Y12 – Chapter 1 Algebraic Expressions

What do I need to be able to do?

By the end of this chapter you should be able to:

- Multiply and divide integer powers
- Expand a single term of brackets and collect like terms
- Expand the product of two or three expressions
- Factorise linear, quadratic and simple cubic expressions
- Know and use the laws of indices
- Simplify and use the rules of surds
- Rationalise denominators

Key words:

- Integer – A number with no fractional part (no decimals)
- Product – The answer when two or more values are multiplied together
- Surd – A number that can't be simplified to remove a square root (or cube root etc)
- Irrational – A real number that can NOT be made by dividing two integers eg π
- Rational – A number that can be made by dividing two integers
- Base – The number that gets multiplied when using an exponent. (index/power)

Expanding and factorising

Expanding and factorising are the inverse of each other

Expanding brackets →

$$4x(2x + y) = 8x^2 + 4xy$$

$$(x + 5)^3 = x^3 + 15x^2 + 75x + 125$$

$$(x + 2y)(x - 5y) = x^2 - 3xy - 10y^2$$

← Factorising

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Surds

Writing surds in their simplest form

If a square root has a perfect square number as a factor, then it can be simplified.

e.g. $\sqrt{20}$ can be re-written as $\sqrt{4} \times \sqrt{5}$ which simplifies to $2\sqrt{5}$

Perfect square ($\sqrt{4}$)

Adding and subtracting surds

Remember to add or subtract like terms (i.e. the rational numbers and the roots (of the same number))

e.g. $(7+3\sqrt{2})+(8-\sqrt{2})=15+2\sqrt{2}$ Add rational parts: $(7+8=15)$ Add roots: $(3\sqrt{2}-1\sqrt{2}=2\sqrt{2})$

Multiplying surds

If there is no rational part then multiplying is easy: e.g. $\sqrt{3}\times\sqrt{5}=\sqrt{15}$

If there is a rational part then multiply out the brackets

e.g.

Remember that $\sqrt{3}\times\sqrt{3}=3$

$(5+\sqrt{3})\times(2-\sqrt{3})=10-5\sqrt{3}+2\sqrt{3}-\sqrt{3}\sqrt{3}$ tidies up to give $7-3\sqrt{3}$

Rationalising the denominator

You rationalise the denominator to get rid of the surd on the bottom of a fraction.

To rationalise the denominator just multiply the top and bottom of the fraction by the bottom of the fraction with the opposite sign in front of the root.

e.g. $\dfrac{3+\sqrt{5}}{2-\sqrt{5}}$

We are just finding an equivalent fraction by multiplying by 1 (just in disguise!)

$$\frac{3+\sqrt{5}}{2-\sqrt{5}} \times \frac{2+\sqrt{5}}{2+\sqrt{5}} = \frac{6+3\sqrt{5}+2\sqrt{5}+\sqrt{5}\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-\sqrt{5}\sqrt{5}} = \frac{11+5\sqrt{5}}{-1} = -11-5\sqrt{5}$$

Notice these are the same – but the sign in front of the root has changed

Changing the sign in front of the root makes the middle parts cancel each other out

Indices

An index (power) tells you how many times to multiply something by itself:

e.g. x^5 means $x \times x \times x \times x \times x$

There is a base and a power eg:

base → a^m ← power

Rule	Meaning
$a^m \times a^n = a^{m+n}$	To multiply 2 numbers with the same base you add the powers.
$\frac{a^m}{a^n} = a^{m-n}$	To divide 2 numbers with the same base you subtract the powers.
$(a^m)^n = a^{mn}$	To simplify a power inside and outside of a bracket you multiply the powers.
$a^{-m} = \frac{1}{a^m}$	A negative power means find the reciprocal ("one over") so send everything to the bottom of a fraction.
$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$	A fractional power means a root. Denominator tells you the root and the numerator tells you the power.
$a^0 = 1$	Anything to the power of zero = 1
$a^1 = a$	Any number to the power of one stays the same

Y12 – Chapter 2 Quadratics

What do I need to be able to do?

By the end of this chapter you should be able to:

- Solve quadratic equations using factorisation, the quadratic formula and completing the square
- Read and use f(x) notation when working with functions
- Sketch the graph and find the turning point of a quadratic function
- Find and interpret the discriminant of a quadratic expression
- Use and apply models that involve quadratic functions

Solving quadratic equations

Remember that to solve a quadratic equation you should collect all the terms on one side so that the other side of the equation is 0.
When you solve the equation, it you have found the roots (ie. where the graph of the quadratic function crosses the $\boldsymbol{x}$-axis).

Factorising
Put the quadratic into brackets. If the product of two expressions is zero one or both of them must be equal to zero.

