

Topic 1 - Regression, Correlation and Hypothesis Testing

Bronze, Silver and Gold
Worksheets for
A Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis.

They are drawn from the latest specification questions and legacy questions. The papers are between approximately 25 and 45 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel A Level Mathematics: Statistics and Mechanics Year 2' textbook.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- [Bronze Questions](#)
- [Bronze Mark Scheme](#)
- [Silver Questions](#)
- [Silver Mark Scheme](#)
- [Gold Questions](#)
- [Gold Mark Scheme](#)

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found [here](#) on the Pearson Edexcel Website, or [here](#) on the Maths Emporium



Bronze Questions

Calculator

The total mark for this section is 27

Q1

A scientist is researching whether or not birds of prey exposed to pollutants lay eggs with thinner shells. He collects a random sample of egg shells from each of 6 different nests and tests for pollutant level, p , and measures the thinning of the shell, t . The results are shown in the table below.

p	3	8	30	25	15	12
t	1	3	9	10	5	6

[You may use $\sum p^2 = 1967$ and $\sum pt = 694$]

- (a) On graph paper, draw a scatter diagram to represent these data. (2)
- (b) Explain why a linear regression model may be appropriate to describe the relationship between p and t . (1)
- (c) The scientist reviews similar studies and finds that pollutant levels above 16 are likely to result in an increased risk of infection soon after hatching.

Given that $t = a + bp$.

Estimate the minimum thinning of the shell that is likely to result in an increased risk of infection. (2)

(Total for Question 1 is 5 marks)

Q2

A random sample of 15 days is taken from the large data set for Perth in June and July 1987. The scatter diagram in Figure 1 displays the values of two of the variables for these 15 days.

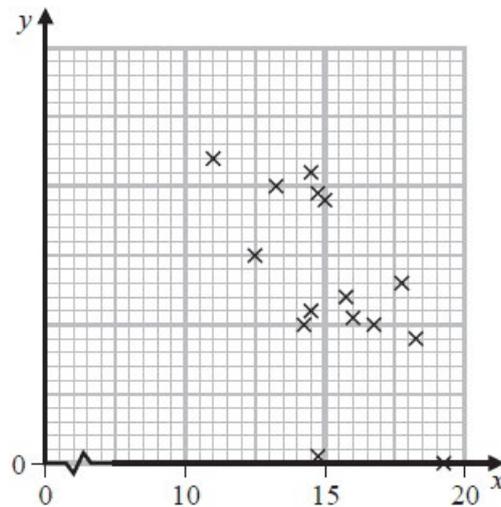


Figure 1

(a) Describe the correlation.

(1)

The variable on the x -axis is Daily Mean Temperature measured in $^{\circ}\text{C}$.

(b) Using your knowledge of the large data set,

(i) suggest which variable is on the y -axis,

(ii) state the units that are used in the large data set for this variable.

(2)

Stav believes that there is a correlation between Daily Total Sunshine and Daily Maximum Relative Humidity at Heathrow.

He calculates the product moment correlation coefficient between these two variables for a random sample of 30 days and obtains $r = -0.377$.

(c) Carry out a suitable test to investigate Stav's belief at a 5% level of significance.

State clearly

- your hypotheses
- your critical value

(3)

On a random day at Heathrow the Daily Maximum Relative Humidity was 97%

(d) Comment on the number of hours of sunshine you would expect on that day, giving a reason for your answer.

(1)

(Total for Question 2 is 7 marks)

Q3

A meteorologist believes that there is a relationship between the daily mean windspeed, w kn, and the daily mean temperature, t °C. A random sample of 9 consecutive days is taken from past records from a town in the UK in July and the relevant data is given in the table below.

t	13.3	16.2	15.7	16.6	16.3	16.4	19.3	17.1	13.2
w	7	11	8	11	13	8	15	10	11

The meteorologist calculated the product moment correlation coefficient for the 9 days and obtained $r = 0.609$

- (a) Explain why a linear regression model based on these data is unreliable on a day when the mean temperature is 24 °C (1)
- (b) State what is measured by the product moment correlation coefficient. (1)
- (c) Stating your hypotheses clearly test, at the 5% significance level, whether or not the product moment correlation coefficient for the population is greater than zero. (3)

Using the same 9 days a location from the large data set gave $\bar{t} = 27.2$ and $\bar{w} = 3.5$.

- (d) Using your knowledge of the large data set, suggest, giving your reason, the location that gave rise to these statistics. (1)

(Total for Question 3 is 6 marks)

Q4

Barbara is investigating the relationship between average income (GDP per capita), x US dollars, and average annual carbon dioxide (CO_2) emissions, y tonnes, for different countries.

She takes a random sample of 24 countries and finds the product moment correlation coefficient between average annual CO_2 emissions and average income to be 0.446.

- (a) Stating your hypotheses clearly, test, at the 5% level of significance, whether or not the product moment correlation coefficient for all countries is greater than zero.

(3)

Barbara believes that a non-linear model would be a better fit to the data.

She codes the data using the coding $m = \log_{10} x$ and $c = \log_{10} y$ and obtains the model $c = -1.82 + 0.89m$.

The product moment correlation coefficient between c and m is found to be 0.882.

- (b) Explain how this value supports Barbara's belief.

(1)

- (c) Show that the relationship between y and x can be written in the form $y = ax^n$ where a and n are constants to be found.

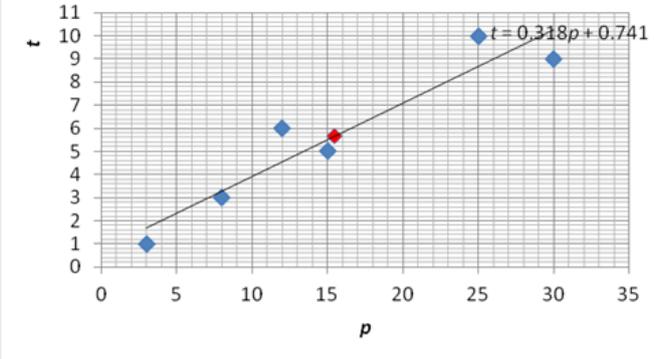
(5)

(Total for Question 4 is 9 marks)

End of Questions

Bronze Mark Scheme

Q1

Question	Scheme	Marks
(a)		Use overlay B1 B1 (2)
(b)	Points (appear to) lie close to a (straight) line <u>or</u> “strong /high correlation”	B1 (1)
(c)	$t = "0.741" + "0.318" \times 16$ $= 5.825\dots$	M1 A1 (2) awrt 5.8 5

	Scheme	Marks	AO
(a)	Negative	B1 (1)	1.2
(b)(i)	Rainfall	B1	2.2b
(ii)	mm <u>or</u> Pressure hPa or Pascals or hectopascals or mb or millibars	B1ft (2)	1.1b
(c)	$H_0 : \rho = 0$ $H_1 : \rho \neq 0$ Critical value: $-0.361(0)$ $r < -0.3610$ so significant result and there is evidence of a correlation between Daily Total <u>Sunshine</u> and Daily Maximum Relative <u>Humidity</u>	B1 M1 A1 (3)	2.5 1.1b 2.2b
(d)	Humidity is high and there is evidence of correlation and $r < 0$ So expect amount of sunshine to be <u>lower</u> than the <u>average</u> for Heathrow(oe)	B1 (1)	2.2b
		(7 marks)	
Notes			
(a)	B1 for stating negative. "Negative skew" is B0 though		
(b)(i)	B1 for mentioning "rainfall" (allow "rain" <u>or</u> "precipitation") <u>or</u> "pressure" (if more than 1 answer both must be correct) NB the other quantitative variable for Perth is: Daily Mean Wind Speed and scores B0 [Not allowed "wind speed" since $r = +0.15$ and in winter might expect wind to raise temp]		
(ii)	B1ft for giving the correct units. If Daily Mean Wind Speed (kn) or knots "Wind speed" and "knots" would score B0B1 but any other variable scores B0B0		
(c)	B1 for both hypotheses correct in terms of ρ M1 for the correct critical value compatible with their H_1 : allow $\pm 0.361(0)$ If the hypotheses are 1-tail then allow cv of ± 0.3061 e.g. Alternative hypothesis with $r < \pm 0.377$ implies a one-tail test <u>or</u> H_0 and H_1 in words saying " H_0 : there is no correlation, H_1 : there is correlation" is two-tail If there are no hypotheses (or they are nonsensical) assume 2-tail so M1 for $\pm 0.361(0)$		
	A1 for a correct conclusion in context based on comparing -0.377 with their cv. Condone incorrect inequality e.g. $-0.3610 < -0.377$ as long as they reject H_0 Do not accept contradictory statements such as "accept H_0 so there is evidence of ..." Can say "support for Stav's <u>belief</u> "(o.e.e.g. "claim") or "evidence of a correlation between <u>sunshine</u> and <u>humidity</u> " condone "negative correlation" or comments such as "if humidity is high amount of sunshine will be low"		
(d)	B1 for stating <u>low</u> amount of sunshine (o. e.) and some reference to $r < 0$ or fog Check for the following 2 features: (i) low sunshine: allow ≤ 5 hrs (LDS mean for 2015 is 5.3, humidity 97% is 4.1, $\geq 97\%$ is 3.1) (ii) negative correlation may be described in words e.g. "high humidity gives low sunshine" <u>or</u> fog (LDS says $>95\%$ humidity is foggy) so less sunshine		

Q3

Question	Scheme	Marks	AOs
(a)	e.g. It requires extrapolation so will be unreliable (o.e.)	B1	1.2
		(1)	
(b)	e.g. Linear association between w and t	B1	1.2
		(1)	
(c)	$H_0: \rho = 0$ $H_1: \rho > 0$	B1	2.5
	Critical value 0.5822	M1	1.1a
	Reject H_0		
	There is evidence that the product moment correlation coefficient is greater than 0	A1	2.2b
		(3)	
(d)	Higher \bar{t} suggests overseas and not Perth...lower wind speed so perhaps not close to the sea so suggest Beijing	B1	2.4
		(1)	
(6 marks)			
Notes:			
(a)			
B1: for a correct statement (unreliable) with a suitable reason			
(b)			
B1: for a correct statement			
(c)			
B1: for both hypotheses in terms of ρ			
M1: for selecting a suitable 5% critical value compatible with their H_1			
A1: for a correct conclusion stated			
(d)			
B1: for suggesting Beijing with some supporting reason based on t or w Allow Jacksonville with a reason based just on higher \bar{t}			

Q4

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$H_0 : \rho = 0$ $H_1 : \rho > 0$	B1	This mark is given for both hypotheses in terms of ρ found correctly
	For sample size 24 at the 5% level of significance, the critical value = 0.3438	M1	This mark is given for selecting a suitable critical value compatible with H_1
	$0.446 > 0.3438$, so reject H_0 There is evidence that the product moment correlation coefficient (pmcc) is greater than 0	A1	This mark is given for a correct conclusion stated
(b)	The value of the pmcc is close to 1 so there is a strong positive correlation	B1	This mark is given for a correct explanation about the strength of the correlation
(c)	$\log_{10} y = -1.82 + 0.89 \log_{10} x$	M1	This mark is given for a correct substitution of both c and m
	$y = 10^{-1.82 + 0.89 \log x}$	M1	This mark is given for dealing with logs to find an expression in terms of y
	$y = 10^{-1.82} \times 10^{0.89 \log x}$ $y = 10^{-1.82} \times 10^{(\log x)^{0.89}}$	M1	This mark is given for a method to find values for a and n
	$y = 0.015 \times x^{0.89}$	A1	This mark is given for find a correct value of $a = 0.015$
		A1	This mark is given for find a correct value of $n = 0.89$
			(Total 9 marks)



Silver Questions

Calculator

The total mark for this section is 27

Q1

In a controlled experiment, the number of microbes, N , present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form

$$N = aT^b, \quad \text{where } a \text{ and } b \text{ are constants.}$$

(a) Show that this relationship can be expressed in the form

$$\log_{10}N = m\log_{10}T + c$$

giving m and c in terms of the constants a and/or b .

