: $\begin{Bmatrix} u = \ln(x^2+1) & v = x^2+1 \\ \frac{du}{dx} = \frac{2x}{x^2+1} & \frac{dv}{dx} = 2x \end{Bmatrix}$		
	$\frac{dy}{dx} = \frac{\left(\frac{2x}{x^2+1}\right)(x^2+1) - 2x\ln(x^2+1)}{(x^2+1)^2}$	Applying $\frac{vu' - uv'}{v^2}$	M1
		Correct differentiation with correct bracketing but allow recovery.	A1
			(4)
	$\left\{\frac{dy}{dx} = \frac{2x - 2x\ln(x^2+1)}{(x^2+1)^2}\right\}$	{Ignore subsequent working.}	

Question Number	Scheme		Marks
(ii)	$y=\sqrt{4x+1},\ x>-\frac{1}{4}$		
	At P, $y=\sqrt{4(2)+1}=\sqrt{9}=3$	At P, $y=\sqrt{9}$ or 3	B1
	$\frac{dy}{dx}=\frac{1}{2}(4x+1)^{-\frac{1}{2}}(4)$	$\pm k(4x+1)^{-\frac{1}{2}}$	M1*
		$2(4x+1)^{-\frac{1}{2}}$	A1 aef
	$\frac{dy}{dx}=\frac{2}{(4x+1)^{\frac{1}{2}}}$		
	At P, $\frac{dy}{dx}=\frac{2}{(4(2)+1)^{\frac{1}{2}}}$	Substituting $x=2$ into an equation involving $\frac{dy}{dx}$;	M1
	Hence $m(\mathbf{T})=\frac{2}{3}$		
	Either $\mathbf{T}$: $y-3=\frac{2}{3}(x-2)$; or $y=\frac{2}{3}x+c$ and $3=\frac{2}{3}(2)+c\ \Rightarrow\ c=3-\frac{4}{3}=\frac{5}{3}$;	$y-y_1=m(x-2)$ or $y-y_1=m(x-\text{their stated }x)$ with 'their TANGENT gradient' and their y_1; or uses $y=mx+c$ with 'their TANGENT gradient', their x and their y_1.	dM1*;
	Either $\mathbf{T}$: $3y-9=2(x-2)$;		
	$\mathbf{T}$: $3y-9=2x-4$		
	$\mathbf{T}$: $2x-3y+5=0$	$2x-3y+5=0$ Tangent must be stated in the form $ax+by+c=0$, where a, b and c are integers.	A1
			(6)
	or $\mathbf{T}$: $y=\frac{2}{3}x+\frac{5}{3}$		
	$\mathbf{T}$: $3y=2x+5$		
	$\mathbf{T}$: $2x-3y+5=0$		
			[13]

Q2

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$6 = \pi r^2 h + \frac{2}{3}\pi r^3$	B1	This mark is given for a method to find the volume of the cylinder and the semi-hemisphere
	$A = 3\pi r^2 + 2\pi\left(\frac{6 - \frac{2}{3}\pi r^3}{\pi r}\right)$	M1	This mark is given for a method to find the surface area of the tank
		A1	This mark is given for finding an expression for the surface area of the tank
	$A = 3\pi r^2 + \frac{12}{r} - \frac{4\pi r^2}{3} = \frac{12}{r} + \frac{5\pi r^2}{3}$	A1	This mark is given for a fully correct proof to show the surface area of the tank as required
(b)	$A = \frac{12}{r} + \frac{5\pi r^2}{3} \Rightarrow \frac{dA}{dr} = -\frac{12}{r^2} + \frac{10\pi r}{3}$	M1	This mark is given for a method to differentiate to find r
		A1	This mark is given for accurately differentiating to find r
	When $\frac{dA}{dr} = 0$, $-\frac{12}{r^2} + \frac{10\pi r}{3} = 0$ $r^3 = \frac{18}{5\pi}$	M1	This mark is given for a method to set $\frac{dA}{dr} = 0$ to find a value for r
	$r = 1.046$	A1	This mark is given for finding the radius for which the surface area is a minimum

Q3

Question Number	Scheme	Marks
	$\frac{d}{dx}(2^x) = \ln 2 . 2^x$	B1
	$\ln 2 . 2^x + 2y\frac{dy}{dx} = 2y + 2x\frac{dy}{dx}$	M1 A1= A1
	Substituting $(3, 2)$	
	$8\ln 2 + 4\frac{dy}{dx} = 4 + 6\frac{dy}{dx}$	M1
	$\frac{dy}{dx} = 4\ln 2 - 2$ Accept exact equivalents	M1 A1 (7)
		[7]

Q4

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{dx}{dy} = 8\cos 2y \Rightarrow \frac{dy}{dx} = \frac{1}{8\cos 2y}$	M1	This mark is given for differentiating and inverting
	At (0, 0), $\frac{dy}{dx} = \frac{1}{8}$	A1	This mark is given for finding $\frac{dy}{dx}$ when $y = 0$
(b)(i)	$\sin 2y \approx 2y \Rightarrow x \approx 8y$	B1	This mark is given for finding an approximation for x
(b)(ii)	When x and y are small, $x = 4\sin 2y$ approximates to the line $x = 8y$	B1	This mark is given for a valid explanation of the relationship between x and y when both are small
(c)	$\sin^2 2y + \cos^2 2y = 1$ $\Rightarrow \cos^2 2y = 1 - \sin^2 2y$ $x = 4\sin 2y \Rightarrow \sin^2 2y = \left(\frac{x}{4}\right)^2$	M1	This mark is given for a method to use find an expression for $\sin^2 2y$ in terms of x
	$\frac{dy}{dx} = \frac{1}{8\cos 2y} = \frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$	A1	This mark is given for an unsimplified expression for $\frac{dy}{dx}$
	$\frac{dy}{dx} = \frac{1}{2\sqrt{16-x^2}}$	A1	This mark is given for a fully correct answer with $a = 2$ and $b = 16$

Platinum Questions

Non-calculator

The total mark for this section is 22

Q1

(*a*) Given that $y = \ln[t + \sqrt{(1 + t^2)}]$, show that $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{1}{\sqrt{(1+t^2)}}$.

(3)

The curve C has parametric equations

$$x = \frac{1}{\sqrt{(1+t^2)}}, \quad y = \ln[t + \sqrt{(1 + t^2)}], \qquad t \in \mathbb{R}.$$

A student was asked to prove that, for $t > 0$, the gradient of the tangent to C is negative.

The attempted proof was as follows:

$$y = \ln\left(t + \frac{1}{x}\right)$$

$$= \ln\left(\frac{tx+1}{x}\right)$$

$$= \ln(tx + 1) - \ln x$$

$$\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{t}{tx + 1} - \frac{1}{x}$$

$$= \frac{\frac{t}{x}}{t + \frac{1}{x}} - \frac{1}{x}$$

$$= \frac{t\sqrt{(1+t^2)}}{t + \sqrt{(1+t^2)}} - \sqrt{(1 + t^2)}$$

$$= -\frac{(1+t^2)}{t + \sqrt{(1+t^2)}}$$

As $(1 + t^2) > 0$, and $t + \sqrt{(1 + t^2)} > 0$ for $t > 0$, $\frac{\mathrm{d}y}{\mathrm{d}x} < 0$ for $t > 0$.

(*b*) (i) Identify the error in this attempt.

(ii) Give a correct version of the proof.

(6)

(c) Prove that $\ln [-t + \sqrt{(1 + t^2)}] = -\ln [t + \sqrt{(1 + t^2)}]$.

(3)

(d) Deduce that C is symmetric about the x-axis and sketch the graph of C.

(3)

(Total for Question 1 is 15 marks)

Q2

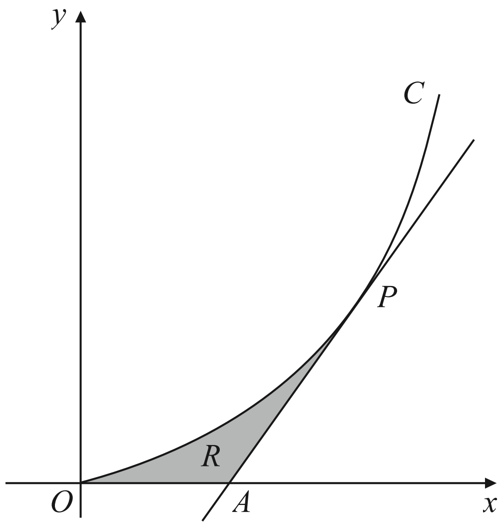


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 2\sin t, \quad y = \ln(\sec t), \quad 0 < t < \frac{\pi}{2}$$

The tangent to C at the point P, where $t = \frac{\pi}{3}$, cuts the x-axis at A.

(a) Show that the x-coordinate of A is $\frac{\sqrt{3}}{3}(3 - \ln 2)$.

(6)

The shaded region R lies between C, the positive x-axis and the tangent AP as shown in Figure 2.

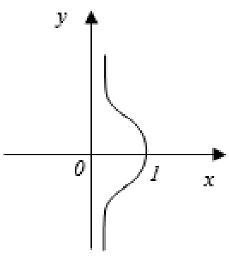
(b) Show that the area of R is $\sqrt{3}\,(1+\ln 2) - 2\ln(2+\sqrt{3}) - \frac{\sqrt{3}}{6}(\ln 2)^2$. **(11)**

(Total for Question 2 is 17 marks)

End of Questions

Platinum Mark Scheme

Q1

Question Number	Scheme	Marks
5. (a)	Attempt at $\frac{dy}{dt}=\frac{f'(u)}{u}$;	M1
	$f'(u)=1+\frac{t}{\sqrt{(1+t^2)}}$ $\left[\ \frac{t+\sqrt{(1+t^2)}}{\sqrt{(1+t^2)}}\ \right]$	B1
	Completion: $\frac{dy}{dt}=\frac{1}{\sqrt{(1+t^2)}}$ AG	A1 (3) (cso)
(b) (i)	$\frac{dy}{dx}=\frac{t}{tx+1}-\frac{1}{x}$ [t in numerator should be $t+x\frac{dt}{dx}$]	B1
(ii)	$\frac{dx}{dt}=-\frac{t}{(1+t^2)^{\frac{3}{2}}}$	M1 A1
	$\frac{dy}{dx}=\frac{dy}{dt}.\frac{dt}{dx}=\frac{1}{\sqrt{(1+t^2)}}.-\frac{(1+t^2)^{\frac{3}{2}}}{t}$ $\left[=-\frac{(1+t^2)}{t}\right]$	M1 A1
	Correct complete argument	A1 (6)
(c)	$\ln\{-t+\sqrt{(1+t^2)}\}+\ln\{t+\sqrt{(1+t^2)}\}=\ln\{(1+t^2)-t^2\}$ or equiv	M1 M1
	$\ln\{-t+\sqrt{(1+t^2)}\}+\ln\{t+\sqrt{(1+t^2)}\}=0\Rightarrow$ result	A1 (3)
	As $t\to -t$, $(x, y)\to(x, -y)$	
(d)	 [Accept that as enough, if fuller explanation not given]	B1
	Asymptotic to y-axis, symmetric in x-axis	M1
	Correct curve, (1, 0) and no cusp	A1 (3)
		(15 marks)

Q2

(a)	P is $(\sqrt{3}, \ln 2)$	B1	Score anywhere.
	$\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}} = \frac{\tan t}{2\cos t}$	M1 A1	M1 attempt $\frac{dy}{dx}$ A1 correct
	When $t = \frac{\pi}{3}$ $\quad m = \sqrt{3}$	A1	
	Equation of tangent at P is: $y - \ln 2 = \sqrt{3}(x - \sqrt{3})$	M1	Attempt tangent at P. √ their P and m
	A is where $y = 0$ ∴ $-\frac{\ln 2}{\sqrt{3}} + \sqrt{3} = x \Rightarrow$ $(x =)\frac{\sqrt{3}}{3}(3 - \ln 2)$	A1 cso (6)	Allow $\frac{3 - \ln 2}{\sqrt{3}}$
(b)	Area under curve = $\int_{t=0}^{t=\pi/3} y\,dx = \int_{(0)}^{(\pi/3)} \ln \sec t . 2\cos t\,dt$	M1	Attempt $\int y\dot{x}\,dt$ √ $\dot{x}$ condone missing 2
	$= [2 \sin t \ln \sec t] - \int 2 \sin t \tan t \; dt$	M1 A1	Attempt parts. Both parts correct.
	$= [\quad] - \int 2\frac{(1-\cos^2 t)}{\cos t}dt$	M1	Use of $s^2 = 1 - c^2$
	$= [\quad] - 2\int \sec t\; dt + 2 \int \cos t \; dt$	M1	Split
	$= [2 \sin t \ln \sec t] - 2\ln\lvert\sec t + \tan t\rvert + 2\sin t$	A1, A1	Accept cos t tan t
	$= \sqrt{3} \ln 2 - (2 \ln [2 + \sqrt{3}] - 0) + (2\frac{\sqrt{3}}{2} - 0)$	M1	Use of correct limits on all 3 integrals
	$[= \sqrt{3}(\ln 2 + 1) - 2\ln(2 + \sqrt{3})]$		
	Area of Δ $= \frac{1}{2}\left[\sqrt{3} - \frac{\sqrt{3}}{3}(3 - \ln 2)\right]\ln 2$ $\left\{= \frac{\sqrt{3}}{6}(\ln 2)^2\right\}$	B1	Any correct expression.

	Area of R = are under curve – area of Δ	M1	Strategy must be $\int$ or area
	$= \sqrt{3}(\ln 2 + 1) - 2\ln(2+\sqrt{3}) - \frac{\sqrt{3}}{6}(\ln 2)^2$ (*)	A1 cso	
		(11)	
		[17]	
	ALT Area = $-\frac{1}{2}\int \ln(1 - \frac{x^2}{4})\,dx$ o.e.	M1	Condone missing $-\frac{1}{2}$
	$= \left[-\frac{1}{2}x\ln(1-\frac{x^2}{4})\right] + \int \frac{-x^2}{4-x^2}\,dx$	M1 A1	Parts correct
	$= [\qquad] + \int 1\,dx - \int \frac{4}{4-x^2}\,dx$	M1	Split
	$= [\qquad] + x - \int\left(\frac{1}{2-x} + \frac{1}{2+x}\right)dx$	M1	Partial Fractions
	$= \left[-\frac{1}{2}x\ln\left(1-\frac{x^2}{4}\right)\right] + x + \ln\left(\frac{2-x}{2+x}\right)$	A1, A1 o.e.	
	Then use of limits etc as before.		

Pearson

Topic 10: Numerical methods

Bronze, Silver, and Gold for

A Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 45 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel A Level Mathematics: Pure Mathematics Year 2' textbook.

Please note the questions in this topic are calculator questions.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculator

The total mark for this section is 34

Q1

$$f(x) = \ln(x + 2) - x + 1, \quad x > -2, x \in \mathbb{R}.$$

(a) Show that there is a root of $f(x) = 0$ in the interval $2 < x < 3$.

(2)

(b) Use the iterative formula

$$x_{n+1} = \ln(x_n + 2) + 1, \; x_0 = 2.5$$

to calculate the values of x_1 , x_2 and x_3 giving your answers to 5 decimal places.

(3)

(c) Show that $x = 2.505$ is a root of $f(x) = 0$ correct to 3 decimal places.

(2)

(Total for Question 1 is 7 marks)

Q2

$$f(x) = -x^3 + 3x^2 - 1.$$

(a) Show that the equation $f(x) = 0$ can be rewritten as

$$x = \sqrt{\left(\frac{1}{3 - x}\right)}$$

(2)

(b) Starting with $x_1 = 0.6$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{1}{3 - x_n}\right)}$$

to calculate the values of x_2, x_3 and x_4, giving all your answers to 4 decimal places.

(2)

(c) Show that $x = 0.653$ is a root of $f(x) = 0$ correct to 3 decimal places.

(3)

(Total for Question 2 is 7 marks)

Q3

$$f(x) = x^3 + 3x^2 + 4x - 12$$

(a) Show that the equation f $(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{4(3-x)}{(3+x)}\right)}, \qquad x \neq -3$$

(3)

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2.

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{(3+x_n)}\right)}, n \geqslant 0$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1, x_2 and x_3.

(3)

The root of f$(x) = 0$ is α.

(c) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places.

(3)

(Total for Question 3 is 9 marks)

Q4

$$f(x) = 3xe^x - 1$$

The curve with equation $y = f(x)$ has a turning point P.

(a) Find the exact coordinates of P.

(5)

The equation f $(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$

(b) Use the iterative formula

$$x_{n+1} = \frac{1}{3}e^{-x_n}$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1, x_2 and x_3.

(3)

(c) By choosing a suitable interval, show that a root of f $(x) = 0$ is $x = 0.2576$ correct to 4 decimal places.