E.g. Solve $x^2+6x+8=0$

$$(x+4)(x+2)=0$$

We need two numbers that add to make the coefficient of x and multiply to give the constant term

$x+4=0$ or $x+2=0$

Therefore: $x=-4$ or $x=-2$

The quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

E.g. Solve $3x^2-7x-1=0$

a = 3 b = -7 c = -1

Substitute into the formula:

$$x=\frac{-(-7)\pm\sqrt{(-7)^2-4\times(3)\times(-1)}}{2\times(3)}$$

Put each number in a bracket to avoid any sign errors

Therefore: $x=\frac{7+\sqrt{61}}{6}$ or $x=\frac{7-\sqrt{61}}{6}$

Make sure you give your answer in the form asked for. If they want exact leave in surd for like this. If they say 3sf or 1dp then make sure you give the decimal form of the answer

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Key words:

- Quadratic – Where the highest exponent (index/power) of the variable is a square (2)
- Function – A special relationship where each input has a single output. It is often written as "f(x)" where x is the input value
- Domain – All the values that go into a function
- Range – The set of all output values of a function
- Discriminant – The expression b2 – 4ac used when solving Quadratic Equations. It can "discriminate" between the possible types of answer

The general shape of a quadratic graph:

$y=x^2$  $y=-x^2$ 

Completing the square

Completing the square can be used to solve a quadratic equation but it is also very useful in determining the turning point of a quadratic function

The completed square form looks like this:

$$A(x+B)^2+C=0$$

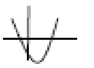
Where the turning point is (-B, C)

Remember! If you need to solve the quadratic to find the roots and it is already in the completed square form, you don't need to factorise or use the formula you can just rearrange to find $\boldsymbol{x}$.

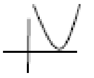
The discriminant

The expression inside the square root sign is called the discriminant and tells you what type of roots to expect.

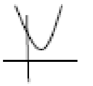
If $b^2-4ac>0$ there are 2 real roots (ie. the curve crosses the $\boldsymbol{x}$-axis in 2 places)



If $b^2-4ac=0$ there is 1 real root (ie. the curve touches the $\boldsymbol{x}$-axis in 1 place)



If $b^2-4ac<0$ there are no real roots (ie. the curve does not cross the $\boldsymbol{x}$-axis)



What do I need to be able to do?

By the end of this chapter you should be able to:

- Solve linear simultaneous equations using elimination or substitution
- Solve simultaneous equations: one linear and one quadratic
- Interpret algebraic solutions of equations graphically
- Solve linear and quadratic inequalities
- Interpret inequalities graphically
- Represent linear and quadratic inequalities graphically

Y12 – Chapter 3 Equations and inequalities

Key words:

- Simultaneous equations – Two or more equations that share variables
- Equation – a mathematical statement containing an equals sign, to show that two expressions are equal. An equation will have a finite set of solutions
- Inequality – An inequality compares two values, showing if one is less than, greater than, or simply not equal to another value

Solving simultaneous equations

Method	Explanation	Works for
Elimination	Make the coefficients of one of the unknowns the same. (whichever seems easier) Add or subtract the equations to eliminate one unknown Solve the new equation to find the first unknown. Substitute back into one of the original equations to find the other unknown.	Linear simultaneous equations
Substitution	Rearrange one of the equations (if necessary) to make either x or y the subject. Substitute into the other equation. Solve the new equation to find x or y. Substitute back into your rearranged equation to find the value of the other letter. *If after substituting you get a quadratic equation you can use the discriminant to determine the number of solutions	Linear only and one linear and one quadratic simultaneous equations
Graphically	On the same set of axes draw the graphs of both simultaneous equations The points of intersection will give you the solutions	Linear only and one linear and one quadratic simultaneous equations

Linear inequalities

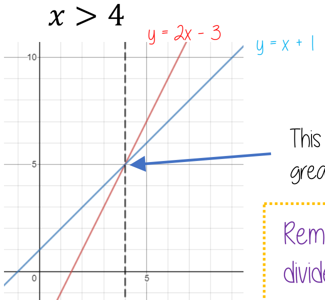
We solve linear inequalities the same way we would solve equations, except you get a range of solutions instead of one particular solution.

Eg Solve the inequality $2x - 3 > x + 1$ and sketch the outcome on a graph.

$2x - 3 > x + 1$

$2x > x + 4$

$x > 4$



This is the point where 2x-3 becomes greater than x +1

Remember! If you multiply or divide an inequality by a negative number you have to reverse the inequality sign

Quadratic inequalities

To solve a quadratic inequality: always do a quick sketch (you will need to know the shape and the roots) then look for the appropriate part of the graph (ie. <0 (below the x-axis) or >0 (above the x-axis) depending on what you are looking for).

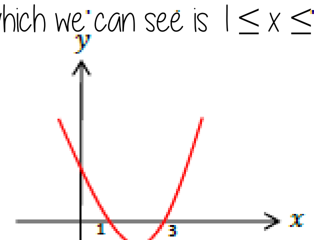
Eg Solve the inequality $x^2 + 4x + 3 \leq 0$

$x^2 + 4x + 3 = 0$

$(x + 3)(x + 1) = 0$

$x = -3$ or $x = -1$ ← These are the roots

We want the graph to be ≤ 0 so we want to describe the x values that represent the part of the curve under the x axis which we can see is $1 \leq x \leq 3$



Y12 – Chapter 4 Graphs and transformations

What do I need to be able to do?

