

Q4

$$f(x) = 4x^3 - 12x^2 + 2x - 6$$

Hence show that 3 is the only real root of the equation $f(x) = 0$

(4)

(Total for Question 4 is 4 marks)

Q5

Express $\frac{5}{(x-1)(3x+2)}$ in partial fractions.

(3)

(Total for Question 5 is 3 marks)

Q6

Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4.

(4)

(Total for Question 6 is 4 marks)

End of Questions



Silver Questions



Non-calculator

The total mark for this section is 28

Q1

Show that $\frac{3x+6}{x^2-3x-10} \div \frac{x+5}{x^3-25x}$ simplifies to ax where a is an integer.

(4)

(Total for Question 1 is 4 marks)

Q2

Express $\frac{4x}{x^2-9} - \frac{2}{x+3}$ as a single fraction in its simplest form.

(4)

(Total for Question 2 is 4 marks)

Q3

Express

$$\frac{3x+5}{x^2+x-12} - \frac{2}{x-3}$$

as a single fraction in its simplest form.

(4)

(Total for Question 3 is 4 marks)

Q4

Express in partial fractions

$$\frac{5x+3}{(2x+1)(x+1)^2}$$

(4)

(Total for Question 4 is 4 marks)

Q5

Express

$$\frac{3}{2x+3} - \frac{1}{2x-3} + \frac{6}{4x^2-9}$$

as a single fraction in its simplest form.

(4)

(Total for Question 5 is 4 marks)

Q6

Given that

$$\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 - 4}, \quad x \neq \pm 2$$

find the values of the constants a , b , c , d and e .

(4)

(Total for Question 6 is 4 marks)

Q7

Use algebra to prove that the square of any natural number is **either** a multiple of 3 **or** one more than a multiple of 3.

(4)

(Total for Question 4 is 10 marks)

End of Questions



Gold Questions



Non-calculator

The total mark for this section is 31

Q1

$$\frac{9x^2}{(x-1)^2(2x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(2x+1)}$$

Find the values of the constants A , B and C .

(4)

(Total for Question 1 is 4 marks)

Q2

(a) Simplify fully

$$\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}$$

(3)

Given that

$$\ln(2x^2 + 9x - 5) = 1 + \ln(x^2 + 2x - 15), \quad x \neq -5,$$

(b) find x in terms of e .

(4)

(Total for Question 2 is 7 marks)

Q3

Express $\frac{9x^2 + 20x - 10}{(x+2)(3x-1)}$ in partial fractions.

(4)

(Total for Question 3 is 4 marks)

Q4

Given that

$$\frac{2x^4 - 3x^2 + x + 1}{(x^2 - 1)} \equiv (ax^2 + bx + c) + \frac{dx + e}{(x^2 - 1)},$$

find the values of the constants a , b , c , d and e .

(4)

(Total for Question 4 is 4 marks)

Q5

Kayden claims that $3^x \geq 2^x$.

(i) Determine whether Kayden's claim is always true, sometimes true or never true, justifying your answer.

(2)

(ii) Prove that $\sqrt{3}$ is an irrational number.

(6)

(Total for Question 5 is 8 marks)

Q6

Prove by contradiction that there are no positive integers p and q such that

$$4p^2 - q^2 = 25$$

(4)

(Total for Question 6 is 4 marks)

End of Questions



Bronze Questions



Non-calculator

The total mark for this section is 32

Q1

The function f is defined by

$$f(x) = \frac{3x - 7}{x - 2} \quad x \in \mathbb{R}, x \neq 2$$

(a) Find $f^{-1}(7)$.

(2)

(b) Show that $ff(x) = \frac{ax + b}{x - 3}$ where a and b are integers to be found.

(3)

(Total for Question 1 is 5 marks)

Q2

$$g(x) = \frac{2x + 5}{x - 3} \quad x \geq 5$$

(a) Find $gg(5)$.

(2)

(b) State the range of g .

(1)

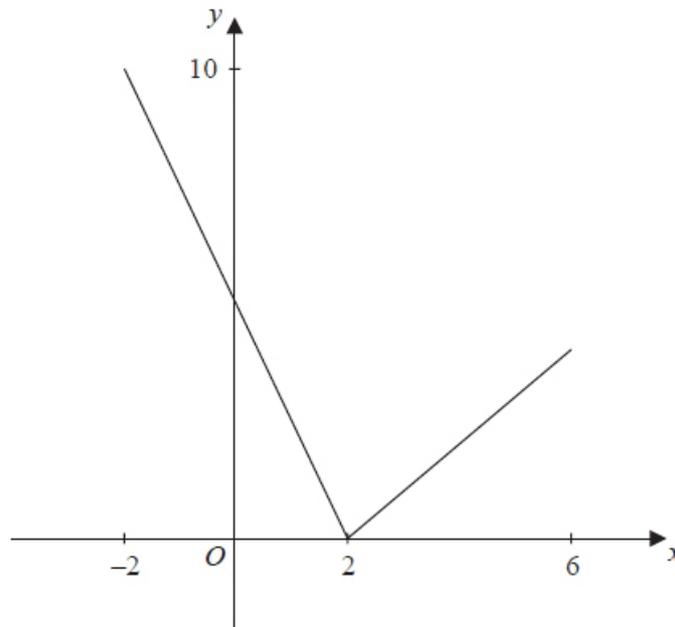
(c) Find $g^{-1}(x)$, stating its domain.

(3)

(Total for Question 2 is 6 marks)

Q3

The function f has domain $-2 \leq x \leq 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = f(x)$ is shown in Figure 1.

**Figure 1**

(a) Write down the range of f .

(1)

(b) Find $ff(0)$.

(2)

The function g is defined by

$$g : x \rightarrow \frac{4 + 3x}{5 - x}, \quad x \in \mathbb{R}, \quad x \neq 5$$

(c) Find $g^{-1}(x)$.

(3)

(d) Solve the equation $gf(x) = 16$.

(5)

(Total for Question 3 is 11 marks)

Q4

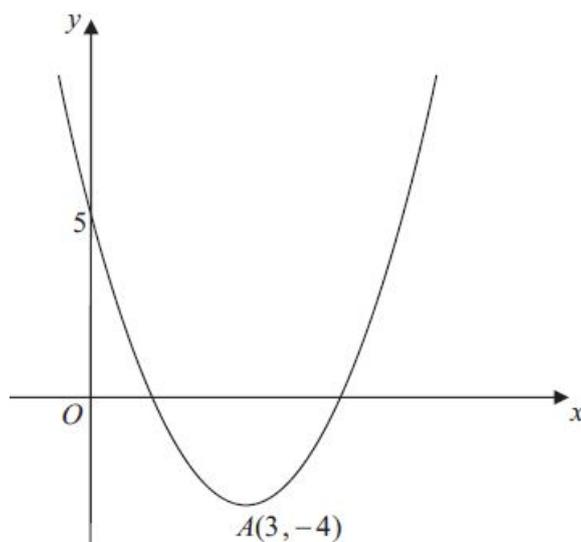


Figure 2

Figure 2 shows a sketch of the curve with the equation $y = f(x)$, $x \in \mathbb{R}$.

The curve has a turning point at $A(3, -4)$ and also passes through the point $(0, 5)$.

(a) Write down the coordinates of the point to which A is transformed on the curve with equation

(i) $y = |f(x)|$,

(ii) $y = 2f\left(\frac{1}{2}x\right)$.

(4)

(b) Sketch the curve with equation

$$y = f(|x|)$$

On your sketch show the coordinates of all turning points and the coordinates of the point at which the curve cuts the y -axis.

(3)

The curve with equation $y = f(x)$ is a translation of the curve with equation $y = x^2$.

(c) Find $f(x)$.

(2)

(d) Explain why the function f does not have an inverse.

(1)

(Total for Question 4 is 10 marks)

End of Questions



Silver Questions



Non-calculator

The total mark for this section is 31

Q1

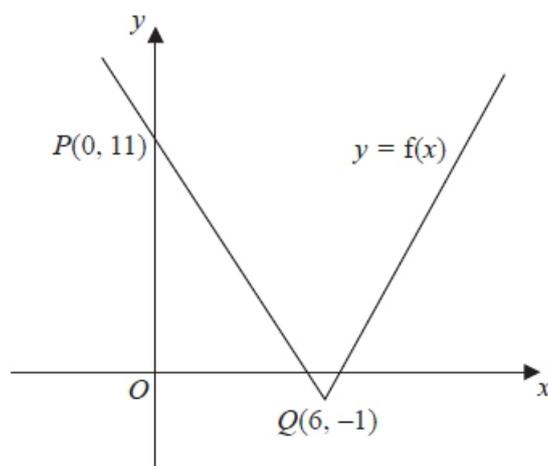


Figure 1

Figure 1 shows part of the graph with equation $y = f(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point $Q(6, -1)$.

The graph crosses the y -axis at the point $P(0, 11)$.

Sketch, on separate diagrams, the graphs of

(a) $y = |f(x)|$

(2)

(b) $y = 2f(-x) + 3$

(3)

(Total for Question 1 is 5 marks)

Q2

The function f is defined by

$$f: x \mapsto 4 - \ln(x+2), \quad x \in \mathbb{R}, \quad x \geq -1$$

(a) Find $f^{-1}(x)$. (3)

(b) Find the domain of $f^{-1}(x)$. (1)

The function g is defined by

$$g: x \mapsto e^{x^2} - 2, \quad x \in \mathbb{R}$$

(c) Find $fg(x)$, giving your answer in its simplest form. (3)

(d) Find the range of fg . (1)

(Total for Question 2 is 8 marks)

Q3

The function f is defined by

$$f: x \mapsto |2x - 5|, \quad x \in \mathbb{R}$$

(a) Sketch the graph with equation $y = f(x)$, showing the coordinates of the points where the graph cuts or meets the axes. (2)

(b) Solve $f(x) = 15 + x$. (3)

The function g is defined by

$$g: x \mapsto x^2 - 4x + 1, \quad x \in \mathbb{R}, \quad 0 \leq x \leq 5$$

(c) Find $fg(2)$. (2)

(d) Find the range of g . (3)

(Total for Question 3 is 10 marks)

Q4

Given that a and b are positive constants,

(a) on separate diagrams, sketch the graph with equation

(i) $y = |2x - a|$

(ii) $y = |2x - a| + b$

Show, on each sketch, the coordinates of each point at which the graph crosses or meets the axes.

(4)

Given that the equation

$$|2x - a| + b = \frac{3}{2}x + 8$$

has a solution at $x = 0$ and a solution at $x = c$,

(b) find c in terms of a .

(4)

(Total for Question 4 is 8 marks)

End of Questions



Gold Questions



Non-calculator

The total mark for this section is 35

Q1

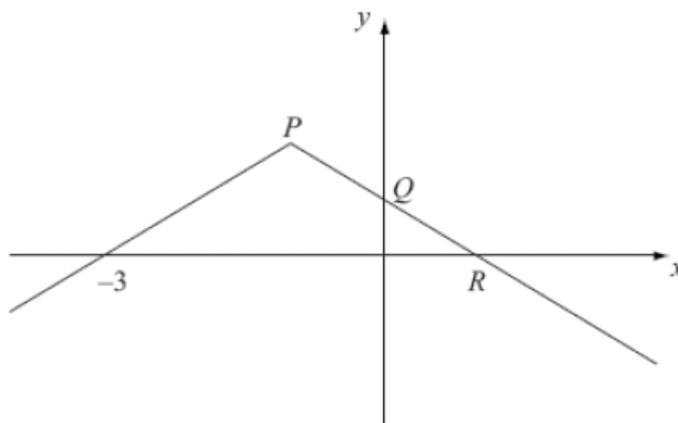


Figure 1

Figure 1 shows the graph of $y = f(x)$, $x \in \mathbb{R}$,

The graph consists of two line segments that meet at the point P .

The graph cuts the y -axis at the point Q and the x -axis at the points $(-3, 0)$ and R .

Sketch, on separate diagrams, the graphs of

(a) $y = |f(x)|$,

(2)

(b) $y = f(-x)$.

(2)

Given that $f(x) = 2 - |x + 1|$,

(c) find the coordinates of the points P , Q and R ,

(3)

(d) solve $f(x) = \frac{1}{2}x$.

(5)

(Total for Question 1 is 12 marks)

Q2

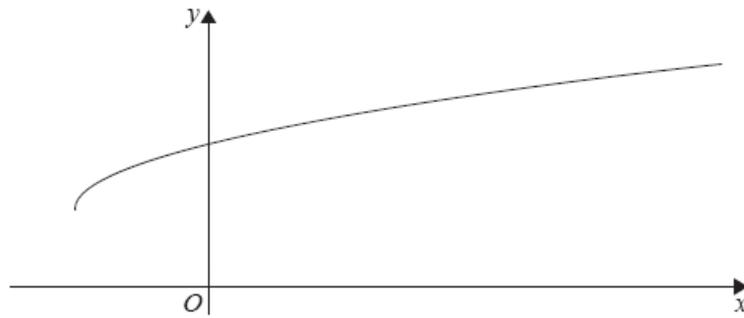


Figure 1

Figure 1 shows a sketch of part of the graph of $y = g(x)$, where

$$g(x) = 3 + \sqrt{x+2}, \quad x \geq -2$$

(a) Find $g^{-1}(x)$ and state its domain.

(3)

(b) Find the exact value of x for which

$$g(x) = x$$

(4)

(c) Hence state the value of a for which

$$g(a) = g^{-1}(a)$$

(1)

(Total for Question 2 is 8 marks)

Q3

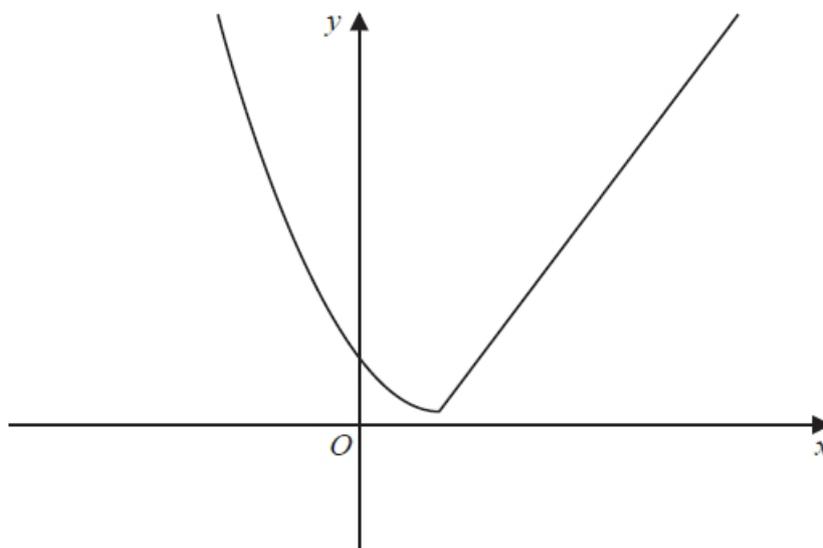


Figure 4

Figure 4 shows a sketch of the graph of $y = g(x)$, where

$$g(x) = \begin{cases} (x-2)^2 + 1 & x \leq 2 \\ 4x - 7 & x > 2 \end{cases}$$

(a) Find the value of $gg(0)$.

(2)

(b) Find all values of x for which

$$g(x) > 28$$

(4)

The function h is defined by

$$h(x) = (x-2)^2 + 1 \quad x \leq 2$$

(c) Explain why h has an inverse but g does not.