(2)

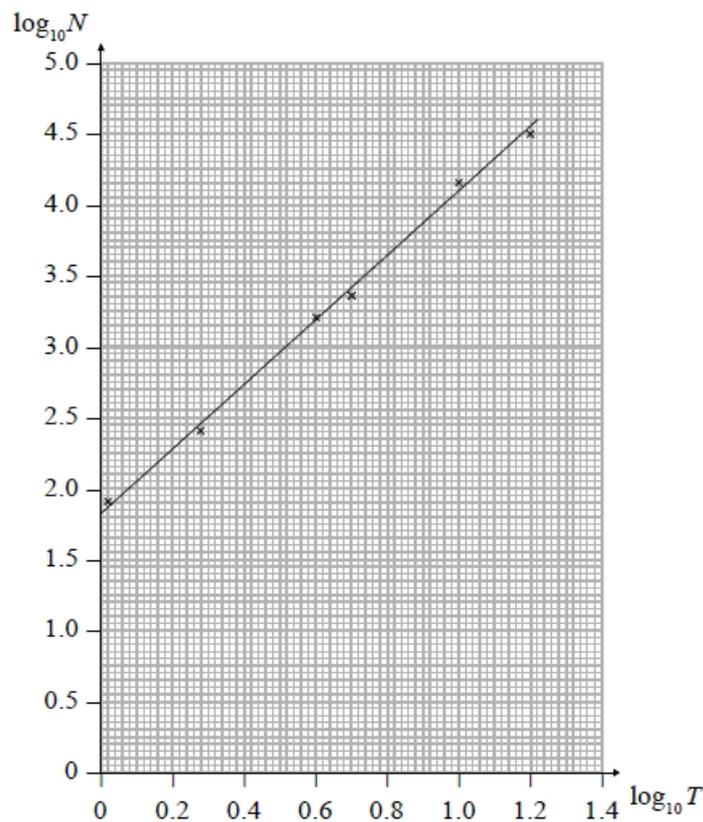


Figure 3

Figure 3 shows the line of best fit for values of $\log_{10}N$ plotted against values of $\log_{10}T$.

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(4)

(c) Explain why the information provided could not reliably be used to estimate the day when the number of microbes in the culture first exceeds 1 000 000.

(2)

(d) With reference to the model, interpret the value of the constant a .

(1)

(Total for Question 1 is 9 marks)

Q2

The value, £ V , of a vintage car t years after it was first valued on 1st January 2001, is modelled by the equation

$$V = Ap^t \quad \text{where } A \text{ and } p \text{ are constants.}$$

Given that the value of the car was £32 000 on 1st January 2005 and £50 000 on 1st January 2012

(a) (i) find p to 4 decimal places,

(ii) show that A is approximately 24 800.

(4)

(b) With reference to the model, interpret

(i) the value of the constant A ,

(ii) the value of the constant p .

(2)

Using the model,

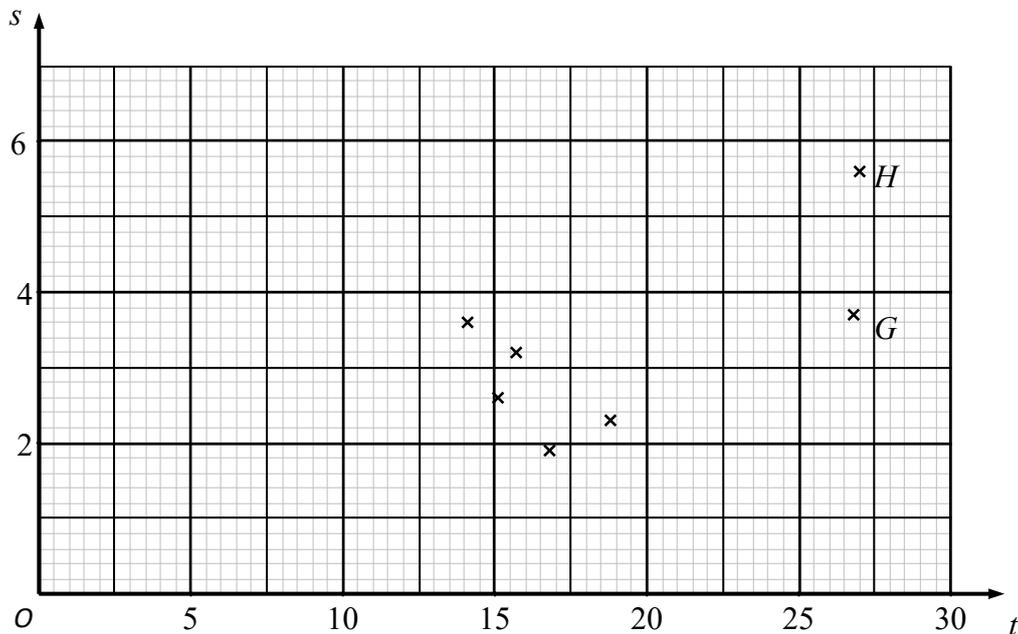
(c) find the year during which the value of the car first exceeds £100 000.

(4)

(Total for Question 2 is 10 marks)

Q3

A researcher believes that there is a linear relationship between daily mean temperature and daily total rainfall. The 7 places in the northern hemisphere from the large data set are used. The mean of the daily mean temperatures, t °C, and the mean of the daily total rainfall, s mm, for the month of July in 2015 are shown on the scatter diagram below.



- (a) With reference to the scatter diagram, explain why a linear regression model may not be suitable for the relationship between t and s . (1)

The researcher calculated the product moment correlation coefficient for the 7 places and obtained $r = 0.658$.

- (b) Stating your hypotheses clearly, test at the 10% level of significance, whether or not the product moment correlation coefficient for the population is greater than zero. (3)
- (c) Using your knowledge of the large data set, suggest the names of the 2 places labelled G and H . (1)
- (d) Using your knowledge from the large data set, and with reference to the locations of the two places labelled G and H , give a reason why these places have the highest temperatures in July. (2)
- (e) Suggest how you could make better use of the large data set to investigate the relationship between daily mean temperature and daily total rainfall. (1)

(Total for Question 3 is 7 marks)

End of questions

Silver Mark Scheme

Q1

Question	Scheme	Marks	AOs
(a)	$N = aT^b \Rightarrow \log_{10} N = \log_{10} a + b \log_{10} T$	M1	2.1
	$\Rightarrow \log_{10} N = \log_{10} a + b \log_{10} T$ so $m = b$ and $c = \log_{10} a$	A1	1.1b
		(2)	
(b)	Uses the graph to find either a or b $a = 10^{\text{intercept}}$ or $b = \text{gradient}$	M1	3.1b
	Uses the graph to find both a and b $a = 10^{\text{intercept}}$ and $b = \text{gradient}$	M1	1.1b
	Uses $T = 3$ in $N = aT^b$ with their a and b	M1	3.1b
	Number of microbes ≈ 800	A1	1.1b
		(4)	
(c)	$N = 1000000 \Rightarrow \log_{10} N = 6$	M1	3.4
	We cannot 'extrapolate' the graph and assume that the model still holds	A1	3.5b
		(2)	
(d)	States that ' a ' is the number of microbes 1 day after the start of the experiment	B1	3.2a
		(1)	
(9 marks)			

Notes:**(a)****M1:** Takes logs of both sides and shows the addition law**M1:** Uses the power law, writes $\log_{10} N = \log_{10} a + b \log_{10} T$ and states $m = b$ and $c = \log_{10} a$ **(b)****M1:** Uses the graph to find either a or b $a = 10^{\text{intercept}}$ or $b = \text{gradient}$. This would be implied by the sight of $b = 2.3$ or $a = 10^{1.8} \approx 63$ **M1:** Uses the graph to find both a and b $a = 10^{\text{intercept}}$ and $b = \text{gradient}$. This would be implied by the sight of $b = 2.3$ and $a = 10^{1.8} \approx 63$ **M1:** Uses $T = 3 \Rightarrow N = aT^b$ with their a and b . This is implied by an attempt at $63 \times 3^{2.3}$ **A1:** Accept a number of microbes that are approximately 800. Allow 800 ± 150 following correct work.

There is an alternative to this using a graphical approach.

M1: Finds the value of $\log_{10} T$ from $T=3$. Accept as $T = 3 \Rightarrow \log_{10} T \approx 0.48$ **M1:** Then using the line of best fit finds the value of $\log_{10} N$ from their "0.48"Accept $\log_{10} N \approx 2.9$ **M1:** Finds the value of N from their value of $\log_{10} N$ $\log_{10} N \approx 2.9 \Rightarrow N = 10^{2.9}$ **A1:** Accept a number of microbes that are approximately 800. Allow 800 ± 150 following correct work**(c)****M1** For using $N = 1000000$ and stating that $\log_{10} N = 6$ **A1:** Statement to the effect that "we only have information for values of $\log N$ between 1.8 and 4.5 so we cannot be certain that the relationship still holds". "We cannot extrapolate with any certainty, we could only interpolate"

There is an alternative approach that uses the formula.

M1: Use $N = 1000000$ in their $N = 63 \times T^{2.3} \Rightarrow \log_{10} T = \frac{\log_{10} \left(\frac{1000000}{63} \right)}{2.3} \approx 1.83$.**A1:** The reason would be similar to the main scheme as we only have $\log_{10} T$ values from 0 to 1.2. We cannot 'extrapolate' the graph and assume that the model still holds**(d)****B1:** Allow a numerical explanation $T = 1 \Rightarrow N = a1^b \Rightarrow N = a$ giving a is the value of N at $T=1$

Q2

Question	Scheme	Marks	AOs
(a)	(i) Method to find p Eg. Divides $32000 = Ap^4$ by $50000 = Ap^{11}$ $p^7 = \frac{50000}{32000} \Rightarrow p = \sqrt[7]{\frac{50000}{32000}} = \dots$	M1	3.1a
	$p = 1.0658$	A1	1.1b
	(ii) Substitutes their $p = 1.0658$ into either equation and finds A $A = \frac{32000}{1.0658^4} \text{ or } A = \frac{50000}{1.0658^{11}}$	M1	1.1b
	$A = 24795 \rightarrow 24805 \approx 24800^*$	A1*	1.1b
		(4)	
(b)	$A / (\pounds)24800$ is the value of the car on 1st January 2001	B1	3.4
	$p/1.0658$ is the factor by which the value rises each year. Accept that the value rises by 6.6% a year (ft on their p)	B1	3.4
		(2)	
(c)	Attempts $100000 = '24800 \times '1.0658^t$		
	$'1.0658^t = \frac{100000}{24800}$	M1	3.4
	$t = \log_{1.0658} \left(\frac{100000}{24800} \right)$	dM1	1.1b
	$t = 21.8 \text{ or } 21.9$	A1	1.1b
	cso 2022	A1	3.2a
	(4)		
(10 marks)			

(a) (i)	<p>M1: Attempts to use both pieces of information within $V = Ap^t$, eliminates A correctly and solves an equation of the form $p^n = k$ to reach a value for p.</p> <p>Allow for slips on the 32 000 and 50 000 and the values of t.</p> <p>A1: $p = \text{awrt } 1.0658$</p> <p>Both marks can be awarded from incorrect but consistent interpretations of t. Eg. $32000 = Ap^5$, $50000 = Ap^{12}$</p>
(a) (ii)	<p>M1: Substitutes their $p = 1.0658$ into either of their equations and finds A</p> <p>Eg $A = \frac{32000}{1.0658^4}$ or $A = \frac{50000}{1.0658^7}$ but you may follow through on incorrect equations from part (i)</p> <p>A1*: Shows that A is between 24 795 and 24 805 before you see '≈ 24800' or '≈ 24800'. Accept with or without units.</p> <p>An alternative to (ii) is to start with the given answer.</p> <p>M1: Attempts $24800 \times '1.0658^t = (32000.34)$</p>

A1: 24800×1.0658^4 , achieves a value between 31095 and 32005 followed by $\approx 32\ 000$ hence A must be $\approx 24\ 800$

(b)

B1: States that A is the value of the car on 1st January 2001.

The statement must reference **the car**, its **cost/value**, and **"0" time**

Allow 'it is the initial value of the car' "it is the cost of the car at $t = 0$ " "it is the cars starting value"

B1: States that p is the rate at which the value of the car rises each year.

The statement must reference a **yearly rate** and an **increase in value or multiplier**.

They could reference the 1.0658 Eg "The cars value rises by 6.5 % each year."