By the end of this chapter you should be able to:

- Sketch cubic, quartic and reciprocal graphs
- Use intersection points to solve equations
- Translate graphs
- Stretch graphs
- Transform graphs of unfamiliar functions

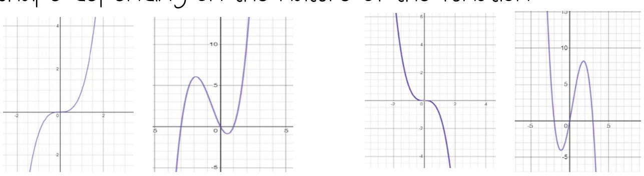
Key words:

- Cubic function– A function where the highest exponent (index/power) of the variable is a cube (3)
- Quartic function – A function where the highest exponent (index/power) of the variable is 4
- Reciprocal function – A function where the highest exponent (index/power) of the variable is negative
- Asymptote – A line that a curve approaches, as it heads towards infinity

Cubic graphs

Have the form: $ax^3 + bx^2 + cx + d$ where a, b, c and d are real numbers and a is non-zero

A cubic graph can have varying forms of the same basic shape depending on the nature of the function



For these two function a is positive   For these two function a is negative

Finding the roots and y intercept of the function helps sketch the function.

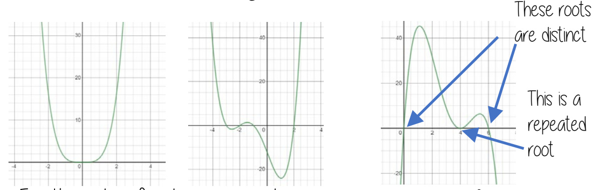
To find the roots substitute y = 0 into the function and solve

To find the y intercept substitute x=0 into the function and solve

Quartic graphs

Have the form: $ax^4 + bx^3 + cx^2 + dx + e$ where a, b, c, d and e are real numbers and a is non-zero

A quartic graph can have varying forms of the same basic shape depending on the nature of the function



For these two function a is positive   For these two function a is negative

Finding the roots and y intercept of the function helps sketch the function.

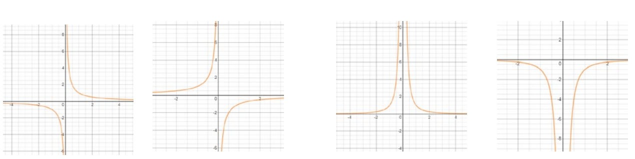
To find the roots substitute y = 0 into the function and solve

To find the y intercept substitute x=0 into the function and solve

Reciprocal graphs

Have the form: $\frac{k}{x}$ or $\frac{k}{x^2}$ where k is a real constant.

Reciprocal graphs will have asymptotes. Reciprocal graphs in the form $\frac{k}{x}$ or $\frac{k}{x^2}$ will have asymptotes as x=0 and y =0



$y = \frac{k}{x}$ with k>0 $y = \frac{k}{x}$ with k<0 $y = \frac{k}{x^2}$ with k>0 $y = \frac{k}{x^2}$ with k<0

Transformations of functions

Function	Transformation	Explanation
f(x)	None - original function	n/a
f(x) + a	Translation	Graph moves along y axis by the vector $\binom{0}{a}$
f(x+a)	Translation	Graph moves along x axis by the vector $\binom{-a}{0}$
af(x)	Stretch	Scale factor a in the vertical direction
f(ax)	Stretch	Scale factor $\frac{1}{a}$ in the horizontal direction
-f(x)	Reflection	Reflection of f(x) in the x-axis
f(-x)	Reflection	Reflection of f(x) in the y-axis

Y12 – Chapter 5 Straight line graphs

What do I need to be able to do?

By the end of this chapter you should be able to:

- Calculate the gradient of a line
- Understand the link between the equation of a line and its gradient and y-intercept
- Find the equation of a line
- Find the points of intersection of straight lines
- Know and use the rules for parallel and perpendicular gradients
- Solve length and area problems
- Use straight line graphs to construct mathematical models

Key words:

- Gradient – How steep a line is
- Y-intercept – The point where a line or curve crosses the y-axis of a graph
- Parallel – Always the same distance apart and never touching
- Perpendicular – At right angles (90°) to
- Linear equation – An equation that makes a straight line when it is graphed

Parallel or perpendicular?

Parallel lines – have the same gradient

Perpendicular lines – the product of the gradients is -1 (the gradients are negative reciprocals of each other)

The equation of a straight line

There are several ways you can write an equation of a straight line:

Form	Why it's useful
$y=mx+c$	The most commonly used form where m is the gradient and c the y-intercept
$y-y_1=m(x-x_1)$	When you have the gradient and a single point on the line, substitute them in for m, y_1 and x_1 - rearrange if necessary
$ax+by+c=0$	Useful when the gradient is a fraction and you want integer values

Finding the distance between two point

Find the distance between (x_1, y_1) and (x_2, y_2) - Pythagoras' theorem

$$\text{Distance} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Sketching a straight line

If you are given two points on the line, plot them and draw a line going through them

If you are given the equation in the form y=mx+c plot the y intercept and then use the gradient to find additional points and join up

If you are given the equation in the form ax+by+c=0, find the x intercept (sub in y=0) and the y intercept (x=0), plot and join

Finding the gradient of a straight line

The gradient (m) of the line that joins the points (x_1, y_1) and (x_2, y_2) use the formula:

$$m=\frac{y_2-y_1}{x_2-x_1}$$

Mathematical modelling

ALWAYS interpret your gradient and y intercept in the context of the question!