(3)

(Total for Question 4 is 11 marks)

End of Questions

Bronze Mark Scheme

Q1

Question Number	Scheme		Marks	
	(a) $f(2) = 0.38\ \ldots$			
	$f(3) = -0.39\ \ldots$		M1	
	Change of sign (and continuity) $\Rightarrow$ root in $(2, 3)$ ∗	cso	A1	(2)
	(b) $x_1 = \ln 4.5 + 1 \approx 2.50408$		M1	
	$x_2 \approx 2.50498$		A1	
	$x_3 \approx 2.50518$		A1	(3)
	(c) Selecting $[2.5045, 2.5055]$, or appropriate tighter range, and evaluating at both ends.		M1	
	$f(2.5045) \approx 6 \times 10^{-4}$			
	$f(2.5055) \approx -2 \times 10^{-4}$			
	Change of sign (and continuity) $\Rightarrow$ root $\in (2.5045, 2.5055)$			
	$\Rightarrow$ root $= 2.505$ to 3 dp ∗	cso	A1	(2)
				[7]
	Note: The root, correct to 5 dp, is 2.50524			

Q2

Question Number	Scheme	Marks
(*a*)	$x^2(3-x)-1=0$ o.e. (e.g. $x^2(-x+3)=1$) $x=\sqrt{\dfrac{1}{3-x}}$ (✱) Note(✱), answer is given: need to see appropriate working and A1 is cso [Reverse process: Squaring and non-fractional equation M1, form f(x) A1]	M1 A1 (cso) (2)
(*b*)	$x_2 = 0.6455$, $x_3 = 0.6517$, $x_4 = 0.6526$ 1st B1 is for one correct, 2nd B1 for other two correct If all three are to greater accuracy, award B0 B1	B1; B1 (2)
(*c*) Alt (i) Alt (ii)	Choose values in interval (0.6525, 0.6535) or tighter and evaluate both f(0.6525) = –0.0005 (372… f(0.6535) = 0.002 (101… At least one correct "up to bracket", i.e. -0.0005 or 0.002 **Change of sign,** ∴ $x = 0.653$ is a root (correct) to 3 d.p. Requires both correct "up to bracket" and conclusion as above Continued iterations at least as far as x_6 M1 $x_5 = 0.65268$, $x_6 = 0.6527$, $x_7 =$ … two correct to at least 4 s.f. A1 Conclusion : Two values correct to 4 d.p., so 0.653 is root to 3 d.p. A1 If use g(0.6525) = 0.6527..>0.6525 and g(0.6535) = 0.6528..<0.6535 M1A1 Conclusion : Both results correct, so 0.653 is root to 3 d.p. A1	M1 A1 A1 (3) **(7 marks)**

Q3

Question Number	Scheme	Marks
	(a) $x^3+3x^2+4x-12=0 \Rightarrow x^3+3x^2=12-4x$ $\Rightarrow x^2(x+3)=12-4x$ $\Rightarrow x^2=\dfrac{12-4x}{(x+3)} \Rightarrow x=\sqrt{\dfrac{4(3-x)}{(x+3)}}$	 M1 dM1A1* (3)
	(b) $x_1 = 1.41$, awrt $x_2 = 1.20$ $x_3 = 1.31$	M1A1,A1 (3)
	(c) Choosing (1.2715,1.2725) or tighter containing root 1.271998323 f (1.2725) = (+)0.00827… f (1.2715) = –0.00821…. Change of sign⇒α=1.272	M1 M1 A1 (3) (9 marks)

Pearson

Q4

Question Number	Scheme	Marks
(a)	$f'(x) = 3e^x + 3xe^x$	M1 A1
	$3e^x + 3xe^x = 3e^x(1+x) = 0$	
	$x = -1$	M1 A1
	$f(-1) = -3e^{-1} - 1$	B1 (5)
(b)	$x_1 = 0.2596$	B1
	$x_2 = 0.2571$	B1
	$x_3 = 0.2578$	B1 (3)
(c)	Choosing $(0.257\,55, 0.257\,65)$ or an appropriate tighter interval.	M1
	$f(0.257\,55) = -0.000\,379\ ...$	
	$f(0.257\,65) = 0.000\,109\ \ldots$	A1
	Change of sign (and continuity) $\Rightarrow$ root $\in (0.257\,55, 0.257\,65)$ ✱ cso	A1
	($\Rightarrow x = 0.2576$, is correct to 4 decimal places)	(3) [11]
	Note: $x = 0.257\,627\,65\ \ldots$ is accurate	

Silver Questions

Calculator

The total mark for this section is 39

Q1

$$g(x) = e^{x-1} + x - 6$$

(a) Show that the equation $g(x) = 0$ can be written as

$$x = \ln(6 - x) + 1, \qquad x < 6$$

(2)

The root of $g(x) = 0$ is a.

The iterative formula

$$x_{n+1} = \ln(6 - x_n) + 1, \qquad x_0 = 2$$

is used to find an approximate value for a.

(b) Calculate the values of x_1, x_2 and x_3 to 4 decimal places.

(3)

(c) By choosing a suitable interval, show that $a = 2.307$ correct to 3 decimal places.

(3)

(Total for Question 1 is 8 marks)

Q2

$$f(x) = x^2 - 3x + 2\cos\left(\frac{1}{2}x\right), \quad 0 \leq x \leq \pi$$

(a) Show that the equation f $(x) = 0$ has a solution in the interval $0.8 < x < 0.9$

(2)

The curve with equation $y = f(x)$ has a minimum point P.

(b) Show that the x-coordinate of P is the solution of the equation

$$x = \frac{3 + \sin\left(\frac{1}{2}x\right)}{2}$$

(4)

(c) Using the iteration formula

$$x_{n+1} = \frac{3 + \sin\left(\frac{1}{2}x_n\right)}{2}, \quad x_0 = 2$$

find the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(d) By choosing a suitable interval, show that the x-coordinate of P is 1.9078 correct to 4 decimal places.

(3)

(Total for Question 2 is 12 marks)

Q3

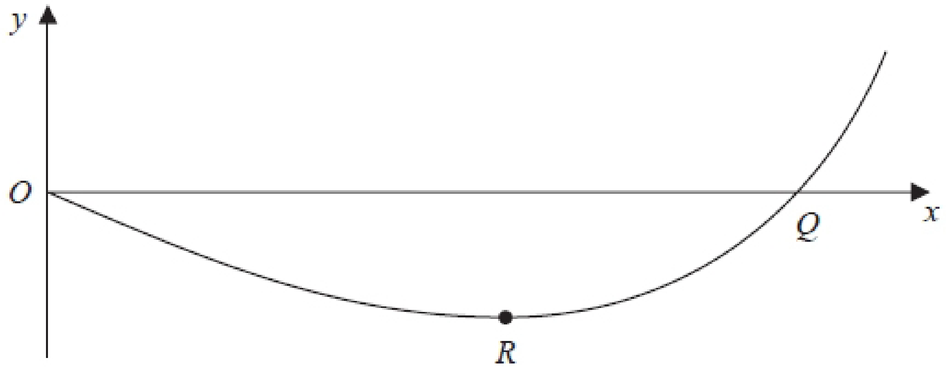


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 2\cos\left(\frac{1}{2}x^2\right) + x^3 - 3x - 2$$

The curve crosses the x-axis at the point Q and has a minimum turning point at R.

(a) Show that the x coordinate of Q lies between 2.1 and 2.2.

(2)

(b) Show that the x coordinate of R is a solution of the equation

$$x = \sqrt{1 + \frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$$

(4)

Using the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{2}{3}x_n\sin\left(\frac{1}{2}x_n^{\,2}\right)}, \quad x_0 = 1.3$$

(c) find the values of x_1 and x_2 to 3 decimal places.

(2)

(Total for Question 3 is 8 marks)

Q4

$$f(x) = 3x^3 - 2x - 6.$$

(a) Show that $f(x) = 0$ has a root, α, between $x = 1.4$ and $x = 1.45$.

(2)

(b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}, \quad x \neq 0.$$

(3)

(c) Starting with $x_0 = 1.43$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(d) By choosing a suitable interval, show that $\alpha = 1.435$ is correct to 3 decimal places.

(3)

(Total for Question 4 is 11 marks)

End of Questions

Silver Mark Scheme

Q1

Question Number	Scheme	Marks
	(a) $0 = e^{x-1} + x - 6 \Rightarrow x = \ln(6-x)+1$	M1A1*
		(2)
	(b) Sub $x_0 = 2$ into $x_{n+1} = \ln(6-x_n)+1 \Rightarrow x_1 = 2.3863$	M1, A1
	AWRT 4 dp. $x_2 = 2.2847$ $x_3 = 2.3125$	A1
		(3)
	(c) Chooses interval [2.3065,2.3075]	M1
	$g(2.3065)$=-0.0002(7), $g(2.3075)$=0.004(4)	dM1
	Sign change , hence root (correct to 3dp)	A1
		(3)
		(8 marks)

Q2

Question No	Scheme	Marks
6	(a) f(0.8) = 0.082, f(0.9)= -0.089	M1
	Change of sign $\Rightarrow$ root (0.8,0.9)	A1
		(2)
	(b) $f'(x) = 2x - 3 - \sin(\frac{1}{2}x)$	M1 A1
	Sets $f'(x) = 0 \Rightarrow$ $x = \frac{3+\sin(\frac{1}{2}x)}{2}$	M1A1*
		(4)
	(c) Sub x_0=2 into $x_{n+1} = \frac{3+\sin(\frac{1}{2}x_n)}{2}$	M1
	x_1=awrt 1.921, x_2=awrt 1.91(0) and x_3=awrt 1.908	A1,A1
		(3)
	(d) [1.90775,1.90785]	M1
	f'(1.90775)=-0.00016.. AND f'(1.90785)= 0.0000076..	M1
	Change of sign $\Rightarrow$ x=1.9078	A1
		(3)
		(12 marks)

Q3

Question Number	Scheme	Marks
(a)	$y_{2.1} = -0.224$, $y_{2.2} = (+)0.546$	M1
	Change of sign$\Rightarrow Q$ lies between	A1
		(2)
(b)	At R $\dfrac{dy}{dx} = -2x\sin\left(\dfrac{1}{2}x^2\right) + 3x^2 - 3$	M1A1
	$-2x\sin\left(\dfrac{1}{2}x^2\right) + 3x^2 - 3 = 0 \Rightarrow$ $x = \sqrt{1 + \dfrac{2}{3}x\sin\left(\dfrac{1}{2}x^2\right)}$ cso	M1A1*
		(4)
(c)	$x_1 = \sqrt{1 + \dfrac{2}{3} \times 1.3\sin\left(\dfrac{1}{2} \times 1.3^2\right)}$	M1
	$x_1 =$ awrt 1.284 $x_2 =$ awrt 1.276	A1
		(2)
		(8 marks)

Q4

Question Number	Scheme	Marks
(a)	$f(1.4) = -0.568\ \ldots < 0$	
	$f(1.45) = 0.245\ \ldots > 0$	M1
	Change of sign (and continuity) $\Rightarrow\ \alpha \in (1.4, 1.45)$	A1 (2)
(b)	$3x^3 = 2x + 6$	
	$x^3 = \dfrac{2x}{3} + 2$	
	$x^2 = \dfrac{2}{3} + \dfrac{2}{x}$	M1 A1
	$x = \sqrt{\left(\dfrac{2}{x} + \dfrac{2}{3}\right)}$ *	A1 cso (3)
(c)	$x_1 = 1.4371$	B1
	$x_2 = 1.4347$	B1
	$x_3 = 1.4355$	B1 (3)
(d)	Choosing the interval $(1.4345, 1.4355)$ or appropriate tighter interval.	M1
	$f(1.4345) = -0.01\ \ldots$	
	$f(1.4355) = 0.003\ \ldots$	M1
	Change of sign (and continuity) $\Rightarrow\ \alpha \in (1.4345, 1.4355)$	
	$\Rightarrow \alpha = 1.435$, correct to 3 decimal places * cso	A1 (3)
		(11 marks)

Gold Questions

47 Marks

Calculator

The total mark for this section is 47

Q1

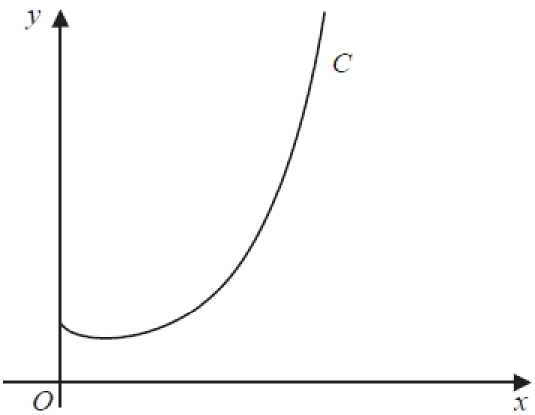


Figure 8

Figure 8 shows a sketch of the curve C with equation $y = x^x$, $x > 0$

(a) Find, by firstly taking logarithms, the x coordinate of the turning point of C.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

The point $P(\alpha, 2)$ lies on C.

(b) Show that $1.5 < \alpha < 1.6$

(2)

A possible iteration formula that could be used in an attempt to find α is

$$x_{n+1} = 2x_n^{\,1-x_n}$$

Using this formula with $x_1 = 1.5$

(c) find x_4 to 3 decimal places,

(2)

(d) describe the long-term behaviour of x_n.

(2)

(Total for Question 1 is 11 marks)

Q2

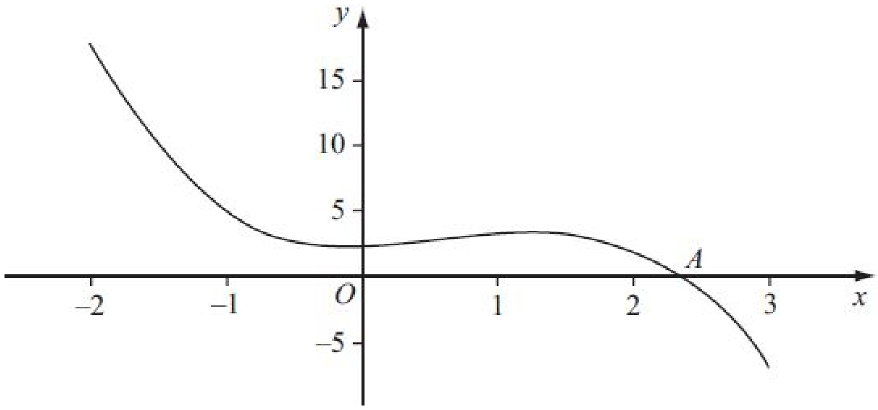


Figure 1

Figure 1 shows part of the curve with equation $y = -x^3 + 2x^2 + 2$, which intersects the x-axis at the point A where $x = \alpha$.

To find an approximation to α, the iterative formula

$$x_{n+1} = \frac{2}{(x_n)^2} + 2$$

is used.

(a) Taking $x_0 = 2.5$, find the values of x_1, x_2, x_3 and x_4.
Give your answers to 3 decimal places where appropriate.

(3)

(b) Show that $\alpha = 2.359$ correct to 3 decimal places.

(3)

(Total for Question 2 is 6 marks)

Q3

The curve with equation $y = f(x)$ where

$$f(x) = x^2 + \ln(2x^2 - 4x + 5)$$

has a single turning point at $x = \alpha$.

(a) Show that α is a solution of the equation

$$2x^3 - 4x^2 + 7x - 2 = 0$$

(4)

The iterative formula

$$x_{n+1} = \frac{1}{7}(2 + 4x_n^{\ 2} - 2x_n^{\ 3})$$

is used to find an approximate value for α.

Starting with $x_1 = 0.3$

(b) calculate, giving each answer to 4 decimal places,

(i) the value of x_2

(ii) the value of x_4

(3)

Using a suitable interval and a suitable function that should be stated,

(c) show that α is 0.341 to 3 decimal places.

(2)

(Total for Question 3 is 9 marks)

Q4

The curve with equation $y = 2\ln(8 - x)$ meets the line $y = x$ at a single point, $x = \alpha$.

(a) Show that $3 < \alpha < 4$

(2)

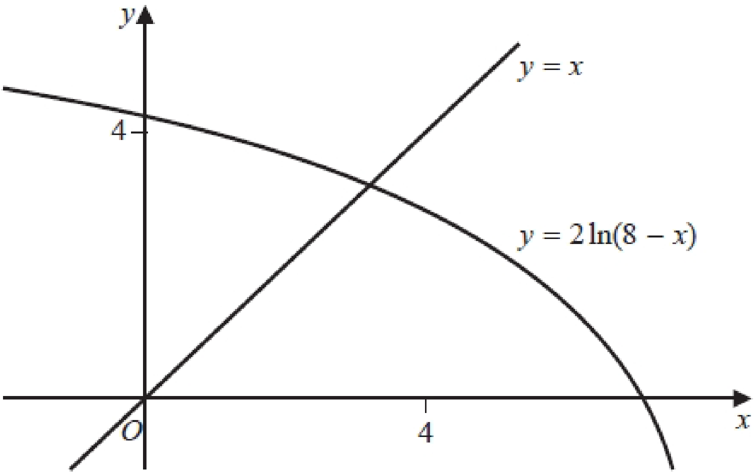


Figure 2

Figure 2 shows the graph of $y = 2\ln(8 - x)$ and the graph of $y = x$.

A student uses the iteration formula

$$x_{n+1} = 2\ln(8 - x_n), \quad n \in \mathbb{N}$$

in an attempt to find an approximation for α.

Using the graph and starting with $x_1 = 4$

(b) determine whether or not this iteration formula can be used to find an approximation for α, justifying your answer.

(2)

(Total for Question 4 is 4 marks)

Pearson

Q5

$$f(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5$$

(a) Show that $f(x) = 0$ has a root α in the interval [3.5, 4]

(2)

A student takes 4 as the first approximation to α.

Given $f(4) = 3.099$ and $f'(4) = 16.67$ to 4 significant figures,

(b) apply the Newton-Raphson procedure once to obtain a second approximation for α, giving your answer to 3 significant figures.

(2)

(c) Show that α is the only root of $f(x) = 0$

(2)

(Total for Question 5 is 6 marks)

Q6

$$f(x) = 25x^2e^{2x} - 16, \qquad x \in \mathbb{R}$$

(a) Using calculus, find the exact coordinates of the turning points on the curve with equation $y = f(x)$.

(5)

(b) Show that the equation $f(x) = 0$ can be written as $x = \pm\frac{4}{5}e^{-x}$.