(1)

(d) Solve the equation

$$h^{-1}(x) = -\frac{1}{2}$$

(3)

(Total for Question 3 is 10 marks)

Q4

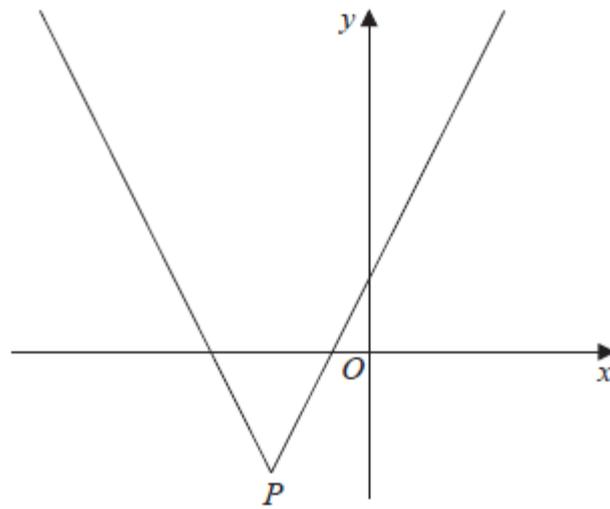


Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = 2|x + 4| - 5$$

The vertex of the graph, at point P , is $(-4, -5)$.

(a) Solve the equation

$$3x + 40 = 2|x + 4| - 5 \quad (2)$$

A line l has equation $y = ax$, where a is a constant.

Given that l intersects $y = 2|x + 4| - 5$ at least once,

(b) Find the range of possible values of a , writing your answer in set notation. (3)

(Total for Question 4 is 5 marks)

End of Questions



Platinum Questions



Non-calculator

The total mark for this section is 28

Q1

The functions f and g are defined by

$$f(x) = 2\sqrt{1 - e^{-x}} \quad x \in \mathbb{R}, x \geq 0$$

$$g(x) = \ln(4 - x^2) \quad x \in \mathbb{R}, -2 < x < 2$$

- (a) (i) Explain why fg cannot be formed as a composite function.
- (ii) Explain why gf can be formed as a composite function. (2)
- (b) (i) Find $gf(x)$, giving the answer in the form $gf(x) = a + bx$, where a and b are constants.
- (ii) State the domain and range of gf . (5)
- (c) Sketch the graph of the function gf .

On your sketch, you should show the coordinates of any points where the graph meets or crosses the coordinate axes. (2)

The circle C with centre $(0, -\ln 9)$ touches the line with equation $y = gf(x)$ at precisely one point.

- (d) Find an equation of the circle C . (3)
- (+S1)

(Total for Question 1 is 13 marks)

Q2

The function f is given by

$$f(x) = \sqrt{x+2} \quad \text{for } x \in \mathbb{R}, x \geq 0$$

- (a) Find $f^{-1}(x)$ and state the domain of f^{-1} (2)

The function g is given by

$$g(x) = x^2 - 4x + 5 \quad \text{for } x \in \mathbb{R}, x \geq 0$$

- (b) Find the range of g . (2)
- (c) Solve the equation $fg(x) = x$. (3)

(Total for Question 2 is 7 marks)

Q3

- (a) On the same diagram, sketch

$$y = (x+1)(2-x) \quad \text{and} \quad y = -x^2 + 2|x|.$$

Mark clearly the coordinates of the points where these curves cross the coordinate axes. (3)

- (b) Find the x -coordinates of the points of intersection of these two curves. (5)

(Total for Question 3 is 8 marks)

End of Questions



Bronze Questions



Non-calculator

The total mark for this section is 29

Q1

A girl saves money over a period of 200 weeks. She saves 5p in Week 1, 7p in Week 2, 9p in Week 3, and so on until Week 200. Her weekly savings form an arithmetic sequence.

- (a) Find the amount she saves in Week 200. (3)
- (b) Calculate her total savings over the complete 200 week period. (3)

(Total for Question 1 is 6 marks)

Q2

A sequence of positive numbers is defined by

$$a_{n+1} = \sqrt{(a_n^2 + 3)}, \quad n \geq 1,$$
$$a_1 = 2$$

- (a) Find a_2 and a_3 , leaving your answers in surd form. (2)
- (b) Show that $a_5 = 4$ (2)

(Total for Question 2 is 4 marks)

Q3

Each year, Andy pays into a savings scheme. In year one he pays in £600. His payments increase by £120 each year so that he pays £720 in year two, £840 in year three and so on, so that his payments form an arithmetic sequence.

(a) Find out how much Andy pays into the savings scheme in year ten.

(2)

Kim starts paying money into a different savings scheme at the same time as Andy. In year one she pays in £130. Her payments increase each year so that she pays £210 in year two, £290 in year three and so on, so that her payments form a different arithmetic sequence.

At the end of year N , Andy has paid, in total, twice as much money into his savings scheme as Kim has paid, in total, into her savings scheme.

(b) Find the value of N .

(5)

(Total for Question 3 is 7 marks)

Q4

The third term of a geometric sequence is 324 and the sixth term is 96.

(a) Show that the common ratio of the sequence is $\frac{2}{3}$.

(2)

(b) Find the first term of the sequence.

(2)

(c) Find the sum to infinity of the sequence.

(2)

(Total for Question 4 is 6 marks)

Q5

A sequence x_1, x_2, x_3, \dots is defined by

$$\begin{aligned}x_1 &= 1, \\x_{n+1} &= ax_n - 3, \quad n \geq 1,\end{aligned}$$

where a is a constant.

(a) Find an expression for x_2 in terms of a .

(1)

(b) Show that $x_3 = a^2 - 3a - 3$.

(2)

Given that $x_3 = 7$,

(c) find the possible values of a .

(3)

(Total for Question 5 is 6 marks)

End of Questions



Silver Questions



Non-calculator

The total mark for this section is 35

Q1

A boy saves some money over a period of 60 weeks. He saves 10p in week 1, 15p in week 2, 20p in week 3 and so on until week 60. His weekly savings form an arithmetic sequence.

(a) Find how much he saves in week 15

(2)

(b) Calculate the total amount he saves over the 60 week period.

(3)

The boy's sister also saves some money each week over a period of m weeks. She saves 10p in week 1, 20p in week 2, 30p in week 3 and so on so that her weekly savings form an arithmetic sequence. She saves a total of £63 in the m weeks.

(c) Show that

$$m(m + 1) = 35 \times 36$$

(4)

(d) Hence write down the value of m .

(1)

(Total for Question 1 is 10 marks)

Q2

A sequence of numbers a_1, a_2, a_3, \dots is defined by

$$a_{n+1} = \frac{k(a_n + 2)}{a_n} \quad n \in \mathbb{N}$$

where k is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 2$

(a) show that

$$k^2 + k - 2 = 0 \tag{3}$$

(b) For this sequence explain why $k \neq 1$ (1)

(c) Find the value of

$$\sum_{r=1}^{80} a_r \tag{3}$$

(Total for Question 2 is 7 marks)

Q3

An arithmetic sequence has first term a and common difference d . The sum of the first 10 terms of the sequence is 162.

(a) Show that $10a + 45d = 162$ (2)

Given also that the sixth term of the sequence is 17,

(b) write down a second equation in a and d , (1)

(c) find the value of a and the value of d . (4)

(Total for Question 3 is 7 marks)

Q4

The first term of an arithmetic series is a and the common difference is d .

The 18th term of the series is 25 and the 21st term of the series is $32\frac{1}{2}$.

(a) Use this information to write down two equations for a and d . (2)

(b) Show that $a = -17.5$ and find the value of d . (2)

The sum of the first n terms of the series is 2750.

(c) Show that n is given by

$$n^2 - 15n = 55 \times 40. \quad (4)$$

(d) Hence find the value of n . (3)

(Total for Question 4 is 11 marks)

End of Questions



Gold Questions



Non-calculator

The total mark for this section is 29

Q1

Lewis played a game of space invaders. He scored points for each spaceship that he captured.

Lewis scored 140 points for capturing his first spaceship.

He scored 160 points for capturing his second spaceship, 180 points for capturing his third spaceship, and so on.

The number of points scored for capturing each successive spaceship formed an arithmetic sequence.

(a) Find the number of points that Lewis scored for capturing his 20th spaceship.

(2)

(b) Find the total number of points Lewis scored for capturing his first 20 spaceships.

(3)

Sian played an adventure game. She scored points for each dragon that she captured. The number of points that Sian scored for capturing each successive dragon formed an arithmetic sequence.

Sian captured n dragons and the total number of points that she scored for capturing all n dragons was 8500.

Given that Sian scored 300 points for capturing her first dragon and then 700 points for capturing her n th dragon,

(c) find the value of n .

(3)

(Total for Question 1 is 8 marks)

Q2

In the year 2000 a shop sold 150 computers. Each year the shop sold 10 more computers than

$\sqrt{\frac{1+4x}{1-x}} \approx 1 + \frac{5}{2}x - \frac{5}{8}x^2$ the year before, so that the shop sold 160 computers in 2001, 170 computers in 2002, and so on forming an arithmetic sequence.

(a) Show that the shop sold 220 computers in 2007.

(2)

(b) Calculate the total number of computers the shop sold from 2000 to 2013 inclusive.

(3)

In the year 2000, the selling price of each computer was £900. The selling price fell by £20 each year, so that in 2001 the selling price was £880, in 2002 the selling price was £860, and so on forming an arithmetic sequence.

(c) In a particular year, the selling price of each computer in £s was equal to three times the number of computers the shop sold in that year. By forming and solving an equation, find the year in which this occurred.

(4)

(Total for Question 2 is 9 marks)

Q3

A geometric series has common ratio r and first term a .

Given $r \neq 1$ and $a \neq 0$

(a) prove that

$$S_n = \frac{a(1 - r^n)}{1 - r} \quad (4)$$

Given also that S_{10} is four times S_5

(b) find the exact value of r .

(4)

(Total for Question 3 is 8 marks)

Q4

In a geometric series the common ratio is r and sum to n terms is S_n

Given that

$$S_\infty = \frac{8}{7} \times S_6$$

show that $r = \pm \frac{1}{\sqrt{k}}$, where k is an integer to be found.

(4)

(Total for Question 4 is 4 marks)

End of Questions



Platinum Questions



Non-calculator

The total mark for this section is 26

Q1

The sum of the first p terms of an arithmetic series is q and the sum of the first q terms of the same arithmetic series is p , where p and q are positive integers and $p \neq q$.

Giving simplified answers in terms of p and q , find

(a) the common difference of the terms in this series, (5)

(b) the first term of the series, (3)

(c) the sum of the first $(p + q)$ terms of the series. (3)

(Total for Question 1 is 11 marks)

Q2

- (a) The sides of the triangle ABC have lengths $BC = a$, $AC = b$ and $AB = c$, where $a < b < c$. The sizes of the angles A , B and C form an arithmetic sequence.

(i) Show that the area of triangle ABC is $ac \frac{\sqrt{3}}{4}$. (4)

Given that $a = 2$ and $\sin A = \frac{\sqrt{15}}{5}$, find

(ii) the value of b , (2)

(iii) the value of c . (4)

- (b) The internal angles of an n -sided polygon form an arithmetic sequence with first term 143° and common difference 2° .

Given that all of the internal angles are less than 180° , find the value of n .

(5)

(Total for Question 2 is 15 marks)

End of Questions



Bronze Questions



Non-calculator

The total mark for this section is 25

Q1

Find the first 3 terms, in ascending powers of x , of the binomial expansion of $(3 - 2x)^5$, giving each term in its simplest form.

(4)

(Total for Question 1 is 4 marks)

Q2

Find the first 4 terms, in ascending powers of x , of the binomial expansion of

$$\left(1 + \frac{3x}{2}\right)^8$$

Giving each term in its simplest form.

(4)

(Total for Question 2 is 4 marks)

Q3

$$f(x) = \frac{1}{\sqrt{4+x}}, \quad |x| < 4$$

Find the binomial expansion of $f(x)$ in ascending powers of x , up to and including the term in x^3 . Give each coefficient as a simplified fraction.

(6)

(Total for Question 3 is 6 marks)

Q4

$$f(x) = \frac{1}{\sqrt{(9+4x^2)}}, \quad |x| < \frac{3}{2}$$

Find the first three non-zero terms of the binomial expansion of $f(x)$ in ascending powers of x .
Give each coefficient as a simplified fraction.

(6)

(Total for Question 4 is 6 marks)

Q5

(a) Use the binomial expansion, in ascending powers of x , to show that

$$\sqrt{(4-x)} = 2 - \frac{1}{4}x + kx^2 + \dots$$

Where k is a rational constant to be found.

(4)

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

(b) State, giving a reason, if the expansion is valid for this value of x .

(1)

(Total for Question 5 is 5 marks)

End of Questions



Silver Questions



Non-calculator

The total mark for this section is 35

Q1

- (a) Find the first 3 terms, in ascending powers of x , of the binomial expansion of

$$(3 + bx)^5$$

where b is a non-zero constant. Give each term in its simplest form.

(4)

Given that, in this expansion, the coefficient of x^2 is twice the coefficient of x ,

- (b) find the value of b .

(2)

(Total for Question 1 is 6 marks)

Q2

- (a) Find the first 3 terms, in ascending powers of x , of the binomial expansion of

$$(2 - 9x)^4$$

Giving each term in its simplest form.

(4)

$$f(x) = (1 + kx)(2 - 9x)^4, \text{ where } k \text{ is a constant}$$

The expansion, in ascending powers of x , of $f(x)$ up to and including the term in x^2 is

$$A - 232x + Bx^2$$

Where A and B are constants.

- (b) Write down the value of A .

(1)

- (c) Find the value of k .

(2)

- (d) Hence find the value of B .

(2)

(Total for Question 2 is 9 marks)

Q3

$$\frac{2x^2 + 5x - 10}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}$$

(a) Find the values of the constants A , B and C . **(4)**

(b) Hence, or otherwise, expand $\frac{2x^2 + 5x - 10}{(x-1)(x+2)}$ in ascending powers of x , as far as the term in x^2 . Give each coefficient as a simplified fraction. **(7)**

(Total for Question 3 is 11 marks)

Q4

(a) Use the binomial expansion to show that

$$\sqrt{\left(\frac{1+x}{1-x}\right)} \approx 1 + x + \frac{1}{2}x^2, \quad |x| < 1$$

(6)

(b) Substitute $x = \frac{1}{26}$ into

$$\sqrt{\left(\frac{1+x}{1-x}\right)} = 1 + x + \frac{1}{2}x^2$$

To obtain an approximation to $\sqrt{3}$. Give your answer in the form $\frac{a}{b}$ where a and b are integers.

(3)

(Total for Question 4 is 9 marks)

End of Questions



Gold Questions



Non-calculator

The total mark for this section is 35

Q1

$$f(x) = \frac{27x^2 + 32x + 16}{(3x + 2)^2(1 - x)}, \quad |x| < \frac{2}{3}$$

Given that $f(x)$ can be expressed in the form

$$f(x) = \frac{A}{(3x + 2)} + \frac{B}{(3x + 2)^2} + \frac{C}{(1 - x)},$$

(a) Find the values of B and C and show that $A = 0$.

(4)

(b) Hence, or otherwise, find the series expansion of $f(x)$, in ascending powers of x , up to and including the term in x^2 .
Simplify each term.