Finding the point of intersection

Use simultaneous equations either by elimination or substitution

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Y12 – Chapter 6 Circles

What do I need to be able to do?

By the end of this chapter you should be able to:

- Find the midpoint of a line segment
- Find the equation of the perpendicular bisector to a line segment
- Know how to find the equation of a circle
- Solve geometric problems involving straight lines and circles
- Use circle properties to solve problems
- Solve problems involving circles and triangles

Key words:

- Line segment – a finite part of a straight line with two distinct end points
- Perpendicular bisector– A line which cuts a line segment into two equal parts at 90°
- Tangent – A line that just touches a curve at a point, matching the curve's slope there
- Chord –A line segment connecting two points on a curve
- Circumcircle – a circle touching all the vertices of a triangle or polygon
- Circumcentre – The center of a triangle's circumcircle

Finding midpoint of a line segment

Midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Equation of a circle

The equation of a circle with centre (a, b) and radius r is:

$$(x-a)^2 + (y-b)^2 = r^2$$

You may be given the equation of a circle in the form :

$$x^2 + y^2 - 2ax - 2by + a^2 + b^2 - r^2 = 0$$

In this case you need to complete the square for the x and y terms to find the radius and centre of the circle

Eg

$$x^2 + y^2 - 14x + 16y - 12 = 0$$

$$x^2 - 14x + y^2 + 16y - 12 = 0$$

Half the coefficient of x

Half the coefficient of y

$$(x-7)^2 - 7^2 + (y+8)^2 - 8^2 - 12 = 0$$

Subtract back off

Subtract back off

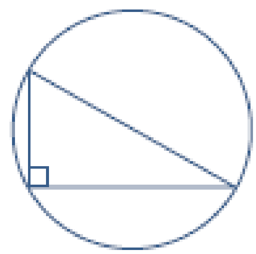
$$(x-7)^2 + (y+8)^2 = 7^2 + 8^2 + 12$$

$$(x-7)^2 + (y+8)^2 = 125$$

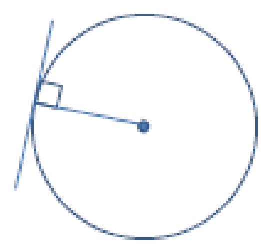
Centre (7, -8); radius $\sqrt{125} = 5\sqrt{5}$

Circle properties

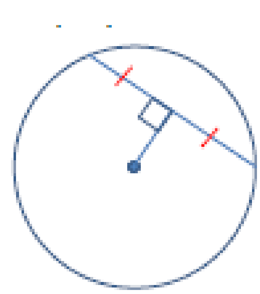
The angle in a semi circle is always a right angle



A tangent to a circle is perpendicular to the radius at the point of intersection



The perpendicular bisector of a chord will go through the centre of the circle



Y12 – Chapter 7 Algebraic Methods

What do I need to be able to do?

By the end of this chapter you should be able to:

- Cancel factors in algebraic fractions
- Divide a polynomial by a linear factor
- Use the factor theorem to factorise a cubic expression
- Construct mathematical proofs using algebra
- Use proof by exhaustion and disproof by counter example

Key words:

- Polynomial – A polynomial can have constants, variables (and exponents that can be combined using addition, subtraction, multiplication and division, but:
  - no division by a variable.
  - a variable's exponents can only be 0 or a positive integer.
  - not an infinite number of terms.
- Proof – Logical mathematical arguments used to show the truth of a mathematical statement.

In a proof we can use:

- axioms (self-evident truths) such as "we can join any two points with a straight-line segment" (one of Euclid's Axioms)
- existing theorems, that have themselves been proven.

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Algebraic fractions

Algebraic fractions behave, and follow the same rules as numerical fractions.
When simplifying algebraic fractions, where possible factorise the numerator and denominator and then cancel out common factors

Eg Simplify

$$\frac{2x^2+11x+12}{x^2+7x+12}$$

$2x^2+11x+12$ factorises to $(2x+3)(x+4)$

$x^2+7x+12$ factorises to $(x+3)(x+4)$

So, the fraction can be written as:

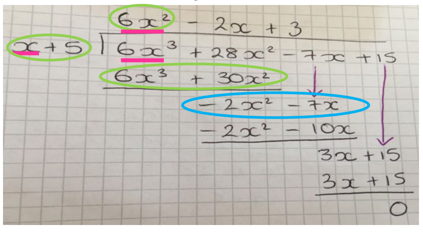
$$\frac{(2x+3)\cancel{(x+4)}}{(x+3)\cancel{(x+4)}}$$

(x+4) is the common factor so it cancels

Polynomial division

$$(6x^3+28x^2-7x+15)\div(x+5)$$

Method 1 – Long division



Divide the first term of the polynomial by x ($6x^3 \div x = 6x^2$)
Multiply (x+5) by $6x^2$ and write under polynomial
Subtract and bring down -7x