(1)

The equation $f(x) = 0$ has a root α, where $\alpha = 0.5$ to 1 decimal place.

(c) Starting with $x_0 = 0.5$, use the iteration formula

$$x_{n+1} = \frac{4}{5}e^{-x_n}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(d) Give an accurate estimate for α to 2 decimal places and justify your answer.

(2)

(Total for Question 6 is 11 marks)

End of Questions

Pearson

Gold Mark Scheme

Q1

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$y = x^x \Rightarrow \ln y = x \ln x$	M1	This mark is for a method to find the x-coordinate of the turning point of C by taking logarithms
	$\ln y = x \ln x \Rightarrow \frac{1}{y}\frac{dy}{dx} = \ln x + 1$	M1	This mark is given for a method using implicit differentiation
		A1	This mark is given for a correct expression for $\frac{1}{y}\frac{dy}{dx}$
	Setting $\frac{dy}{dx} = 0$, $\ln x + 1 = 0$	M1	This mark is given for a method for finding the turning point of C by setting $\frac{dy}{dx} = 0$
	$x = e^{-1}$	A1	This mark is given for correctly finding a value for the x-coordinate of the turning point of C
(b)	$1.5^{1.5} = 1.837\ldots$, $1.6^{1.6} = 2.121\ldots$	M1	This mark is given for substituting 1.5 and 1.6 into $y = x^x$
	The curve C contains the points (1.5, 1.8) and (1.6, 2.1). At P, $y = 2$ Since C is continuous, $1.5 < \alpha < 1.6$	A1	This mark is given for a valid explanation that C contains the points (1.5, 1.8) and (1.6, 2.1) and is continuous
(c)	$x_1 = 1.5$ $x_2 = 2 \times 1.5^{-0.5} = 1.633$	M1	This mark is given for finding a correct value for x_2
	$x_3 = 2 \times 1.633^{-0.633} = 1.466$ $x_4 = 2 \times 1.466^{-0.466} = 1.673$	A1	This mark is given for finding a correct value for x_4
(d)	For example: x_n oscillates is periodic is non-convergent	B1	This mark is given for a valid statement about the long-term behaviour of x_n
	between 1 and 2	B1	This mark is given for stating that the behaviour is between 1 and 2
			(Total 11 marks)

Q2

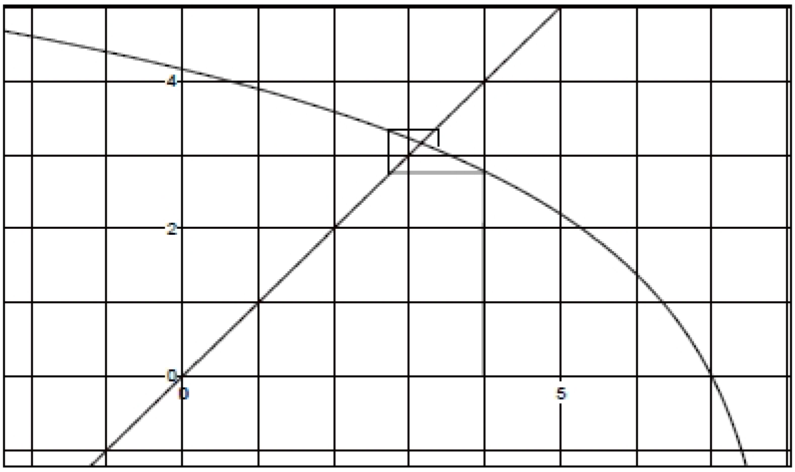
Question Number	Scheme		Marks
Q (a)	Iterative formula: $x_{n+1} = \frac{2}{(x_n)^2} + 2$, $x_0 = 2.5$		
	$x_1 = \frac{2}{(2.5)^2} + 2$	An attempt to substitute $x_0 = 2.5$ into the iterative formula. Can be implied by $x_1 = 2.32$ or 2.320	M1
	$x_1 = 2.32$ $x_2 = 2.371581451...$	Both $x_1 = 2.32(0)$ and $x_2 =$ awrt 2.372	A1
	$x_3 = 2.355593575...$ $x_4 = 2.360436923...$	Both $x_3 =$ awrt 2.356 and $x_4 =$ awrt 2.360 or 2.36	A1 cso
			(3)
(b)	Let $f(x) = -x^3 + 2x^2 + 2 = 0$		
	$f(2.3585) = 0.00583577...$ $f(2.3595) = -0.00142286...$	Choose suitable interval for x, e.g. [2.3585, 2.3595] or tighter	M1
		any one value awrt 1 sf or truncated 1 sf	dM1
	Sign change (and $f(x)$ is continuous) therefore a root α is such that $\alpha \in (2.3585, 2.3595) \Rightarrow \alpha = 2.359$ (3 dp)	both values correct, sign change and conclusion	A1
		At a minimum, both values must be correct to 1sf or truncated 1sf, candidate states "change of sign, hence root".	(3)
			[6]

Q3

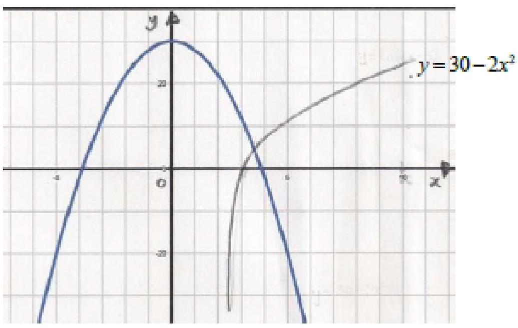
Question	Scheme	Marks	AOs
(a)	$f'(x) = 2x + \dfrac{4x-4}{2x^2-4x+5}$	M1 A1	1.1b 1.1b
	$2x + \dfrac{4x-4}{2x^2-4x+5} = 0 \Rightarrow 2x(2x^2-4x+5)+4x-4=0$	dM1	1.1b
	$2x^3-4x^2+7x-2=0$ *	A1*	2.1
		(4)	
(b)	(i) $x_2 = \dfrac{1}{7}\left(2+4(0.3)^2-2(0.3)^3\right)$	M1	1.1b
	$x_2 = 0.3294$	A1	1.1b
	(ii) $x_4 = 0.3398$	A1	1.1b
		(3)	
(c)	$h(x) = 2x^3-4x^2+7x-2$ $h(0.3415) = 0.00366...$ $h(0.3405) = -0.00130...$	M1	3.1a
	States: • there is a change of sign • $f'(x)$ is continuous • $\alpha = 0.341$ to 3dp	A1	2.4
		(2)	
			(9 marks)
Notes			

Q4

Question	Scheme	Marks	AOs
(a)	Attempts $f(3) =$ and $f(4) =$ where $f(x) = \pm(2\ln(8-x)-x)$	M1	2.1
	$f(3) = (2\ln(5)-x) = (+)0.22$ and $f(4) = (2\ln(4)-4) = -1.23$ Change of sign and function continuous in interval $[3,4] \Rightarrow$ Root *	A1*	2.4
		(2)	
(b)	For annotating the graph by drawing a cobweb diagram starting at $x_1 = 4$ It should have at least two spirals	M1	2.4
	Deduces that the iteration formula **can be used** to find an approximation for α because **the cobweb spirals inwards** for the cobweb diagram	A1	2.2a
		(2)	
			(4 marks)



Q5

Question	Scheme	Marks	AOs
(a)	$\mathrm{f}(3.5) = -4.8,\ \mathrm{f}(4) = (+)3.1$	M1	1.1b
	Change of sign and function continuous in interval $[3.5, 4] \Rightarrow$ Root *	A1*	2.4
		(2)	
(b)	Attempts $x_1 = x_0 - \dfrac{\mathrm{f}(x_0)}{\mathrm{f}'(x_0)} \Rightarrow\ x_1 = 4 - \dfrac{3.099}{16.67}$	M1	1.1b
	$x_1 = 3.81$	A1	1.1b
	$y = \ln(2x-5)$	(2)	
(c)	 Attempts to sketch both $y = \ln(2x-5)$ and $y = 30 - 2x^2$	M1	3.1a
	States that $y = \ln(2x-5)$ meets $y = 30 - 2x^2$ in just one place, therefore $y = \ln(2x-5) = 30 - 2x$ has just one root $\Rightarrow \mathrm{f}(x) = 0$ has just one root	A1	2.4
		(2)	
			(6 marks)

Q6

Question Number	Scheme	Marks
(a)	$f'(x) = 50x^2e^{2x} + 50xe^{2x}$ oe.	M1A1
	Puts $f'(x) = 0$ to give $x = -1$ and $x = 0$ or one coordinate	dM1A1
	Obtains $(0,-16)$ and $(-1, 25e^{-2}-16)$ CSO	A1
		(5)
(b)	Puts $25x^2e^{2x} - 16 = 0 \Rightarrow x^2 = \frac{16}{25}e^{-2x} \Rightarrow x = \pm\frac{4}{5}e^{-x}$	B1*
		(1)
(c)	Subs $x_0 = 0.5$ into $x = \frac{4}{5}e^{-x} \Rightarrow x_1 = \text{awrt } 0.485$	M1A1
	$\Rightarrow x_2 = \text{awrt } 0.492,\ x_3 = \text{awrt } 0.489$	A1
		(3)
(d)	$\alpha = 0.49$	B1
	$f(0.485) = -0.487,\ f(0.495) = (+)0.485$, sign change and deduction	B1
		(2)
		(11 marks)

Notes for Question

No marks can be scored in part (a) unless you see differentiation as required by the question.

(a)

M1 Uses $vu' + uv'$. If the rule is quoted it must be correct.
It can be implied by their $u = \ldots, v = \ldots, u' = \ldots, v' = \ldots$ followed by their $vu' + uv'$
If the rule is not quoted nor implied only accept answers of the form $Ax^2e^{2x} + Bxe^{2x}$

A1 $f'(x) = 50x^2e^{2x} + 50xe^{2x}$.
Allow un simplified forms such as $f'(x) = 25x^2 \times 2e^{2x} + 50x \times e^{2x}$

dM1 Sets $f'(x) = 0$, factorises out/ or cancels the e^{2x} leading to at least one solution of x
This is dependent upon the first M1 being scored.

A1 Both $x = -1$ and $x = 0$ or one complete coordinate . Accept $(0,-16)$ and $(-1, 25e^{-2}-16)$ or $(-1, \text{awrt} -12.6)$

A1 CSO. Obtains both solutions from differentiation. Coordinates can be given in any way.
$x = -1, 0 \quad y = \frac{25}{e^2} - 16, -16$ or linked together by coordinate pairs $(0,-16)$ and $(-1, 25e^{-2}-16)$ but the 'pairs' must be correct and exact.

Pearson

Topic 11a: Basic Integration

Bronze, Silver, Gold and Platinum Worksheets for A Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel A Level Mathematics: Pure Mathematics Year 2' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high-level problem-solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Non-calculator

The total mark for this section is 32

Q1

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 6x^{-\frac{1}{2}} + x\sqrt{x}, \quad x > 0$$

Given that $y = 37$ at $x = 4$, find y in terms of x, giving each term in its simplest form.

(7)

(Total for Question 1 is 7 marks)

Q2

The gradient of a curve C is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x^2+3)^2}{x^2}, \quad x \neq 0.$$

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 + 6 + 9x^{-2}$.

(2)

The point (3, 20) lies on C.

(b) Find an equation for the curve C in the form $y = \mathrm{f}(x)$.

(6)

(Total for Question 2 is 8 marks)

Q3

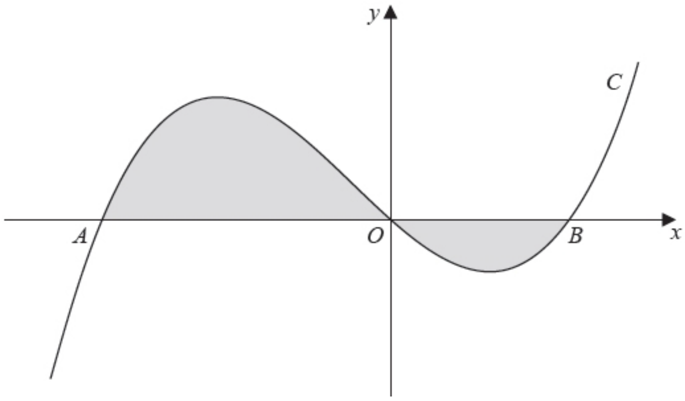


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = x(x + 4)(x - 2).$$

The curve C crosses the x-axis at the origin O and at the points A and B.

(a) Write down the x-coordinates of the points A and B.

(1)

The finite region, shown shaded in Figure 3, is bounded by the curve C and the x-axis.

(b) Use integration to find the total area of the finite region shown shaded in Figure 3.

(7)

(Total for Question 3 is 8 marks)

Q4

(a) Find

$$\int 10x(x^{\frac{1}{2}} - 2)\,dx$$

giving each term in its simplest form.

(4)

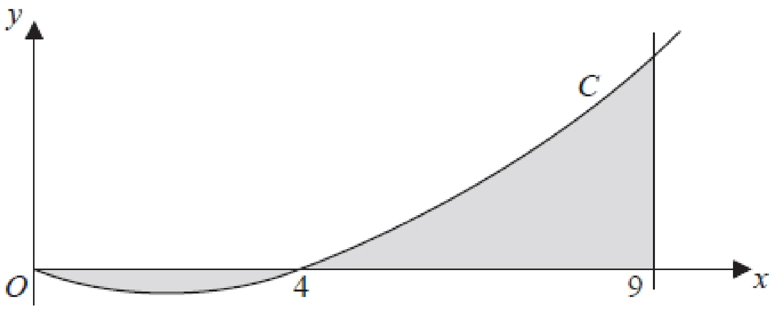


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = 10x(x^{\frac{1}{2}} - 2), \qquad x \geqslant 0$$

The curve C starts at the origin and crosses the x-axis at the point (4, 0).

The area, shown shaded in Figure 2, consists of two finite regions and is bounded by the curve C, the x-axis and the line $x = 9$.

(b) Use your answer from part (a) to find the total area of the shaded regions.

(5)

(Total for Question 4 is 9 marks)

End of Questions

Bronze Mark Scheme

Q1

Question Number	Scheme	Marks
	$\frac{dy}{dx} = 6x^{-\frac{1}{2}} + x\sqrt{x}$	
	$x\sqrt{x} = x^{\frac{3}{2}}$	B1
	$x^n \to x^{n+1}$	M1
	$y = \frac{6}{\frac{1}{2}}x^{\frac{1}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}}(+c)$	A1, A1
	Use $x=4$, $y=37$ to give equation in c, $37 = 12\sqrt{4} + \frac{2}{5}(\sqrt{4})^5 + c$	M1
	$\Rightarrow c = \frac{1}{5}$ or equivalent eg. 0.2	A1
	$(y) = 12x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{5}$	A1
		(7 marks)

B1 $x\sqrt{x} = x^{\frac{3}{2}}$. This may be implied by $+\frac{x^{\frac{5}{2}}}{\frac{5}{2}}$ oe in the subsequent work.

M1 $x^n \to x^{n+1}$ in at least one case so see either $x^{\frac{1}{2}}$ or $x^{\frac{5}{2}}$ or both

A1 One term integrated correctly. It does not have to be simplified Eg. $\frac{6}{\frac{1}{2}}x^{\frac{1}{2}}$ or $+\frac{x^{\frac{5}{2}}}{\frac{5}{2}}$.

No need for $+c$

A1 Other term integrated correctly. See above. No need to simplify nor for $+c$. Need to see $\frac{6}{\frac{1}{2}}x^{\frac{1}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}}$ or a simplified correct version

M1 Substitute $x = 4$, $y = 37$ to produce an equation in c.