Allow " p is the rate the cars value is rising each year" "it is the proportional increase in value of the car each year" "the factor by which the value of the car is rising each year" 'its value appreciates by 6.5% per year' Allow 'the value of the car multiplies by p each year'

Do not allow "by how much the value of the car rises each year" or "it is the rate of inflation"

(c)

M1: Uses the model $100000 = '24800' \times '1.0658'^t$ and proceeds to their ' $1.0658^t = k$ '

Allow use of any inequality here.

dM1: For the complete method of (i) using the information given with their equation of the model and (ii) translating the situation into a correct method to find ' t '

A1: (t) = awrt 21.8 or 21.9 or $\log_{1.0658} \left(\frac{100000}{24800} \right)$ oe

A1: States in the year 2022. A candidate using a GP formula can be awarded full marks

Allow different methods in part (c).

Eg Via GP a formula

M1: $24800 \times 1.0658^{n-1} = 100000 \Rightarrow 1.0658^{n-1} = k$

dM1: Uses a correct method to find n .

A2: 2022

Via (trial and improvement)

M1: Uses the model by substituting integer values of t into their $V = Ap^t$ so that for $t = n, V < 100\ 000$ or

$t = n+1, V > 100\ 000$

(So for the correct A and p this would be scored for $t = 21, V \approx \pounds 95\ 000$ or $t = 21, V \approx \pounds 101\ 000$)

dM1: For a complete method showing that this is the least value. So both of the above values

A1: Allow for 22 following correct and accurate results (awrt nearest $\pounds 1000$ is sufficient accuracy)

A1: As before

Q3

Question	Scheme	Marks	AOs
(a)	Not suitable with a correct reason eg the points do not lie close to a straight line. there appear to be two populations if G and H were removed it appears to be a negative correlation	B1	1.2
		(1)	
(b)	$H_0 : \rho = 0$ $H_1 : \rho > 0$	B1	2.5
	Critical value 0.5509	M1	1.1a
	Reject H_0		
	There is evidence that pmcc is greater than zero	A1	2.2b
		(3)	
(c)	Beijing and Jacksonville	B1	2.2a
		(1)	
(d)	Beijing and Jacksonville are the closest to the equator	B1	2.4
		(1)	
(e)	Use data from one place.	B1	2.4
		(1)	
			(7 marks)



Gold Questions

Calculator

The total mark for this section is 24 marks

Q1

An ornithologist believes that there is a relationship between the tail length, t mm, and the wing length, w mm, of female hook-billed kites. A random sample of size 10 is taken from a database of these kites and the relevant data is given in the table below.

t (mm)	191	197	208	180	188	210	196	191	179	208
w (mm)	284	285	288	273	280	283	288	271	257	289

The ornithologist plans to use a linear regression model based on these data and interpolate or extrapolate as necessary to estimate the wing length of other female hook-billed kites from their tail length.

- (a) (i) Explain what is meant by extrapolation. (1)
- (ii) Explain the dangers of extrapolation. (1)

The ornithologist attempts to calculate the product moment correlation coefficient, r , and obtains a value of 1.3.

- (b) Explain how she should be able to identify that this is incorrect without carrying out any further calculations. (1)
- (c) Use your calculator to find the correct value of the product moment correlation coefficient, r . (1)
- (d) Stating your hypotheses clearly test, at the 1% significance level, whether or not there is evidence that the product moment correlation coefficient for the population is positive. (3)
- (e) Explain what your test in part (d) suggests about female hook-billed kites. (1)

(Total for Question 1 is 8 marks)

Q2

Tessa owns a small clothes shop in a seaside town. She records the weekly sales figures, £ w , and the average weekly temperature, t °C, for 8 weeks during the summer.

The product moment correlation coefficient for these data is -0.915 .

(a) Stating your hypotheses clearly and using a 5% level of significance, test whether or not the correlation between sales figures and average weekly temperature is negative.

(3)

(b) Suggest a possible reason for this correlation.

(1)

Tessa suggests that a linear regression model could be used to model these data.

(c) State, giving a reason, whether or not the correlation coefficient is consistent with Tessa's suggestion.

(1)

(d) State, giving a reason, which variable would be the explanatory variable.

(1)

Tessa calculated the linear regression equation as $w = 10\,755 - 171t$

(e) Give an interpretation of the gradient of this regression equation.

(1)

(Total for Question 2 is 7 marks)

Q3

A research engineer is testing the effectiveness of the braking system of a car when it is driven in wet conditions.

The engineer measures and records the braking distance, d metres, when the brakes are applied from a speed of V km h⁻¹.

Graphs of d against V and $\log_{10} d$ against $\log_{10} V$ were plotted.

The results are shown below together with a data point from each graph.

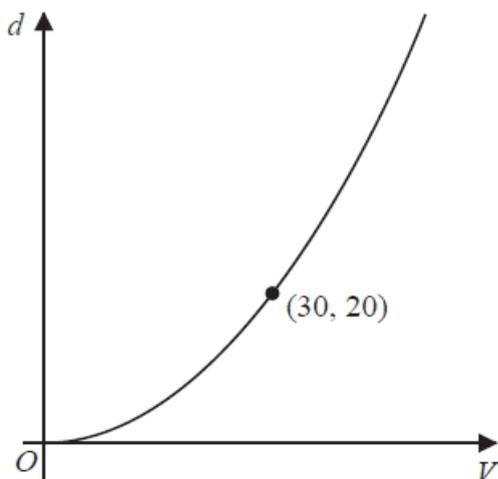


Figure 5

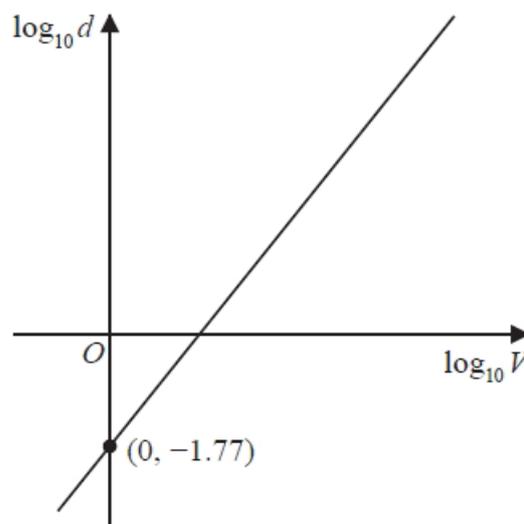


Figure 6

(a) Explain how Figure 6 would lead the engineer to believe that the braking distance should be modelled by the formula

$$d = kV^n \quad \text{where } k \text{ and } n \text{ are constants}$$

with $k \approx 0.017$.

(3)

Using the information given in Figure 5, with $k = 0.017$,

(b) find a complete equation for the model giving the value of n to 3 significant figures.

(3)

Sean is driving this car at 60 km h⁻¹ in wet conditions when he notices a large puddle in the road 100 m ahead. It takes him 0.8 seconds to react before applying the brakes.

(c) Use your formula to find out if Sean will be able to stop before reaching the puddle.

(3)

(Total for Question 3 is 9 marks)

End of questions

Gold Mark Scheme

Q1

Question	Scheme	Marks	AOs
(a)(i)	Extrapolation is making predictions outside the original data range.	B1	1.2
(a)(ii)	This is unreliable as the trend may not continue.	B1	2.4
		(2)	
(b)	The product moment correlation coefficient cannot be greater than 1	B1	1.2
		(1)	
(c)	$r = 0.76279 \dots$ awrt 0.763	B1	1.1b
		(1)	
(d)	$H_0: \rho = 0$ $H_1: \rho > 0$	B1	2.5
	Critical value 0.7155	M1	1.1a
	Reject H_0		
	There is evidence that the product moment correlation coefficient is greater than 0	A1ft	2.2b
		(3)	
(e)	This suggests that on average (female hook-billed) kites with longer tails have longer wings.	B1	3.2a
		(1)	
			(8 marks)

Q2

Qu	Scheme	Marks	AO
(a)	$H_0 : \rho = 0$ $H_1 : \rho < 0$ Critical value: -0.6215 (Allow any cv in range $0.5 < cv < 0.75$) $r < -0.6215$ so significant result and there is evidence of a negative correlation between w and t	B1 M1 A1 (3)	2.5 1.1a 2.2b
(b)	e.g. As temperature increases people spend more time on the beach and less time shopping (o.e.)	B1 (1)	2.4
(c)	Since r is close to -1 , it is consistent with the suggestion	B1 (1)	2.4
(d)	t will be the explanatory variable since sales are likely to depend on the temperature	B1 (1)	2.4
(e)	Every degree rise in temperature leads to a drop in weekly earnings of £171	B1 (1)	3.4
		(7 marks)	
Notes			
(a)	B1 for both hypotheses in terms of ρ M1 for the critical value: sight of ± 0.6215 or any cv such that $0.5 < cv < 0.75$ A1 must reject H_0 on basis of comparing -0.915 with -0.6215 (if $-0.915 < -0.6215$ is seen then A0 but may use $ r $ o.e. which is fine) <u>and mention "negative", "correlation/relationship" and at least "w" and "t"</u>		
(b)	B1 for a suitable <u>reason to explain</u> negative correlation using the context given. e.g. "As temperature drops people are more likely to go shopping (than to the beach)" e.g. "As temperature increases people will be outside rather than in shops" A mere description in context of negative correlation is B0 SO e.g. "As temperature increases people don't want to go shopping/buy clothes" is B0 e.g. "Less clothes needed as temp increases" is B0		
(c)	B1 for a suitable reason e.g. "strong"/"significant"/"near perfect" "correlation", $ r $ close to 1 and saying it is consistent with the suggestion. Allow "yes" followed by the reason.		
(d)	B1 For identifying t <u>and</u> giving a suitable reason. Need idea that " w <u>depends</u> on t " or " w <u>responds</u> to t " or " t <u>affects</u> w " (o.e.) Allow t (temperature) <u>affects</u> the other variable etc Just saying " t is the independent variable" or " t <u>explains</u> change in w " is B0 N. B. Suggesting causation is B0 e.g. " t causes w to decrease"		
(e)	B1 for a description that conveys the idea of rate per degree Celsius. Must have 171, condone missing "£" sign.		

Q3

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	If $d = kV^n$, then $\log_{10} d = \log_{10} k + n \log_{10} V$	M1	This mark is given for
	Plotting $\log_{10} d$ against $\log_{10} V$ will result in a straight line with gradient n and intercept $\log_{10} k$	A1	This mark is given for an explanation of why the second graph shows that $d = kV^n$
	$\log_{10} k = -1.77$ $k = 10^{-1.77} = 0.01698\dots \approx 0.017$	A1	This mark is for showing fully that $k \approx 0.017$
(b)	$d = kV^n$ When $V = 30$, $d = 20$ and $k = 0.17$ then $20 = 0.017 \times 30^n$	M1	This mark is given for substituting in the formula as a method to find the value of n
	$n \log 30 = \log \left(\frac{20}{0.017} \right)$	M1	This mark is given for a correct expression for n
	$n = 2.08$ to 3 significant figures $d = 0.017 \times V^{2.08}$	A1	This mark is given for finding a correct value of n to 3 significant figures and writing a complete equation for the model
(c)	$\frac{60}{3600} \times 0.8 \times 1000 = 13.33$ m	M1	This mark is given for a method to find the distance, in metres, covered in the reaction time of 0.8 seconds
	$d = 0.017 \times 60^{2.08} = 84.92$ m	M1	This mark is given for a method to use the formula to find the stopping distance
	13.33 m + 84.92 m = 98.25 m Sean will be able to stop before reaching the puddle	A1	This mark is given for finding a correct value of the total stopping distance and giving a valid conclusion
			(Total 9 marks)

Topic 2 - Conditional Probability

Bronze, Silver and Gold
Worksheets for
A Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis.

They are drawn from the latest specification questions and legacy questions. The papers are between approximately 25 and 45 marks.

The topic number on this worksheet relates to the corresponding chapter in the 'Pearson Edexcel A Level Mathematics: Statistics and Mechanics Year 2' textbook.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- [Bronze Questions](#)
- [Bronze Mark Scheme](#)
- [Silver Questions](#)
- [Silver Mark Scheme](#)
- [Gold Questions](#)
- [Gold Mark Scheme](#)

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found [here](#) on the Pearson Edexcel Website, or [here](#) on the Maths Emporium



Bronze Questions

Calculator

The total mark for this section is 37

Q1

Three bags, *A*, *B* and *C*, each contain 1 red marble and some green marbles.