Repeat for each term of the polynomial

Method 2 – Box method

$6x^3 + 28x^2 - 7x + 15 \quad \div \quad x+5$

	$6x^2$	$-2x$	3
x	$6x^3$	$-2x^2$	$3x$
$+5$	$30x^2$	$-10x$	15

Divide the first term of the polynomial by x ($6x^3 \div x = 6x^2$)
Multiply +5 by $6x^2$ and write in box ($30x^2$)
Subtract $30x^2$ from x^2 term in the polynomial and complete box ($28x^2 - 30x^2 = -2x^2$ and write in box
Dived $-2x^2$ by x ($-2x$)
Multiply +5 by $-2x$ and write in box ($-10x$)
Subtract $-10x$ from the x term in the polynomial ($-7x - -10x = 3x$) and write in box
Divide 3x by x (3)
Multiply +5 by 3 and write in box (15)
If you collect the terms in your boxes it should match your polynomial

Proof

In a mathematical proof you must:

- State any information or assumptions you are using
- Show every step clearly
- Each step should follow logically from the previous step
- Make sure you have covered all possible cases
- Write a statement of proof at the end of your working

To prove an identity you should:

- Start with one side of the identity
- Manipulate it ro match the other side
- Show every step of your working

Proof continued…

Proof by exhaustion – break the statement into smaller cases and prove each one separately
Proof by counter example – give one example that does not work

Factor Theorem

If f(p) = 0, then (x – p) is a factor of f(x)
If (x – p) is a factor of f(x), then f(p) = 0

What do I need to be able to do?

By the end of this chapter you should be able to:

- Use Pascal's triangle to identify binomial coefficients and use them to expand simple binomial expressions
- Use combinations and factorial notation
- Use the binomial expansion to expand brackets
- Find individual coefficients in a binomial expansion
- Make approximations using the binomial expansion

Y12 – Chapter 8 The Binomial Expansion

Key words:

- Binomial expansion – shows us what happens when we multiply a binomial (like a+b) by itself as many times as we want.
- Binomial – A polynomial with two terms
- Factorial – to multiply all whole numbers from the chosen number down to one. The symbol is !
- Combinations – Any of the ways we can combine things, when the order does not matter

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Pascal's triangle

$$(a+b)^0 = 1$$
$$(a+b)^1 = a + b$$
$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$
$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

Use Pascal's triangle to find the coefficients

- The 1st term in the brackets starts with the power of $\boldsymbol{n}$ and decreases to 0
- The 2nd term in the brackets starts with the power of 0 and increases to $\boldsymbol{n}$

The Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n$$

$$(n \in \mathbb{N})$$

Finding Coefficients

Find the coefficient of x^4 in the binomial expansion:
$(2 + 3x)^{10}$

$$x^4 \text{ term} = \binom{10}{4}2^6(3x)^4$$
$$= 210 \times 64 \times 81x^4$$
$$= 1088640x^4$$

So the coefficient of x^4 in the binomial expansion of $(2+3x)^{10}$ is 1088640

Approximations using the Binomial Expansion

The first four terms of the binomial expansion of $\mathbf{(1-\frac{x}{4})^{10}}$ in ascending order are:

$$\mathbf{1 - 2.5x + 2.8125x^2 - 1.875x^3}$$

Use this expansion to estimate the value of 0.975^{10}

$$1 - \frac{x}{4} = 0.975$$
$$x = 0.1$$

$$\mathbf{0.975^{10} \approx 1 - 2.5(0.1) + 2.8125(0.1)^2 - 1.875(0.1)^3}$$
$$\mathbf{0.975^{10} \approx 0.7763\ (4sf)}$$

Y12 – Chapter 9 Trigonometric Ratios

What do I need to be able to do?

By the end of this chapter you should be able to:

- Use the cosine rule to find a missing side or angle
- Use the sine rule to find a missing side or angle
- Find the are of a triangle using an appropriate formula
- Solve problems involving triangles
- Sketch the graphs of the sine, cosine and tangent functions
- Sketch simple transformations of these graphs

Key words:

- Periodic function – a function (like Sine and Cosine) that repeats forever

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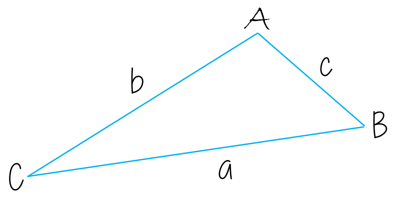
The Cosine Rule



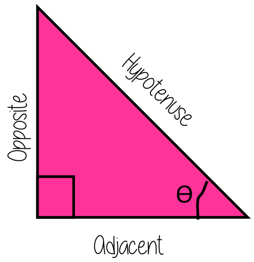
To find a missing side: $a^2 = b^2 + c^2 - 2bc\cos A$

To find a missing angle: $\cos A = \frac{b^2+c^2-a^2}{2bc}$

Use the cosine rule when you either:

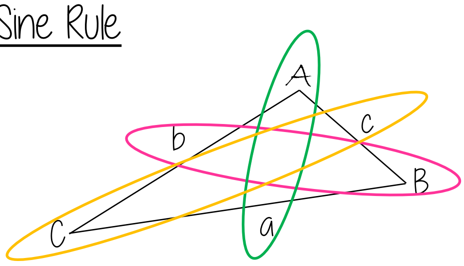
- Know two sides and the angle between them and want to know the third side
- Know three sides and want to find an angle