A1 Correctly calculates $c = \frac{1}{5}$ or equivalent e.g. 0.2

A1 cso $y = 12x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{5}$. Allow $5y = 60x^{\frac{1}{2}} + 2x^{\frac{5}{2}} + 1$ and accept fully simplified equivalents.

e.g. $y = \frac{1}{5}(60x^{\frac{1}{2}} + 2x^{\frac{5}{2}} + 1)$, $y = 12\sqrt{x} + \frac{2}{5}\sqrt{x^5} + \frac{1}{5}$

Q2

Question Number	Scheme	Marks
(a)	$\left(x^2+3\right)^2 = x^4+3x^2+3x^2+3^2$	M1
	$\dfrac{\left(x^2+3\right)^2}{x^2} = \dfrac{x^4+6x^2+9}{x^2} = x^2+6+9x^{-2}$ (*)	A1 cso (2)
(b)	$y = \dfrac{x^3}{3}+6x+\dfrac{9}{-1}x^{-1}(+c)$	M1 A1 A1
	$20 = \dfrac{27}{3}+6\times 3-\dfrac{9}{3}+c$	M1
	$c = -4$	A1
	$[y =]\dfrac{x^3}{3}+6x-9x^{-1}-4$	A1 ft (6)
		(8 marks)

Q3

Question Number	Scheme	Marks
(a)	Seeing -4 and 2.	B1
		(1)
(b)	$x(x+4)(x-2) = \underline{x^3+2x^2-8x}$ or $\underline{x^3-2x^2+4x^2-8x}$ (without simplifying)	B1
	$\int (x^3+2x^2-8x)\,dx = \frac{x^4}{4}+\frac{2x^3}{3}-\frac{8x^2}{2}\{+c\}$ or $\frac{x^4}{4}-\frac{2x^3}{3}+\frac{4x^3}{3}-\frac{8x^2}{2}\{+c\}$	M1A1ft
	$\left[\frac{x^4}{4}+\frac{2x^3}{3}-\frac{8x^2}{2}\right]_{-4}^{0} = (0)-\left(64-\frac{128}{3}-64\right)$ or $\left[\frac{x^4}{4}+\frac{2x^3}{3}-\frac{8x^2}{2}\right]_{0}^{2} = \left(4+\frac{16}{3}-16\right)-(0)$	dM1
	One integral $=\pm 42\frac{2}{3}$ (42.6 or awrt 42.7) or other integral $=\pm 6\frac{2}{3}$ (6.6 or awrt 6.7)	A1
	Hence Area = "$their\,42\frac{2}{3}$"+ "$their\,6\frac{2}{3}$" or Area = "$their\,42\frac{2}{3}$"− "−$their\,6\frac{2}{3}$"	dM1
	$= 49\frac{1}{3}$ or 49.3 or $\frac{148}{3}$ (NOT $-\frac{148}{3}$)	A1
	(An answer of $= 49\frac{1}{3}$ may not get the final two marks – check solution carefully)	(7)
		[8]
	Notes for Question	
(a)	B1: Need both –4 and 2. May see (-4,0) and (2,0) (correct) but allow (0,-4) and (0, 2) or A = -4, B = 2 or indeed any indication of -4 and 2 – check graph also	
(b)	B1: Multiplies out cubic correctly (terms may not be collected, but if they are, mark collected terms here) M1: Tries to integrate their expansion with $x^n \to x^{n+1}$ for at least one of the terms A1ft: completely correct integral **following through** from their CUBIC expansion (if only quadratic or quartic this is A0) dM1: (dependent on previous M) substituting EITHER -a and 0 and subtracting either way round OR similarly for 0 and b. **If their limits –a and b are used in ONE integral, apply the Special Case below.** A1: Obtain **either** $\pm 42\frac{2}{3}$ (or 42.6 or awrt 42.7)*from the integral from -4 to 0* **or** $\pm 6\frac{2}{3}$ (6.6 or awrt 6.7) *from the integral from 0 to 2;* NO follow through on their cubic (allow decimal or improper equivalents $\frac{128}{3}$ **or** $\frac{20}{3}$) isw such as subtracting from rectangles. This will be penalized in the next two marks, which will be M0A0. dM1 (depends on first method mark) **Correct method to obtain shaded area** so adds two positive numbers (areas) together or uses their **positive** value minus **their negative** value, **obtained from** two separate **definite integrals**. A1: Allow 49.3, 49.33, 49.333 etc. Must follow correct logical work with **no errors** seen. For full marks on this question there must be **two** definite integrals, from -4 to 0 and from 0 to 2, though the evaluations for 0 may not be seen. (Trapezium rule **gets no marks after first two B marks**)	
(b)	**Special Case: one integral only from –a to b**: B1M1A1 available as before, then $\left[\frac{x^4}{4}+\frac{2x^3}{3}-\frac{8x^2}{2}\right]_{-4}^{2} = (4+\frac{16}{3}-16)-\left(64-\frac{128}{3}-64\right) = -6\frac{2}{3}+42\frac{2}{3} = ...$ **dM1** for correct use of their limits –a and b and **subtracting** either way round. A1 for 36: NO follow through. Final M and A marks not available. Max 5/7 for part (b)	

Q4

Question Number	Scheme	Marks
	May mark (a) and (b) together	
(a)	Expands to give $10x^{\frac{3}{2}} - 20x$	B1
	Integrates to give $\frac{10}{"\frac{5}{2}"}x^{"\frac{5}{2}"} + \frac{-"20"x^2}{2}$ $(+c)$	M1 A1ft
	Simplifies to $4x^{\frac{5}{2}} - 10x^2 (+c)$	A1cao **(4)**
(b)	Use limits 0 and 4 either way round on their integrated function (may only see 4 substituted)	M1
	Use limits 4 and 9 either way round on their integrated function	dM1
	Obtains either ± -32 or $\pm$ 194 needs at least one of the previous M marks for this to be awarded	A1
	(So area = $\left\|\int_0^4 ydx\right\| + \int_4^9 ydx$) i.e. 32 + 194, = 226	ddM1,A1 **(5)** **[9]**

Notes

(a) **B1**: Expands the bracket correctly

M1: Correct integration process **on at least one term** after attempt at multiplication. (Follow correct expansion or one slip resulting in $10x^k - 20x$ where k may be $\frac{1}{2}$ or $\frac{5}{2}$ or resulting in $10x^{\frac{3}{2}} - Bx$, where B may be 2 or 5)

So $x^{\frac{3}{2}} \to \frac{x^{\frac{5}{2}}}{5/2}$ or $x^{\frac{1}{2}} \to \frac{x^{\frac{3}{2}}}{3/2}$ or $x^{\frac{5}{2}} \to \frac{x^{\frac{7}{2}}}{7/2}$ and/or $x \to \frac{x^2}{2}$.

A1: Correct unsimplified follow through for both terms of their integration. Does not need $(+c)$

A1: Must be simplified and correct– allow answer in scheme or $4x^{2\frac{1}{2}} - 10x^2$. Does not need $(+c)$

(b) **M1**: (does not depend on first method mark) Attempt to substitute 4 into their integral (however obtained but must not be differentiated) or seeing their evaluated number (usually 32) is enough – do not need to see minus zero.

dM1: (depends on first method mark in (a)) Attempt to subtract either way round using the limits 4 and 9 $A\times 9^{\frac{5}{2}} - B\times 9^2$ with $A\times 4^{\frac{5}{2}} - B\times 4^2$ is enough – or seeing 162 –(-32) {but not 162 – 32 }

A1: At least one of the values (32 and 194) correct (needs just one of the two previous M marks in (b)) or may see 162 + 32 + 32 or 162 + 64 or may be implied by correct final answer if not evaluated until last line of working

ddM1: Adds 32 and 194 (may see 162 + 32 + 32 or may be implied by correct final answer if not evaluated until last line of working). This depends on everything being correct to this point.

A1cao: Final answer of 226 not (- 226)

Common errors: $4\times 4^{\frac{5}{2}} - 10\times 4^2 + 4\times 9^{\frac{5}{2}} - 10\times 9^2 - \ 4\times 4^{\frac{5}{2}} - 10\times 4^2 = \pm$ 162 obtains M1 M1 A0 (neither 32 nor 194 seen and final answer incorrect) then M0 A0 so 2/5

Uses correct limits to obtain -32 +162 +32 = +/-162 is M1 M1 A1 (32 seen) M0 A0 so 3/5

Special case: In part (b) Uses limits 9 and 0 = 972 – 810 - 0 = 162 M0 M1 A0 M0A0 scores 1/5
This also applies if 4 never seen.

Silver Questions

Non-calculator

The total mark for this section is 33

Q1

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -x^3 + \frac{4x-5}{2x^3}, \quad x \neq 0$$

Given that $y = 7$ at $x = 1$, find y in terms of x, giving each term in its simplest form.

(6)

(Total for Question 1 is 6 marks)

Q2

Given that a is a positive constant and

$$\int_a^{2a} \frac{t+1}{t}\,\mathrm{d}t = \ln 7$$

show that $a = \ln k$, where k is a constant to be found.

(4)

(Total for Question 2 is 4 marks)

Q3

A curve C has equation $y = \mathrm{f}(x)$.

Given that

- $\mathrm{f}'(x) = 6x^2 + ax - 23$ where a is a constant
- the y intercept of C is -12
- $(x + 4)$ is a factor of $\mathrm{f}(x)$

find, in simplest form, $\mathrm{f}(x)$.

(6)

(Total for Question 3 is 6 marks)

Q4

Given that $k \in \mathbb{R}^{+}$.

(a) show that $\int_{k}^{3k} \frac{2}{(3x-k)} \mathrm{d}x$ is independent of k,

(4)

(b) show that $\int_{k}^{2k} \frac{2}{(2x-k)^2} \mathrm{d}x$ is inversely proportional to k.

(3)

(Total for Question 4 is 7 marks)

Q5

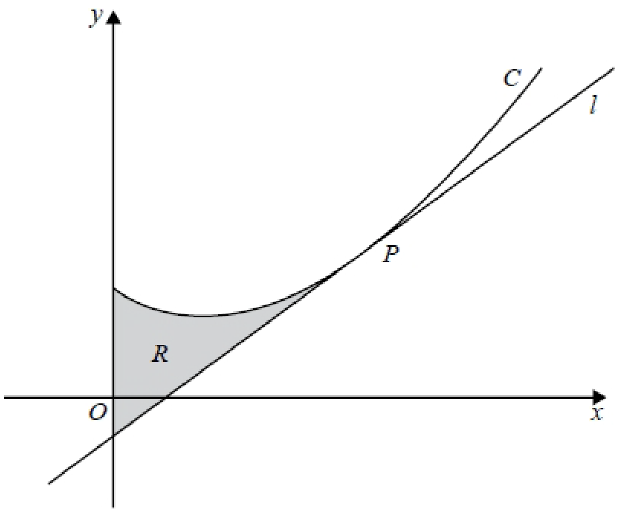


Figure 4

Figure 4 shows a sketch of the curve C with equation

$$y = 5x^{\frac{3}{2}} - 9x + 11, x \geqslant 0$$

The point P with coordinates (4, 15) lies on C.

The line l is the tangent to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the curve C, the line l and the y-axis.

Show that the area of R is 24, making your method clear.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(10)

(Total for Question 5 is 10 marks)

End of Questions

Silver Mark Scheme

Q1

Question Number	Scheme		Marks
	$\left(\frac{dy}{dx}=\right) \quad -x^3+"2"x^{-2}-"\left(\frac{5}{2}\right)"x^{-3}$		M1
	$(y=) \quad -\frac{1}{4}x^4+\frac{"2"x^{-1}}{(-1)}-"\left(\frac{5}{2}\right)"\frac{x^{-2}}{(-2)}(+c)$	Raises power correctly on any one term. Any two follow through terms correct.	M1 A1ft
	$(y=) \quad -\frac{1}{4}x^4+\frac{2x^{-1}}{(-1)}-\frac{5}{2}\frac{x^{-2}}{(-2)}(+c)$	This is not follow through – must be correct	A1
	Given that $y=7$, at $x=1$, then $7=-\frac{1}{4}-2+\frac{5}{4}+c \Rightarrow c=$		M1
	So, $(y=) \quad -\frac{1}{4}x^4-2x^{-1}+\frac{5}{4}x^{-2}+c,\ c=8$ or $(y=)-\frac{1}{4}x^4-2x^{-1}+\frac{5}{4}x^{-2}+8$		A1 [6]
			6 marks
	Notes		
	M1: Expresses as three term polynomial with powers 3, -2 and –3. Allow slips in coefficients. This may be implied by later integration having all three powers 4, -1 and -2. **M1**: An attempt to integrate at least one term so $x^n \to x^{n+1}$ (not a term in the numerator or denominator) **A1ft**: Any two integrations are correct – coefficients may be unsimplified (follow through errors in **coefficients only here**) so should have two of the powers 4, -1 and –2 after integration – depends on 2nd method mark only. There should be a maximum of three terms here. **A1**: Correct three terms – coefficients **may be unsimplified**- do not need constant for this mark Depends on both Method marks **M1**: **Need constant for this mark.** Uses $y = 7$ and $x = 1$ in their changed expression in order to find c, and attempt to find c. *This mark is available even after there is suggestion of differentiation.* **A1**: Need **all four** correct terms to be **simplified** and need $c = 8$ here.		

Q2

Question	Scheme	Marks	AOs
	Writes $\int \frac{t+1}{t}\,dt = \int 1+\frac{1}{t}\,dt$ and attempts to integrate	M1	2.1
	$= t + \ln t \ (+c)$	M1	1.1b
	$(2a+\ln 2a)-(a+\ln a)=\ln 7$	M1	1.1b
	$a=\ln\frac{7}{2}$ with $k=\frac{7}{2}$	A1	1.1b
			(4 marks)

Notes:

M1: Attempts to divide each term by t or alternatively multiply each term by t^{-1}

M1: Integrates each term and knows $\int \frac{1}{t}dt = \ln t$. The + c is not required for this mark

M1: Substitutes in both limits, subtracts and sets equal to ln7

A1: Proceeds to $a=\ln\frac{7}{2}$ and states $k=\frac{7}{2}$ or exact equivalent such as 3.5

Q3

Via firstly integrating

Question	Scheme	Marks	AOs
	$f'(x)=6x^2+ax-23 \Rightarrow f(x)=2x^3+\frac{1}{2}ax^2-23x+c$	M1 A1	1.1b 1.1b
	$"c"=-12$	B1	2.2a
	$f(-4)=0 \Rightarrow 2\times(-4)^3+\frac{1}{2}a(-4)^2-23(-4)-12=0$	dM1	3.1a
	$a=\ldots\ (6)$	dM1	1.1b
	$(f(x)=)2x^3+3x^2-23x-12$ Or Equivalent e.g. $(f(x)=)(x+4)(2x^2-5x-3)$ $(f(x)=)(x+4)(2x+1)(x-3)$	A1cso	2.1
		(6)	
			(6 marks)

Notes:

M1: Integrates $f'(x)$ with two correct indices. There is no requirement for the $+c$

A1: Fully correct integration (may be unsimplified). The $+c$ must be seen (or implied by the -12)

B1: Deduces that the constant term is -12

dM1: Dependent upon having done some integration. It is for setting up a linear equation in a by using $f(-4)=0$

May also see long division attempted for this mark. Need to see a complete method leading to a remainder in terms of a which is then set $=0$.

For reference, the quotient is $2x^2+\left(\frac{a}{2}-8\right)x+9-2a$ and the remainder is $8a-48$

May also use $(x+4)(px^2+qx+r)=2x^3+\frac{1}{2}ax^2-23x-12$ and compare coefficients to find p, q and r and hence a. Allow this mark if they solve for p, q and r

Note that some candidates use 2f(x) which is acceptable and gives the same result if executed correctly.

dM1: Solves the linear equation in a or uses p, q and r to find a.

It is dependent upon having attempted some integration and used $f(\pm 4)=0$ or long division/comparing coefficients with $(x+4)$ as a factor.

A1cso: For $(f(x)=)2x^3+3x^2-23x-12$ oe. Note that "f(x) =" does not need to be seen and ignore any "= 0"

Via firstly using factor

Question	Scheme	Marks	AOs
Alt	$\mathrm{f}(x)=(x+4)(Ax^2+Bx+C)$	M1 A1	1.1b 1.1b
	$\mathrm{f}(x)=Ax^3+(4A+B)x^2+(4B+C)x+4C \Rightarrow C=-3$	B1	2.2a
	$\mathrm{f}'(x)=3Ax^2+2(4A+B)x+(4B+C)$ and $\mathrm{f}'(x)=6x^2+ax-23$ $\Rightarrow A=\ldots$	dM1	3.1a
	Full method to get A, B and C	dM1	1.1b
	$\mathrm{f}(x)=(x+4)(2x^2-5x-3)$	A1cso	2.1
		(6)	
			(6 marks)

Notes:

M1: Uses the fact that $\mathrm{f}(x)$ is a cubic expression with a factor of $(x+4)$
A1: For $\mathrm{f}(x)=(x+4)(Ax^2+Bx+C)$
B1: Deduces that $C=-3$

dM1: Attempts to differentiate either by product rule or via multiplication and compares to $\mathrm{f}'(x)=6x^2+ax-23$ to find A.
dM1: Full method to get A, B and C
A1cso: $\mathrm{f}(x)=(x+4)(2x^2-5x-3)$ or $\mathrm{f}(x)=(x+4)(2x+1)(x-3)$

Q4

Question	Scheme	Marks	AOs
(a)	$\int \frac{2}{(3x-k)}\mathrm{d}x = \frac{2}{3}\ln(3x-k)$	M1 A1	1.1a 1.1b
	$\int_k^{3k} \frac{2}{(3x-k)}\mathrm{d}x = \frac{2}{3}\ln(9k-k) - \frac{2}{3}\ln(3k-k)$	dM1	1.1b
	$= \frac{2}{3}\ln\left(\frac{8\not{k}}{2\not{k}}\right) = \frac{2}{3}\ln 4$ oe	A1	2.1
		(4)	
(b)	$\int \frac{2}{(2x-k)^2}\mathrm{d}x = -\frac{1}{(2x-k)}$	M1	1.1b
	$\int_k^{2k} \frac{2}{(2x-k)^2}\mathrm{d}x = -\frac{1}{(4k-k)} + \frac{1}{(2k-k)}$	dM1	1.1b
	$= \frac{2}{3k} \quad \left(\propto \frac{1}{k}\right)$	A1	2.1
		(3)	
			(7 marks)

(a)

M1: $\int \frac{2}{(3x-k)}\mathrm{d}x = A\ln(3x-k)$ Condone a missing bracket

A1: $\int \frac{2}{(3x-k)}\mathrm{d}x = \frac{2}{3}\ln(3x-k)$

Allow recovery from a missing bracket if in subsequent work $A\ln 9k - k \rightarrow A\ln 8k$

dM1: For substituting k and $3k$ into their $A\ln(3x-k)$ and subtracting either way around

A1: Uses correct ln work and notation to show that I $= \frac{2}{3}\ln\left(\frac{8}{2}\right)$ or $\frac{2}{3}\ln 4$ oe (ie independent of k)

(b)

M1: $\int \frac{2}{(2x-k)^2}\mathrm{d}x = \frac{C}{(2x-k)}$

dM1: For substituting k and $2k$ into their $\frac{C}{(2x-k)}$ and subtracting

A1: Shows that it is inversely proportional to k Eg proceeds to the answer is of the form A/k with $A = 2/3$

There is no need to perform the whole calculation. Accept from $-\frac{1}{(3k)} + \frac{1}{(k)} = \left(-\frac{1}{3}+1\right)\times\frac{1}{k} \propto \frac{1}{k}$

If the calculation is performed it must be correct.