Bag *A* contains 1 red marble and 9 green marbles only

Bag *B* contains 1 red marble and 4 green marbles only

Bag *C* contains 1 red marble and 2 green marbles only

Sasha selects at random one marble from bag *A*.

If he selects a red marble, he stops selecting.

If the marble is green, he continues by selecting at random one marble from bag *B*.

If he selects a red marble, he stops selecting.

If the marble is green, he continues by selecting at random one marble from bag *C*.

- (a) Draw a tree diagram to represent this information. (2)
- (b) Find the probability that Sasha selects 3 green marbles. (2)
- (c) Find the probability that Sasha selects at least 1 marble of each colour. (2)
- (d) Given that Sasha selects a red marble, find the probability that he selects it from bag *B*. (2)

(Total for Question 1 is 8 marks)

Q2

A survey of the reading habits of some students revealed that, on a regular basis, 25% read quality newspapers, 45% read tabloid newspapers and 40% do not read newspapers at all.

- (a) Find the proportion of students who read both quality and tabloid newspapers. (3)
- (b) Draw a Venn diagram to represent this information. (3)

A student is selected at random. Given that this student reads newspapers on a regular basis,

- (c) find the probability that this student only reads quality newspapers. (3)

(Total for Question 2 is 9 marks)

Q3

On a randomly chosen day the probability that Bill travels to school by car, by bicycle or on foot is $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$ respectively. The probability of being late when using these methods of travel is $\frac{1}{5}$, $\frac{2}{5}$ and $\frac{1}{10}$ respectively.

- (a) Draw a tree diagram to represent this information. (3)
- (b) Find the probability that on a randomly chosen day
- (i) Bill travels by foot and is late,
 - (ii) Bill is not late. (4)
- (c) Given that Bill is late, find the probability that he did not travel on foot. (4)

(Total for Question 3 is 11 marks)

Q4

A and B are two events such that

$$P(B) = \frac{1}{2} \quad P(A | B) = \frac{2}{5} \quad P(A \cup B) = \frac{13}{20}$$

(a) Find $P(A \cap B)$.

(2)

(b) Draw a Venn diagram to show the events A , B and all the associated probabilities.

(3)

Find

(c) $P(A)$

(1)

(d) $P(B | A)$

(2)

(e) $P(A' \cap B)$

(1)

(Total for Question 4 is 9 marks)

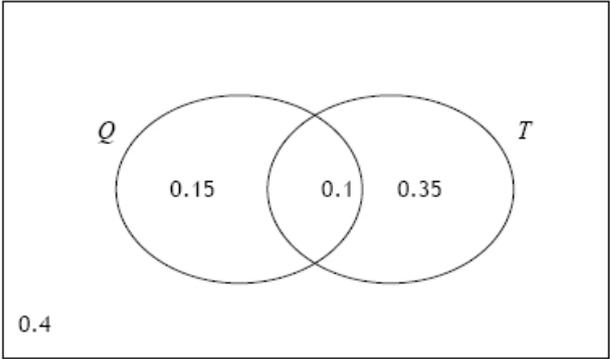
End of questions

Bronze Mark Scheme

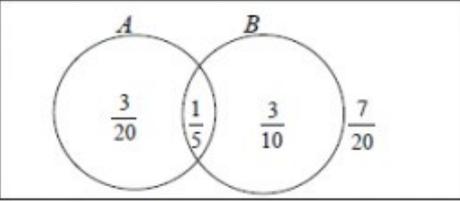
Q1

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)		B1	This mark is given for a correct shape and labels for a tree diagram
		B1	This mark is given for the correct probabilities shown
(b)	$\frac{9}{10} \times \frac{4}{5} \times \frac{2}{3}$	M1	This mark is given for a multiplication of three probabilities
	$= \frac{12}{25}$	A1	This mark is given for the correct probability that Sasha selects three marbles
(c)	$\frac{9}{10} \times \frac{1}{5} + \frac{4}{5} \times \frac{1}{3}$	M1	This mark is given for the addition of two products
	$= \frac{21}{50}$	A1	This mark is given for the correct probability that Sasha selects at least one marble of each colour
(d)	$P(\text{red from } B \mid \text{red selected}) =$ $\frac{\frac{9}{10} \times \frac{1}{5}}{1 - \frac{12}{25}} = \frac{9}{50} \times \frac{25}{13}$	M1	This mark is given for determining the correct ratio of probabilities
	$= \frac{9}{26}$	A1	This mark is given for the correct probability that Sasha selects a red marble from bag B
(Total 8 marks)			

Q2

Question Number	Scheme	Marks
<p>(a)</p> <p>(b)</p> <p>(c)</p>	$P(Q \cup T) = 0.6$ $P(Q) + P(T) - P(Q \cap T) = 0.6$ $P(Q \cap T) = 0.1$  <p>Venn 0.15, 0.35 0.4 and box</p> $P(Q \cap T Q \cup T) = \frac{0.15}{0.60} = \frac{1}{4} \text{ or } 0.25 \text{ or } 25\%$	<p>B1 M1 A1 (3)</p> <p>M1 A1 ✓ B1 (3)</p> <p>M1A1 ✓ A1 (3) Total 9 marks</p>
<p>Notes:</p> <p>(a)</p> <p>(b)</p> <p>(c)</p>	<p>B1 for 0.6 M1 for use of $P(Q) + P(T) - P(Q \cap T) = P(Q \cup T)$ 0.1 Correct answer only for A1 Alternative: (25+45+40=)110% B1 110-100=10% M1A1 0.1 stated clearly as the final answer with no working gets 3/3</p> <p>Two intersecting closed curves for M1, no box required. At least one label (Q or T) required for first A1. Follow through (0.25-'their 0.1') and (0.45-'their 0.1') for A1. 0.4 and box required, correct answer only for B1 Using %, whole numbers in Venn diagram without % sign, whole numbers in correct ratio all OK</p> <p>Require fraction with denominator 0.6 or their equivalent from Venn diagram for M1 Follow through their values in fraction for A1 Final A1 is correct answer only. <u>No working no marks.</u></p>	

Q4

Question Number	Scheme	Marks
<p>(a)</p> $P(A \cap B) = P(A B) \times P(B)$ $P(A \cap B) = \frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$ <p>(b)</p>  <p>(c)</p> $P(A) = \frac{3}{20} + \frac{1}{5} = \frac{7}{20} \text{ or } 0.35$ <p>(d)</p> $P(B A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{5}}{\frac{7}{20}} = \frac{4}{7}$ <p>(e)</p> <p>0.3</p>	<p>2 intersecting circles and 'P(A∩B)'</p> $\frac{3}{20} \text{ and } \frac{3}{10}$ <p>Box and $\frac{7}{20}$</p>	<p>M1 A1 (2)</p> <p>B1ft B1 B1 (3)</p> <p>B1ft (1)</p> <p>M1 A1 cao (2)</p> <p>B1ft (1)</p> <p>[Total 9]</p>
Notes		
	<p>(a) M1 for $\frac{2}{5} \times \frac{1}{2}$ or a correct probability product expression and one correct prob. Ans only 2/2</p> <p>(b) 1st B1 for 2 intersecting circles labelled A and B and ft their prob. for intersection Condone missing labels for 2nd and 3rd B marks</p> <p>(c) B1ft for 0.35 (o.e.) if no Venn diagram or correct follow through from their diagram or allow 0.35 (or correct ft) from correct working e.g. $0.65 - 0.5 + (a)$ B0 for 0.35 if their diagram does not give 0.35 unless it comes from correct work Don't insist on $P(A) = \dots$ but do not award for $P(A' \cap B') = \frac{7}{20}$</p> <p>(d) M1 for $\frac{\text{their (a)}}{\text{their (c)}}$ or a correct ratio of probabilities from their diagram NB incorrect use of $P(A' \cap B') = \frac{7}{20}$ scores M0 and num \geq denom scores M0 A1 for $\frac{4}{7}$ only</p> <p>(e) B1ft for 0.3 or correct ft from their Venn diagram or ft from $\frac{13}{20} - \text{their (c)}$</p>	



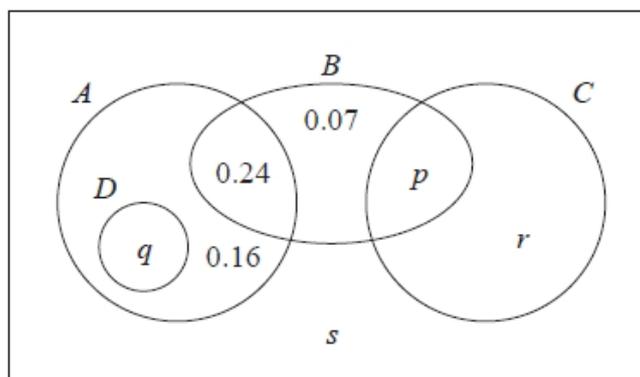
Silver Questions

Calculator

The total mark for this section is 34

Q1

The Venn diagram shows the probabilities associated with four events, A , B , C and D .



(a) Write down any pair of mutually exclusive events from A , B , C and D .

(1)

Given that $P(B) = 0.4$

(b) find the value of p .

(1)

Given also that A and B are independent

(c) find the value of q .

(2)

Given further that $P(B' | C) = 0.64$

(d) find

(i) the value of r ,

(ii) the value of s .

(4)

(Total for Question 1 is 8 marks)

Q2

For the events A and B ,

$$P(A' \cap B) = 0.22 \text{ and } P(A' \cap B') = 0.18 .$$

(a) Find $P(A)$. (1)

(b) Find $P(A \cup B)$. (1)

Given that $P(A | B) = 0.6$

(c) find $P(A \cap B)$. (3)

(d) Determine whether or not A and B are independent. (2)

(Total for Question 2 is 7 marks)

Q3

A disease is known to be present in 2% of a population. A test is developed to help determine whether or not someone has the disease.

Given that a person has the disease, the test is positive with probability 0.95.

Given that a person does not have the disease, the test is positive with probability 0.03.

(a) Draw a tree diagram to represent this information. (3)

A person is selected at random from the population and tested for this disease.

(b) Find the probability that the test is positive. (3)

A doctor randomly selects a person from the population and tests him for the disease. Given that the test is positive,

(c) find the probability that he does not have the disease. (2)

(d) Comment on the usefulness of this test. (1)

(Total for Question 3 is 9 marks)

Q4

The Venn diagram in Figure 1 shows the number of students in a class who read any of 3 popular magazines *A*, *B* and *C*.

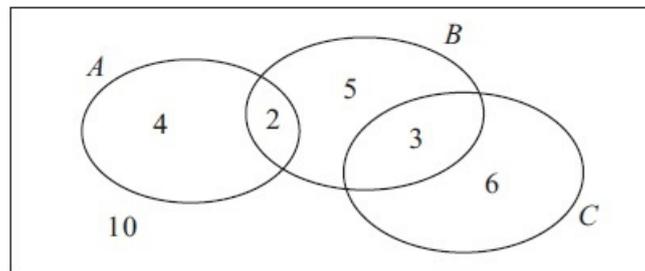


Figure 1

One of these students is selected at random.