Sine, Cosine and Tangent



SInθ= Opposite/Hypotenuse
Cosθ= Adjacent/Hypotenuse
Tanθ= Opposite/Adjacent

The Sine Rule



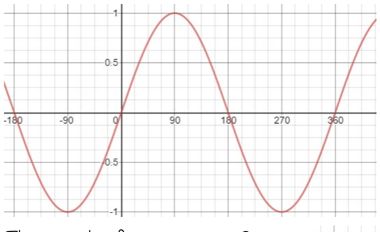
To find a missing side: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

To find a missing angle: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Use the sine rule when you have opposite pairs of angles and sides

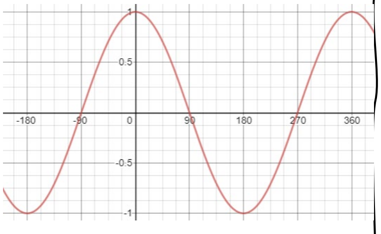
The sine rule sometimes produces two possible solutions for a missing angle: $\sin\theta = \sin(180 - \theta)$

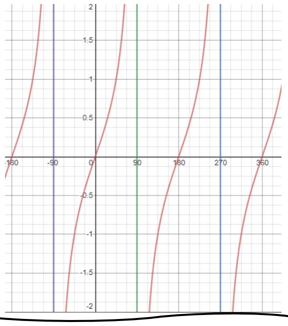
Graphs of sine, cosine and tangent



The graph of $y = \sin\theta$
Repeats every 360°
Crosses the x axis every 180°
Has a maximum value of 1 and a minimum value of -1

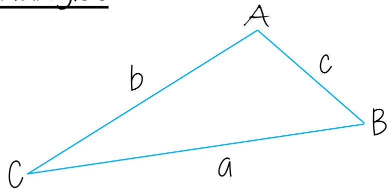
The graph of $y = \cos\theta$
Repeats every 360°
Crosses the x axis at -90°, 90°, 270°…
Has a maximum value of 1 and a minimum value of -1





The graph of $y = \tan\theta$
Repeats every 180°
Crosses the x axis at -180°, 0, 180°, 360°…
Has vertical asymptotes at x=-90°, x=90°, x=270°…

Areas of triangles



$$Area = \frac{1}{2}ab\sin C$$

Y12 – Chapter 10 Trigonometric identities and equations

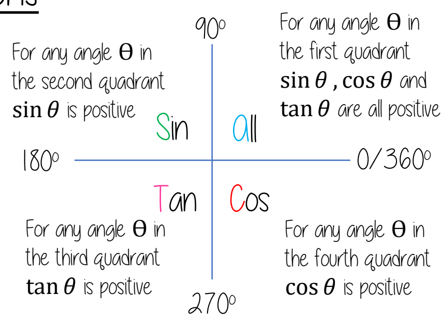
What do I need to be able to do?

By the end of this chapter you should be able to:

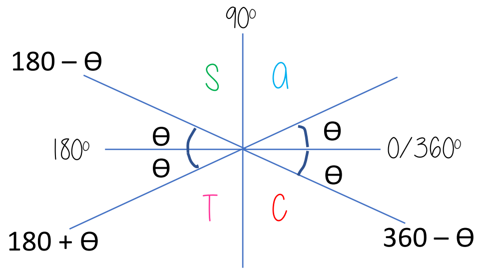
- Calculate the sine, cosine and tangent of any angle
- Know the exact trigonometric ratios for 30º, 45º and 60º
- Know and use the identities $\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$ and $sin^2\theta + cos^2\theta \equiv 1$
- Solve trigonometric equations

Solving trig equations

CAST diagram



You can use the diagram to find sin, cos or tan of any positive or negative angle using the corresponding acute angle made with the x axis



$$\sin\theta = \sin(180 - \theta)$$
$$\cos\theta = \cos(360 - \theta)$$
$$\tan\theta = \tan(180 + \theta)$$
$$-\sin\theta = \sin(180 + \theta) = \sin(360 - \theta)$$
$$-\cos\theta = \cos(180 - \theta) = \cos(180 + \theta)$$
$$-\tan\theta = \tan(180 - \theta) = \tan(360 - \theta)$$

When you use the inverse trigonometric functions on a calculator, the angle you get is the principal value. Your calculator gives principal values in the ranges:

$$\sin^{-1} -90° \leq \theta \leq 180°$$
$$\cos^{-1}\ 0° \leq \theta \leq 90°$$
$$\tan^{-1} -90° \leq \theta \leq 90°$$sin

sinθ = k and cosθ= k only have solutions when $-1 \leq k \leq 1$
tanθ = p has solutions for all real values of p

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Key words:

- Identity – An identity is an equation which is always true, no matter what values are substituted

Trig Identities

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$$

$$sin^2\theta + cos^2\theta \equiv 1$$

Exact trig values

	30	45	60
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Techniques for solving trig equations