Do not isw here. They should know when they have an expression that is inversely proportional to k.

You may see substitution used but the mark is scored for the same result. See below

$u = 2x - k \rightarrow \left[\frac{C}{u}\right]$ for M1 with limits $3k$ and k used for dM1

Pearson

Q5

Question	Scheme	Marks	AOs
	$\frac{dy}{dx} = \frac{15}{2}x^{\frac{1}{2}} - 9$	M1 A1	3.1a 1.1b
	Substitutes $x = 4 \Rightarrow \frac{dy}{dx} = 6$	M1	2.1
	Uses (4, 15) and gradient $\Rightarrow y - 15 = 6(x-4)$	M1	2.1
	Equation of l is $y = 6x - 9$	A1	1.1b
	Area $R = \int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11\right) - (6x-9)\,dx$	M1	3.1a
	$= \left[2x^{\frac{5}{2}} - \frac{15}{2}x^2 + 20x(+c)\right]_0^4$	A1	1.1b
	Uses both limits of 4 and 0 $\left[2x^{\frac{5}{2}} - \frac{15}{2}x^2 + 20x\right]_0^4 = 2\times 4^{\frac{5}{2}} - \frac{15}{2}\times 4^2 + 20\times 4 - 0$	M1	2.1
	Area of $R = 24$ *	A1*	1.1b
	Correct notation with good explanations	A1	2.5
		(10)	
		(10 marks)	

Pearson

Notes:

M1: Differentiates $5x^{\frac{3}{2}} - 9x + 11$ to a form $Ax^{\frac{1}{2}} + B$

A1: $\frac{dy}{dx} = \frac{15}{2}x^{\frac{1}{2}} - 9$ but may not be simplified

M1: Substitutes $x = 4$ in their $\frac{dy}{dx}$ to find the gradient of the tangent

M1: Uses their gradient and the point (4, 15) to find the equation of the tangent

A1: Equation of l is $y = 6x - 9$

M1: Uses Area $R = \int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11\right) - (6x - 9)\,dx$ following through on their $y = 6x - 9$

Look for a form $Ax^{\frac{5}{2}} + Bx^2 + Cx$

A1: $= \left[2x^{\frac{5}{2}} - \frac{15}{2}x^2 + 20x(+c)\right]_0^4$ This must be correct but may not be simplified

M1: Substitutes in both limits and subtracts

A1*: Correct area for $R = 24$

A1: Uses correct notation and produces a well explained and accurate solution. Look for

- Correct notation used consistently and accurately for both differentiation and integration
- Correct explanations in producing the equation of l. See scheme.
- Correct explanation in finding the area of R. In way 2 a diagram may be used.

Alternative method for the area using area under curve and triangles. (Way 2)

M1: Area under curve $= \int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11\right) = \left[Ax^{\frac{5}{2}} + Bx^2 + Cx\right]_0^4$

A1: $= \left[2x^{\frac{5}{2}} - \frac{9}{2}x^2 + 11x\right]_0^4 = 36$

M1: This requires a full method with all triangles found using a correct method

Look for Area $R = \text{their } 36 - \frac{1}{2} \times 15 \times \left(4 - \text{their } \frac{3}{2}\right) + \frac{1}{2} \times \text{their } 9 \times \text{their } \frac{3}{2}$

Gold Questions

Non-calculator

The total mark for this section is 34

Q1

(i) Find $\int \ln\left(\frac{x}{2}\right)\mathrm{d}x$.

(4)

(ii) Find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \,\mathrm{d}x$.

(5)

(Total for Question 1 is 9 marks)

Q2

Given that A is constant and

$$\int_1^4 \left(3\sqrt{x} + A\right)\mathrm{d}x = 2A^2$$

show that there are exactly two possible values for A.

(5)

(Total for Question 2 is 5 marks)

Q3

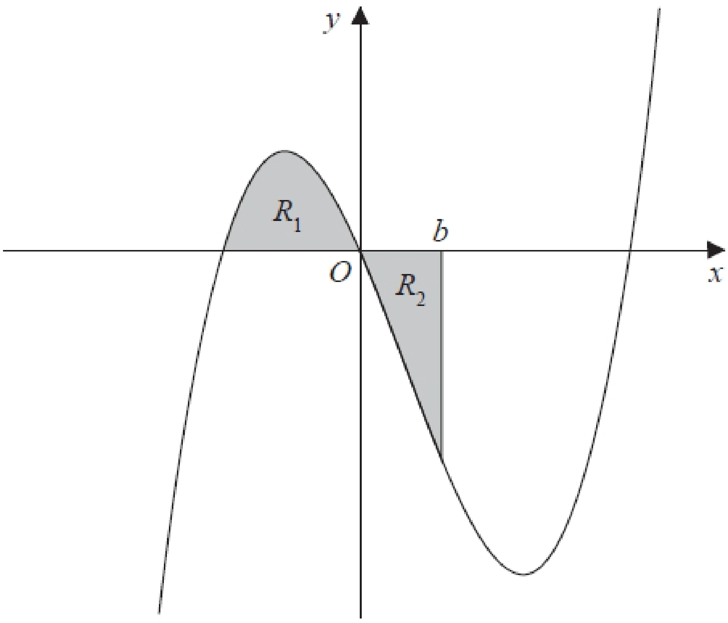


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = x(x + 2)(x - 4)$.

The region R_1 shown shaded in Figure 2 is bounded by the curve and the negative x-axis.

(a) Show that the exact area of R_1 is $\dfrac{20}{3}$.

(4)

The region R_2 also shown shaded in Figure 2 is bounded by the curve, the positive x-axis and the line with equation $x = b$, where b is a positive constant and $0 < b < 4$

Given that the area of R_1 is equal to the area of R_2

(b) verify that b satisfies the equation

$$(b + 2)^2 (3b^2 - 20b + 20) = 0$$

(4)

The roots of the equation $3b^2 - 20b + 20 = 0$ are 1.225 and 5.442 to 3 decimal places.
The value of b is therefore 1.225 to 3 decimal places.

(c) Explain, with the aid of a diagram, the significance of the root 5.442

(2)

(Total for Question 3 is 10 marks)

Q4

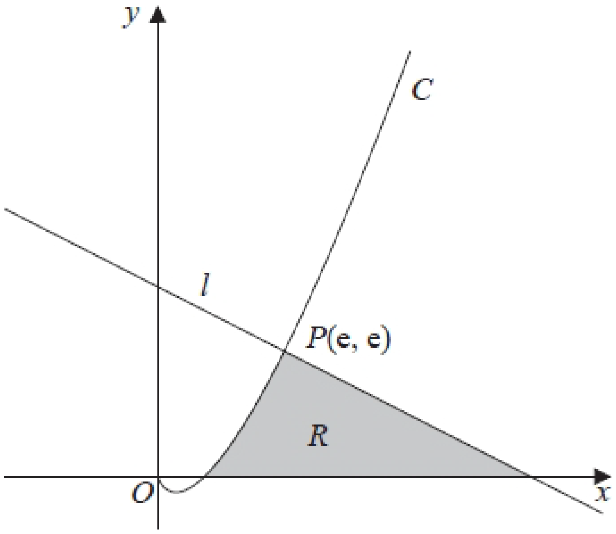


Figure 2

Figure 2 shows a sketch of part of the curve C with equation $y = x \ln x, \quad x > 0$

The line l is the normal to C at the point $P(\mathrm{e}, \mathrm{e})$

The region R, shown shaded in Figure 2, is bounded by the curve C, the line l and the x-axis.

Show that the exact area of R is $A\mathrm{e}^2 + B$ where A and B are rational numbers to be found.

(10)

(Total for Question 4 is 10 marks)

End of Questions

Gold Mark Scheme

Q1

Question Number	Scheme		Marks
(i)	$\int \ln\left(\frac{x}{2}\right)\,dx = \int 1.\ln\left(\frac{x}{2}\right)\,dx \Rightarrow \left\{ \begin{array}{ll} u = \ln\left(\frac{x}{2}\right) & \Rightarrow \frac{du}{dx} = \frac{\frac{1}{2}}{\frac{x}{2}} = \frac{1}{x} \\ \frac{dv}{dx} = 1 & \Rightarrow v = x \end{array} \right\}$		
	$\int \ln\left(\frac{x}{2}\right)\,dx = x\ln\left(\frac{x}{2}\right) - \int x.\frac{1}{x}\,dx$	Use of 'integration by parts' formula in the correct direction.	M1
		Correct expression.	A1
	$= x\ln\left(\frac{x}{2}\right) - \int \underline{1}\,dx$	An attempt to multiply x by a candidate's $\frac{a}{x}$ or $\frac{1}{bx}$ or $\frac{1}{x}$.	<u>dM1</u>
	$= x\ln\left(\frac{x}{2}\right) - x + c$	Correct integration with $+\,c$	A1 aef
			[4]
(ii)	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\,dx$		
	$\left[\text{NB}: \underline{\cos 2x = \pm 1 \pm 2\sin^2 x} \text{ gives } \sin^2 x = \frac{1-\cos 2x}{2}\right]$	Consideration of double angle formula for $\sin^2 x$	M1
	$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1-\cos 2x}{2}\,dx = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\underline{1-\cos 2x}\right)\,dx$		
	$= \frac{1}{2}\left[\underline{x - \frac{1}{2}\sin 2x}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$	<u>Integrating to give $\pm ax \pm b\sin 2x$</u>;	dM1
		Correct result of anything equivalent to $\frac{1}{2}x - \frac{1}{4}\sin 2x$	A1
	$= \frac{1}{2}\left[\left(\frac{\pi}{2} - \frac{\sin(\pi)}{2}\right) - \left(\frac{\pi}{4} - \frac{\sin(\frac{\pi}{2})}{2}\right)\right]$	Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round.	ddM1
	$= \frac{1}{2}\left[\left(\frac{\pi}{2} - 0\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right)\right]$		
	$= \frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right) \quad = \frac{\pi}{8} + \frac{1}{4}$	$\underline{\frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right)}$ or $\underline{\frac{\pi}{8} + \frac{1}{4}}$ Candidate must collect their π term and constant term together for A1	A1 aef
			[5]
			9 marks

Question Number	Scheme		Marks
Aliter (i) **Way 2**	$\int \ln\left(\frac{x}{2}\right)\,dx = \int (\ln x - \ln 2)\,dx = \int \ln x\,dx - \int \ln 2\,dx$		
	$\int \ln x\,dx = \int 1.\ln x\,dx \Rightarrow \begin{Bmatrix} u = \ln x & \Rightarrow & \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = 1 & \Rightarrow & v = x \end{Bmatrix}$		
	$\int \ln x\,dx = x\ln x - \int x.\frac{1}{x}\,dx$	Use of 'integration by parts' formula in the correct direction.	M1
	$= x\ln x - x + c$	Correct integration of $\ln x$ with or without $+ c$	A1
	$\int \ln 2\,dx = x\ln 2 + c$	Correct integration of $\ln 2$ with or without $+ c$	M1
	Hence, $\int \ln\left(\frac{x}{2}\right)\,dx = x\ln x - x - x\ln 2 + c$	Correct integration with $+ c$	A1 aef
			[4]

Question Number	Scheme		Marks
Aliter (ii) **Way 2**	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\,dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x.\sin x\,dx$ and $I = \int \sin^2 x\,dx$		
	$\begin{Bmatrix} u = \sin x & \Rightarrow & \frac{du}{dx} = \cos x \\ \frac{dv}{dx} = \sin x & \Rightarrow & v = -\cos x \end{Bmatrix}$		
	$\therefore I = \underline{\left\{-\sin x\cos x + \int \cos^2 x\,dx\right\}}$	An attempt to use the correct by parts formula.	M1
	$\therefore I = \left\{-\sin x\cos x + \int (1 - \sin^2 x)\,dx\right\}$		
	$\int \sin x\,dx = \left\{-\sin x\cos x + \int 1\,dx - \int \sin^2 x\,dx\right\}$		
	$2\int \sin^2 x\,dx = \left\{-\sin x\cos x + \int 1\,dx\right\}$	For the LHS becoming $2I$	dM1
	$2\int \sin^2 x\,dx = \{-\sin x\cos x + x\}$		
	$\int \sin^2 x\,dx = \left\{\underline{-\frac{1}{2}\sin x\cos x + \frac{x}{2}}\right\}$	<u>Correct integration</u>	A1
	$\therefore \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\,dx = \left[\left(-\frac{1}{2}\sin(\frac{\pi}{2})\cos(\frac{\pi}{2}) + \frac{(\frac{\pi}{2})}{2}\right) - \left(-\frac{1}{2}\sin(\frac{\pi}{4})\cos(\frac{\pi}{4}) + \frac{(\frac{\pi}{4})}{2}\right)\right]$ $= \left[(0 + \frac{\pi}{4}) - (-\frac{1}{4} + \frac{\pi}{8})\right]$	Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round.	ddM1
	$= \frac{\pi}{8} + \frac{1}{4}$	$\underline{\frac{1}{8}(\pi + 2)}$ or $\underline{\frac{\pi}{8} + \frac{1}{4}}$ Candidate must collect their pi term and constant term together for A1	A1 aef [5]

Q2

Question	Scheme		Marks	AOs
	$\int\left(3x^{0.5}+A\right)\mathrm{d}x=2x^{1.5}+Ax\left(+c\right)$		M1 A1	3.1a 1.1b
	Uses limits and sets $= 2A^2 \Rightarrow \left(2\times8+4A\right)-\left(2\times1+A\right)=2A^2$		M1	1.1b
	Sets up quadratic and attempts to solve	Sets up quadratic and attempts b^2-4ac	M1	1.1b
	$\Rightarrow A=-2,\frac{7}{2}$ and states that there are two roots	States $b^2-4ac=121>0$ and hence there are two roots	A1	2.4
				(5 marks)

Notes:

M1: Integrates the given function and achieves an answer of the form $kx^{1.5}+Ax\left(+c\right)$ where k is a non- zero constant

A1: Correct answer but may not be simplified

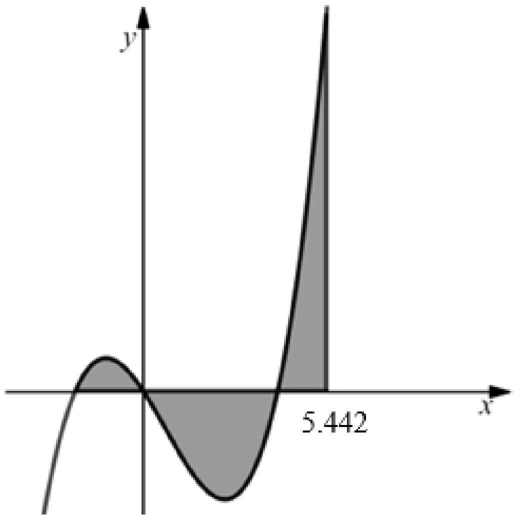
M1: Substitutes in limits and subtracts. This can only be scored if $\int A\,\mathrm{d}x=Ax$ and not $\frac{A^2}{2}$

M1: Sets up quadratic equation in A and either attempts to solve or attempts b^2-4ac

A1: Either $A=-2,\frac{7}{2}$ and states that there are two roots

Or states $b^2-4ac=121>0$ and hence there are two roots

Q3

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$y = x(x+2)(x-4) = x^3 - 2x^2 - 8x$	B1	This mark is given for expanding brackets as a first step to a solution
	$\int_{-2}^{0} x^3 - 2x^2 - 8x \; dx$	M1	This mark is given for a method to find the exact are of R_1
	$= \left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2}^{0}$	M1	This mark is given for a method to evaluate the integral
	$= 0 - (4 - \frac{-16}{3} - 16) = \frac{20}{3}$	A1	This mark is given for a full method to show the exact value of R_1
(b)	$\frac{1}{4}b^4 - \frac{2}{3}b^3 - 4b^2 = -\frac{20}{3}$	M1	This mark is given for deducing the area of $R_2 = -\frac{20}{3}$
	$3b^4 - 8b^3 - 48b^2 + 80 = 0$	A1	This mark is given for rearranging the equation to a quartic
	$(b+2)^2(3b^2 - 20b + 20)$ $= (b^2 + 4b + 4)(3b^2 - 20b + 20)$	M1	This mark is given for expanding the equation given
	$= 3b^4 - 8b^3 - 48b^2 + 80 = 0$ The two equations are the same, so verified	A1	This mark is for showing, and stating, that the equations are the same
(c)		B1	This mark is given for a sketch of the curve with $b = 5.442$ shown
	Between $x = -2$ and $b = 5.442$, the area above the x-axis is the same as the area below the x-axis	B1	This mark is given for a valid explanation of the significance of the root 5.442