- (a) Show that the probability that the student reads more than one magazine is $\frac{1}{6}$. (2)
- (b) Find the probability that the student reads *A* or *B* (or both). (2)
- (c) Write down the probability that the student reads both *A* and *C*. (1)
- Given that the student reads at least one of the magazines,
- (d) find the probability that the student reads *C*. (2)
- (e) Determine whether or not reading magazine B and reading magazine C are statistically independent. (3)

(Total for Question 4 is 10 marks)

End of questions

Silver Mark Scheme

Q1

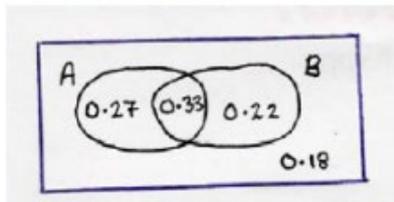
	Scheme	Marks	AO
(a)	A, C <u>or</u> D, B <u>or</u> D, C	B1	1.2
(b)	$[p = 0.4 - 0.07 - 0.24 =]$ <u>0.09</u>	B1	1.1b
(c)	A and B independent implies $P(A) \times 0.4 = 0.24$ <u>or</u> $(q + 0.16 + 0.24) \times 0.4 = 0.24$	M1	1.1b
	so $P(A) = 0.6$ and $q =$ <u>0.20</u>	A1 also	1.1b
		(2)	
(d)(i)	$P(B' C) = 0.64$ gives $\frac{r}{r+p} = 0.64$ <u>or</u> $\frac{r}{r+0.09} = 0.64$ $r = 0.64r + 0.64p$ so $0.36r = 0.0576$ so $r =$ <u>0.16</u>	M1	3.1a
		A1	1.1b
(ii)	Using sum of probabilities = 1 e.g. “0.6” + 0.07 + “0.25” + $s = 1$ so $s =$ <u>0.08</u>	M1	1.1b
		A1	1.1b
		(4)	
		(8 marks)	

	Notes
(a)	B1 for one correct pair. If more than one pair they must all be correct. Condone in a correct probability statement such as $P(A \cap C) = 0$ or correct use of set notation e.g. $A \cap C = \emptyset$ BUT e.g. “ $P(A)$ and $P(C)$ are mutually exclusive” alone is B0
(b)	B1 for $p = 0.09$ (Maybe stated in Venn Diagram [VD]) [If values in VD and text conflict, take text or a value <u>used</u> in a later part]
(c)	M1 for a correct equation in one variable for $P(A)$ or q using independence <u>or</u> for seeing both $P(A \cap B) = P(A) \times P(B)$ <u>and</u> $0.24 = 0.6 \times 0.4$ A1 also for $q = 0.20$ or exact equivalent (dep on correct use of independence)
Beware	Use of $P(A) = 1 - P(B) = 0.6$ leading to $q = 0.2$ scores M0A0
(d)(i)	1 st M1 for use of $P(B' C) = 0.64$ leading to a correct equation in r and possibly p . Can fit their p provided $0 < p < 1$ 1 st A1 for $r = 0.16$ or exact equivalent
(ii)	2 nd M1 for use of total probability = 1 to form a linear equation in s . Allow p, q, r etc Can follow through their values provided each of p, q, r are in $[0, 1)$ 2 nd A1 for $s = 0.08$ or exact equivalent

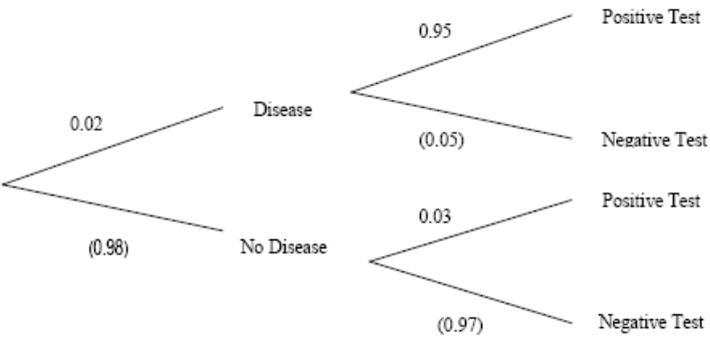
Q2

Question Number	Scheme	Marks			
(a)	$[P(A) = 1 - 0.18 - 0.22] = 0.6$ (or exact equivalent)	B1 (1)			
(b)	$P(A \cup B) = "0.6" + 0.22 = 0.82$ (or exact equivalent)	B1ft (1)			
(c)	<table border="0" style="width: 100%;"> <tr> <td style="width: 30%; vertical-align: top;"> $x = P(A \cap B)$ $\frac{x}{x+0.22} = 0.6$ $x = 0.6x + 0.132$ $0.4x = 0.132$ </td> <td style="width: 30%; vertical-align: top;"> Use $P(B)P(A' B) = P(A' \cap B)$ $P(B) \times [1 - 0.6] = 0.22$ Use $P(A \cap B) = P(A B)P(B)$ $P(A \cap B) = 0.6 \times 0.55$ $x = 0.33$ (or exact equivalent) </td> <td style="width: 30%; vertical-align: top;"> Establish independence before or after 1st M1 and score marks for (d) (RH ver) Find $P(B)$ Use $P(B)P(A) = P(A \cap B)$ $P(A \cap B) = 0.6 \times 0.55$ </td> </tr> </table>	$x = P(A \cap B)$ $\frac{x}{x+0.22} = 0.6$ $x = 0.6x + 0.132$ $0.4x = 0.132$	Use $P(B)P(A' B) = P(A' \cap B)$ $P(B) \times [1 - 0.6] = 0.22$ Use $P(A \cap B) = P(A B)P(B)$ $P(A \cap B) = 0.6 \times 0.55$ $x = 0.33$ (or exact equivalent)	Establish independence before or after 1 st M1 and score marks for (d) (RH ver) Find $P(B)$ Use $P(B)P(A) = P(A \cap B)$ $P(A \cap B) = 0.6 \times 0.55$	M1 dM1 A1cso (3)
$x = P(A \cap B)$ $\frac{x}{x+0.22} = 0.6$ $x = 0.6x + 0.132$ $0.4x = 0.132$	Use $P(B)P(A' B) = P(A' \cap B)$ $P(B) \times [1 - 0.6] = 0.22$ Use $P(A \cap B) = P(A B)P(B)$ $P(A \cap B) = 0.6 \times 0.55$ $x = 0.33$ (or exact equivalent)	Establish independence before or after 1 st M1 and score marks for (d) (RH ver) Find $P(B)$ Use $P(B)P(A) = P(A \cap B)$ $P(A \cap B) = 0.6 \times 0.55$			
(d)	<table border="0" style="width: 100%;"> <tr> <td style="width: 45%; vertical-align: top;"> $P(B) = 0.55$ $P(B) \times P(A) = 0.55 \times 0.6 = 0.33$ $P(B) \times P(A) = P(A \cap B)$ therefore (statistically) independent </td> <td style="width: 55%; vertical-align: top;"> or stating $P(A) = P(A B) [= 0.6]$ or $P(A) = P(A B)$ therefore (statistically) independent </td> </tr> </table>	$P(B) = 0.55$ $P(B) \times P(A) = 0.55 \times 0.6 = 0.33$ $P(B) \times P(A) = P(A \cap B)$ therefore (statistically) independent	or stating $P(A) = P(A B) [= 0.6]$ or $P(A) = P(A B)$ therefore (statistically) independent	M1 A1cso (2)	
$P(B) = 0.55$ $P(B) \times P(A) = 0.55 \times 0.6 = 0.33$ $P(B) \times P(A) = P(A \cap B)$ therefore (statistically) independent	or stating $P(A) = P(A B) [= 0.6]$ or $P(A) = P(A B)$ therefore (statistically) independent				
Total 7					

Notes	
(b)	B1ft for their (a) + 0.22 or $1 - P(A' \cap B')$ Do not fit their (a) if it is > 0.78 NB 3 versions for (c). Check carefully that Ms are genuinely scored. Look out for <u>assuming independence</u> and if you see $P(B) = 0.55$ check it is <u>derived</u> properly
(c)	1 st M1 for a correct equation for x e.g. $\frac{x}{x+0.22} = 0.6$ <u>or</u> a correctly derived equation for $P(B)$ 2 nd dM1 for solving to get in form $kx = L$ <u>or</u> <u>correct</u> use of $P(B)$ to find $P(A \cap B)$ [2 nd or 3 rd ver] <u>or</u> $P(A \cap B) = P(B) - 0.22$ A1cso for 0.33 Dep. on <u>both</u> Ms and no incorrect working seen.
(d)	M1 for finding $P(B) \times P(A) = 0.33$ (values needed) <u>or</u> stating $P(A) = P(A B)$ (= 0.6 not needed) A1cso for a correct statement: $P(B) \times P(A) = P(A \cap B)$ or $P(A) = P(A B)$ <u>and</u> stating independent NB The M1 in (d) using $P(A \cap B)$ requires $P(B) = 0.55$ There is no fit of an incorrect $P(B)$ Full marks in (d) is OK even if 0/3 in (c) {This Venn diagram may be helpful.}



Q3

Question Number	Scheme	Marks
(a)	 <p style="text-align: right;">Tree without probabilities or labels</p> <p style="text-align: right;">0.02(Disease), 0.95(Positive) on correct branches</p> <p style="text-align: right;">0.03(Positive) on correct branch.</p>	<p>M1</p> <p>A1</p> <p>A1 (3)</p>
(b)	$P(\text{Positive Test}) = 0.02 \times 0.95 + 0.98 \times 0.03$ $= 0.0484$	<p>M1A1ft</p> <p>A1 (3)</p>
(c)	$P(\text{Do not have disease} \text{Positive test}) = \frac{0.98 \times 0.03}{0.0484}$ $= 0.607438..$ <p style="text-align: right;">awrt</p>	<p>M1</p> <p>A1 (2)</p>
(d)	<p>Test not very useful or High probability of not having the disease for a person with a positive test</p>	<p>B1 (1)</p> <p>(9 marks)</p>

Q4

Question Number	Scheme	Marks
(a)	$\frac{2+3}{\text{their total}} = \frac{5}{\text{their total}} = \frac{1}{6}$ (** given answer**)	M1 A1cso (2)
(b)	$\frac{4+2+5+3}{\text{total}}, = \frac{14}{30} \text{ or } \frac{7}{15} \text{ or } 0.4\bar{6}$	M1 A1 (2)
(c)	$P(A \cap C) = 0$	B1 (1)
(d)	$P(C \text{reads at least one magazine}) = \frac{6+3}{20} = \frac{9}{20}$	M1 A1 (2)
(e)	$P(B) = \frac{10}{30} = \frac{1}{3}, P(C) = \frac{9}{30} = \frac{3}{10}, P(B \cap C) = \frac{3}{30} = \frac{1}{10} \text{ or } P(B C) = \frac{3}{9}$	M1
	$P(B) \times P(C) = \frac{1}{3} \times \frac{3}{10} = \frac{1}{10} = P(B \cap C) \text{ or } P(B C) = \frac{3}{9} = \frac{1}{3} = P(B)$	M1
	So yes they are statistically independent	A1cso (3)
		Total 10
(a)	M1 for $\frac{2+3}{\text{their total}}$ or $\frac{5}{30}$	
(b)	M1 for adding at least 3 of "4, 2, 5, 3" and dividing by their total to give a probability Can be written as separate fractions substituted into the completely correct Addition Rule	
(c)	B1 for 0 or 0/30	
(d)	M1 for a denominator of 20 or $\frac{20}{30}$ leading to an answer with denominator of 20 $\frac{9}{20}$ only, 2/2	
(e)	1 st M1 for attempting all the required probabilities for a suitable test 2 nd M1 for use of a correct test - must have attempted all the correct probabilities. Equality can be implied in line 2. A1 for fully correct test carried out with a comment	



Gold Questions

Calculator

The total mark for this section is 30

Q1

Given that

$$P(A) = 0.35 \quad P(B) = 0.45 \quad \text{and} \quad P(A \cap B) = 0.13$$

find

- (a) $P(A' | B')$. (2)
- (b) Explain why the events A and B are not independent. (1)

The event C has $P(C) = 0.20$.

The events A and C are mutually exclusive and the events B and C are statistically independent.

- (c) Draw a Venn diagram to illustrate the events A , B and C , giving the probabilities for each region. (5)
- (d) Find $P([B \cup C]')$. (2)

(Total for Question 1 is 10 marks)

Q2

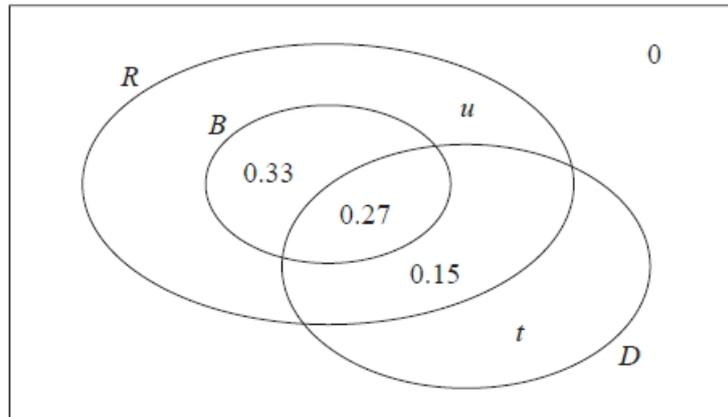
The Venn diagram shows the probabilities of customer bookings at Harry's hotel.