(6)

(Total for Question 1 is 10 marks)

Q2

(a) Find the first three terms, in ascending powers of x , of the binomial expansion of

$$\frac{1}{\sqrt{4-x}}$$

Giving each coefficient in its simplest form.

(4)

The expansion can be used to find an approximation to $\sqrt{2}$. Possible values of x that could be substituted into this expansion are:

- $x = -14$ because $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{18}} = \frac{\sqrt{2}}{6}$
- $x = 2$ because $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
- $x = -\frac{1}{2}$ because $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{\frac{9}{2}}} = \frac{\sqrt{2}}{3}$

(b) Without evaluating your expansion,

(i) state, giving a reason, which of the three values of x should not be used

(1)

(ii) state, giving a reason, which of the three values of x would lead to the most accurate approximation to $\sqrt{2}$.

(1)

(Total for Question 2 is 6 marks)

Q3

$$f(x) = \frac{6}{\sqrt{9-4x}}, \quad |x| < \frac{9}{4}.$$

- (a) Find the binomial expansion of $f(x)$ in ascending powers of x , up to and including the term in x^3 . Give each coefficient in its simplest form. (6)

Use your answer to part (a) to find the binomial expansion in ascending powers of x , up to and including the term in x^3 , of

(b) $g(x) = \frac{6}{\sqrt{9+4x}}, \quad |x| < \frac{9}{4},$ (1)

(c) $h(x) = \frac{6}{\sqrt{9-8x}}, \quad |x| < \frac{9}{8}.$ (2)

(Total for Question 3 is 9 marks)

Q4

- (a) Use the binomial theorem to expand

$$(2-3x)^{-2}, \quad |x| < \frac{2}{3},$$

In ascending powers of x , up to and including the term in x^3 . Give each coefficient as a simplified fraction.

(5)

$$f(x) = \frac{a+bx}{(2-3x)^2}, \quad |x| < \frac{2}{3}, \quad \text{where } a \text{ and } b \text{ are constants.}$$

In the binomial expansion of $f(x)$, in ascending powers of x , the coefficient of x is 0 and the coefficient of x^2 is $\frac{9}{16}$. Find

- (b) The value of a and the value of b .

(5)

(Total for Question 4 is 10 marks)

End of Questions



Platinum Questions



Non-calculator

The total mark for this section is 25

Q1

(a) (i) Write down the binomial series expansion of

$$\left(1 + \frac{2}{n}\right)^n \quad n \in \square, n > 2$$

in powers of $\left(\frac{2}{n}\right)$ up to and including the term in $\left(\frac{2}{n}\right)^3$.

(ii) Hence prove that, for $n \in \square, n \geq 3$

$$\left(1 + \frac{2}{n}\right)^n \geq \frac{19}{3} - \frac{6}{n} \quad (3)$$

(b) Use the binomial series expansion of $\left(1 - \frac{x}{4}\right)^{\frac{1}{2}}$ to show that $\sqrt{3} < \frac{7}{4}$ (4)

$$f(x) = \left(1 + \frac{2}{x}\right)^x - 3^{\frac{x}{6}} \quad x \in \square, x > 0$$

Given that the function $f(x)$ is continuous and that $\sqrt[6]{3} > \frac{6}{5}$

(c) prove that $f(x) = 0$ has a root in the interval $[9, 10]$ (5)
(+S1)

(Total for Question 1 is 13 marks)

Q2

- (a) Find the binomial series expansion for $(4 + y)^{\frac{1}{2}}$ in ascending powers of y up to and including the term in y^3 . Simplify the coefficient of each term.

(3)

- (b) Hence show that the binomial series expansion for $(4 + 5x + x^2)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^3 is

$$2 + \frac{5x}{4} - \frac{9x^2}{64} + \frac{45x^3}{512}$$

(3)

- (c) Show that the binomial series expansion of $(4 + 5x + x^2)^{\frac{1}{2}}$ will converge for $-\frac{1}{2} \leq x \leq \frac{1}{2}$.

(6)**(Total for Question 2 is 12 marks)**

End of Questions



Bronze Questions

Calculator

The total mark for this section is 25

Q1

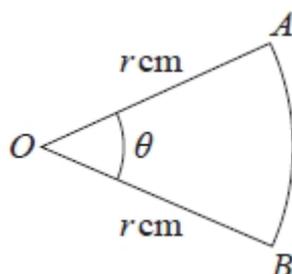


Figure 1

Figure 1 shows a sector AOB of a circle with centre O and radius r cm.

The angle AOB is θ radians.

The area of the sector AOB is 11 cm^2 .

Given that the perimeter of the sector is 4 times the length of the arc AB , find the exact value of r .

(4)

(Total for Question 1 is 4 marks)

Q2

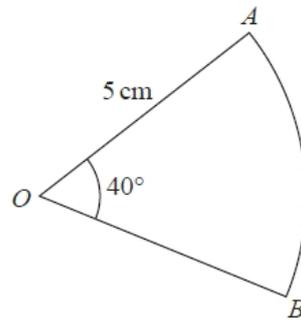


Figure 1

Figure 1 shows a sector AOB of a circle with centre O , radius 5 cm and angle $AOB = 40^\circ$. The attempt of a student to find the area of the sector is shown below.

$$\begin{aligned} \text{Area of sector} &= \frac{1}{2} r^2 \theta \\ &= \frac{1}{2} \times 5^2 \times 40 \\ &= 500 \text{ cm}^2 \end{aligned}$$

(a) Explain the error made by this student.

(1)

(b) Write out a correct solution.

(2)

(Total for Question 2 is 3 marks)

Q3

(a) Sketch, for $0 \leq x \leq 2\pi$, the graph of $y = \sin\left(x + \frac{\pi}{6}\right)$.

(2)

(b) Write down the exact coordinates of the points where the graph meets the coordinate axes.

(3)

(c) Solve, for $0 \leq x \leq 2\pi$, the equation

$$\sin\left(x + \frac{\pi}{6}\right) = 0.65,$$

giving your answers in radians to 2 decimal places.

(5)

(Total for Question 3 is 10 marks)

Q4

(i) Solve, for $0 \leq \theta < 180^\circ$, the equation

$$\frac{\sin 2\theta}{(4 \sin 2\theta - 1)} = 1$$

giving your answers to 1 decimal place.

(3)

(ii) Solve, for $0 \leq x < 2\pi$, the equation

$$5\sin^2x - 2\cos x - 5 = 0$$

giving your answers to 2 decimal places.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5)

(Total for Question 4 is 8 marks)

End of Questions



Silver Questions

20 Marks

Calculator

The total mark for this section is 20

Q1

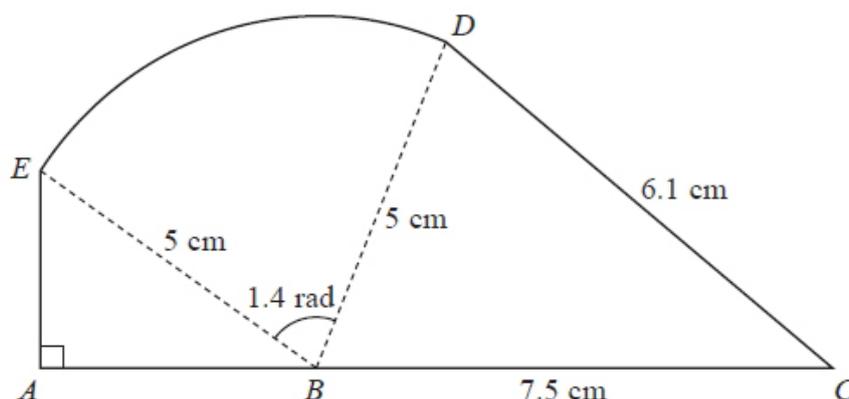


Figure 2

The shape $ABCDEA$, as shown in Figure 2, consists of a right-angled triangle EAB and a triangle DBC joined to a sector BDE of a circle with radius 5 cm and centre B .

The points A , B and C lie on a straight line with $BC = 7.5$ cm.

Angle $EAB = \frac{\pi}{2}$ radians, angle $EBD = 1.4$ radians and $CD = 6.1$ cm.

(a) Find, in cm^2 , the area of the sector BDE .

(2)

(b) Find the size of the angle DBC , giving your answer in radians to 3 decimal places.

(2)

(c) Find, in cm^2 , the area of the shape $ABCDEA$, giving your answer to 3 significant figures.

(5)

(Total for Question 1 is 9 marks)

Q2

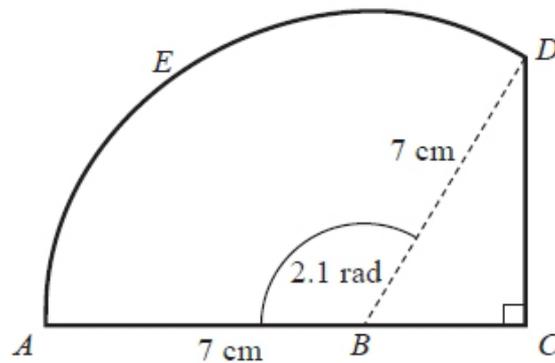


Figure 2

Figure 2 shows the shape $ABCDEA$ which consists of a right-angled triangle BCD joined to a sector $ABDEA$ of a circle with radius 7 cm and centre B .

A , B and C lie on a straight line with $AB = 7$ cm.

Given that the size of angle ABD is exactly 2.1 radians,

(a) find, in cm, the length of the arc DEA ,

(2)

(b) find, in cm, the perimeter of the shape $ABCDEA$, giving your answer to 1 decimal place.

(4)

(Total for Question 2 is 6 marks)

Q3

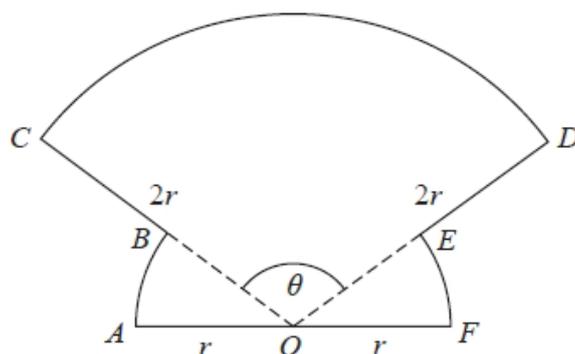


Figure 1

The shape $OABCDEFO$ shown in Figure 1 is a design for a logo.

In the design

- OAB is a sector of a circle centre O and radius r
- sector OFE is congruent to sector OAB
- ODC is a sector of a circle centre O and radius $2r$
- AOF is a straight line

Given that the size of angle COD is θ radians,

(a) write down, in terms of θ , the size of angle AOB

(1)

(b) Show that the area of the logo is

$$\frac{1}{2}r^2(3\theta + \pi)$$

(2)

(c) Find the perimeter of the logo, giving your answer in simplest form in terms of r , θ and π .

(2)

(Total for Question 3 is 5 marks)

End of Questions



Gold Questions

Calculator

The total mark for this section is 30

Q1

A circle C has centre $M(6, 4)$ and radius 3.

(a) Write down the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = r^2.$$

(2)

Figure 3

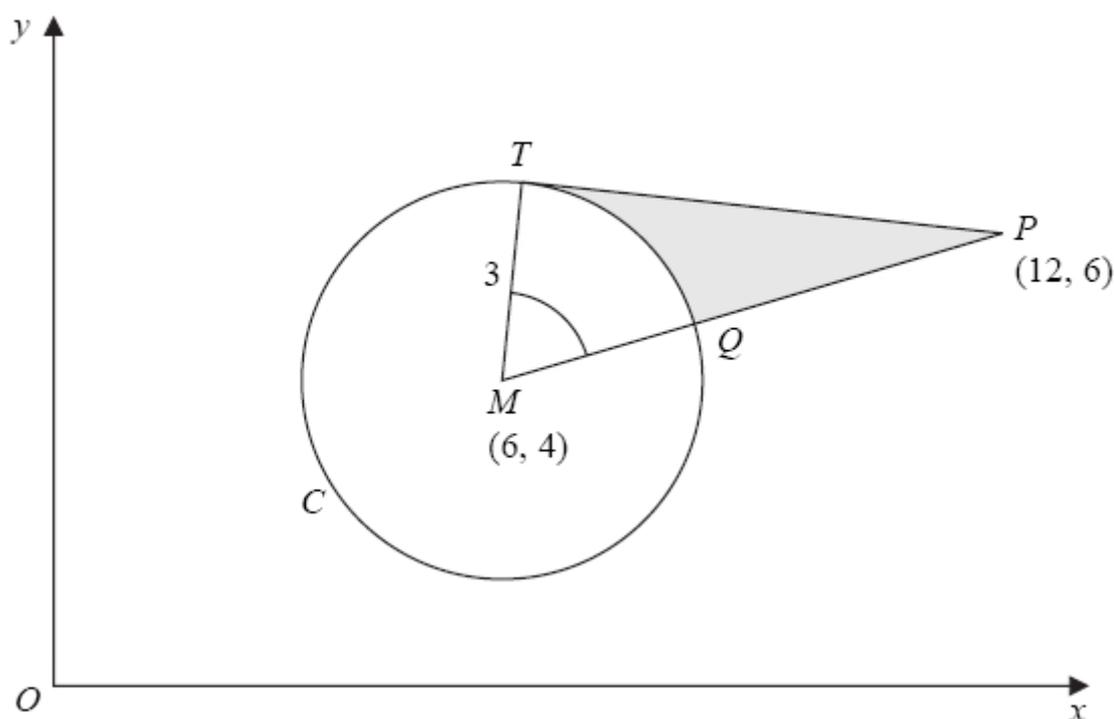


Figure 3 shows the circle C . The point T lies on the circle and the tangent at T passes through the point $P(12, 6)$. The line MP cuts the circle at Q .

(b) Show that the angle TMQ is 1.0766 radians to 4 decimal places.

(4)

The shaded region TPQ is bounded by the straight lines TP , QP and the arc TQ , as shown in Figure 3.

(c) Find the area of the shaded region TPQ . Give your answer to 3 decimal places.

(5)

(Total for Question 1 is 11 marks)

Q2

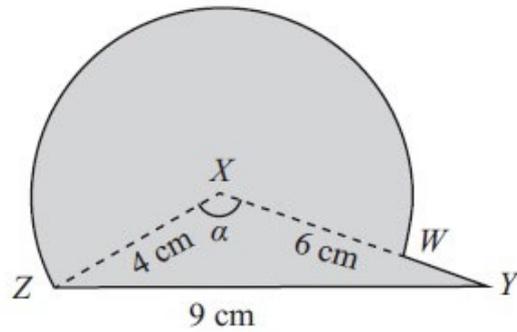


Figure 1

The triangle XYZ in Figure 1 has $XY = 6$ cm, $YZ = 9$ cm, $ZX = 4$ cm and angle $ZXY = a$. The point W lies on the line XY .

The circular arc ZW , in Figure 1 is a major arc of the circle with centre X and radius 4 cm.

(a) Show that, to 3 significant figures, $a = 2.22$ radians.

(2)

(b) Find the area, in cm^2 , of the major sector $XZWX$.

(3)

The region enclosed by the major arc ZW of the circle and the lines WY and YZ is shown shaded in Figure 1.

Calculate

(c) the area of this shaded region,

(3)

(d) the perimeter $ZWYX$ of this shaded region.