Q4

Question	Scheme	Marks	AOs
	$C: y = x\ln x$; l is a normal to C at $P(\mathrm{e}, \mathrm{e})$ Let x_A be the x-coordinate of where l cuts the x-axis		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \ln x + x\left(\frac{1}{x}\right)$ $\{= 1 + \ln x\}$	M1 A1	2.1 1.1b
	$x = \mathrm{e},\ m_T = 2 \Rightarrow m_N = -\frac{1}{2} \Rightarrow y - \mathrm{e} = -\frac{1}{2}(x - \mathrm{e})$ $y = 0 \Rightarrow -\mathrm{e} = -\frac{1}{2}(x - \mathrm{e}) \Rightarrow x = \ldots$	M1	3.1a
	l meets x-axis at $x = 3\mathrm{e}$ (allow $x = 2\mathrm{e} + \mathrm{e}\ln\mathrm{e}$)	A1	1.1b
	{Areas:} **either** $\int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ \ldots\ ]_1^{\mathrm{e}} = \ldots$ **or** $\frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$	M1	2.1
	$\left\{\int x\ln x\,\mathrm{d}x = \right\}$ $\frac{1}{2}x^2\ln x - \int \frac{1}{x}\cdot\left(\frac{x^2}{2}\right)\{\mathrm{d}x\}$	M1	2.1
	$\left\{= \frac{1}{2}x^2\ln x - \int \frac{1}{2}x\,\{\mathrm{d}x\}\right\} = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2$	dM1 A1	1.1b 1.1b
	$\text{Area}(R_1) = \int_1^{\mathrm{e}} x\ln x\,\mathrm{d}x = [\ \ldots\ ]_1^{\mathrm{e}} = \ldots$; $\text{Area}(R_2) = \frac{1}{2}((\text{their } x_A) - \mathrm{e})\mathrm{e}$ and so, $\text{Area}(R) = \text{Area}(R_1) + \text{Area}(R_2)$ $\{= \frac{1}{4}\mathrm{e}^2 + \frac{1}{4} + \mathrm{e}^2\}$	M1	3.1a
	$\text{Area}(R) = \frac{5}{4}\mathrm{e}^2 + \frac{1}{4}$	A1	1.1b
		(10)	

Notes for Question	
M1:	Differentiates by using the product rule to give $\ln x + x(\text{their } g'(x))$, where $g(x) = \ln x$
A1:	Correct differentiation of $y = x\ln x$, which can be un-simplified or simplified
M1:	Complete strategy to find the x coordinate where their normal to C at $P(\text{e}, \text{e})$ meets the x-axis i.e. Sets $y=0$ in $y - \text{e} = m_N(x - \text{e})$ to find $x = ...$
Note:	m_T is found by using calculus and $m_N \neq m_T$
A1:	l meets x-axis at $x = 3\text{e}$, allowing un-simplified values for x such as $x = 2\text{e} + \text{e}\ln\text{e}$
Note:	Allow $x =$ awrt 8.15
M1:	Scored for either • Area under curve $= \int_1^{\text{e}} x\ln x \, dx = [\ ...\ ]_1^{\text{e}} = ...$, with limits of e and 1 and some attempt to substitute these and subtract • or Area under line $= \frac{1}{2}((\text{their } x_A) - \text{e})\text{e}$, with a valid attempt to find x_A
M1:	Integration by parts the correct way around to give $Ax^2\ln x - \int B\left(\frac{x^2}{x}\right)\{dx\}$; $A \neq 0, B > 0$
dM1:	**dependent on the previous M mark** Integrates the second term to give $\pm\lambda x^2$; $\lambda \neq 0$
A1:	$\frac{1}{2}x^2\ln x - \frac{1}{4}x^2$
M1:	Complete strategy of finding the area of R by finding the sum of two key areas. See scheme.
A1:	$\frac{5}{4}\text{e}^2 + \frac{1}{4}$
Note:	$\text{Area}(R_2)$ can also be found by integrating the line l between limits of e and their x_A i.e. $\text{Area}(R_2) = \int_{\text{e}}^{\text{their } x_A} \left(-\frac{1}{2}x + \frac{3}{2}\text{e}\right) dx = [\ ...\ ]_{\text{e}}^{\text{their } x_A} = ...$
Note:	<u>**Calculator approach with no algebra, differentiation or integration seen:**</u> • Finding l cuts through the x-axis at awrt 8.15 is 2nd M1 2nd A1 • Finding area between curve and the x-axis between $x=1$ and $x=\text{e}$ to give awrt 2.10 is 3rd M1 • Using the above information (must be seen) to apply $\text{Area}(R) = 2.0972... + 7.3890... = 9.4862...$ is final M1 Therefore, a maximum of 4 marks out of the 10 available.

Platinum Questions

Calculator

The total mark for this section is 27

Q1

(*a*) On the same diagram, sketch $y = x$ and $y = \sqrt{x}$, for $x \geq 0$, and mark clearly the coordinates of the points of intersection of the two graphs.

(2)

(*b*) With reference to your sketch, explain why there exists a value a of x ($a > 1$) such that

$$\int_0^a x \; \mathrm{d}x = \int_0^a \sqrt{x} \; \mathrm{d}x .$$

(2)

(*c*) Find the exact value of a.

(4)

(*d*) Hence, or otherwise, find a non-constant function $\mathrm{f}(x)$ and a constant b ($b \neq 0$) such that

$$\int_{-b}^{b} \mathrm{f}(x) \; \mathrm{d}x = \int_{-b}^{b} \sqrt{[\mathrm{f}(x)]} \; \mathrm{d}x .$$

(2)

(Total for Question 1 is 10 marks)

Q2

$$f(x) = x - [x], \qquad x \geq 0$$

where $[x]$ is the largest integer $\leq x$.

For example, f(3.7) = 3.7 – 3 = 0.7; f(3) = 3 – 3 = 0.

(*a*) Sketch the graph of $y = f(x)$ for $0 \leq x < 4$.

(3)

(*b*) Find the value of p for which $\int_2^p f(x)\,dx = 0.18$.

(3)

Given that $$g(x) = \frac{1}{1+kx}, \quad x \geq 0, \quad k > 0,$$

and that $x_0 = \frac{1}{2}$ is a root of the equation $f(x) = g(x)$,

(*c*) find the value of k.

(2)

(*d*) Add a sketch of the graph of $y = g(x)$ to your answer to part (*a*).

(1)

The root of $f(x) = g(x)$ in the interval $n < x < n + 1$ is x_n, where n is an integer.

(*e*) Prove that

$$2x_n^2 - (2n - 1)x_n - (n + 1) = 0.$$

(4)

(*f*) Find the smallest value of n for which $x_n - n < 0.05$.

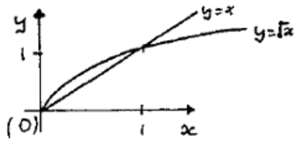
(4)

(Total for Question 2 is 17 marks)

Platinum Mark Scheme

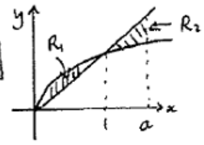
Q1

2. (a)



	Relative shapes	B1	
	0 or (0,0) implied and (1,1). On axes is OK.	B1	(2)

(b) $[R_1 = \frac{1}{6}]$



As a increases from 1 R_2 increases. Choose a so that $R_2 = R_1$ then areas are the same.

	Diagram with regions or mention of areas.	B1g	
	Full argument	B1h	(2)

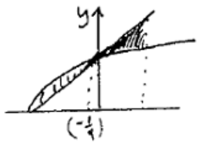
(c) $\int_0^a x\,dx = \int_0^a x^{\frac{1}{2}}\,dx \Rightarrow \left[\frac{x^2}{2}\right]_0^a = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^a$ — Attempt both integrals – one correct — M1

$\Rightarrow \frac{a^2}{2} = \frac{2}{3}a^{3/2}$ — A correct equation in a — A1

$\Rightarrow a^{3/2}(3a^{\frac{1}{2}} - 4) = 0 \rightarrow a^{1/2} = 4/3$ o.e. — Attempt to solve $\rightarrow a^{\wedge} = k$ — M1

$a = \underline{\underline{\frac{16}{9}}}$ — A1 — (4)

(d)



Translate $\frac{1}{2}a \leftarrow$.

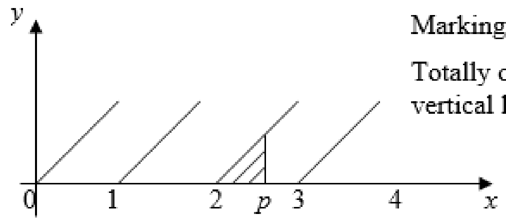
$f(x) = \underline{\underline{x + \frac{8}{9}}}$ — $x + \frac{a}{2} = f(x)$ (Any suitable $f(x) \ne b$) — B1√

$b = \underline{\underline{\frac{8}{9}}}$ — $\frac{a}{2} = b$ — B1√ — (2)

√ their a. — (10)

S.C. if $b = \beta$ and $f(x) = x + \beta$ score B1 only

Q2

Question Number	Scheme	Marks
6. (*a*)	 General shape	M1
	Marking 1 on y-axis	A1
	Totally correct (allow dotted vertical line but not full)	A1 (3)
(*b*)	$\int_2^p \mathrm{f}(x)\mathrm{d}x =$ area of shaded triangle or $\int_2^p (x-2)\mathrm{d}x$	M1
	$= \frac{1}{2}(p-2)^2$	A1
	Setting equal to 0.18 to give $p = 2.6$	A1 (3)
(*c*)	Smallest root occurs where $\frac{1}{1+kx} = x$	M1
	Setting $x = ½ \Rightarrow k = 2$ (allow with no working)	A1 (2)
(*d*)	Sketch of $y = \mathrm{g}(x)$ superimposed on $y = \mathrm{f}(x)$ – see above	B1 (1)
(*e*)	Solution in $n < x < n+1$: $\frac{1}{1+2x_n} = x_n - n$	M1 A1√ on k
	$\Rightarrow 2x^2 - (2n-1)x - (n+1) = 0$ c.s.o. * [M1 even if [x] for n]	M1 A1 (4)
(*f*)	Method using (e)	
	$x = \frac{2n-1+\sqrt{(2n-1)^2+8(n+1)}}{4} = \frac{2n-1+\sqrt{4n^2+4n+9}}{4}$	M1
	$\frac{2n-1+\sqrt{4n^2+4n+9}}{4} < n + 0.05$ [$\sqrt{4n^2+4n+9} < 2n + 1.2$]	A1
	$\Rightarrow 0.8n > 7.56 \Rightarrow n = 10$	M1 A1 (4)
Alt	*Alternative*: $\frac{1}{1+2x_n} = x_n - n \Rightarrow \frac{1}{1+2x_n} < 0.05$	M1 A1
	$\therefore 0.1\, x_n > 0.95 \quad \therefore x_n > 9.5$; $(n > 9.45)$ $n = 10$	M1; A1
	[Equality throughout lose final A1]	**(17 marks)**

Pearson

Topic 12: Vectors

Bronze, Silver, Gold and Platinum Worksheets for

A Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between approximately 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel A Level Mathematics: Pure Mathematics Year 2' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Non-calculator

The total mark for this section is 20

Q1

[*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively*.]

A stone slides horizontally across ice.

Initially the stone is at the point $A(-24\mathbf{i} - 10\mathbf{j})$ m relative to a fixed point O.

After 4 seconds the stone is at the point $B(12\mathbf{i} + 5\mathbf{j})$ m relative to the fixed point O.

The motion of the stone is modelled as that of a particle moving in a straight line at constant speed.

Using the model prove that the stone passes through O.

(2)

(Total for Question 1 is 2 marks)

Q2

Given that the point A has position vector $3\mathbf{i} - 7\mathbf{j}$ and the point B has position vector $8\mathbf{i} + 3\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$.

(2)

(b) Find $|\overrightarrow{AB}|$. Give your answer as a simplified surd.

(2)

(Total for Question 2 is 4 marks)

Q3

Given that the point A has position vector $4\mathbf{i} - 5\mathbf{j}$ and the point B has position vector $-5\mathbf{i} - 2\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$,

(2)

(b) find $\left|\overrightarrow{AB}\right|$. Give your answer as a simplified surd.

(2)

(Total for Question 3 is 4 marks)

Q4

Relative to a fixed origin O, the point A has position vector $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and the point B has position vector $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$. The points A and B lie on a straight line l.

(a) Find $\overrightarrow{AB}$.

(2)

(b) Find a vector equation of l.

(2)

The point C has position vector $2\mathbf{i} + p\mathbf{j} - 4\mathbf{k}$ with respect to O, where p is a constant. Given that AC is perpendicular to l, find

(c) the value of p,

(4)

(d) the distance AC.

(2)

(Total for Question 4 is 10 marks)

End of Questions

Bronze Mark Scheme

Q1

Question	Scheme	Marks	AOs
(a)	Attempts to compare the two position vectors. Allow an attempt using two of $\overrightarrow{AO}, \overrightarrow{OB}$ or $\overrightarrow{AB}$ E.g. $(-24\mathbf{i}-10\mathbf{j}) = -2\times(12\mathbf{i}+5\mathbf{j})$	M1	1.1b
	Explains that as $\overrightarrow{AO}$ is parallel to $\overrightarrow{OB}$ (and the stone is travelling in a straight line) the stone passes through the point O.	A1	2.4
		(2)	

Q2

Question	Scheme	Marks	AOs
(a)	Attempts $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or similar	M1	1.1b
	$\overrightarrow{AB} = 5\mathbf{i} + 10\mathbf{j}$	A1	1.1b
		(2)	
(b)	Finds length using 'Pythagoras' $\lvert AB\rvert = \sqrt{(5)^2+(10)^2}$	M1	1.1b
	$\lvert AB\rvert = 5\sqrt{5}$	A1ft	1.1b
		(2)	
		(4 marks)	

Notes

(a) M1: Attempts subtraction but may omit brackets

A1: cao (allow column vector notation)

(b) M1: Correct use of Pythagoras theorem or modulus formula using their answer to (a)

A1ft: $\lvert AB\rvert = 5\sqrt{5}$ ft from their answer to (a)

Note that the correct answer implies M1A1 in each part of this question

Q3

Question	Scheme	Marks	AOs
(a)	Attempts $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$ or similar	M1	1.1b
	$\overrightarrow{AB}=-9\mathbf{i}+3\mathbf{j}$	A1	1.1b
		(2)	
(b)	Finds length using 'Pythagoras' $\lvert AB\rvert=\sqrt{(-9)^2+(3)^2}$	M1	1.1b
	$\lvert AB\rvert=3\sqrt{10}$	A1ft	1.1b
		(2)	
			(4 marks)

Notes

(a)

M1: Attempts subtraction either way around.

This may be implied by one correct component $\overrightarrow{AB}=\pm 9\mathbf{i}\pm 3\mathbf{j}$

There must be some attempt to write in vector form.

A1: cao (allow column vector notation but not the coordinate)

Correct notation should be used. Accept $-9\mathrm{i}+3\mathrm{j}$ or $\begin{pmatrix}-9\\3\end{pmatrix}$ but not $\begin{pmatrix}-9\mathrm{i}\\3\mathrm{j}\end{pmatrix}$

(b)

M1: Correct use of Pythagoras theorem or modulus formula using their answer to (a)

Note that $\lvert AB\rvert=\sqrt{(9)^2+(3)^2}$ is also correct.

Condone missing brackets in the expression $\lvert AB\rvert=\sqrt{-9^2+(3)^2}$

Also allow a restart usually accompanied by a diagram.

A1ft: $\lvert AB\rvert=3\sqrt{10}$ ft from their answer to (a) as long as it has both an **i** and **j** component.

It must be simplified, if appropriate. Note that $\pm 3\sqrt{10}$ would be M1 A0

Note that, in cases where there is no working, the correct answer implies M1A1 in each part of this question

Q4

Question Number	Scheme	Marks
(a)	$\overrightarrow{AB} = -2\mathbf{i}+2\mathbf{j}-\mathbf{k}-(\mathbf{i}-3\mathbf{j}+2\mathbf{k}) = -3\mathbf{i}+5\mathbf{j}-3\mathbf{k}$	M1 A1 (2)
(b)	$\mathbf{r} = \mathbf{i}-3\mathbf{j}+2\mathbf{k}+\lambda(-3\mathbf{i}+5\mathbf{j}-3\mathbf{k})$ or $\mathbf{r} = -2\mathbf{i}+2\mathbf{j}-\mathbf{k}+\lambda(-3\mathbf{i}+5\mathbf{j}-3\mathbf{k})$	M1 A1ft (2)
(c)	$\overrightarrow{AC} = 2\mathbf{i}+p\mathbf{j}-4\mathbf{k}-(\mathbf{i}-3\mathbf{j}+2\mathbf{k})$ $= \mathbf{i}+(p+3)\mathbf{j}-6\mathbf{k}$ or $\overrightarrow{CA}$	 B1
	$\overrightarrow{AC}.\overrightarrow{AB} = \begin{pmatrix}1\\p+3\\-6\end{pmatrix}.\begin{pmatrix}-3\\5\\-3\end{pmatrix} = 0$	M1
	$-3+5p+15+18=0$ Leading to $p=-6$	 M1 A1 (4)
(d)	$AC^2 = (2-1)^2+(-6+3)^2+(-4-2)^2 \quad (=46)$	M1
	$AC = \sqrt{46}$ accept awrt 6.8	A1 (2) **[10]**

Silver Questions

Non-calculator

The total mark for this section is 34

Q1

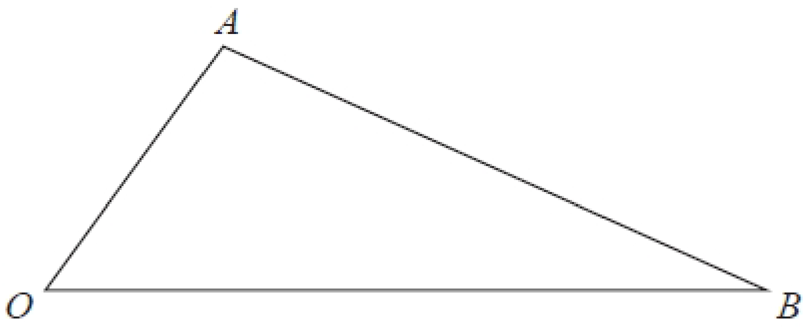


Figure 7

Figure 7 shows a sketch of triangle *OAB*.