R is the event that a customer books a room

B is the event that a customer books breakfast

D is the event that a customer books dinner

u and t are probabilities.



(a) Write down the probability that a customer books breakfast but does not book a room.

(1)

Given that the events B and D are independent

(b) find the value of t ,

(4)

(c) hence find the value of u .

(2)

(d) Find

(i) $P(D|R \cap B)$,

(ii) $P(D|R \cap B')$.

(4)

A coach load of 77 customers arrive at Harry's hotel.

Of these 77 customers

40 have booked a room and breakfast

37 have booked a room without breakfast

(e) Estimate how many of these 77 customers will book dinner.

(2)

(Total for Question 2 is 13 marks)

Q3

(a) Given that $P(A) = a$ and $P(B) = b$ express $P(A \cup B)$ in terms of a and b when

- (i) A and B are mutually exclusive,
- (ii) A and B are independent.

(2)

Two events R and Q are such that

$$P(R \cap Q') = 0.15, \quad P(Q) = 0.35 \text{ and } P(R|Q) = 0.1$$

Find the value of

(b) $P(R \cup Q)$,

(1)

(c) $P(R \cap Q)$,

(2)

(d) $P(R)$.

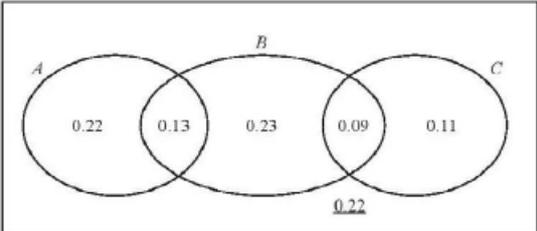
(2)

(Total for Question 3 is 7 marks)

End of questions

Gold Mark Scheme

Q1

Question	Scheme	Marks	AOs
(a)	$P(A' B') = \frac{P(A' \cap B')}{P(B')} \text{ or } \frac{0.33}{0.55}$	M1	3.1a
	$= \frac{3}{5} \text{ or } 0.6$	A1	1.1b
		(2)	
(b)	e.g. $P(A) \times P(B) = \frac{7}{20} \times \frac{9}{20} = \frac{63}{400} \neq P(A \cap B) = 0.13 = \frac{52}{400}$ or $P(A' B') = 0.6 \neq P(A') = 0.65$	B1	2.4
		(1)	
(c)		B1	2.5
		M1	3.1a
		A1	1.1b
		M1	1.1b
		A1	1.1b
	(5)		
(d)	$P(B \cup C)' = 0.22 + 0.22 \text{ or } 1 - [0.56]$ or $1 - [0.13 + 0.23 + 0.09 + 0.11]$ o.e.	M1	1.1b
	$= 0.44$	A1	1.1b
		(2)	
(10 marks)			

Notes:	
(a)	
M1:	for a correct ratio of probabilities formula and at least one correct value.
A1:	a correct answer
(b)	
	for a fully correct explanation: correct probabilities and correct comparisons.
(c)	
B1:	for box with B intersecting A and C but C not intersecting A . (Or accept three intersecting circles, but with zeros entered for $A \cap C$ and $A \cap B \cap C$) No box is B0
M1:	for method for finding $P(B \cap C)$
A1:	for 0.09
M1:	for 0.13 and their 0.09 in correct places and method for their 0.23
A1:	fully correct
(d)	
M1:	for a correct expression – fit their probabilities from their Venn diagram.
A1:	cao

Q2

Question Number	Scheme	Marks
(a)	$[P(B \cap R') =] \underline{0}$	B1 (1)
(b)	$P(B) = 0.27 + 0.33 = 0.6$, $P(D) = 0.27 + 0.15 + t$, $P(B \cap D) = 0.27$ $[P(B) \times P(D) = P(B \cap D) \text{ gives}] \quad 0.6 \times (0.42 + t) = 0.27$ $0.42 + t = \frac{0.27}{0.6} \quad \text{or} \quad 0.6t = 0.018$ $t = \underline{0.03}$	M1 M1 A1 A1 (4)
(c)	$[u =] \quad 1 - (0.6 + 0.15 + t)$ $u = \underline{0.22}$	M1 A1ft (2)
(d)(i)	$\left[\frac{P(D \cap R \cap B)}{P(R \cap B)} \right] = \frac{0.27}{0.27 + 0.33} \quad \text{or} \quad P(D R \cap B) = P(D B) = P(D)$ $= \underline{0.45}$	M1 A1
(ii)	$\left[\frac{P(D \cap [R \cap B'])}{P(R \cap B')} \right] = \frac{0.15}{0.15 + u}$ $= \frac{15}{37}$	M1 A1 (4)
(e)	$40 \times "0.45" \quad \text{and} \quad 37 \times \frac{15}{37}$ $= \underline{33}$	M1 A1 (2)
[13 marks]		

Notes	
(b)	<p>1st M1 for attempting 3 suitable probabilities, one involving t (at least 2 correct) e.g. sight of 0.6, 0.27, $0.42 + t$ correctly labelled in terms of B, D, R <u>or</u> in a correct equation. May see e.g. $P(B D) = \frac{0.27}{0.42 + t}$</p> <p>2nd M1 for using the independence to form a linear equation in t. ft their probs if stated. 1st A1 for solving leading to a correct equation as far as $p + t = q$ <u>or</u> $pt = q$ 2nd A1 for 0.03 or exact equivalent</p>
(c)	<p>M1 for a correct expression for u. Allow their t or just letter t in a correct expression A1ft for 0.22 (or exact equivalent) <u>or</u> ft their t. i.e. $u = 0.25 - t$ provided u & t are probs Can score M1A1ft provided their $u +$ their $t = 0.25$ where u and t are both in $[0, 1]$</p>
(d)(i)	<p>M1 for a correct numerical ratio of probabilities A1 for 0.45 or exact equivalent (Answer only 2/2)</p>
(ii)	<p>M1 for a correct numerical ratio of probabilities, ft their u, provided u is a probability A1 for $\frac{15}{37}$ or 0.405 <u>or</u> allow awrt 0.41 following a correct expression (Ans only 2/2)</p>
(e)	<p>M1 for a correct method for <u>both</u> 18 and 15 ft their 0.45 and their $\frac{15}{37}$ provided both in $[0, 1]$ NB $P(D) \times 77$ is M0 A1 for 33 only NB $\frac{27}{33} \times 40 = 32.7\dots$ which rounds to 33 but scores M0A0. (Ans only send to review)</p>

Q3

Question Number	Scheme	Marks
<p>(a) (i)</p> <p>(ii)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	<p>$P(A \cup B) = a + b$</p> <p>or equivalent</p> <p>$P(R \cup Q) = 0.15 + 0.35 = 0.5$</p> <p>$P(R \cap Q) = P(R Q) \times P(Q) = 0.1 \times 0.35 = 0.035$</p> <p>OR $P(R) = P(R \cap Q') + P(R \cap Q) = 0.15 + \text{their (c)} = 0.15 + 0.035 = 0.185$</p> <p>$0.5 = P(R) + 0.35 - 0.035$ $P(R) = 0.185$</p>	<p>cao B1</p> <p>B1 (2)</p> <p>0.5 B1 (1)</p> <p>M1</p> <p>0.035 A1 (2)</p> <p>M1</p> <p>0.185 A1 (2)</p> <p>[7]</p>
<p>Notes</p>	<p>7(a) (i) Accept $a + b - 0$ for B1</p> <p>Special Case</p> <p>If answers to (i) and (ii) are</p> <p>(i) $P(A)+P(B)$ and (ii) $P(A)+P(B)-P(A)P(B)$</p> <p>award B0B1</p> <p>7(a)(i) and (ii) answers must be clearly labelled or in correct order for marks to be awarded.</p>	

Topic 3 - Normal Distribution

Bronze, Silver and Gold Worksheets for A Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis.

They are drawn from the latest specification questions and legacy questions. The papers are between approximately 25 and 45 marks.

The topic number on this worksheet relates to the corresponding chapter in the 'Pearson Edexcel A Level Mathematics: Statistics and Mechanics Year 2' textbook.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- [Bronze Questions](#)
- [Bronze Mark Scheme](#)
- [Silver Questions](#)
- [Silver Mark Scheme](#)
- [Gold Questions](#)
- [Gold Mark Scheme](#)

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found [here](#) on the Pearson Edexcel Website, or [here](#) on the Maths Emporium



Bronze Questions

Calculator

The total mark for this section is 33

Q1

The random variable X has a normal distribution with mean 20 and standard deviation 4.

(a) Find $P(X > 25)$.

(3)

(b) Find the value of d such that $P(20 < X < d) = 0.4641$.

(4)

(Total for Question 1 is 7 marks)

Q2

The time taken for a randomly selected person to complete a test is M minutes, where $M \sim N(14, \sigma^2)$

Given that 10% of people take less than 12 minutes to complete the test,

(a) find the value of σ .

(3)

Graham selects 15 people at random.

(b) Find the probability that fewer than 2 of these people will take less than 12 minutes to complete the test.

(3)

Jovanna takes a random sample of n people.

Using a normal approximation, the probability that fewer than 9 of these n people will take less than 12 minutes to complete the test is 0.3085 to 4 decimal places.

(c) Find the value of n .

(8)

(Total for Question 2 is 14 marks)

Q3

A machine cuts strips of metal to length L cm, where L is normally distributed with standard deviation 0.5 cm.

Strips with length either less than 49 cm or greater than 50.75 cm **cannot** be used.

Given that 2.5% of the cut lengths exceed 50.98 cm,

(a) find the probability that a randomly chosen strip of metal **can** be used.

(5)

Ten strips of metal are selected at random.

(b) Find the probability fewer than 4 of these strips **cannot** be used.

(2)

A second machine cuts strips of metal of length X cm, where X is normally distributed with standard deviation 0.6 cm

A random sample of 15 strips cut by this second machine was found to have a mean length of 50.4 cm

(c) Stating your hypotheses clearly and using a 1% level of significance, test whether or not the mean length of all the strips, cut by the second machine, is greater than 50.1 cm

(5)

(Total for Question 3 is 12 marks)

End of questions

Bronze Mark Scheme

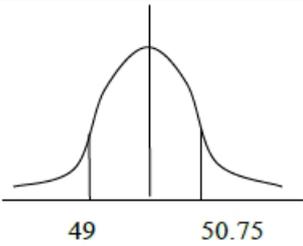
Q1

Question Number	Scheme	Marks
(a)	$P(X > 25) = P\left(Z > \frac{25-20}{4}\right)$ $= P(Z > 1.25)$ $= 1 - 0.8944$ $= 0.1056$	<p>M1</p> <p>M1</p> <p>A1 (3)</p>
(b)	$P(X < 20) = 0.5 \text{ so } P(X < d) = 0.5 + 0.4641 = 0.9641$ $P(Z < z) = 0.9641, z = 1.80$ $\frac{d-20}{4} = 1.80$ $d = 27.2$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1 (4)</p> <p>Total 7 marks</p>
<p>Notes:</p> <p>(a) Standardise with 20 and 4 for M1, allow numerator 20-25 1- probability for second M1 Anything that rounds to 0.106 for A1. Correct answer with no working award 3/3</p> <p>(b) 0.9641 seen or implied by 1.80 for B1 1.80 seen for B1 Standardise with 20 and 4 and equate to z value for M1 Z=0.8315 is M0 Anything that rounds to 27.2 for final A1. Correct answer with no working 4/4</p>		