(4)

(Total for Question 2 is 12 marks)

Q3

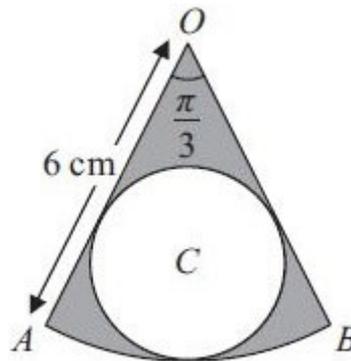


Figure 1

The shape shown in Figure 1 is a pattern for a pendant. It consists of a sector OAB of a circle centre O , of radius 6 cm, and angle $AOB = \frac{\pi}{3}$. The circle C , inside the sector, touches the two straight edges, OA and OB , and the arc AB as shown.

Find

- (a) the area of the sector OAB , (2)
- (b) the radius of the circle C . (3)

The region outside the circle C and inside the sector OAB is shown shaded in Figure 1.

- (c) Find the area of the shaded region. (2)

(Total for Question 3 is 7 marks)

End of Questions



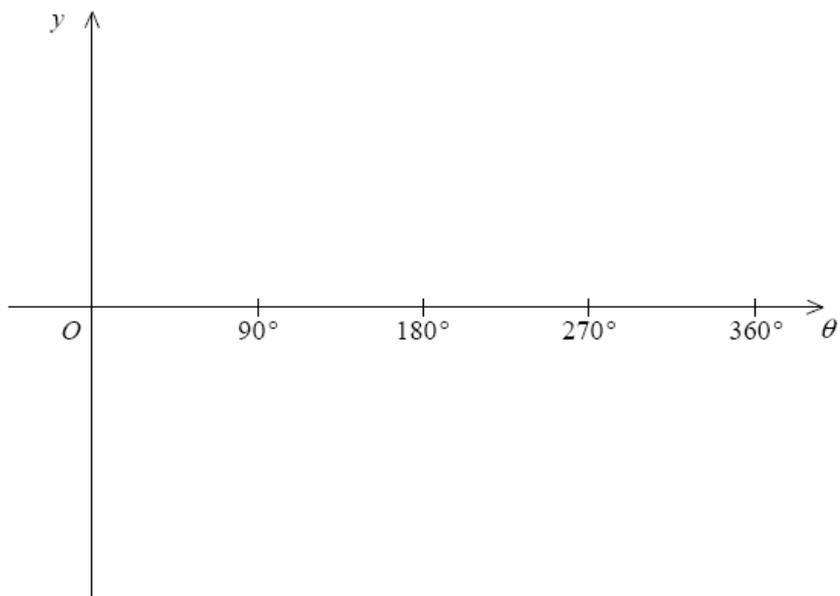
Bronze Questions

Non-calculator

The total mark for this section is 27

Q1.

On the axes below, sketch the graph of $y = 2 \operatorname{cosec} 2\theta$ for $0^\circ < \theta < 360^\circ$.



(2)

(Total for Question 1 is 2 marks)

Q2

Solve

$$\sec^2 \theta = 4$$

giving your answers in terms of π .

(3)

(Total for Question 2 is 3 marks)

Q3

(i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta. \quad (4)$$

(ii) Hence, or otherwise, for $0 < \theta < 60^\circ$, solve

$$8 \sin^3 \theta - 6 \sin \theta + 1 = 0. \quad (5)$$

(Total for Question 3 is 9 marks)

Q4

Given that

$$\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} \equiv 2 \cot 2\theta \quad \theta \neq (90n)^\circ, n \in \mathbb{Z}$$

Solve, for $90^\circ < \theta < 180^\circ$, the equation

$$\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} = 2. \quad (3)$$

(Total for Question 4 is 3 marks)

Q5

Solve, for $0 \leq \theta \leq 180^\circ$,

$$2 \cot^2 3\theta = 3 \operatorname{cosec} 3\theta \quad (10)$$

(Total for Question 5 is 10 marks)

End of Questions



Silver Questions



Non-calculator

The total mark for this section is 27

Q1

(a) Use the identity $\cos^2\theta + \sin^2\theta = 1$ to prove that $\tan^2\theta = \sec^2\theta - 1$.

(2)

(b) Solve, for $0 \leq \theta < 360^\circ$, the equation

$$2 \tan^2\theta + 4 \sec \theta + \sec^2\theta = 2$$

(6)

(Total for Question 1 is 8 marks)

Q2

Solve

$$\operatorname{cosec}^2 2x - \cot 2x = 1$$

for $0 \leq x \leq 180^\circ$.

(7)

(Total for Question 2 is 7 marks)

Q3

Solve, for $0 \leq \theta < 2\pi$, the equation

$$3\sec^2\theta + 3\sec\theta = 2 \tan^2\theta$$

You must show all of your working. Give your answers in terms of π .

(6)

(Total for Question 2 is 6 marks)

Q4

Given that

$$2 \cot 2x + \tan x \equiv \cot x \quad x \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

Hence, solve, for $-\pi \leq x < \pi$,

$$4 \cot 2x + 2 \tan x = \operatorname{cosec}^2 x$$

Give your answers in exact form.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(6)

(Total for Question 4 is 6 marks)

End of Questions



Gold Questions



Non-calculator

The total mark for this section is 29

Q1

Given that

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = 2 \operatorname{cosec} 2\theta, \quad \theta \neq 90n^\circ.$$

Solve, for $0^\circ < \theta < 360^\circ$, the equation

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = 1$$

giving your answers in exact form.

(6)

(Total for Question 1 is 6 marks)

Q2

Given that $\sin 2x = 2 \sin x \cos x$, find, for $0 < x < \pi$, all the solutions of the equation

$$\operatorname{cosec} x - 4 \cos x = 0$$

(5)

(Total for Question 2 is 5 marks)

Q3

Given that

$$\operatorname{cosec} 2x + \cot 2x = \cot x, \quad x \neq 90n^\circ, \quad n \in \mathbb{R}.$$

Hence, or otherwise, solve, for $0 \leq \theta < 180^\circ$,

$$\operatorname{cosec} (4\theta + 10^\circ) + \cot (4\theta + 10^\circ) = \sqrt{3}$$

You must show your working.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5)

(Total for Question 3 is 5 marks)

Q4

In this question you must show all stages of your working.

(a) Show that

$$\operatorname{cosec} \theta - \sin \theta \equiv \cos \theta \cot \theta \quad \theta \neq (180n)^\circ \quad n \in \mathbb{Z} \quad (3)$$

(b) Hence, or otherwise, solve for $0 < x < 180^\circ$

$$\operatorname{cosec} x - \sin x = \cos x \cot (3x - 50^\circ) \quad (5)$$

(Total for Question 4 is 8 marks)

Q5

(a) For $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, sketch the graph of $y = g(x)$ where

$$g(x) = \arcsin x \quad -1 \leq x \leq 1 \quad (2)$$

(b) Find the exact value of x for which

$$3g(x + 1) + \pi = 0 \quad (3)$$

(Total for Question 5 is 5 marks)

End of Questions



Platinum Questions



Non-calculator

The total mark for this section is 28

Q1

Solve, for $0 \leq \theta \leq 180^\circ$,

$$\tan(\theta + 35^\circ) = \cot(\theta - 53^\circ).$$

(4)

(Total for Question 1 is 4 marks)

Q2

The angle θ , $0 < \theta < \frac{\pi}{2}$, satisfies

$$\tan \theta \tan 2\theta = \sum_{r=0}^{\infty} 2 \cos^r 2\theta.$$

(a) Show that $\tan \theta = 3^p$, where p is a rational number to be found.

(8)

(b) Hence show that $\frac{\pi}{6} < \theta < \frac{\pi}{4}$.

(2)

(Total for Question 2 is 10 marks)

Q3

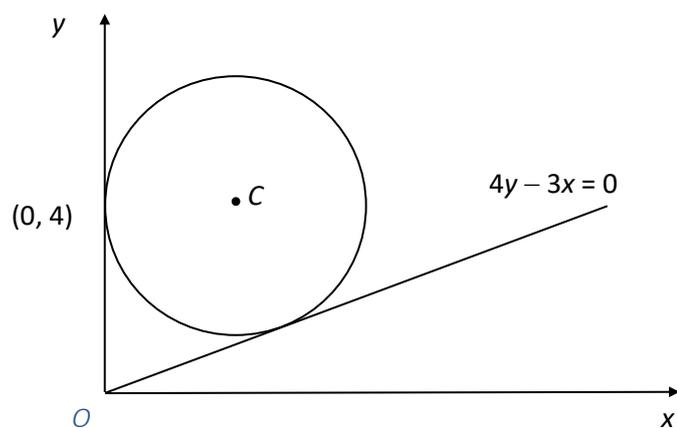


Figure 1

The circle, with centre C and radius r , touches the y -axis at $(0, 4)$ and also touches the line with equation $4y - 3x = 0$, as shown in Figure 1.

(a) (i) Find the value of r .

(ii) Show that $\arctan\left(\frac{3}{4}\right) + 2 \arctan\left(\frac{1}{2}\right) = \frac{1}{2}\pi$.

(8)

The line with equation $4x + 3y = q$, $q > 12$, is a tangent to the circle.

(b) Find the value of q .

(4)

(Total for Question 3 is 12 marks)

End of Questions



Bronze Questions



Non-calculator

The total mark for this section is 32

Q1

Prove that

$$1 - \cos 2\theta \equiv \tan \theta \sin 2\theta, \quad \theta \neq \frac{(2n+1)\pi}{2}, \quad n \in \mathbb{Z} \quad (3)$$

(Total for Question 1 is 3 marks)

Q2

Prove that

$$\sin 2x - \tan x \equiv \tan x \cos 2x, \quad x \neq (2n+1)90^\circ, \quad n \in \mathbb{Z} \quad (4)$$

(Total for Question 2 is 4 marks)

Q3

Given that

$$\tan \theta^\circ = p, \text{ where } p \text{ is a constant, } p \neq \pm 1$$

use standard trigonometric identities, to find in terms of p ,

(a) $\tan 2\theta^\circ$ (2)

(b) $\cos \theta^\circ$ (2)

(c) $\cot(\theta - 45)^\circ$ (2)

Write each answer in its simplest form.

(Total for Question 3 is 6 marks)

Q4

- (a) Use the double angle formulae and the identity

$$\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$$

to obtain an expression for $\cos 3x$ in terms of powers of $\cos x$ only.

(4)

- (b) (i) Prove that

$$\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} \equiv 2 \sec x, \quad x \neq (2n+1)\frac{\pi}{2}.$$

(4)

- (ii) Hence find, for $0 < x < 2\pi$, all the solutions of

$$\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} = 4.$$

(3)

(Total for Question 4 is 11 marks)

Q5

- (a) Show that

$$\operatorname{cosec} \theta - \sin \theta \equiv \cos \theta \cot \theta \quad \theta \neq (180n)^\circ \quad n \in \mathbb{Z}$$

(3)

- (b) Hence, or otherwise, solve for $0 < x < 180^\circ$

$$\operatorname{cosec} x - \sin x = \cos x \cot (3x - 50^\circ)$$

(5)

(Total for Question 5 is 8 marks)

End of Questions



Silver Questions



Non-calculator

The total mark for this section is 34

Q1

- (i) Use an appropriate double angle formula to show that

$$\operatorname{cosec} 2x = \lambda \operatorname{cosec} x \sec x,$$

and state the value of the constant λ .

(3)

- (ii) Solve, for $0 \leq \theta < 2\pi$, the equation

$$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta$$

You must show all your working. Give your answers in terms of π .

(6)

(Total for Question 1 is 9 marks)

Q2

- (a) Starting from the formulae for $\sin(A + B)$ and $\cos(A + B)$, prove that

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

(4)

- (b) Deduce that

$$\tan\left(\theta + \frac{\pi}{6}\right) = \frac{1 + \sqrt{3} \tan \theta}{\sqrt{3} - \tan \theta}$$

(3)

- (c) Hence, or otherwise, solve, for $0 \leq \theta \leq \pi$,

$$1 + \sqrt{3} \tan \theta = (\sqrt{3} - \tan \theta) \tan(\pi - \theta)$$

Give your answers as multiples of π .

(6)

(Total for Question 2 is 13 marks)

Q3

(a) Prove that

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \tan \theta, \quad \theta \neq 90n^\circ, \quad n \in \mathbb{Z} \quad (4)$$

(b) Hence, or otherwise,

(i) show that $\tan 15^\circ = 2 - \sqrt{3}$, (3)

(ii) solve, for $0 < x < 360^\circ$,

$$\operatorname{cosec} 4x - \cot 4x = 1 \quad (5)$$

(Total for Question 3 is 12 marks)

End of Questions



Gold Questions



Non-calculator

The total mark for this section is 33

Q1

Show that

$$\operatorname{cosec} 2x + \cot 2x = \cot x, \quad x \neq 90n^\circ, \quad n \in \mathbb{Z} . \quad (5)$$

(Total for Question 1 is 5 marks)

Q2

Prove

$$\frac{\cos 3\theta}{\sin \theta} + \frac{\sin 3\theta}{\cos \theta} \equiv 2 \cot 2\theta \quad \theta \neq (90n)^\circ, \quad n \in \mathbb{Z} \quad (4)$$

(Total for Question 2 is 4 marks)

Q3

(i) Solve, for $0 \leq \theta < \pi$, the equation

$$\sin 3\theta - \sqrt{3} \cos 3\theta = 0$$

giving your answers in terms of π .

(3)

(ii) Given that

$$4 \sin^2 x + \cos x = 4 - k, \quad 0 \leq k \leq 3$$

find $\cos x$ in terms of k .

(3)

(Total for Question 3 is 6 marks)

Q4

Prove that

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A}, \quad A \neq \frac{(2n+1)\pi}{4}, \quad n \in \mathbb{Z} \quad (5)$$

(Total for Question 4 is 5 marks)

Q5

(a) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta. \quad (4)$$

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8 \sin^3 \theta - 6 \sin \theta + 1 = 0.$$

Give your answers in terms of π . (5)

(b) Using $\sin(\theta - \alpha) = \sin \theta \cos \alpha - \cos \theta \sin \alpha$, or otherwise, show that

$$\sin 15^\circ = \frac{1}{4}(\sqrt{6} - \sqrt{2}). \quad (4)$$

(Total for Question 5 is 13 marks)

End of Questions



Platinum Questions



Non-calculator

The total mark for this section is 33

Q1

(a) Prove the identity

$$(\sin x + \cos y) \cos(x - y) \equiv (1 + \sin(x - y))(\cos x + \sin y) \quad (5)$$

(b) Hence, or otherwise, show that

$$\frac{\sin 5\theta + \cos 3\theta}{\cos 5\theta + \sin 3\theta} \equiv \frac{1 + \tan \theta}{1 - \tan \theta} \quad (6)$$

(c) Given that $k > 1$, show that the equation $\frac{\sin 5\theta + \cos 3\theta}{\cos 5\theta + \sin 3\theta} = k$ has a unique solution

$$\text{in the interval } 0 < \theta < \frac{\pi}{4} \quad (4)$$

(+S2)

(Total for Question 1 is 17 marks)

Q2

Solve, for $0 \leq x \leq 360^\circ$,

$$\sin 47^\circ \cos^3 x + \cos 47^\circ \sin x \cos^2 x = \frac{1}{2} \cos^2 x. \quad (7)$$

(Total for Question 2 is 7 marks)

Q3

Solve for $0 < x < 360^\circ$

$$\cot 2x - \tan 78^\circ = \frac{(\sec x)(\sec 78^\circ)}{2}$$

where x is not an integer multiple of 90° .