Technique	Example
Rearrange to make sin, cos or tan the subject	sin(x) – 0.3 = 0 sin(x) = 0.3 x = 17º (2sf)
Factorise if possible	3cos(x)sin(x) + sin(x) = 0 sin(x)(3cos(x) + 1) = 0 sin(x) = 0 or 3cos(x)+1=0 x = 0º or x=110º (2sf)
If it is a mixture of sin(x) and cos(x) use the identity $\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$	sin(x) = 3cos(x) sin(x)/cos(x) = 3 tan(x) = 3 x = 72º (2sf)
If you have a mixture of sin and cos in a quadratic use the identity $sin^2\theta + cos^2\theta \equiv 1$ then solve	cos^2(x) = sin(x) + 1 1 – sin^2(x) = sin(x) + 1 sin^2(x) - sin(x) = 0 Factorise and solve
Solve multiples of the unknown angle	tan(2x) = 5 2x = 79º x = 39.5º

Y12 – Chapter 11 Vectors

What do I need to be able to do?

By the end of this chapter you should be able to:

- Use vectors in two dimensions
- Use column vectors and carry out arithmetic operations on vectors
- Calculate the magnitude and direction of a vector
- Understand and use position vectors
- Use vectors to solve geometric problems
- Understand vector magnitude and use vectors in speed and distance calculations
- Use vectors to solve problems in context

Vector forms

Form	Uses	Examples	Meaning
Component	i and j	2i – 3j	Go along 2 to the right and down 3
Column	digits	$\begin{pmatrix} -4 \\ 5 \end{pmatrix}$	Go along 4 to the left and up 5

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Magnitude of a vector

$\underline{a}$ = xi + yj = $\begin{pmatrix} x \\ y \end{pmatrix}$ magnitude of $\underline{a}$ is given by:

$$|a| = \sqrt{x^2 + y^2}$$

Position vectors

A position vector starts at the origin
eg a point A (4, -5) has position vector $\overrightarrow{OA}$ = 4i – 5j

$\overrightarrow{AO} = -\overrightarrow{OA}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

Midpoint

$\overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$

Key words:

- Magnitude – The magnitude of a vector is its length (ignoring direction)
- Resultant – the vector sum of two or more vectors
- Scalars – A single number (used when dealing with vectors or matrices)

Adding and multiplying vectors

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

To add vectors algebraically you add the i and j components eg:

$\underline{a}$ = 3i + 5j $\underline{b}$ = i – 7j

$\underline{a} + \underline{b}$ = 4i – 2j

To multiply by a scalar, multiply each component eg:
$\underline{a}$ = 3i + 5j

4$\underline{a}$ = 4(3i + 5j) = 12i + 20j

$\underline{b} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$
$3\underline{b} = 3\begin{pmatrix} -2 \\ 6 \end{pmatrix} = \begin{pmatrix} -6 \\ 18 \end{pmatrix}$

Unit vectors

A unit vector in the direction of $\underline{a}$ is $\frac{a}{|a|}$

Parallel vectors

Any vector parallel to $\underline{a}$ can be written as $\lambda\underline{a}$ where λ is a non-zero scalar

If $\underline{a}$ and $\underline{b}$ are two non parallel vectors and $p\underline{a} + q\underline{b} = r\underline{a} + s\underline{b}$ then p=r and q=s

Y12 – Chapter 12 Differentiation

What do I need to be able to do?

By the end of this chapter you should be able to:

- Find the derivative of a simple function
- Use the derivative to solve problems involving gradients, tangents and normal
- Identify increasing and decreasing functions
- Find the second order derivative
- Find stationary points of functions and determine their nature
- Sketch the gradient function of a given function
- Model real life situations with differentiation

Differentiating from first principles

It's a proof so you have to show ALL steps use the formula, substituting in the function

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Differentiating

If $y = ax^n$ then $\frac{dy}{dx} = anx^{n-1}$

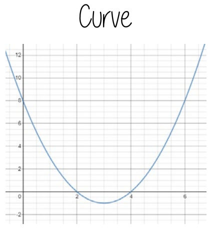
If $f(x) = ax^n$ then $f'(x) = anx^{n-1}$

When differentiating you multiply each term by its power and then reduce its power by 1

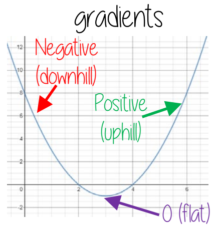
Sketching gradient functions

To sketch the gradient function, think about what is happening to the gradient at various points on the curve and sketch them

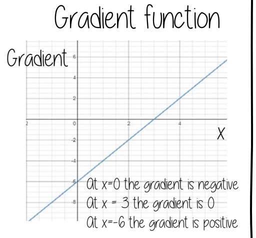
Curve



Thinking about the gradients



Gradient function



Key words:

- Derivative – a way to show rate of change: that is, the amount by which a function is changing at one given point
- Stationary point – A point on a curve where the slope is zero. This can be where the curve reaches a minimum or maximum

Notation and definitions

The gradient of a curve at a given point is defined as the gradient to the tangent to the curve at that point

The gradient function or derivative of the curve y = f(x) is written as f'(x) or $\frac{dy}{dx}$ or y' or $\frac{\delta y}{\delta x}$

The gradient function ($\frac{dy}{dx}$) measures the rate of change of y with respect to x