Q2

Question Number	Scheme		Marks
(a)	$P(M < 10) = P\left(Z < \frac{12-14}{\sigma}\right) = 0.1$		
	$\Rightarrow \frac{12-14}{\sigma} = -1.2816$	M1 standardising (\pm) with 12, 14 and σ and setting equal to a z value where $ z > 1$ B1 ± 1.2816 or better	M1 B1
	$\sigma = 1.5605\dots = \text{awrt } 1.56 \text{ minutes}$	A1 awrt 1.56 Do not allow answer written as an exact fraction.	A1
			(3)
(b)	T represents number less than 12 minutes. $T \sim B(15, 0.1)$	B1 Writing or using $B(15, 0.1)$.	B1
	$P(T \leq 1)$	M1 writing $P(T \leq 1)$ or $P(T < 2)$ any letter may be used.	M1
	$= 0.549$	A1 awrt 0.549	A1
		NB 0.549 gets B1 M1 A1	(3)
(c)	$[T \sim \text{number of people who take less than 12 mins to complete the test}] T \sim B(n, 0.1)$		
	T can be approximated by $N(0.1n, 0.09n)$	B1 mean = $0.1n$ and Var = $0.09n$ oe may be seen in an attempt at standardisation	B1
	$P\left(Z < \frac{8.5-0.1n}{\sqrt{0.09n}}\right) = 0.3085$	M1 using a continuity correction either 8.5 or 7.5 in an attempt at standardised form. Allow 0.09 for sd.	M1
		B1 a z value of awrt ± 0.5	B1
	$\frac{8.5-0.1n}{\sqrt{0.09n}} = -0.5$ or $\frac{8.5-0.1x^2}{0.3x} = -0.5$	M1 standardising using their mean and sd. (If these have not been given then they must be correct here) and one of 7.5, 8, 8.5, 9 or 9.5 and equal to a z value where $ z > 0.4$. Allow any form	M1
		A1 A correct equation in any form . ISW. Do not allow if they have $0.3n$ rather than $0.3\sqrt{n}$	A1
	$0.1n - 0.15\sqrt{n} - 8.5 = 0$ $\sqrt{n} = 10$	M1 using either the quadratic formula or completing the square or factorising or any correct method to solve their 3 term quadratic . If they write the quadratic formula down then allow one slip. If no formula written down then it must be correct for their equation. May be implied by seeing 10 or 8.5. They must show working if the equation used is not correct. 2nd A1 awrt 10.0 – do not need to see n or \sqrt{n} . Allow $n = 10$ May be implied by 100	M1A1
$n = 100$	3rd A1 cso 100 If they have a second answer of 72.25 they must reject it to get this final mark.	A1cso	
			(8)
			(Total 14)

Q3

Question	Scheme	Marks	AOs
(a)			
	$P(L > 50.98) = 0.025$	B1cao	3.4
	$\therefore \frac{50.98 - \mu}{0.5} = 1.96$	M1	1.1b
	$\therefore \mu = 50$	A1cao	1.1b
	$P(49 < L < 50.75)$	M1	3.4
	$= 0.9104\dots$ awrt <u>0.910</u>	A1ft	1.1b
	(5)		
(b)	$S =$ number of strips that cannot be used so $S \sim B(10, 0.090)$	M1	3.3
	$= P(S \leq 3) = 0.991166\dots$ awrt 0.991	A1	1.1b
	(2)		
(c)	$H_0 : \mu = 50.1$ $H_1 : \mu > 50.1$	B1	2.5
	$\bar{X} \sim N\left(50.1, \frac{0.6^2}{15}\right)$ and $\bar{X} > 50.4$	M1	3.3
	$P(\bar{X} > 50.4) = 0.0264$	A1	3.4
	$p = 0.0264 > 0.01$ or $z = 1.936\dots < 2.3263$ and not significant	A1	1.1b
	There is insufficient evidence that the <u>mean length</u> of strips is <u>greater than 50.1</u>	A1	2.2b
	(5)		
(12 marks)			

Notes:
<p>(a)</p> <p>1st M1: for standardizing with μ and 0.5 and setting equal to a z value ($z > 1$)</p> <p>2nd M1: for attempting the correct probability for strips that can be used</p> <p>2nd A1ft: awrt 0.910 (allow ft of their μ)</p>
<p>(b)</p> <p>M1: for identifying a suitable binomial distribution</p> <p>A1: awrt 0.991 (from calculator)</p>
<p>(c)</p> <p>B1: hypotheses stated correctly</p> <p>M1: for selecting a correct model (stated or implied)</p> <p>1st A1: for use of the correct model to find $p = \text{awrt } 0.0264$ (allow $z = \text{awrt } 1.94$)</p> <p>2nd A1: for a correct calculation, comparison and correct statement</p> <p>3rd A1: for a correct conclusion in context mentioning “mean length” and 50.1</p>



Silver Questions

Calculator

The total mark for this section is 37

Q1

A manufacturer fills jars with coffee. The weight of coffee, W grams, in a jar can be modelled by a normal distribution with mean 232 grams and standard deviation 5 grams.

(a) Find $P(W < 224)$.

(3)

(b) Find the value of w such that $P(232 < W < w) = 0.20$

(4)

Two jars of coffee are selected at random.

(c) Find the probability that only one of the jars contains between 232 grams and w grams of coffee.

(3)

(Total for Question 1 is 10 marks)

Q2

A shopkeeper knows, from past records, that 15% of customers buy an item from the display next to the till. After a refurbishment of the shop, he takes a random sample of 30 customers and finds that only 1 customer has bought an item from the display next to the till.

(a) Stating your hypotheses clearly, and using a 5% level of significance, test whether or not there has been a change in the proportion of customers buying an item from the display next to the till.

(6)

During the refurbishment a new sandwich display was installed. Before the refurbishment 20% of customers bought sandwiches. The shopkeeper claims that the proportion of customers buying sandwiches has now increased. He selects a random sample of 120 customers and finds that 31 of them have bought sandwiches.

(b) Using a suitable approximation and stating your hypotheses clearly, test the shopkeeper's claim. Use a 10% level of significance.

(8)

(Total for Question 2 is 14 marks)

Q3

A machine puts liquid into bottles of perfume. The amount of liquid put into each bottle, D ml, follows a normal distribution with mean 25 ml.

Given that 15% of bottles contain less than 24.63 ml

(a) find, to 2 decimal places, the value of k such that $P(24.63 < D < k) = 0.45$.

(5)

A random sample of 200 bottles is taken.

(b) Using a normal approximation, find the probability that fewer than half of these bottles contain between 24.63 ml and k ml.

(3)

The machine is adjusted so that the standard deviation of the liquid put in the bottles is now 0.16 ml.

Following the adjustments, Hannah believes that the mean amount of liquid put in each bottle is less than 25 ml.

She takes a random sample of 20 bottles and finds the mean amount of liquid to be 24.94 ml.

(c) Test Hannah's belief at the 5% level of significance.

You should state your hypotheses clearly.

(5)

(Total for Question 3 is 13 marks)

End of questions

Silver Mark Scheme

Q1

Question Number	Scheme	Marks
(a)	$P(W < 224) = P\left(z < \frac{224 - 232}{5}\right)$ $= P(z < -1.6)$ $= 1 - 0.9452$ $= 0.0548$	M1 M1 A1 awrt 0.0548 (3)
(b)	$0.5 - 0.2 = 0.3$ $\frac{w - 232}{5} = 0.5244$ $w = 234.622$	0.3 or 0.7 seen M1 0.5244 seen awrt 235 B1; M1 A1 (4)
(c)	$0.2 \times (1 - 0.2)$ $2 \times 0.8 \times (1 - 0.8) = 0.32$	M1 M1 A1 (3)
Total 10		
NOTES		
(a)	M1 for standardising with 232 and 5. (i.e. not 5^2 or $\sqrt{5}$). Accept $\pm \frac{w - 232}{5}$. M1 for finding (1 - a probability > 0.5) A1 awrt 0.0548	
(b)	M1 Can be implied by use of ± 0.5244 or $\pm (0.52 \text{ to } 0.53)$ B1 for ± 0.5244 only. Second M1 standardise with 232 and 5 and equate to z value of (0.52 to 0.53) or (0.84 to 0.85) 1 - z used award second M0. Require consistent signs i.e. $\frac{232 - w}{5} = -0.5244$ or negative z value for M1. A1 dependent upon second M mark for awrt 235 but see note below. Common errors involving probabilities and not z values: $P(Z < 0.2) = 0.5793$ used instead of z value gives awrt 235 but award M0B0M0A0 $P(Z < 0.8) = 0.7881$ used instead of z value award M0B0M0A0. M1B0M0A0 for 0.6179, M1B0M0A0 for 0.7580	
(c)	M1 for 0.16 seen M1 for ' $2 \times p(1 - p)$ ' A1 0.32 correct answer only	

Q2

Question Number	Scheme	Marks
(a)	$H_0 : p = 0.15 \quad H_1 : p \neq 0.15$ $X \sim B(30, 0.15)$ $P(X \leq 1) = 0.0480$ or CR: $X = 0$ (0.0480 > 0.025) not a significant result or do not reject H_0 or not in CR there is no evidence of a <u>change</u> in the <u>proportion of customers buying an item from the display</u> .	B1 B1 M1 A1 M1 A1ft (6)
(b)	$H_0 : p = 0.2 \quad H_1 : p > 0.2$ Let S = the number who buy sandwiches, $S \sim B(120, 0.2)$, $S \approx W \sim N\left(24, \sqrt{19.2^2}\right)$ $P(S \geq 31) = P(W \geq 30.5)$ $= P\left(Z > \frac{30.5 - 24}{\sqrt{19.2}}\right)$ or $\frac{x - 0.5 - 24}{\sqrt{19.2}} = 1.2816$ $[= P(Z > 1.48..)]$ $= 1 - 0.9306$ $= 0.0694$ $x = 30.1$ < 0.10 so a significant result, there is evidence that more customers are purchasing sandwiches or the shopkeepers claim is correct.	B1 M1 A1 M1 M1 M1 A1 B1ft (8)

14

Notes:		
(a)	1 st B1 for H_0 must use p 2 nd B1 for H_1 must use p 1 st M1 for writing or using $B(30, 0.15)$ – may be implied by correct CR 1 st A1 0.0480 or $X = 0$. Allow $X \leq 0$. Ignore upper CR. NB Allow CR $X \leq 1$ if using one tail test. 2 nd M1 A correct statement (see table below) Do not allow non-contextual conflicting statements eg “significant” and “accept H_0 ”. Ignore comparisons 2 nd A1 for a correct statement in context. For context we need idea of <u>change/decrease in number of customers buying from display</u> – may use different words. NB A correct contextual statement on its own scores M1A1	
	Two tail $0.025 < p < 0.975$ or One tail $0.05 < p < 0.95$	Two tail $p < 0.025$ or $p > 0.975$ or One tail $p < 0.05$ or $p > 0.95$
2 nd M1	not significant/ accept H_0 / Not in CR or contextual	significant/ reject H_0 / In CR or contextual
2 nd A1	There is no evidence of a <u>change/decrease</u> in the <u>proportion of customers buying an item from the display</u>	There is evidence of a <u>change/decrease</u> in the <u>proportion of customers buying an item from the display</u> .
(b)	1 st B1 both hypotheses correct – must use p . 1 st M1 for a normal approx 1 st A1 for correct mean and sd 2 nd M1 for use of continuity correction, either 30.5 or 31.5 or $(x \pm 0.5)$ seen 3 rd M1 standardising with their mean and their sd and 30.5, 31 or 31.5 or x or $(x \pm 0.5)$ 4 th M1 for 1 - tables value or 1.2816 2 nd A1 for awrt 0.069 or $x = 30.1$ 2 nd B1ft For a correct conclusion in context using their probability and 0.1 For context we need idea of <u>more customers buying sandwiches</u> – may use different words	
	One tail $0.1 < p < 0.9$ or Two tail $0.05 < p < 0.95$	One tail $p < 0.1$ or $p > 0.9$ or Two tail $p <$ 0.05 or $p > 0.95$
2 nd M1	not significant/ accept H_0 / Not in CR or contextual	significant/ reject H_0 / In CR or contextual
2 nd A1	There is no evidence of an increase in the proportion of customers buying sandwiches	There is evidence of a <u>change/increase</u> in the proportion of customers buying sandwiches.