The point *C* is such that $\overrightarrow{OC} = 2\overrightarrow{OA}$.

The point *M* is the midpoint of *AB*.

The straight line through *C* and *M* cuts *OB* at the point *N*.

Given $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

(a) Find $\overrightarrow{CM}$ in terms of **a** and **b**

(2)

(b) Show that $\overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$, where λ is a scalar constant.

(2)

(c) Hence prove that *ON* : *NB* = 2 : 1

(2)

(Total for Question 1 is 6 marks)

Q2

Relative to a fixed origin O,

the point A has position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$,

the point B has position vector $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$,

and the point C has position vector $(a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$, where a is a constant and $a < 0$

D is the point such that $\overrightarrow{AB} = \overrightarrow{BD}$.

(a) Find the position vector of D.

(2)

Given $|\overrightarrow{AC}| = 4$

(b) find the value of a.

(3)

(Total for Question 2 is 5 marks)

Q3

Relative to a fixed origin O, the point A has position vector $(10\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$,
and the point B has position vector $(8\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$

(2)

(b) Find a vector equation for the line l.

(2)

The point C has position vector $(3\mathbf{i} + 12\mathbf{j} + 3\mathbf{k})$.

The point P lies on l. Given that the vector $\overrightarrow{CP}$ is perpendicular to l,

(c) find the position vector of the point P.

(6)

(Total for Question 3 is 10 marks)

Q4

With respect to a fixed origin O the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \qquad l_2: \quad \mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$$

where λ and μ are parameters and p and q are constants. Given that l_1 and l_2 are perpendicular,

(a) show that $q = -3$.

(2)

Given further that l_1 and l_2 intersect, find

(b) the value of p,

(6)

(c) the coordinates of the point of intersection.

(2)

The point A lies on l_1 and has position vector $\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}$. The point C lies on l_2.

Given that a circle, with centre C, cuts the line l_1 at the points A and B,

(d) find the position vector of B.

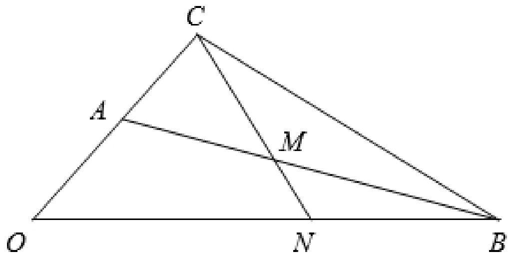
(3)

(Total for Question 4 is 13 marks)

End of Questions

Silver Mark Scheme

Q1

Part	Working or answer an examiner might expect to see	Mark	Notes
			
(a)	$\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} = \overrightarrow{CA} + \frac{1}{2}\overrightarrow{AB}$	M1	This mark is given for a method to find an expression for $\overrightarrow{CM}$
	$\overrightarrow{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) = -\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$	A1	This mark is given for a correct expression for $\overrightarrow{CM}$ in terms of **a** and **b**
(b)	$\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN} = \overrightarrow{OC} + \lambda\overrightarrow{CM}$	M1	This mark is given for a method to find an expression for $\overrightarrow{ON}$
	$\overrightarrow{ON} = 2\mathbf{a} + \lambda\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)$ $= \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \left(\frac{1}{2}\lambda\right)\mathbf{b}$	A1	This mark is given for a correct expression for $\overrightarrow{ON}$ in terms of **a** and **b**
(c)	$\left(2 - \frac{3}{2}\lambda\right) = 0$ so $\lambda = \frac{4}{3}$	M1	This mark is given for deducing that the coefficient of **a** = 0 and finding a value for λ
	$\overrightarrow{ON} = 0 \times \mathbf{a} + \frac{2}{3}\mathbf{b}$ Hence $ON{:}NB = \frac{2}{3} : \frac{1}{3} = 2{:}1$	A1	This mark is given for finding $\overrightarrow{ON}$ and giving a valid conclusion
			(Total 6 marks)

Q2

Question	Scheme	Marks	AOs
	$\overrightarrow{OA} = 2\mathbf{i}+3\mathbf{j}-4\mathbf{k}$, $\overrightarrow{OB} = 4\mathbf{i}-2\mathbf{j}+3\mathbf{k}$, $\overrightarrow{OC} = a\mathbf{i}+5\mathbf{j}-2\mathbf{k}$, $a<0$ $\overrightarrow{AB} = \overrightarrow{BD}$, $\lvert\overrightarrow{AB}\rvert = 4$		
(a)	E.g. $\overrightarrow{OD} = \overrightarrow{OB}+\overrightarrow{BD} = \overrightarrow{OB}+\overrightarrow{AB}$ **or** $\overrightarrow{OD} = \overrightarrow{OB}+\overrightarrow{BD} = \overrightarrow{OB}+\overrightarrow{AB} = \overrightarrow{OB}+\overrightarrow{OB}-\overrightarrow{OA} = 2\overrightarrow{OB}-\overrightarrow{OA}$ **or** $\overrightarrow{OD} = \overrightarrow{OB}+\overrightarrow{BD} = \overrightarrow{OB}+\overrightarrow{AB} = \overrightarrow{OA}+\overrightarrow{AB}+\overrightarrow{AB} = \overrightarrow{OA}+2\overrightarrow{AB}$		
	$=\begin{pmatrix}4\\-2\\3\end{pmatrix}+\begin{pmatrix}4\\-2\\3\end{pmatrix}-\begin{pmatrix}2\\3\\-4\end{pmatrix}\left\{=\begin{pmatrix}4\\-2\\3\end{pmatrix}+\begin{pmatrix}2\\-5\\7\end{pmatrix}\right\}$ **or** $=\begin{pmatrix}2\\3\\-4\end{pmatrix}+2\left(\begin{pmatrix}4\\-2\\3\end{pmatrix}-\begin{pmatrix}2\\3\\-4\end{pmatrix}\right)\left\{=\begin{pmatrix}2\\3\\-4\end{pmatrix}+2\begin{pmatrix}2\\-5\\7\end{pmatrix}\right\}$	M1	3.1a
	$=\begin{pmatrix}6\\-7\\10\end{pmatrix}$ or $6\mathbf{i}-7\mathbf{j}+10\mathbf{k}$	A1	1.1b
		(2)	
(b)	$(a-2)^2+(5-3)^2+(-2--4)^2$	M1	1.1b
	$\left\{\lvert\overrightarrow{AC}\rvert = 4 \Rightarrow\right\} (a-2)^2+(5-3)^2+(-2--4)^2 = (4)^2$ $\Rightarrow (a-2)^2 = 8 \Rightarrow a = \ldots$ **or** $\Rightarrow a^2-4a-4=0 \Rightarrow a = \ldots$	dM1	2.1
	(as $a<0 \Rightarrow$) $a = 2-2\sqrt{2}$ (or $a = 2-\sqrt{8}$)	A1	1.1b
		(3)	
			(5 marks)

Notes for Question	
(a)	
M1:	Complete *applied* strategy to find a vector expression for $\overrightarrow{OD}$
A1:	See scheme
Note:	Give M0 for subtracting the wrong way wrong to give e.g. $(4\mathbf{i}-2\mathbf{j}+3\mathbf{k})+(2\mathbf{i}+3\mathbf{j}-4\mathbf{k})\ -(4\mathbf{i}-2\mathbf{j}+3\mathbf{k})=(4\mathbf{i}-2\mathbf{j}+3\mathbf{k})+(-2\mathbf{i}+5\mathbf{j}-7\mathbf{k})=\ (2\mathbf{i}+3\mathbf{j}-4\mathbf{k})$
Note:	Writing e.g. $\overrightarrow{OD}=\overrightarrow{OB}+\overrightarrow{AB}$ or $\overrightarrow{OD}=2\overrightarrow{OB}-\overrightarrow{OA}$ with no other work is M0
Note:	Finding ***coordinates***, i.e. $(6, -7, 10)$ without reference to the correct position vectors is A0
Note:	Allow M1A1 for writing down $6\mathbf{i}-7\mathbf{j}+10\mathbf{k}$ with no working
Note:	M1 can be implied for at least two correct components in their position vector of D
(b)	
M1:	Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OC}$, then squares and adds each of the 3 components **Note:** Ignore labelling
dM1:	Complete method of *correctly* applying Pythagoras' Theorem on $\left\|\overrightarrow{AC}\right\|=4$ and using a correct method of solving their resulting quadratic equation to find at least one of $a=...$
Note:	Condone at least one of either awrt 4.8 or awrt -0.83 for the dM mark
A1:	Obtains **only one** exact value, $a=2-2\sqrt{2}$
Note:	Writing $a=2\pm2\sqrt{2}$, without evidence of rejecting $a=2+2\sqrt{2}$ is A0
Note:	Allow exact alternatives such as $2-\sqrt{8}$ or $\dfrac{4-\sqrt{32}}{2}$ for A1, and isw can be applied
Note:	Writing $a=-0.828...$, without reference to a correct exact value is A0

Q3

Question Number	Scheme	Marks
	(a) $\overrightarrow{AB} = \begin{pmatrix}8\\3\\4\end{pmatrix} - \begin{pmatrix}10\\2\\3\end{pmatrix} = \begin{pmatrix}-2\\1\\1\end{pmatrix}$	M1 A1 (2)
	(b) $\mathbf{r} = \begin{pmatrix}10\\2\\3\end{pmatrix} + t\begin{pmatrix}-2\\1\\1\end{pmatrix}$ $\quad\quad$ $\mathbf{r} = \begin{pmatrix}8\\3\\4\end{pmatrix} + t\begin{pmatrix}-2\\1\\1\end{pmatrix}$	M1 A1ft (2)
	(c) $\overrightarrow{CP} = \begin{pmatrix}10-2t\\2+t\\3+t\end{pmatrix} - \begin{pmatrix}3\\12\\3\end{pmatrix} = \begin{pmatrix}7-2t\\t-10\\t\end{pmatrix}$	M1 A1
	$\begin{pmatrix}7-2t\\t-10\\t\end{pmatrix} . \begin{pmatrix}-2\\1\\1\end{pmatrix} = -14+4t+t-10+t=0$	M1
	Leading to $\quad t=4$	A1
	Position vector of P is $\begin{pmatrix}10-8\\2+4\\3+4\end{pmatrix} = \begin{pmatrix}2\\6\\7\end{pmatrix}$	M1 A1 (6)
		[10]
	Alternative working for (c)	
	$\overrightarrow{CP} = \begin{pmatrix}8-2t\\3+t\\4+t\end{pmatrix} - \begin{pmatrix}3\\12\\3\end{pmatrix} = \begin{pmatrix}5-2t\\t-9\\t+1\end{pmatrix}$	M1 A1
	$\begin{pmatrix}5-2t\\t-9\\t+1\end{pmatrix} . \begin{pmatrix}-2\\1\\1\end{pmatrix} = -10+4t+t-9+t+1=0$	M1
	Leading to $\quad t=3$	A1
	Position vector of P is $\begin{pmatrix}8-6\\3+3\\4+3\end{pmatrix} = \begin{pmatrix}2\\6\\7\end{pmatrix}$	M1 A1 (6)

Q4

Question Number	Scheme		Marks
(a)	$\mathbf{d}_1 = -2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$, $\mathbf{d}_2 = q\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$		
	As $\left\{\mathbf{d}_1 \bullet \mathbf{d}_2 = \begin{pmatrix}-2\\1\\-4\end{pmatrix} \bullet \begin{pmatrix}q\\2\\2\end{pmatrix}\right\} = \underline{(-2\times q)+(1\times 2)+(-4\times 2)}$	Apply dot product calculation between two direction vectors, ie. $\underline{(-2\times q)+(1\times 2)+(-4\times 2)}$	M1
	$\mathbf{d}_1 \bullet \mathbf{d}_2 = 0 \Rightarrow -2q + 2 - 8 = 0$ $-2q = 6 \Rightarrow \underline{q = -3}$ AG	Sets $\mathbf{d}_1 \bullet \mathbf{d}_2 = 0$ and solves to find $\underline{q = -3}$	A1 cso
			(2)
(b)	Lines meet where:		
	$\begin{pmatrix}11\\2\\17\end{pmatrix} + \lambda\begin{pmatrix}-2\\1\\-4\end{pmatrix} = \begin{pmatrix}-5\\11\\p\end{pmatrix} + \mu\begin{pmatrix}q\\2\\2\end{pmatrix}$		
	First two of $\mathbf{i}: \ 11 - 2\lambda = -5 + q\mu$ (1) $\mathbf{j}: \ 2 + \lambda = 11 + 2\mu$ (2) $\mathbf{k}: \ 17 - 4\lambda = p + 2\mu$ (3)	Need to see equations (1) and (2). Condone one slip. (Note that $q = -3$.)	M1
	(1) + 2(2) gives: $15 = 17 + \mu \Rightarrow \mu = -2$	Attempts to solve (1) and (2) to find one of either λ or μ	dM1
	(2) gives: $2 + \lambda = 11 - 4 \Rightarrow \lambda = 5$	Any one of $\underline{\lambda = 5}$ or $\underline{\mu = -2}$	A1
		Both $\underline{\lambda = 5}$ and $\underline{\mu = -2}$	A1
	(3) $\Rightarrow 17 - 4(5) = p + 2(-2)$	Attempt to substitute their λ and μ into their **k** component to give an equation in p alone.	ddM1
	$\Rightarrow p = 17 - 20 + 4 \Rightarrow \underline{p = 1}$	$\underline{p = 1}$	A1 cso
			(6)
(c)	$\mathbf{r} = \begin{pmatrix}11\\2\\17\end{pmatrix} + 5\begin{pmatrix}-2\\1\\-4\end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix}-5\\11\\1\end{pmatrix} - 2\begin{pmatrix}-3\\2\\2\end{pmatrix}$	Substitutes their value of λ or μ into the correct line l_1 or l_2.	M1
	Intersect at $\mathbf{r} = \underline{\begin{pmatrix}1\\7\\-3\end{pmatrix}}$ or $\underline{(1, 7, -3)}$	$\underline{\begin{pmatrix}1\\7\\-3\end{pmatrix}}$ or $\underline{(1, 7, -3)}$	A1
			(2)

Question Number	Scheme		Marks
(d)	Let $\overrightarrow{OX} = \mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ be point of intersection		
	$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \begin{pmatrix}1\\7\\-3\end{pmatrix} - \begin{pmatrix}9\\3\\13\end{pmatrix} = \begin{pmatrix}-8\\4\\-16\end{pmatrix}$	Finding vector $\overrightarrow{AX}$ by finding the difference between $\overrightarrow{OX}$ and $\overrightarrow{OA}$. Can be ft using candidate's $\overrightarrow{OX}$.	M1√ ±
	$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + 2\overrightarrow{AX}$		
	$\overrightarrow{OB} = \begin{pmatrix}9\\3\\13\end{pmatrix} + 2\begin{pmatrix}-8\\4\\-16\end{pmatrix}$	$\begin{pmatrix}9\\3\\13\end{pmatrix} + 2\left(\text{their } \overrightarrow{AX}\right)$	dM1√
	Hence, $\overrightarrow{OB} = \underline{\begin{pmatrix}-7\\11\\-19\end{pmatrix}}$ or $\overrightarrow{OB} = \underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$	$\underline{\begin{pmatrix}-7\\11\\-19\end{pmatrix}}$ or $\underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ or $\underline{(-7, 11, -19)}$	A1
			(3)
			[13]

Gold Questions

Non-calculator

The total mark for this section is 42

Q1

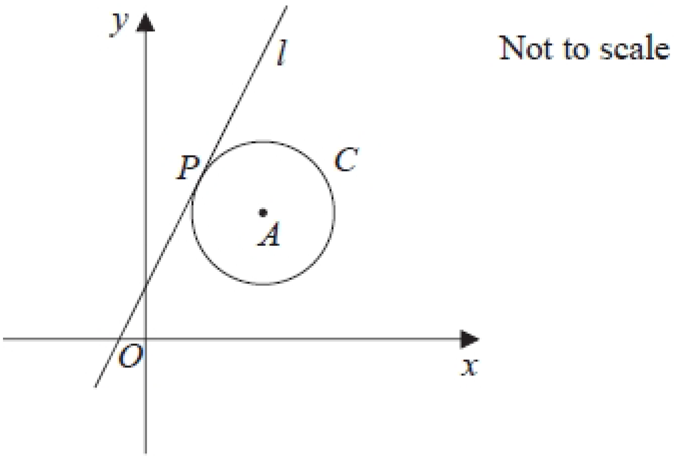


Figure 3

The circle C has centre A with coordinates (7, 5).

The line l, with equation $y = 2x + 1$, is the tangent to C at the point P, as shown in Figure 3.

(a) Show that an equation of the line PA is $2y + x = 17$

(3)

(b) Find an equation for C.

(4)

The line with equation $y = 2x + k$, $k \neq 1$ is also a tangent to C.

(c) Find the value of the constant k.

(3)

(Total for Question 1 is 5 marks)

Q2

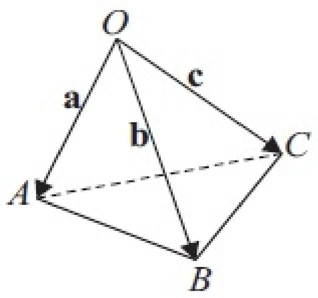


Figure 1

The points A, B and C have position vectors **a**, **b** and **c** respectively, relative to a fixed origin O, as shown in Figure 1.

It is given that

$$\mathbf{a} = \mathbf{i} + \mathbf{j},\ \mathbf{b} = 3\mathbf{i} - \mathbf{j} + \mathbf{k} \text{ and } \mathbf{c} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}.$$

Calculate

(a) $\mathbf{b} \times \mathbf{c}$,

(3)

(b) $\mathbf{a}.(\mathbf{b} \times \mathbf{c})$,

(2)

(c) the area of triangle OBC,

(2)

(d) the volume of the tetrahedron $OABC$.