(9)

(Total for Question 3 is 9 marks)

End of Questions



Bronze Questions



Non-calculator

The total mark for this section is 27

Q1

A curve C has parametric equations

$$x = 2t - 1, \quad y = 4t - 7 + \frac{3}{t}, \quad t \neq 0$$

Show that the Cartesian equation of the curve C can be written in the form

$$y = \frac{2x^2 + ax + b}{x + 1}, \quad x \neq -1$$

Where a and b are integers to be found.

(3)

(Total for Question 1 is 3 marks)

Q2

The curve C has parametric equations

$$x = 3t - 4, \quad y = 5 - \frac{6}{t}, \quad t > 0$$

(a) Find $\frac{dy}{dx}$ in terms of t .

(2)

The point P lies on C where $t = \frac{1}{2}$.

(b) Find the equation of the tangent to C at the point P . Give your answer in the form $y = px + q$, where p and q are integers to be determined.

(3)

(c) Show that the Cartesian equation for C can be written in the form

$$y = \frac{ax + b}{x + 4}, \quad x > -4$$

Where a and b are integers to be determined.

(3)

(Total for Question 2 is 8 marks)

Q3

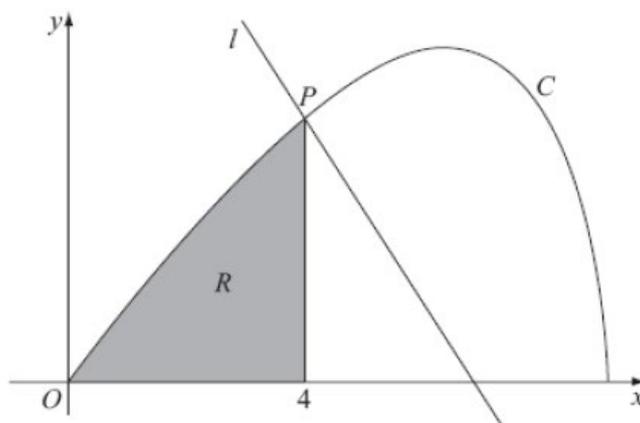


Figure 3

Figure 3 shows the curve C with parametric equations

$$x = 8 \cos t, \quad y = 4 \sin 2t, \quad 0 \leq t \leq \frac{\pi}{2}.$$

The point P lies on C and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of t at the point P .

(2)

The line l is a normal to C at P .

(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$.

(6)

The finite region R is enclosed by the curve C , the x -axis and the line $x = 4$, as shown shaded in Figure 3.

(c) Show that the area of R is given by the integral $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64 \sin^2 t \cos t \, dt$.

(4)

(d) Use this integral to find the area of R , giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(4)

(Total for Question 3 is 16 marks)

End of Questions



Silver Questions



Non-calculator

The total mark for this section is 28

Q1

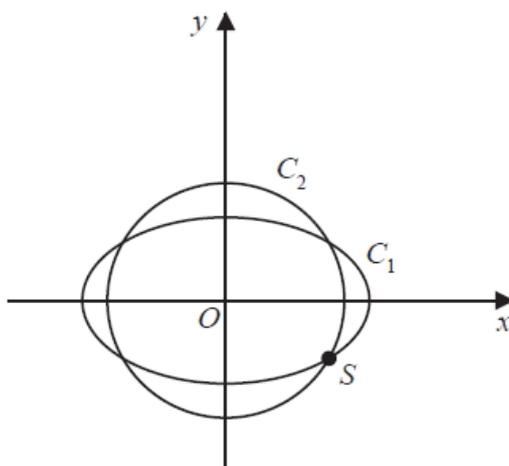


Figure 2

The curve C_1 with parametric equations

$$x = 10\cos t, \quad y = 4\sqrt{2}\sin t, \quad 0 \leq t < 2\pi$$

Meets the circle C_2 with equation

$$x^2 + y^2 = 66$$

At four distinct points as shown in Figure 2.

Given that one of these points, S , lies in the 4th quadrant, find the Cartesian coordinates of S .

(6)

(Total for Question 1 is 6 marks)

Q2

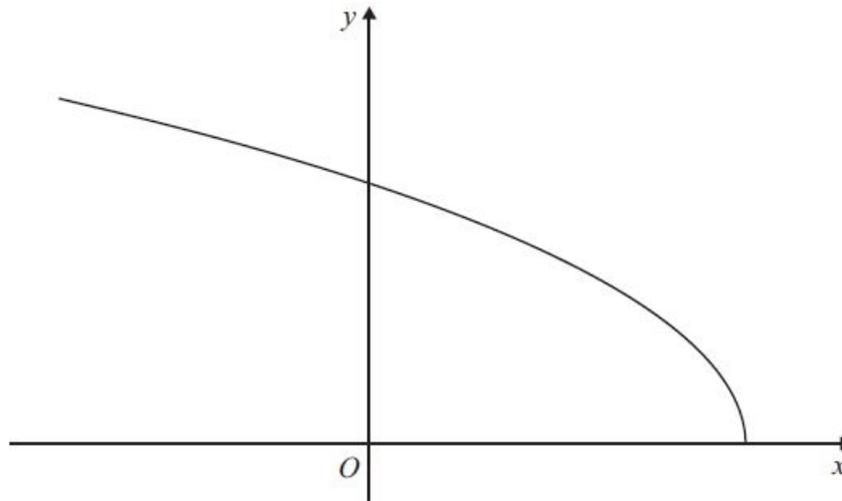


Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = 2 \cos 2t, \quad y = 6 \sin t, \quad 0 \leq t \leq \frac{\pi}{2}$$

(a) Find the gradient of the curve at the point where $t = \frac{\pi}{3}$.

(4)

(b) Find a Cartesian equation of the curve in the form

$$y = f(x), \quad -k \leq x \leq k,$$

Stating the value of the constant k .

(4)

(c) Write down the range of $f(x)$.

(2)

(Total for Question 2 is 10 marks)

Q3

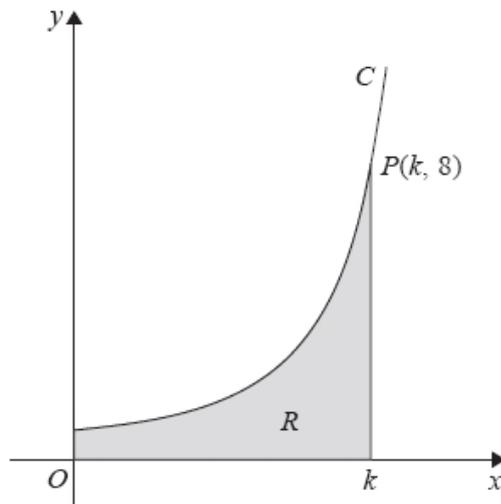


Diagram not
drawn to scale

Figure 4

Figure 4 shows a sketch of part of the curve C with parametric equations

$$x = 3\theta \sin \theta, \quad y = \sec^3 \theta, \quad 0 \leq \theta \leq \frac{\pi}{2}$$

The point $P(k, 8)$ lies on C , where k is a constant.

(a) Find the exact value of k .

(2)

The finite region R , shown shaded in Figure 4, is bounded by the curve C , the y -axis, the x -axis and the line with equation $x = k$.

(b) Show that the area of R can be expressed in the form

$$\lambda \int_{\alpha}^{\beta} (\theta \sec^2 \theta + \tan \theta \sec^2 \theta) d\theta$$

Where λ , α and β are constants to be determined.

(4)

(c) Hence use integration to find the exact value of the area of R .

(6)

(Total for Question 3 is 12 marks)

End of Questions



Gold Questions



Non-calculator

The total mark for this section is 30

Q1

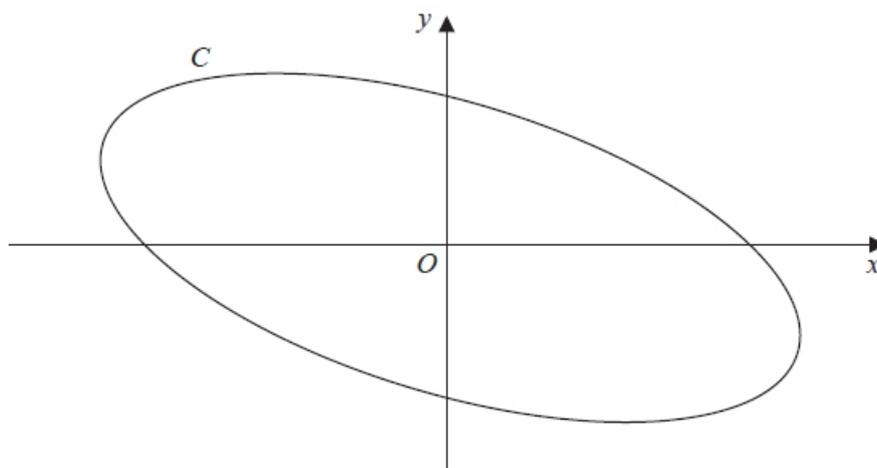


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 4 \cos\left(t + \frac{\pi}{6}\right), \quad y = 2 \sin t, \quad 0 \leq t \leq 2\pi$$

(a) Show that

$$x + y = \sqrt{3} \cos t$$

(3)

(b) Show that a Cartesian equation of C is

$$(x + y)^2 + ay^2 = b$$

Where a and b are integers to be determined.

(2)

(Total for Question 1 is 5 marks)

Q2

A curve C has parametric equations

$$x = 3 + 2 \sin t, \quad y = 4 + 2 \cos 2t, \quad 0 \leq t < 2\pi$$

(a) Show that all points on C satisfy $y = 6 - (x - 3)^2$ (2)

(b) (i) Sketch the curve C .

(ii) Explain briefly why C does not include all points of $y = 6 - (x - 3)^2$, $x \in \mathbb{R}$. (3)

The line with equation $x + y = k$, where k is a constant, intersects C at two distinct points.

(c) State the range of values of k , writing your answer in set notation. (5)

(Total for Question 2 is 10 marks)

Q3

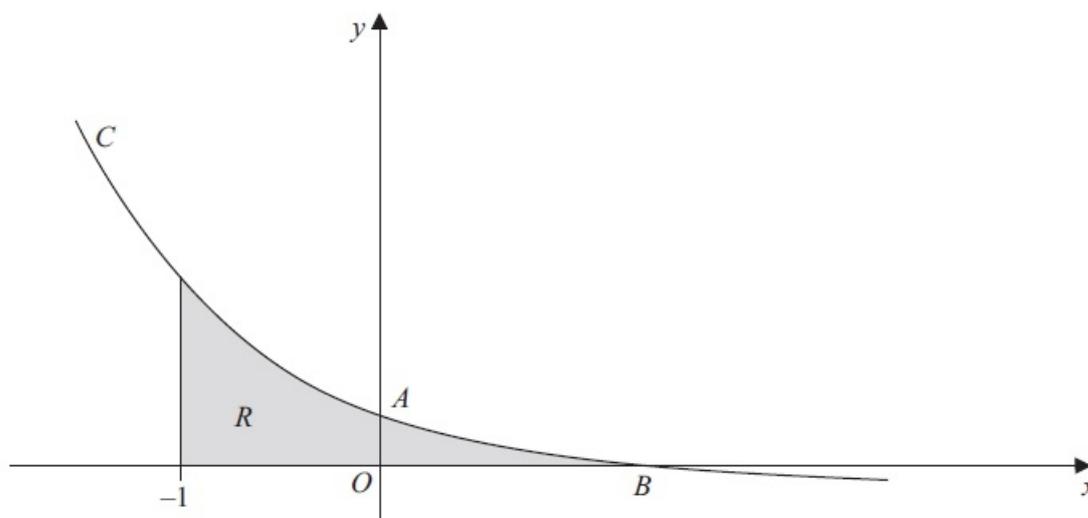


Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$$

The curve crosses the y -axis at the point A and crosses the x -axis at the point B .

(a) Show that A has coordinates $(0, 3)$.

(2)

(b) Find the x coordinate of the point B .

(2)

(c) Find an equation of the normal to C at the point A .

(5)

The region R , as shown shaded in Figure 2, is bounded by the curve C , the line $x = -1$ and the x -axis.

(d) Use integration to find the exact area of R .

(6)

(Total for Question 3 is 15 marks)

End of Questions



Platinum Questions



Non-calculator

The total mark for this section is 13

Q1

The curve C has parametric equations

$$x = \cos^2 t,$$

$$y = \cos t \sin t,$$

where $0 \leq t < \pi$.

(a) Show that C is a circle and find its centre and its radius.

(5)

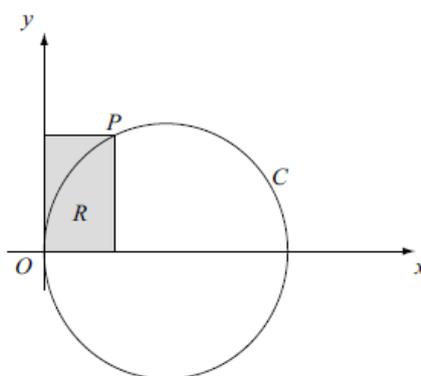


Figure 1

Figure 1 shows a sketch of C . The point P , with coordinates $(\cos^2 \alpha, \cos \alpha \sin \alpha)$, $0 < \alpha < \frac{\pi}{2}$, lies on C . The rectangle R has one side on the x -axis, one side on the y -axis and OP as a diagonal, where O is the origin.

(b) Show that the area of R is $\sin \alpha \cos^3 \alpha$.

(1)

(c) Find the maximum area of R , as α varies.

(7)

(Total for Question 1 is 13 marks)

End of Questions



Bronze Questions



Non-calculator

The total mark for this section is 35

Q1

The curve C has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find

(i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 2$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

(Total for Question 1 is 7 marks)

Q2

Given $y = x(2x + 1)^4$, show that

$$\frac{dy}{dx} = (2x + 1)^n (Ax + B)$$

where n , A and B are constants to be found.

(4)

(Total for Question 2 is 4 marks)

Q3

A curve C has the equation $y^2 - 3y = x^3 + 8$.

(a) Find $\frac{dy}{dx}$ in terms of x and y .

(4)

(b) Hence find the gradient of C at the point where $y = 3$.

(3)

(Total for Question 3 is 7 marks)

Q4

Find the gradient of the curve with equation

$$\ln y = 2x \ln x, \quad x > 0, y > 0$$

at the point on the curve where $x = 2$. Give your answer as an exact value.

(7)

(Total for Question 4 is 7 marks)

Q5

(i) Find, using calculus, the x coordinate of the turning point of the curve with equation

$$y = e^{3x} \cos 4x, \quad \frac{\pi}{4} \leq x < \frac{\pi}{2}$$

Write your answer in the form

$$x = m \arctan(b)$$

Where m and b are constants to be determined.

(5)

(ii) Given that $x = \sin^2 2y$, $0 < y < \frac{\pi}{4}$, find $\frac{dy}{dx}$ as a function of y .

Write your answer in the form

$$\frac{dy}{dx} = p \operatorname{cosec}(qy), \quad 0 < y < \frac{\pi}{4}$$

where p and q are constants to be determined.