Q3

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{24.63 - 25}{\sigma} = -1.0364$	M1	This mark is given for standardising as part of a method to find σ
	$\sigma = 0.357$	A1	This mark is given for a correct value of σ
	$P(D > K) = 0.4$ or $P(D < K) = 0.6$	B1	This mark is given for
	$\frac{k - 25}{\sigma} = \frac{k - 25}{0.357} = 0.2533$	M1	This mark is given for using a normal model to find the probability
	$k = 25.09$	A1	This mark is given for a correct value for k
(b)	$Y \sim B(200, 0.45)$ so $W \sim N(90, 49.5)$	B1	This mark is given for setting up the normal distribution approximation of the binomial
	$P(Y < 100) \approx P(W < 99.5) = P\left(Z < \frac{99.5 - 90}{\sqrt{49.5}}\right)$	M1	This mark is given for using the normal model with a continuity correction
	$= 0.912$	A1	This mark is given for finding a correct value of the probability
(c)	$H_0 : \mu = 25$ $H_1 : \mu < 25$	B1	This mark is given for both hypotheses in terms of μ found correctly
	$\bar{D} \sim N\left(25, \frac{0.16^2}{20}\right)$	M1	This mark is given for a method to set up the normal distribution
	$P(\bar{D} < 24.94) = 0.0468$	A1	This mark is given for using the model to find a correct p -value
	$p = 0.0468 < 0.05$, so reject H_0	M1	This mark is given for a correct comparison and non-contextual conclusion
	There is sufficient evidence to support Hannah's belief	A1	This mark is given for a correct conclusion in context stated
			(Total 13 marks)



Gold Questions

Calculator

The total mark for this section is 35

Q1

The lifetimes of bulbs used in a lamp are normally distributed.

A company X sells bulbs with a mean lifetime of 850 hours and a standard deviation of 50 hours.

(a) Find the probability of a bulb, from company X , having a lifetime of less than 830 hours. (3)

(b) In a box of 500 bulbs, from company X , find the expected number having a lifetime of less than 830 hours. (2)

A rival company Y sells bulbs with a mean lifetime of 860 hours and 20% of these bulbs have a lifetime of less than 818 hours.

(c) Find the standard deviation of the lifetimes of bulbs from company Y . (4)

Both companies sell the bulbs for the same price.

(d) State which company you would recommend. Give reasons for your answer. (2)

(Total for Question 1 is 11 marks)

Q2

The length of time, L hours, that a phone will work before it needs charging is normally distributed with a mean of 100 hours and a standard deviation of 15 hours.

(a) Find $P(L > 127)$. (3)

(b) Find the value of d such that $P(L < d) = 0.10$. (3)

Alice is about to go on a 6 hour journey.

Given that it is 127 hours since Alice last charged her phone,

(c) find the probability that her phone will not need charging before her journey is completed. (4)

(Total for Question 2 is 10 marks)

Q3.

The lifetime, L hours, of a battery has a normal distribution with mean 18 hours and standard deviation 4 hours.

Alice's calculator requires 4 batteries and will stop working when any one battery reaches the end of its lifetime.

- (a) Find the probability that a randomly selected battery will last for longer than 16 hours. **(1)**

At the start of her exams Alice put 4 new batteries in her calculator. She has used her calculator for 16 hours, but has another 4 hours of exams to sit.

- (b) Find the probability that her calculator will not stop working for Alice's remaining exams. **(5)**

Alice only has 2 new batteries so, after the first 16 hours of her exams, although her calculator is still working, she randomly selects 2 of the batteries from her calculator and replaces these with the 2 new batteries.

- (c) Show that the probability that her calculator will not stop working for the remainder of her exams is 0.199 to 3 significant figures. **(3)**

After her exams, Alice believed that the lifetime of the batteries was more than 18 hours. She took a random sample of 20 of these batteries and found that their mean lifetime was 19.2 hours.

- (d) Stating your hypotheses clearly and using a 5% level of significance, test Alice's belief. **(5)**

(Total for Question 3 is 14 marks)

End of questions

Gold Mark Scheme

Q1

Question Number	Scheme	Marks
(a)	Let the random variable X be the lifetime in hours of bulb	
	$P(X < 830) = P(Z < \frac{\pm(830 - 850)}{50})$	Standardising with 850 and 50
	$= P(Z < -0.4)$	
	$= 1 - P(Z < 0.4)$	Using 1-(probability>0.5)
	$= 1 - 0.6554$	
	$= 0.3446$ or 0.344578 by calculator	awrt 0.345
		(3)
(b)	0.3446×500	Their (a) x 500
	$= 172.3$	Accept 172.3 or 172 or 173
		(2)
(c)	Standardise with 860 and σ and equate to z value $\frac{\pm(818 - 860)}{\sigma} = z$ value	M1
	$\frac{818 - 860}{\sigma} = -0.84(16)$ or $\frac{860 - 818}{\sigma} = 0.84(16)$ or $\frac{902 - 860}{\sigma} = 0.84(16)$ or equiv.	A1
		$\pm 0.8416(2)$
	$\sigma = 49.9$	50 or awrt 49.9
		(4)
(d)	Company Y as the <u>mean</u> is greater for Y .	both
	They have (approximately) the same <u>standard deviation</u> or <u>sd</u>	B1 B1
		(2)
		[11]
Notes	<p>8(a) If 1-z used e.g. 1-0.4=0.6 then award second M0</p> <p>8(c) M1 can be implied by correct line 2</p> <p>A1 for completely correct statement or equivalent.</p> <p>Award B1 if 0.8416(2) seen</p> <p>Do not award final A1 if any errors in solution e.g. negative sign lost.</p> <p>8(d) Must use statistical terms as underlined.</p>	

Q2

Question Number	Scheme	Marks
<p>(a)</p> $\frac{127-100}{15}$ <p>So $P(L > 127) = P(Z > 1.8)$ or $1 - P(Z < 1.8)$ o.e. $= 1 - 0.9641 = \underline{0.0359}$ (awrt <u>0.0359</u>)</p> <p>(b)</p> $\frac{d-100}{15} = -1.2816 \quad (\text{Calculator gives } -1.2815515\dots)$ $d = 80.776 \quad (\text{awrt } \underline{80.8})$ <p>(c) Require $P(L > 133 L > 127)$</p> $= \frac{P(L > 133)}{P(L > 127)} = \frac{P(Z > 2.2)}{P(L > 127)}$ $= \frac{1 - 0.9861}{1 - 0.9641} = \frac{0.0139}{0.0359}$ $= 0.3871\dots = \text{awrt } \underline{0.39}$ <p>S.C. An attempt at $P(L < 133 L > 127)$ that leads to awrt 0.61 (M0M1A0A0)</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>(3)</p> <p>M1, B1</p> <p>A1</p> <p>(3)</p> <p>M1</p> <p>dM1</p> <p>A1</p> <p>A1</p> <p>(4)</p>	<p>10</p>
Notes		
<p>(a)</p> <p>(b)</p> <p>Calc</p> <p>(c)</p> <p>ALT</p>	<p>M1 for attempting to standardise with 127, 100 and 15. Allow \pm</p> <p>1st A1 for $Z > 1.8$. Allow a diagram but must have 1.8 and correct area indicated. Must have the Z so $P(L > 127)$ with or without a diagram is insufficient. May be implied by 0.0359</p> <p>2nd A1 for awrt 0.0359 (calc. gives 0.035930266...). Correct ans only 3/3. M1A0A1 not poss.</p> <p>M1 for an attempt to standardise with 100 and 15 and set = \pm any z value ($z > 1$)</p> <p>B1 for $z = \pm 1.2816$ (or better) seen anywhere [May be implied by 80.776(72...) or better seen]</p> <p>A1 for awrt 80.8 (can be scored for using 1.28 but then they get M1B0A1)</p> <p>The 80.8 must follow from correct working.</p> <p>If answer is awrt 80.8 and awrt 80.777 or 80.776... or better seen then award M1B1A1</p> <p>If answer is awrt 80.8 or 80.77 then award M1B0A1 (unless of course $z = 1.2816$ is seen)</p> <p>1st M1 for clear indication of correct conditional probability or attempt at correct ratio</p> <p>So clear attempt at $\frac{P(L > 133)}{P(L > 127)}$ is sufficient for the 1st M1</p> <p>2nd dM1 dependent on 1st M1 for $P(L > 133)$ leading to $P(Z > 2.2)$.</p> <p>1st A1 for 0.0139 or better seen coming from $P(Z > 2.20)$. Dependent on both Ms</p> <p>2nd A1 for awrt 0.39. Both Ms required</p> <p>If they assume Alice did not check that the phone was working you may see: $[P(L < 127).0] + P(L > 127).P(L > 133 L > 127)$ Provided the <u>conditional probability</u> is seen as part of this calculation the 1st M1 can be scored and their final answer will be 0.0139(4/4)</p> <p>An answer of 0.0139 without sight of the conditional probability is 0/4.</p>	

Q3

Qu	Scheme	Marks	AO
(a)	$P(L > 16) = 0.69146...$ awrt 0.691	B1 (1)	1.1b
(b)	$P(L > 20 L > 16) = \frac{P(L > 20)}{P(L > 16)}$ $= \frac{0.308537...}{(a)} \text{ or } \frac{1-(a)}{(a)}, = 0.44621...$ For calc to work require $(0.44621...) ^4 = 0.03964...$ awrt <u>0.0396</u>	M1 A1ft, A1 dM1 A1 (5)	3.1b 1.1b 1.1b 2.1 1.1b
(c)	Require: $[P(L > 4)]^2 \times [P(L > 20 L > 16)]^2$ $= (0.99976...) ^2 \times ("0.44621...")^2$ $= 0.19901...$ awrt <u>0.199</u> (*)	M1 A1ft A1cso* (3)	1.1a 1.1b 1.1b
(d)	$H_0 : \mu = 18 \quad H_1 : \mu > 18$ $\bar{L} \sim N\left(18, \left(\frac{4}{\sqrt{20}}\right)^2\right)$ $P(\bar{L} > 19.2) = P(Z > 1.3416...) = 0.089856...$ (0.0899 > 5%) or (19.2 < 19.5) or 1.34 < 1.6449 so not significant Insufficient evidence to support Alice's claim (or belief)	B1 M1 A1 A1 A1 (5)	2.5 3.3 3.4 1.1b 3.5a
		(14 marks)	

Notes	
(a)	B1 for evaluating probability using their calculator (awrt 0.691) Accept 0.6915
(b)	1 st M1 for a first step of identifying a suitable conditional probability (either form) 1 st A1ft for a ratio of probabilities with numerator = awrt 0.309 or 1 - (a) and denom = their (a) 2 nd A1 for awrt 0.446 (o.e.) Accept 0.4465 (from $\frac{0.3085}{0.691} = 0.44645...$) NB $\frac{P(16 < L < 20)}{P(L > 16)} = 0.5538...$ scores M1A1A1 when they do $1 - 0.5538 = 0.4462...$ 2 nd M1 (dep on 1 st M1) for 2 nd correct step i.e. (their 0.446...) ⁴ or $X \sim B(4, "0.446")$ and $P(X = 4)$ 3 rd A1 for awrt 0.0396
(c)	1 st M1 for a correct approach to solving the problem (May be implied by A1ft) 1 st A1ft for $P(L > 4) = \text{awrt } 0.9998$ used and ft their 0.44621 in correct expression If use $P(L > 20) = 0.3085..$ as 0.446.. in (b) then M1 for $(0.3085..) ^2 \times [P(L > 4)]^2$; A1ft as above * 2 nd A1cso for 0.199 or better with clear evidence of M1 [NB $(0.4662..) ^2 = 0.199...$ is M0A0A0] Must see M1 scored by correct expression in symbols or values (M1A1ft)
(d)	B1 for both hypotheses in terms of μ M1 for selecting a suitable model. Sight of <u>normal</u> , <u>mean</u> 18, <u>sd</u> $\frac{4}{\sqrt{20}}$ (o.e.) or <u>variance</u> = 0.8 1 st A1 for using the model correctly. Allow awrt 0.0899 or 0.09 from correct prob. statement ALT CR $(\bar{L}) > 19.471...$ (accept awrt 19.5) or CV of 1.6449 (or better: calc 1.6448536..) 2 nd A1 for correct non-contextual conclusion. Wrong comparison or contradictions A0 Error giving 2 nd A0 implies 3 rd A0 but just a correct contextual conclusion can score A1A1 3 rd A1 dep on M1 and 1 st A1 for a correct contextual conclusion mentioning <u>Alice's claim</u> /belief or there is insufficient evidence that the mean <u>lifetime</u> is more than 18 hours