(1)

(Total for Question 2 is 8 marks)

Q3

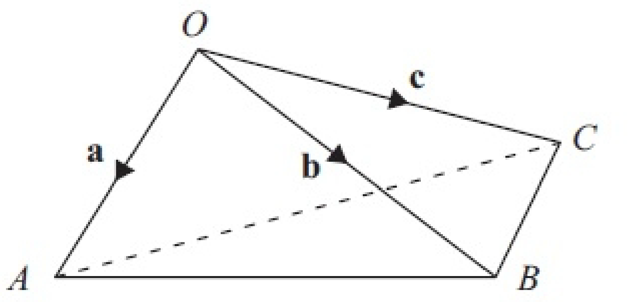


Figure 1

The points A, B and C have position vectors **a**, **b** and **c** respectively, relative to a fixed origin O, as shown in Figure 1.
It is given that

$$\mathbf{a} = \mathbf{i} + \mathbf{j},\ \mathbf{b} = 3\mathbf{i} - \mathbf{j} + \mathbf{k} \text{ and } \mathbf{c} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}.$$

Calculate

(a) $\mathbf{b} \times \mathbf{c}$,

(3)

(b) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$,

(2)

(c) the area of triangle OBC,

(2)

(d) the volume of the tetrahedron $OABC$.

(1)

(Total for Question 3 is 8 marks)

Q4

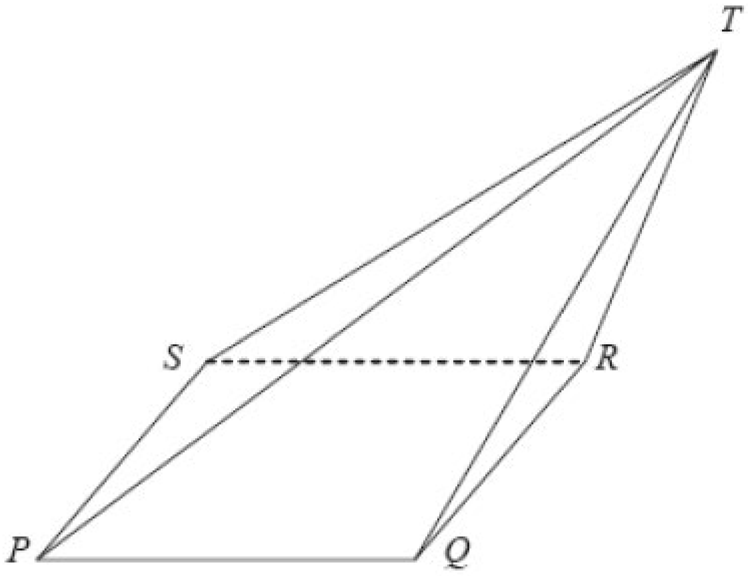


Figure 1

Figure 1 shows a pyramid *PQRST* with base *PQRS*.

The coordinates of *P*, *Q* and *R* are *P* (1, 0, –1), *Q* (2, –1, 1) and *R* (3, –3, 2).

Find

(a) $\overrightarrow{PQ} \times \overrightarrow{PR}$,

(3)

(b) a vector equation for the plane containing the face *PQRS*, giving your answer in the form $\mathbf{r}.\mathbf{n} = d$.

(2)

The plane Π contains the face *PST*. The vector equation of Π is $\mathbf{r}.(\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}) = 6$.

(c) Find cartesian equations of the line through *P* and *S*.

(5)

(d) Hence show that *PS* is parallel to *QR*.

(2)

Given that *PQRS* is a parallelogram and that *T* has coordinates (5, 2, –1),

(e) find the volume of the pyramid *PQRST*.

(3)

(Total for Question 4 is 15 marks)

End of Questions

Gold Mark Scheme

Q1

Question	Scheme	Marks	AOs
(a)	Deduces that gradient of PA is $-\frac{1}{2}$	M1	2.2a
	Finding the equation of a line with gradient "$-\frac{1}{2}$" and point $(7,5)$ $y-5=-\frac{1}{2}(x-7)$	M1	1.1b
	Completes proof $2y+x=17$ *	A1*	1.1b
		(3)	
(b)	Solves $2y+x=17$ and $y=2x+1$ simultaneously	M1	2.1
	$P=(3,7)$	A1	1.1b
	Length $PA=\sqrt{(3-7)^2+(7-5)^2}=\left(\sqrt{20}\right)$	M1	1.1b
	Equation of C is $(x-7)^2+(y-5)^2=20$	A1	1.1b
		(4)	
(c)	Attempts to find where $y=2x+k$ meets C using $\overrightarrow{OA}+\overrightarrow{PA}$	M1	3.1a
	Substitutes their $(11,3)$ in $y=2x+k$ to find k	M1	2.1
	$k=-19$	A1	1.1b
		(3)	
			(10 marks)

Q2

Question Number	Scheme	Marks
(a)	$\mathbf{b}\times\mathbf{c}=0\mathbf{i}+5\mathbf{j}+5\mathbf{k}$	M1 A1 A1 (3)
(b)	$\mathbf{a}.(\mathbf{b}\times\mathbf{c})=0+5=5$	M1 A1 ft (2)
(c)	Area of triangle $OBC=\frac{1}{2}\lvert 5\mathbf{j}+5\mathbf{k}\rvert=\frac{5}{2}\sqrt{2}$	M1 A1 (2)
(d)	Volume of tetrahedron $=\frac{1}{6}\times 5=\frac{5}{6}$	B1 ft (1) [8]

Q3

Question Number	Scheme	Marks
(a)	$\mathbf{b}\times\mathbf{c} = 0\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$	M1 A1 A1 (3)
(b)	$\mathbf{a}.(\mathbf{b}\times\mathbf{c}) = \begin{pmatrix}1\\1\\0\end{pmatrix}.\begin{pmatrix}0\\5\\5\end{pmatrix} = 0+5+0=5$	M1 A1ft (2)
(c)	Area of triangle $OBC = \frac{1}{2}\lvert 5\mathbf{j}+5\mathbf{k}\rvert = \frac{5}{2}\sqrt{2}$ oe	M1 A1 (2)
(d)	Volume of tetrahedron $= \frac{1}{6}\times 5 = \frac{5}{6}$	B1 ft (1) **[8]**

Q4

Number		
(a)	$\overrightarrow{PQ}=\mathbf{i}-\mathbf{j}+2\mathbf{k},\quad \overrightarrow{PR}=2\mathbf{i}-3\mathbf{j}+3\mathbf{k}$	B1
	$\overrightarrow{PQ}\times\overrightarrow{PR}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&-1&2\\2&-3&3\end{vmatrix}=3\mathbf{i}+\mathbf{j}-\mathbf{k}$	M1 A1 (3)
(b)	$\mathbf{r}\,.\,(3\mathbf{i}+\mathbf{j}-\mathbf{k})=(\mathbf{i}-\mathbf{k})\,.\,(3\mathbf{i}+\mathbf{j}-\mathbf{k})$ [may use $\overrightarrow{OQ}$ or $\overrightarrow{OR}$]	M1
	$\mathbf{r}\,.\,(3\mathbf{i}+\mathbf{j}-\mathbf{k})=4$ o.e. ft from (a)	A1ft (2)
(c)	$3x+y-z=4$ (i), $x-2y-5z=6$ (ii)	
	(i) $\times$ 2 + (ii) $7x-7z=14$, $x=z+2$ (M: Eliminate one variable)	M1
	In (ii) $z+2-2y-5z=6$, $y+2=-2z$ (M: Substitute back)	M1
	$\therefore x=z+2$ and $y+2=-2z$ o.e. $(y=2-2x)$ (Two correct '3-term' equations)	A1
	$\dfrac{x-2}{(1)}=\dfrac{y+2}{-2}=\dfrac{z}{(1)}$ o.e. (M: Form cartesian equations)	M1 A1 (5)
(d)	Writing down direction vector of $\overrightarrow{PS}$ from part (c).	M1
	$\overrightarrow{QR}=\mathbf{i}-2\mathbf{j}+\mathbf{k}=\overrightarrow{PS}\quad\therefore PS\,//\,QR$ (or cross-product = 0)	A1 (2)
(e)	$\overrightarrow{PT}=4\mathbf{i}+2\mathbf{j}$ (or $\overrightarrow{QT}=3\mathbf{i}+3\mathbf{j}-2\mathbf{k}$ or $\overrightarrow{RT}=2\mathbf{i}+5\mathbf{j}-3\mathbf{k}$)	
	Volume $=\dfrac{1}{3}\left\|\overrightarrow{PQ}\times\overrightarrow{PR}.\overrightarrow{PT}\right\|=\dfrac{1}{3}\left\|(3\mathbf{i}+\mathbf{j}-\mathbf{k})\,.\,(4\mathbf{i}+2\mathbf{j})\right\|$ ft from (a)	M1 A1ft
	(Instead of $\overrightarrow{PQ}\times\overrightarrow{PR}$, it could be $\overrightarrow{PQ}\times\overrightarrow{QR}$ or $\overrightarrow{PR}\times\overrightarrow{QR}$)	
	$=\dfrac{1}{3}(12+2)$	
	$=4\frac{2}{3}$ o.e.	A1 (3)
		(15)
	(a) If both vectors are 'reversed', B0 M1 A1 is possible	
	(c) <u>Alternative</u>:	
	Direction of line: $\begin{pmatrix}1\\-2\\-5\end{pmatrix}\times\begin{pmatrix}3\\1\\-1\end{pmatrix}=7\begin{pmatrix}1\\-2\\1\end{pmatrix}$ M2 A1	
	Through P $(1,0,-1)$: $\dfrac{x-1}{1}=\dfrac{y}{-2}=\dfrac{z+1}{1}$ M1 A1	
	(e) <u>Alternative</u>:	
	$\dfrac{1}{3}\begin{vmatrix}4&2&0\\1&-1&2\\2&-3&3\end{vmatrix}$ gives M1 A1 directly. Here ft from 1st line of part (a).	
	<u>Special case</u>:	
	$\dfrac{1}{6}$ or $\dfrac{1}{2}$ instead of $\dfrac{1}{3}$, but method otherwise correct: M1 A0 A0	

Platinum Questions

Non-calculator

The total mark for this section is 26

Q1

Points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$, respectively, relative to an origin O, and are such that OAB is a triangle with $OA = a$ and $OB = b$.

The point C, with position vector $\mathbf{c}$, lies on the line through O that bisects the angle AOB.

(a) Prove that the vector $b\mathbf{a} - a\mathbf{b}$ is perpendicular to $\mathbf{c}$.

(4)

The point D, with position vector $\mathbf{d}$, lies on the line AB between A and B.

(b) Explain why $\mathbf{d}$ can be expressed in the form $\mathbf{d} = (1 - \lambda)\mathbf{a} + \lambda\mathbf{b}$ for some scalar λ with $0 < \lambda < 1$

(2)

(c) Given that D is also on the line OC, find an expression for λ in terms of a and b only and hence show that

$$DA : DB = OA : OB$$

(8)

(+S2)

(Total for Question 1 is 16 marks)

Q2

The lines L_1 and L_2 have the equations

$$L_1 : \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 9 \end{pmatrix} + s\begin{pmatrix} 2 \\ p \\ 6 \end{pmatrix} \quad \text{and} \quad L_2 : \mathbf{r} = \begin{pmatrix} -15 \\ 12 \\ -9 \end{pmatrix} + t\begin{pmatrix} 4 \\ -5 \\ 2 \end{pmatrix}$$

where p is a constant.

The acute angle between L_1 and L_2 is θ where $\cos\theta = \dfrac{\sqrt{5}}{3}$

(a) Find the value of p.

(5)

The line L_3 has equation $\mathbf{r}$ $\mathbf{r} = \begin{pmatrix} -15 \\ 12 \\ -9 \end{pmatrix} + u\begin{pmatrix} 8 \\ -6 \\ -5 \end{pmatrix}$ and the lines L_3 and L_2 intersect at the point A.

The point B on L_2 has position vector $\begin{pmatrix} 5 \\ -13 \\ 1 \end{pmatrix}$ and point C lies on L_3 such that $ABDC$ is a rhombus.

(b) Find the two possible position vectors of D.

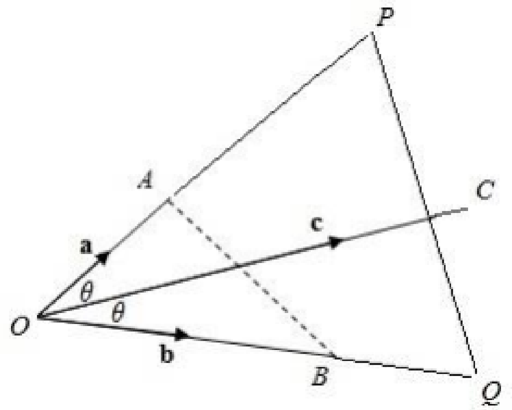
(5)

(Total for Question 2 is 10 marks)

End of Questions

Platinum Mark Scheme

Q1

(a)			(S+ for good diagram sketched)
	Let P and Q be points such that $\overrightarrow{OP} = b\mathbf{a}$ and $\overrightarrow{OQ} = a\mathbf{b}$.	M1	Extends OA and OB (may use unit vectors instead)
	Then $\lvert\overrightarrow{OP}\rvert = b\lvert\mathbf{a}\rvert = ba = ab = a\lvert\mathbf{b}\rvert = \lvert\overrightarrow{OQ}\rvert$ hence OPQ is isosceles.	A1	Deduce isosceles or equivalent.
	Hence the angle bisector from O is perpendicular to PQ.	M1	Use isosceles to deduce perpendicular
	But $\overrightarrow{QP} = \overrightarrow{OP} - \overrightarrow{OQ} = b\mathbf{a} - a\mathbf{b}$ and hence as C is on the angle bisector, so $b\mathbf{a} - a\mathbf{b}$ is perpendicular to $\mathbf{c}$.	A1	Draw correct conclusion.
		(4)	
(b)	$\overrightarrow{OD} = \overrightarrow{OA} + \lambda\overrightarrow{AB} \Rightarrow \mathbf{d} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$	M1	Sets up appropriate equation, either form.
	$\Rightarrow \mathbf{d} = (1 - \lambda)\mathbf{a} + \lambda\mathbf{b}$ ($0 < \lambda < 1$ since D is between A and B)	A1 (S+)	Correctly shown (Reasoning for λ)
		(2)	
(c)	($\overrightarrow{OD} = k\mathbf{c}$ and from (a) $\mathbf{c} = K \times \frac{1}{2}\left(\overrightarrow{OP} + \overrightarrow{OQ}\right)$ hence) $\overrightarrow{OD} = k'\left(\overrightarrow{OP} + \overrightarrow{OQ}\right)$	M1	Makes deduction that **d** is a multiple of **p**+**q**
	Hence $\mathbf{d} = k'\left(\overrightarrow{OP} + \overrightarrow{OQ}\right) = (1-\lambda)\mathbf{a} + \lambda\mathbf{b}$	M1	Equates their **d** to **d** from (b)
	So $k'(b\mathbf{a} + a\mathbf{b}) = (1-\lambda)\mathbf{a} + \lambda\mathbf{b}$	M1	Forms equation in **a** and **b**
	Therefore (since **a** and **b** are not parallel) $k'b = 1-\lambda$ and $k'a = \lambda$ $\Rightarrow \frac{\lambda}{a}b = 1-\lambda \Rightarrow \lambda = \frac{a}{a+b}$	M1 A1	Extracts simultaneous equations and solves for λ. (S+ for non-parallel reasoning)

Q2

(a)	$\begin{pmatrix}2\\p\\6\end{pmatrix}\bullet\begin{pmatrix}4\\-5\\2\end{pmatrix}=8-5p+12$	M1	Attempt suitable scalar product. Intention and at least one correct term.
	$20-5p=\sqrt{40+p^2}\times\sqrt{45}\times\dfrac{\sqrt{5}}{3}$	M1 A1	RHS attempt – numerical slip OK Correct equation
	$4-p=\sqrt{40+p^2} \quad\Rightarrow\quad 16-8p+p^2=40+p^2$	M1	Solving (condone 1 error)
	$p=\underline{-3}$	A1 (5)	
(b)	$\lvert AB\rvert=\sqrt{(5--15)^2+(-13-12)^2+(1--9)^2}$ **or** when $t=5\left[\lvert AB\rvert=15\sqrt{5}\right]$	M1	Attempt to find AB
	$\left[\overrightarrow{AC}=\overrightarrow{BD}=u\begin{pmatrix}8\\-6\\-5\end{pmatrix}\right]$ Length of $\begin{pmatrix}8\\-6\\-5\end{pmatrix}$ is $\sqrt{8^2+6^2+5^2}=5\sqrt{5}$	M1	Attempt to find length of this vector
	So $\overrightarrow{OC}$ is given by $u=\pm 3$ and therefore $\overrightarrow{OD}=\overrightarrow{OB}\pm 3\begin{pmatrix}8\\-6\\-5\end{pmatrix}$	M1	Correct expression for position of D

	So $\overrightarrow{OD}=\begin{pmatrix}5\\-13\\1\end{pmatrix}\pm\begin{pmatrix}24\\-18\\-15\end{pmatrix}=\begin{pmatrix}29\\-31\\-14\end{pmatrix}$ [or from $u=-3$] $\begin{pmatrix}-19\\5\\16\end{pmatrix}$	A1A1 (5) **[10]**	