(5)

(Total for Question 5 is 10 marks)

End of Questions



Silver Questions



Non-calculator

The total mark for this section is 36

Q1

Given that

$$y = 3x^2 + 6x^{\frac{1}{3}} + \frac{2x^3 - 7}{3\sqrt{x}}, \quad x > 0$$

Find $\frac{dy}{dx}$. Give each term in your answer in its simplified form.

(6)

(Total for Question 1 is 6 marks)

Q2

The curve C has the equation $2x + 3y^2 + 3x^2 y = 4x^2$.

The point P on the curve has coordinates $(-1, 1)$.

(a) Find the gradient of the curve at P .

(5)

(b) Hence find the equation of the normal to C at P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(3)

(Total for Question 2 is 8 marks)

Q3

Given that θ is measured in radians, prove, from first principles, that the derivative of $\sin \theta$ is $\cos \theta$

You may assume the formula for $\sin(A \pm B)$ and that as $h \rightarrow 0$, $\frac{\sin h}{h} \rightarrow 1$ and $\frac{\cos h - 1}{h} \rightarrow 0$.

(5)

(Total for Question 3 is 5 marks)

Q4

$$y = \frac{5x^2 + 10x}{(x + 1)^2} \quad x \neq -1$$

(a) Show that $\frac{dy}{dx} = \frac{A}{(x+1)^n}$ where A and n are constants to be found.

(4)

(b) Hence deduce the range of values for x for which $\frac{dy}{dx} < 0$.

(1)

(Total for Question 4 is 5 marks)

Q5

(i) Given that

$$x = \sec^2 2y, \quad 0 < y < \frac{\pi}{4}$$

show that

$$\frac{dy}{dx} = \frac{1}{4x\sqrt{(x-1)}}$$

(4)

(ii) Given that

$$y = (x^2 + x^3)\ln 2x$$

find the exact value of $\frac{dy}{dx}$ at $x = \frac{e}{2}$, giving your answer in its simplest form.

(5)

(iii) Given that

$$f(x) = \frac{3\cos x}{(x+1)^{\frac{1}{3}}}, \quad x \neq -1$$

show that

$$f'(x) = \frac{g(x)}{(x+1)^{\frac{4}{3}}}, \quad x \neq -1$$

where $g(x)$ is an expression to be found.

(3)

(Total for Question 5 is 12 marks)

End of Questions



Gold Questions



Non-calculator

The total mark for this section is 35

Q1

(i) Differentiate with respect to x

(a) $x^2 \cos 3x$

(3)

(b) $\frac{\ln(x^2 + 1)}{x^2 + 1}$

(4)

(i) A curve C has the equation

$$y = \sqrt{4x + 1}, \quad x > -\frac{1}{4}, \quad y > 0$$

The point P on the curve has x -coordinate 2. Find an equation of the tangent to C at P in the form $ax + by + c = 0$, where a , b and c are integers.

(6)

(Total for Question 1 is 13 marks)

Q2

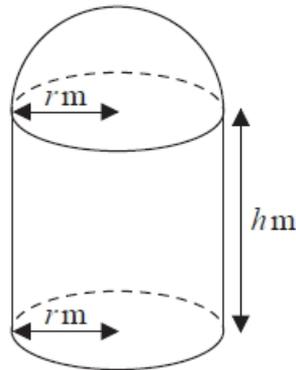


Figure 9

[A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$]

A manufacturer produces a storage tank.

The tank is modelled in the shape of a hollow circular cylinder closed at one end with a hemispherical shell at the other end as shown in Figure 9.

The walls of the tank are assumed to have negligible thickness.

The cylinder has radius r metres and height h metres and the hemisphere has radius r metres.

The volume of the tank is 6 m^3 .

(a) Show that, according to the model, the surface area of the tank, in m^2 , is given by

$$\frac{12}{r} + \frac{5}{3}\pi r^2$$

(4)

The manufacturer needs to minimise the surface area of the tank.

(b) Use calculus to find the radius of the tank for which the surface area is a minimum.

(4)

(Total for Question 2 is 8 marks)

Q3

A curve C has equation

$$2^x + y^2 = 2xy$$

Find the exact value of $\frac{dy}{dx}$ at the point on C with coordinates $(3, 2)$.

(7)

(Total for Question 3 is 7 marks)

Q4

The curve C , in the standard Cartesian plane, is defined by the equation

$$x = 4 \sin 2y \quad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{dy}{dx}$ at the origin.

(2)

(b) (i) Use the small angle approximation for $\sin 2y$ to find an equation linking x and y for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i).

(2)

(c) Show that, for all points (x, y) lying on C ,

$$\frac{dy}{dx} = \frac{1}{a\sqrt{b-x^2}}$$

where a and b are constants to be found.

(3)

(Total for Question 4 is 7 marks)

End of Questions



Platinum Questions



Non-calculator

The total mark for this section is 22

Q1

(a) Given that $y = \ln [t + \sqrt{(1 + t^2)}]$, show that $\frac{dy}{dt} = \frac{1}{\sqrt{(1+t^2)}}$.

(3)

The curve C has parametric equations

$$x = \frac{1}{\sqrt{(1+t^2)}}, \quad y = \ln [t + \sqrt{(1 + t^2)}], \quad t \in \mathbb{R}.$$

A student was asked to prove that, for $t > 0$, the gradient of the tangent to C is negative.

The attempted proof was as follows:

$$\begin{aligned} y &= \ln \left(t + \frac{1}{x} \right) \\ &= \ln \left(\frac{tx+1}{x} \right) \\ &= \ln (tx + 1) - \ln x \\ \therefore \frac{dy}{dx} &= \frac{t}{tx + 1} - \frac{1}{x} \\ &= \frac{\frac{t}{x}}{t + \frac{1}{x}} - \frac{1}{x} \\ &= \frac{t\sqrt{(1+t^2)}}{t + \sqrt{(1+t^2)}} - \sqrt{(1+t^2)} \\ &= -\frac{(1+t^2)}{t + \sqrt{(1+t^2)}} \end{aligned}$$

As $(1 + t^2) > 0$, and $t + \sqrt{(1 + t^2)} > 0$ for $t > 0$, $\frac{dy}{dx} < 0$ for $t > 0$.

- (b) (i) Identify the error in this attempt.
(ii) Give a correct version of the proof.

(6)

(c) Prove that $\ln [-t + \sqrt{1 + t^2}] = -\ln [t + \sqrt{1 + t^2}]$. (3)

(d) Deduce that C is symmetric about the x -axis and sketch the graph of C . (3)

(Total for Question 1 is 15 marks)

Q2

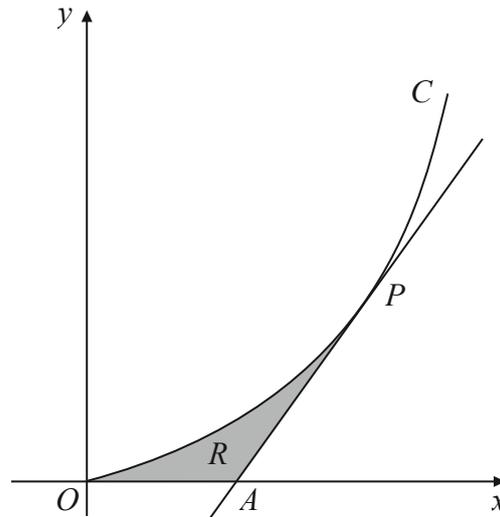


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 2 \sin t, \quad y = \ln(\sec t), \quad 0 < t < \frac{\pi}{2}$$

The tangent to C at the point P , where $t = \frac{\pi}{3}$, cuts the x -axis at A .

(a) Show that the x -coordinate of A is $\frac{\sqrt{3}}{3}(3 - \ln 2)$. (6)

The shaded region R lies between C , the positive x -axis and the tangent AP as shown in Figure 2.

(b) Show that the area of R is $\sqrt{3} (1 + \ln 2) - 2 \ln(2 + \sqrt{3}) - \frac{\sqrt{3}}{6} (\ln 2)^2$. (11)

(Total for Question 2 is 17 marks)

End of Questions



Bronze Questions

Calculator

The total mark for this section is 34

Q1

$$f(x) = \ln(x + 2) - x + 1, \quad x > -2, x \in \mathbb{R}.$$

(a) Show that there is a root of $f(x) = 0$ in the interval $2 < x < 3$.

(2)

(b) Use the iterative formula

$$x_{n+1} = \ln(x_n + 2) + 1, \quad x_0 = 2.5$$

to calculate the values of x_1 , x_2 and x_3 giving your answers to 5 decimal places.

(3)

(c) Show that $x = 2.505$ is a root of $f(x) = 0$ correct to 3 decimal places.

(2)

(Total for Question 1 is 7 marks)

Q2

$$f(x) = -x^3 + 3x^2 - 1.$$

(a) Show that the equation $f(x) = 0$ can be rewritten as

$$x = \sqrt{\left(\frac{1}{3-x}\right)}$$

(2)

(b) Starting with $x_1 = 0.6$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{1}{3-x_n}\right)}$$

to calculate the values of x_2 , x_3 and x_4 , giving all your answers to 4 decimal places.

(2)

(c) Show that $x = 0.653$ is a root of $f(x) = 0$ correct to 3 decimal places.

(3)

(Total for Question 2 is 7 marks)

Q3

$$f(x) = x^3 + 3x^2 + 4x - 12$$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{4(3-x)}{3+x}\right)}, \quad x \neq -3$$

(3)

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2.

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{3+x_n}\right)}, \quad n \geq 0$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1, x_2 and x_3 .

(3)

The root of $f(x) = 0$ is α .

(c) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places.

(3)

(Total for Question 3 is 9 marks)

Q4

$$f(x) = 3xe^x - 1$$

The curve with equation $y = f(x)$ has a turning point P .

(a) Find the exact coordinates of P .

(5)

The equation $f(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$

(b) Use the iterative formula

$$x_{n+1} = \frac{1}{3}e^{-x_n}$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1, x_2 and x_3 .

(3)

(c) By choosing a suitable interval, show that a root of $f(x) = 0$ is $x = 0.2576$ correct to 4 decimal places.

(3)

(Total for Question 4 is 11 marks)

End of Questions



Silver Questions

Calculator

The total mark for this section is 39

Q1

$$g(x) = e^{x-1} + x - 6$$

(a) Show that the equation $g(x) = 0$ can be written as

$$x = \ln(6 - x) + 1, \quad x < 6$$

(2)

The root of $g(x) = 0$ is a .

The iterative formula

$$x_{n+1} = \ln(6 - x_n) + 1, \quad x_0 = 2$$

is used to find an approximate value for a .

(b) Calculate the values of x_1, x_2 and x_3 to 4 decimal places.

(3)

(c) By choosing a suitable interval, show that $a = 2.307$ correct to 3 decimal places.

(3)

(Total for Question 1 is 8 marks)

Q2

$$f(x) = x^2 - 3x + 2 \cos\left(\frac{1}{2}x\right), \quad 0 \leq x \leq \pi$$

(a) Show that the equation $f(x) = 0$ has a solution in the interval $0.8 < x < 0.9$

(2)

The curve with equation $y = f(x)$ has a minimum point P .

(b) Show that the x -coordinate of P is the solution of the equation

$$x = \frac{3 + \sin\left(\frac{1}{2}x\right)}{2}$$

(4)

(c) Using the iteration formula

$$x_{n+1} = \frac{3 + \sin\left(\frac{1}{2}x_n\right)}{2}, \quad x_0 = 2$$

find the values of x_1, x_2 and x_3 , giving your answers to 3 decimal places.

(3)

(d) By choosing a suitable interval, show that the x -coordinate of P is 1.9078 correct to 4 decimal places.

(3)

(Total for Question 2 is 12 marks)

Q3

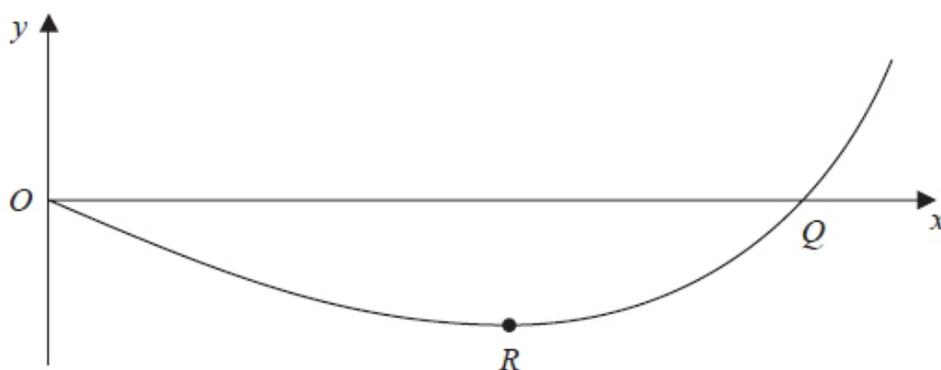


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 2 \cos\left(\frac{1}{2}x^2\right) + x^3 - 3x - 2$$

The curve crosses the x -axis at the point Q and has a minimum turning point at R .

(a) Show that the x coordinate of Q lies between 2.1 and 2.2.

(2)

(b) Show that the x coordinate of R is a solution of the equation

$$x = \sqrt{1 + \frac{2}{3}x \sin\left(\frac{1}{2}x^2\right)}$$

(4)

Using the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{2}{3}x_n \sin\left(\frac{1}{2}x_n^2\right)}, \quad x_0 = 1.3$$

(c) find the values of x_1 and x_2 to 3 decimal places.

(2)

(Total for Question 3 is 8 marks)

Q4

$$f(x) = 3x^3 - 2x - 6.$$

(a) Show that $f(x) = 0$ has a root, α , between $x = 1.4$ and $x = 1.45$.

(2)

(b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}, \quad x \neq 0.$$

(3)

(c) Starting with $x_0 = 1.43$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$$

to calculate the values of x_1, x_2 and x_3 , giving your answers to 4 decimal places.

(3)

(d) By choosing a suitable interval, show that $\alpha = 1.435$ is correct to 3 decimal places.

(3)

(Total for Question 4 is 11 marks)

End of Questions



Gold Questions

47 Marks

Calculator

The total mark for this section is 47

Q1

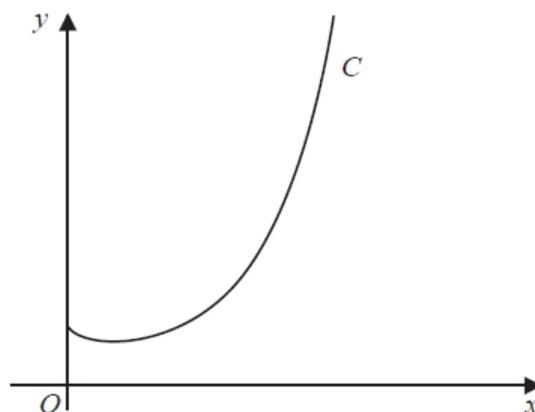


Figure 8

Figure 8 shows a sketch of the curve C with equation $y = x^x$, $x > 0$

- (a) Find, by firstly taking logarithms, the x coordinate of the turning point of C .
(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5)

The point $P(\alpha, 2)$ lies on C .

- (b) Show that $1.5 < \alpha < 1.6$

(2)

A possible iteration formula that could be used in an attempt to find α is

$$x_{n+1} = 2x_n^{1-x_n}$$

Using this formula with $x_1 = 1.5$

- (c) find x_4 to 3 decimal places,

(2)

- (d) describe the long-term behaviour of x_n .

(2)

(Total for Question 1 is 11 marks)

Q2

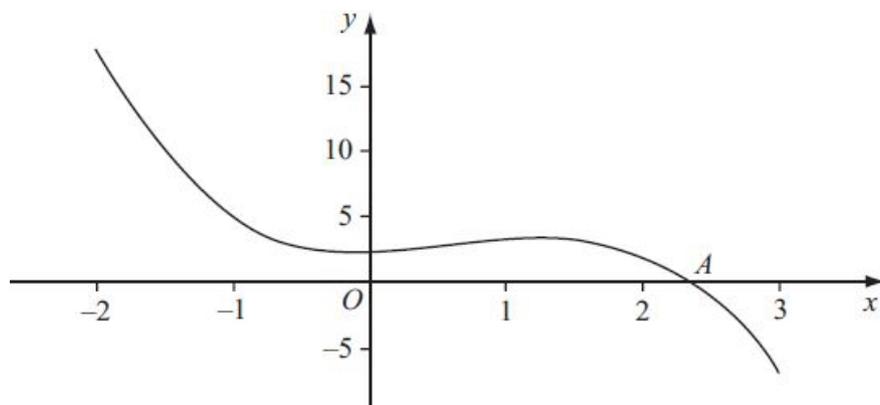


Figure 1

Figure 1 shows part of the curve with equation $y = -x^3 + 2x^2 + 2$, which intersects the x -axis at the point A where $x = \alpha$.

To find an approximation to α , the iterative formula

$$x_{n+1} = \frac{2}{(x_n)^2} + 2$$

is used.

(a) Taking $x_0 = 2.5$, find the values of x_1, x_2, x_3 and x_4 .

Give your answers to 3 decimal places where appropriate.

(3)

(b) Show that $\alpha = 2.359$ correct to 3 decimal places.

(3)

(Total for Question 2 is 6 marks)

Q3

The curve with equation $y = f(x)$ where

$$f(x) = x^2 + \ln(2x^2 - 4x + 5)$$

has a single turning point at $x = \alpha$.

(a) Show that α is a solution of the equation

$$2x^3 - 4x^2 + 7x - 2 = 0$$

(4)

The iterative formula

$$x_{n+1} = \frac{1}{7}(2 + 4x_n^2 - 2x_n^3)$$

is used to find an approximate value for α .

Starting with $x_1 = 0.3$

(b) calculate, giving each answer to 4 decimal places,

- (i) the value of x_2
- (ii) the value of x_4

(3)

Using a suitable interval and a suitable function that should be stated,

(c) show that α is 0.341 to 3 decimal places.

(2)

(Total for Question 3 is 9 marks)

Q4

The curve with equation $y = 2 \ln(8 - x)$ meets the line $y = x$ at a single point, $x = \alpha$.

(a) Show that $3 < \alpha < 4$

(2)

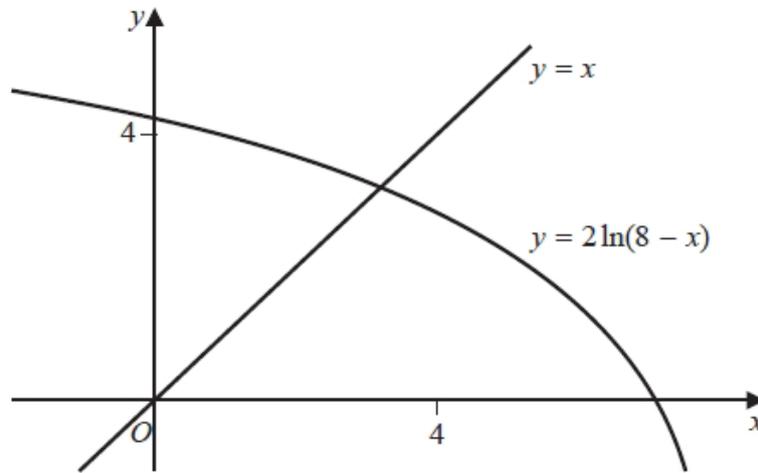


Figure 2

Figure 2 shows the graph of $y = 2 \ln(8 - x)$ and the graph of $y = x$.

A student uses the iteration formula

$$x_{n+1} = 2 \ln(8 - x_n), \quad n \in \mathbb{N}$$

in an attempt to find an approximation for α .

Using the graph and starting with $x_1 = 4$

(b) determine whether or not this iteration formula can be used to find an approximation for α , justifying your answer.

(2)

(Total for Question 4 is 4 marks)

Q5

$$f(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5$$

- (a) Show that $f(x) = 0$ has a root α in the interval $[3.5, 4]$ (2)

A student takes 4 as the first approximation to α .

Given $f(4) = 3.099$ and $f'(4) = 16.67$ to 4 significant figures,

- (b) apply the Newton-Raphson procedure once to obtain a second approximation for α , giving your answer to 3 significant figures. (2)
- (c) Show that α is the only root of $f(x) = 0$ (2)

(Total for Question 5 is 6 marks)

Q6

$$f(x) = 25x^2e^{2x} - 16, \quad x \in \mathbb{R}$$

- (a) Using calculus, find the exact coordinates of the turning points on the curve with equation $y = f(x)$. (5)

- (b) Show that the equation $f(x) = 0$ can be written as $x = \pm \frac{4}{5}e^{-x}$. (1)

The equation $f(x) = 0$ has a root α , where $\alpha = 0.5$ to 1 decimal place.

- (c) Starting with $x_0 = 0.5$, use the iteration formula

$$x_{n+1} = \frac{4}{5}e^{-x_n}$$

- to calculate the values of x_1, x_2 and x_3 , giving your answers to 3 decimal places. (3)

- (d) Give an accurate estimate for α to 2 decimal places and justify your answer. (2)

(Total for Question 6 is 11 marks)

End of Questions



Bronze Questions



Non-calculator

The total mark for this section is 32

Q1

$$\frac{dy}{dx} = 6x^{-\frac{1}{2}} + x\sqrt{x}, \quad x > 0$$

Given that $y = 37$ at $x = 4$, find y in terms of x , giving each term in its simplest form.

(7)

(Total for Question 1 is 7 marks)

Q2

The gradient of a curve C is given by

$$\frac{dy}{dx} = \frac{(x^2 + 3)^2}{x^2}, \quad x \neq 0.$$

(a) Show that $\frac{dy}{dx} = x^2 + 6 + 9x^{-2}$.

(2)

The point $(3, 20)$ lies on C .

(b) Find an equation for the curve C in the form $y = f(x)$.

(6)

(Total for Question 2 is 8 marks)

Q3

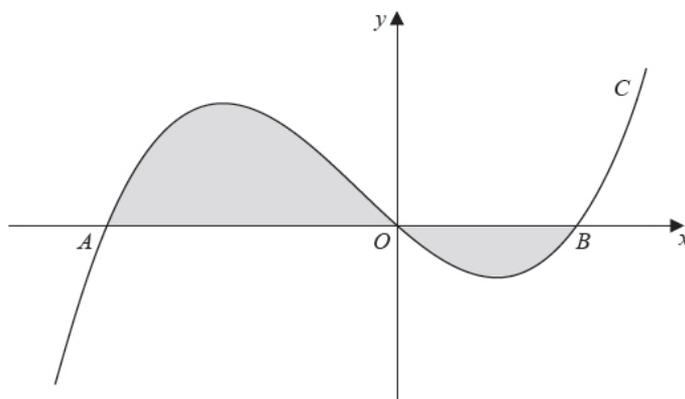


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = x(x + 4)(x - 2).$$

The curve C crosses the x -axis at the origin O and at the points A and B .

(a) Write down the x -coordinates of the points A and B .

(1)

The finite region, shown shaded in Figure 3, is bounded by the curve C and the x -axis.

(b) Use integration to find the total area of the finite region shown shaded in Figure 3.

(7)

(Total for Question 3 is 8 marks)

Q4

(a) Find

$$\int 10x(x^{\frac{1}{2}} - 2)dx$$

giving each term in its simplest form.

(4)

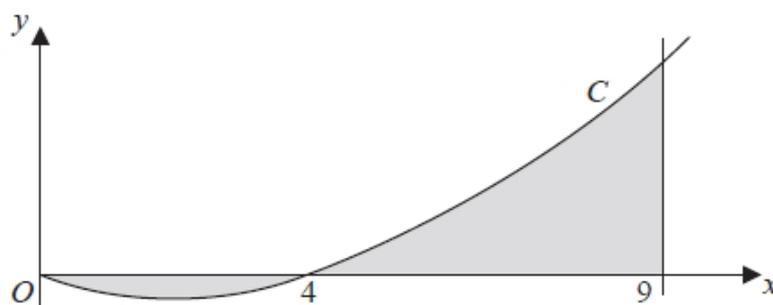


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = 10x(x^{\frac{1}{2}} - 2), \quad x \geq 0$$

The curve C starts at the origin and crosses the x -axis at the point $(4, 0)$.

The area, shown shaded in Figure 2, consists of two finite regions and is bounded by the curve C , the x -axis and the line $x = 9$.

(b) Use your answer from part (a) to find the total area of the shaded regions.

(5)

(Total for Question 4 is 9 marks)

End of Questions



Silver Questions



Non-calculator

The total mark for this section is 33

Q1

$$\frac{dy}{dx} = -x^3 + \frac{4x-5}{2x^3}, \quad x \neq 0$$

Given that $y = 7$ at $x = 1$, find y in terms of x , giving each term in its simplest form.

(6)

(Total for Question 1 is 6 marks)

Q2

Given that a is a positive constant and

$$\int_a^{2a} \frac{t+1}{t} dt = \ln 7$$

show that $a = \ln k$, where k is a constant to be found.

(4)

(Total for Question 2 is 4 marks)

Q3

A curve C has equation $y = f(x)$.

Given that

- $f'(x) = 6x^2 + ax - 23$ where a is a constant
- the y intercept of C is -12
- $(x + 4)$ is a factor of $f(x)$

find, in simplest form, $f(x)$.

(6)

(Total for Question 3 is 6 marks)

Q4

Given that $k \in \mathbb{R}^+$.

(a) show that $\int_k^{3k} \frac{2}{(3x-k)} dx$ is independent of k , (4)

(b) show that $\int_k^{2k} \frac{2}{(2x-k)^2} dx$ is inversely proportional to k . (3)

(Total for Question 4 is 7 marks)

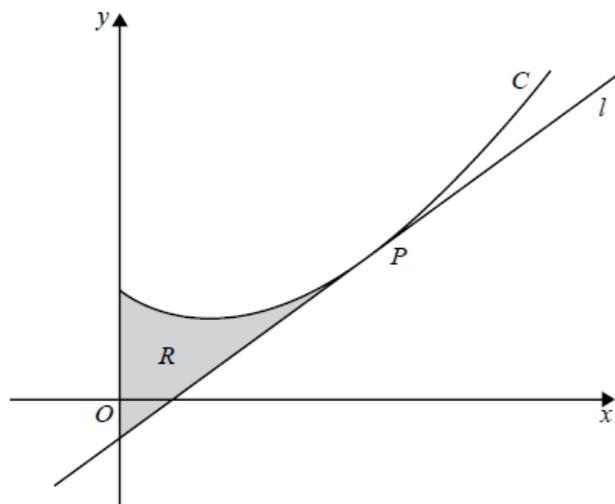
Q5

Figure 4

Figure 4 shows a sketch of the curve C with equation

$$y = 5x^2 - 9x + 11, x \geq 0$$

The point P with coordinates $(4, 15)$ lies on C .

The line l is the tangent to C at the point P .

The region R , shown shaded in Figure 4, is bounded by the curve C , the line l and the y -axis.

Show that the area of R is 24, making your method clear.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(10)

(Total for Question 5 is 10 marks)

End of Questions



Gold Questions



Non-calculator

The total mark for this section is 34

Q1

(i) Find $\int \ln\left(\frac{x}{2}\right) dx$.

(4)

(ii) Find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx$.

(5)

(Total for Question 1 is 9 marks)

Q2

Given that A is constant and

$$\int_1^4 (3\sqrt{x} + A) dx = 2A^2$$

show that there are exactly two possible values for A .

(5)

(Total for Question 2 is 5 marks)

Q3

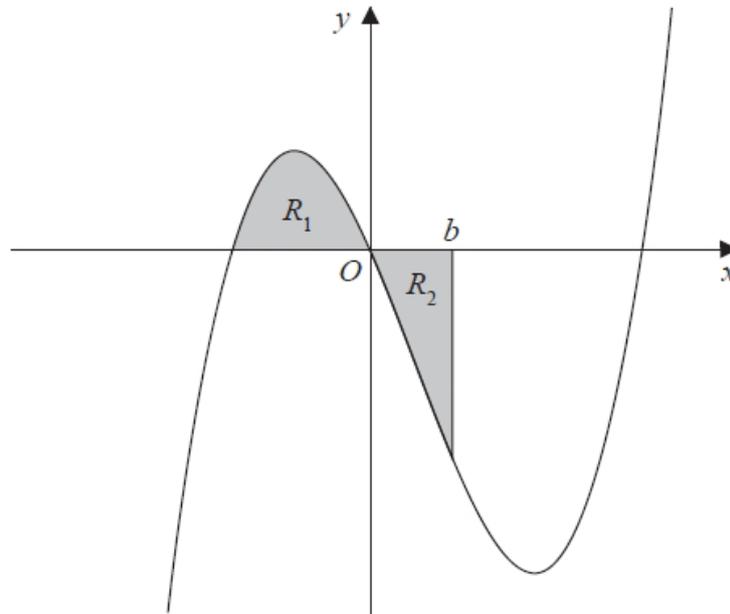


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = x(x + 2)(x - 4)$.

The region R_1 shown shaded in Figure 2 is bounded by the curve and the negative x -axis.

(a) Show that the exact area of R_1 is $\frac{20}{3}$.

(4)

The region R_2 also shown shaded in Figure 2 is bounded by the curve, the positive x -axis and the line with equation $x = b$, where b is a positive constant and $0 < b < 4$

Given that the area of R_1 is equal to the area of R_2

(b) verify that b satisfies the equation

$$(b + 2)^2 (3b^2 - 20b + 20) = 0$$

(4)

The roots of the equation $3b^2 - 20b + 20 = 0$ are 1.225 and 5.442 to 3 decimal places.
The value of b is therefore 1.225 to 3 decimal places.

(c) Explain, with the aid of a diagram, the significance of the root 5.442

(2)

(Total for Question 3 is 10 marks)

Q4

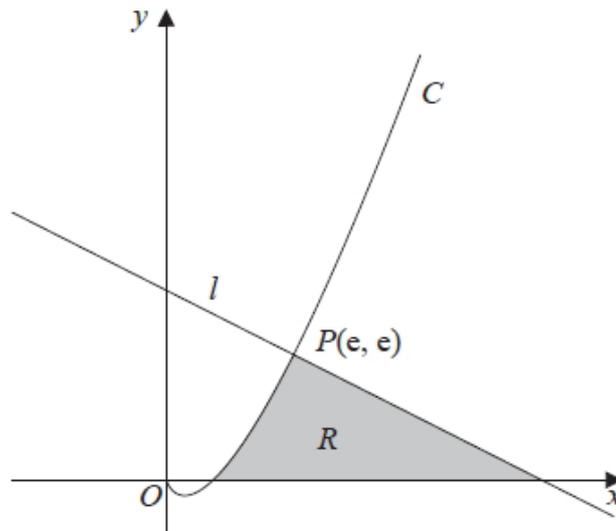


Figure 2

Figure 2 shows a sketch of part of the curve C with equation $y = x \ln x$, $x > 0$

The line l is the normal to C at the point $P(e, e)$

The region R , shown shaded in Figure 2, is bounded by the curve C , the line l and the x -axis.

Show that the exact area of R is $Ae^2 + B$ where A and B are rational numbers to be found.

(10)

(Total for Question 4 is 10 marks)

End of Questions



Platinum Questions



Calculator

The total mark for this section is 27

Q1

- (a) On the same diagram, sketch $y = x$ and $y = \sqrt{x}$, for $x \geq 0$, and mark clearly the coordinates of the points of intersection of the two graphs.

(2)

- (b) With reference to your sketch, explain why there exists a value a of x ($a > 1$) such that

$$\int_0^a x \, dx = \int_0^a \sqrt{x} \, dx.$$

(2)

- (c) Find the exact value of a .

(4)

- (d) Hence, or otherwise, find a non-constant function $f(x)$ and a constant b ($b \neq 0$) such that

$$\int_{-b}^b f(x) \, dx = \int_{-b}^b \sqrt{|f(x)|} \, dx.$$

(2)

(Total for Question 1 is 10 marks)

Q2

$$f(x) = x - [x], \quad x \geq 0$$

where $[x]$ is the largest integer $\leq x$.

For example, $f(3.7) = 3.7 - 3 = 0.7$; $f(3) = 3 - 3 = 0$.

(a) Sketch the graph of $y = f(x)$ for $0 \leq x < 4$.

(3)

(b) Find the value of p for which $\int_2^p f(x) dx = 0.18$.

(3)

Given that

$$g(x) = \frac{1}{1+kx}, \quad x \geq 0, \quad k > 0,$$

and that $x_0 = \frac{1}{2}$ is a root of the equation $f(x) = g(x)$,

(c) find the value of k .

(2)

(d) Add a sketch of the graph of $y = g(x)$ to your answer to part (a).

(1)

The root of $f(x) = g(x)$ in the interval $n < x < n + 1$ is x_n , where n is an integer.

(e) Prove that

$$2x_n^2 - (2n - 1)x_n - (n + 1) = 0.$$

(4)

(f) Find the smallest value of n for which $x_n - n < 0.05$.

(4)

(Total for Question 2 is 17 marks)



Bronze Questions



Non-calculator

The total mark for this section is 20

Q1

[In this question the unit vectors \mathbf{i} and \mathbf{j} are due east and due north respectively.]

A stone slides horizontally across ice.

Initially the stone is at the point $A(-24\mathbf{i} - 10\mathbf{j})$ m relative to a fixed point O .

After 4 seconds the stone is at the point $B(12\mathbf{i} + 5\mathbf{j})$ m relative to the fixed point O .

The motion of the stone is modelled as that of a particle moving in a straight line at constant speed.

Using the model prove that the stone passes through O .

(2)

(Total for Question 1 is 2 marks)

Q2

Given that the point A has position vector $3\mathbf{i} - 7\mathbf{j}$ and the point B has position vector $8\mathbf{i} + 3\mathbf{j}$,

(a) find the vector \overline{AB} .

(2)

(b) Find $|\overline{AB}|$. Give your answer as a simplified surd.

(2)

(Total for Question 2 is 4 marks)

Q3

Given that the point A has position vector $4\mathbf{i} - 5\mathbf{j}$ and the point B has position vector $-5\mathbf{i} - 2\mathbf{j}$,

(a) find the vector \overline{AB} , (2)

(b) find $|\overline{AB}|$. Give your answer as a simplified surd. (2)

(Total for Question 3 is 4 marks)

Q4

Relative to a fixed origin O , the point A has position vector $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and the point B has position vector $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$. The points A and B lie on a straight line l .

(a) Find \overline{AB} . (2)

(b) Find a vector equation of l . (2)

The point C has position vector $2\mathbf{i} + p\mathbf{j} - 4\mathbf{k}$ with respect to O , where p is a constant. Given that AC is perpendicular to l , find

(c) the value of p , (4)

(d) the distance AC . (2)

(Total for Question 4 is 10 marks)

End of Questions



Silver Questions



Non-calculator

The total mark for this section is 34

Q1

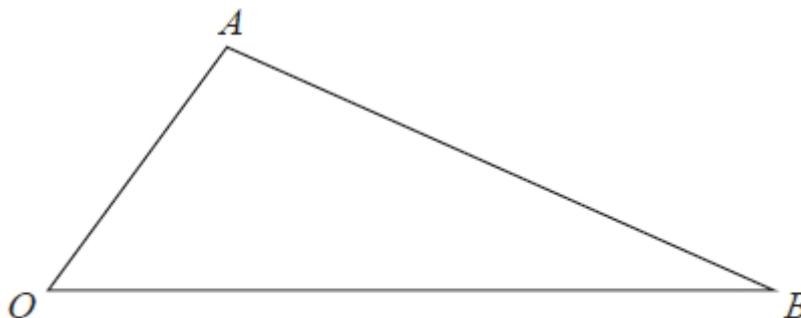


Figure 7

Figure 7 shows a sketch of triangle OAB .

The point C is such that $\overline{OC} = 2\overline{OA}$.

The point M is the midpoint of AB .

The straight line through C and M cuts OB at the point N .

Given $\overline{OA} = \mathbf{a}$ and $\overline{OB} = \mathbf{b}$

(a) Find \overline{CM} in terms of \mathbf{a} and \mathbf{b}

(2)

(b) Show that $\overline{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$, where λ is a scalar constant.

(2)

(c) Hence prove that $ON : NB = 2 : 1$

(2)

(Total for Question 1 is 6 marks)

Q2

Relative to a fixed origin O ,

the point A has position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$,

the point B has position vector $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$,

and the point C has position vector $(a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$, where a is a constant and $a < 0$

D is the point such that $\overline{AB} = \overline{BD}$.

(a) Find the position vector of D .

(2)

Given $|\overline{AC}| = 4$

(b) find the value of a .

(3)

(Total for Question 2 is 5 marks)

Q3

Relative to a fixed origin O , the point A has position vector $(10\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$,

and the point B has position vector $(8\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B .

(a) Find the vector \overline{AB}

(2)

(b) Find a vector equation for the line l .

(2)

The point C has position vector $(3\mathbf{i} + 12\mathbf{j} + 3\mathbf{k})$.

The point P lies on l . Given that the vector \overline{CP} is perpendicular to l ,

(c) find the position vector of the point P .

(6)

(Total for Question 3 is 10 marks)

Q4

With respect to a fixed origin O the lines l_1 and l_2 are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \quad l_2: \mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$$

where λ and μ are parameters and p and q are constants. Given that l_1 and l_2 are perpendicular,

(a) show that $q = -3$.

(2)

Given further that l_1 and l_2 intersect, find

(b) the value of p ,

(6)

(c) the coordinates of the point of intersection.

(2)

The point A lies on l_1 and has position vector $\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}$. The point C lies on l_2 .

Given that a circle, with centre C , cuts the line l_1 at the points A and B ,

(d) find the position vector of B .

(3)

(Total for Question 4 is 13 marks)

End of Questions



Gold Questions



Non-calculator

The total mark for this section is 42

Q1

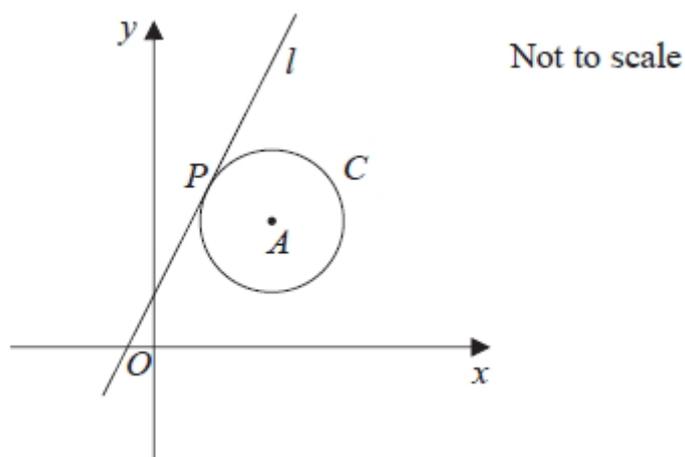


Figure 3

The circle C has centre A with coordinates $(7, 5)$.

The line l , with equation $y = 2x + 1$, is the tangent to C at the point P , as shown in Figure 3.

(a) Show that an equation of the line PA is $2y + x = 17$

(3)

(b) Find an equation for C .

(4)

The line with equation $y = 2x + k$, $k \neq 1$ is also a tangent to C .

(c) Find the value of the constant k .

(3)

(Total for Question 1 is 5 marks)

Q2

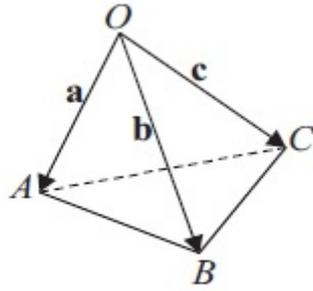


Figure 1

The points A , B and C have position vectors \mathbf{a} , \mathbf{b} and \mathbf{c} respectively, relative to a fixed origin O , as shown in Figure 1.

It is given that

$$\mathbf{a} = \mathbf{i} + \mathbf{j}, \mathbf{b} = 3\mathbf{i} - \mathbf{j} + \mathbf{k} \text{ and } \mathbf{c} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}.$$

Calculate

(a) $\mathbf{b} \times \mathbf{c}$, (3)

(b) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$, (2)

(c) the area of triangle OBC , (2)

(d) the volume of the tetrahedron $OABC$. (1)

(Total for Question 2 is 8 marks)

Q3

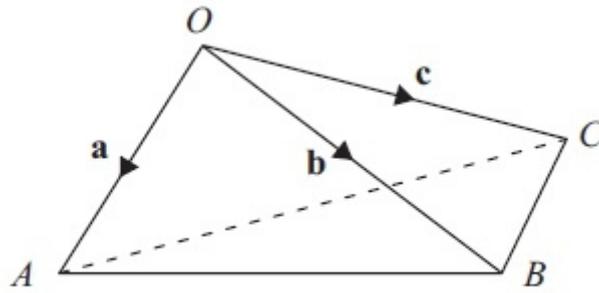


Figure 1

The points A , B and C have position vectors \mathbf{a} , \mathbf{b} and \mathbf{c} respectively, relative to a fixed origin O , as shown in Figure 1.

It is given that

$$\mathbf{a} = \mathbf{i} + \mathbf{j}, \mathbf{b} = 3\mathbf{i} - \mathbf{j} + \mathbf{k} \text{ and } \mathbf{c} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}.$$

Calculate

(a) $\mathbf{b} \times \mathbf{c}$, (3)

(b) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$, (2)

(c) the area of triangle OBC , (2)

(d) the volume of the tetrahedron $OABC$. (1)

(Total for Question 3 is 8 marks)

Q4

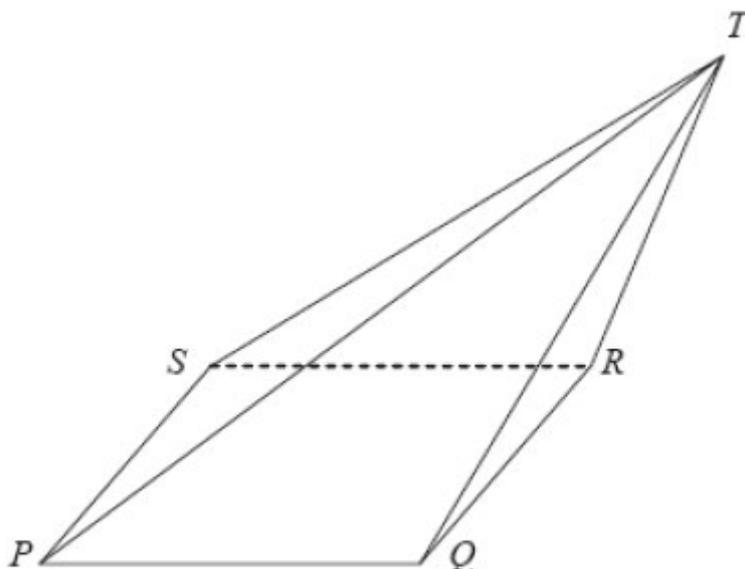


Figure 1

Figure 1 shows a pyramid $PQRST$ with base $PQRS$.

The coordinates of P , Q and R are $P(1, 0, -1)$, $Q(2, -1, 1)$ and $R(3, -3, 2)$.

Find

(a) $\overrightarrow{PQ} \times \overrightarrow{PR}$, (3)

(b) a vector equation for the plane containing the face $PQRS$, giving your answer in the form $\mathbf{r} \cdot \mathbf{n} = d$. (2)

The plane Π contains the face PST . The vector equation of Π is $\mathbf{r} \cdot (\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}) = 6$.

(c) Find cartesian equations of the line through P and S . (5)

(d) Hence show that PS is parallel to QR . (2)

Given that $PQRS$ is a parallelogram and that T has coordinates $(5, 2, -1)$,

(e) find the volume of the pyramid $PQRST$. (3)

(Total for Question 4 is 15 marks)

End of Questions



Platinum Questions



Non-calculator

The total mark for this section is 26

Q1

Points A and B have position vectors \mathbf{a} and \mathbf{b} , respectively, relative to an origin O , and are such that OAB is a triangle with $OA = a$ and $OB = b$.

The point C , with position vector \mathbf{c} , lies on the line through O that bisects the angle AOB .

- (a) Prove that the vector $b\mathbf{a} - a\mathbf{b}$ is perpendicular to \mathbf{c} . (4)

The point D , with position vector \mathbf{d} , lies on the line AB between A and B .

- (b) Explain why \mathbf{d} can be expressed in the form $\mathbf{d} = (1 - \lambda)\mathbf{a} + \lambda\mathbf{b}$ for some scalar λ with $0 < \lambda < 1$ (2)
- (c) Given that D is also on the line OC , find an expression for λ in terms of a and b only and hence show that

$$DA : DB = OA : OB \quad (8)$$

(+S2)

(Total for Question 1 is 16 marks)

Q2

The lines L_1 and L_2 have the equations

$$L_1 : \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 9 \end{pmatrix} + s \begin{pmatrix} 2 \\ p \\ 6 \end{pmatrix} \quad \text{and} \quad L_2 : \mathbf{r} = \begin{pmatrix} -15 \\ 12 \\ -9 \end{pmatrix} + t \begin{pmatrix} 4 \\ -5 \\ 2 \end{pmatrix}$$

where p is a constant.

The acute angle between L_1 and L_2 is θ where $\cos \theta = \frac{\sqrt{5}}{3}$

(a) Find the value of p .

(5)

The line L_3 has equation $\mathbf{r} = \begin{pmatrix} -15 \\ 12 \\ -9 \end{pmatrix} + u \begin{pmatrix} 8 \\ -6 \\ -5 \end{pmatrix}$ and the lines L_3 and L_2 intersect at the point A .

The point B on L_2 has position vector $\begin{pmatrix} 5 \\ -13 \\ 1 \end{pmatrix}$ and point C lies on L_3 such that $ABDC$ is

a rhombus.

(b) Find the two possible position vectors of D .

(5)

(Total for Question 2 is 10 marks)

End of Questions