Tangents and normals

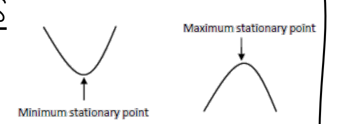
The tangent to the curve y = f(x) at the point (a, f(a)) has the equation:

y – f(a) = f'(a)(x – a)

The normal to the curve y = f(x) at the point (a, f(a)) has the equation:

y – f(a) = 1/f'(a)(x – a)

Stationary points



Solving $\frac{dy}{dx} = 0$ gives the x coordinate of the stationary points. Sub x value into y = f(x) to find the y coordinates

Solving $\frac{d^2y}{dx^2} = 0$ gives the nature of the stationary point. If $\frac{d^2y}{dx^2} > 0$ then it's a minimum. If $\frac{d^2y}{dx^2} < 0$ then it's a maximum

Y12 – Chapter 13 Integration

What do I need to be able to do?

By the end of this chapter you should be able to:

- Find y given $\frac{dy}{dx}$ for x^n
- Integrate polynomials
- Find f(x) given f'(x) on a point on the curve
- Evaluate a definite integral
- Find the area bounded by a curve and the x-axis
- Find areas bounded by curves and straight lines

Indefinite integration

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + c \quad n \neq 1$$

This expression is the integrand

If you are integrating a polynomial function, you integrate each term one at a term

To find the constant of integration, c:

1) Integrate the function
2) Substitute the coordinates of a point on the curve into the integrated function
3) Solve the equation to find c

Definite integration

A definite integral has limits. To evaluate a definite integral you integrate as normal and then substitute the top limit and the bottom limit and subtract

Upper limit

$$\int_b^a gx^n dx = \left[\frac{gx^{n+1}}{n+1}\right]_b^a = \left(\frac{ga^{n+1}}{n+1}\right) - \left(\frac{gb^{n+1}}{n+1}\right) n \neq 1$$

Lower limit

You don't need the +C with definite integration as you are going to subtract so it cancels out

Definite integrals give you the area under the curve between the limits

Key words:

- Integral – the result of integration
- Integrand – The function we want to integrate

Notation and definitions

Integration is the reverse of differentiation

$$\int (3x^5 + 7x^2 - 4x + 2)dx$$

Means integrate the following — With respect to x

Areas under curves

The area between a positive curve, the x-axis and the lines x=a and x=b is given by

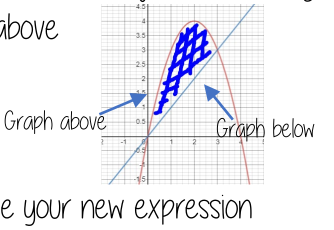
$$Area = \int_b^a y\, dx$$

Where y = f(x) is the equation of the curve
A positive answer means that the area is above the x-axis
A negative answer means that the area is below the x-axis

If there is a mixture of areas above and below the x-axis you have to work out each area separately and add them together (ignoring the negative sign)

To find the area between a curve and a line:

1) Find the x coordinate of the points of intersection
2) Subtract the equation of the graph that is below from the equation of the graph that is above



3) Integrate your new expression
4) Substitute in your x coordinates as limits to find the area

Y12 – Chapter 14 Exponentials and Logarithms

What do I need to be able to do?

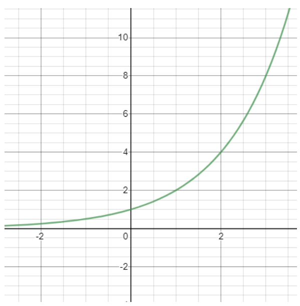
By the end of this chapter you should be able to:

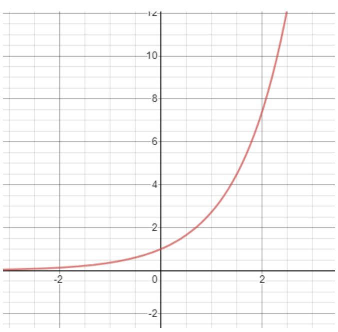
- Sketch graphs of the form $y=a^x$, $y=e^x$, and transformations of these graphs
- Differentiate e^{kx}
- Use and interpret models that use exponential functions
- Recognise the relationship between exponential and logarithms
- Recall and apply the laws of logarithms
- Solve equations in the form $a^x = b$
- Describe and use natural logarithms
- Use logarithms to estimate the values of constants in non-linear models

Exponential functions

$y = a^x$

Always crosses the y-axis at 1
The x-axis is an asymptote





$y = e^x$

The number "e" is one of the most important numbers in mathematics. The first few digits are:

2.718281828459045235360287471352 7

If $y = e^{kx}$ then $\frac{dy}{dx} = ke^{kx}$

Logarithmic graphs

For equations in the form $y = kx^n$ or $y = ab^x$ we can take logs to transform the curves into straight lines

Original	$y = kx^n$	$y = ab^x$
Take logs of both sides	$\log(y) = \log(kx^n)$	$\log(y) = \log(ab^x)$
Use laws of logs to get in the form $y = mx + c$	$\log(y) = n\log(x) + \log(k)$	$\log(y) = x\log(b) + \log(a)$
Gradient	n	$\log(b)$
Y-intercept	$\log(k)$	$\log(a)$

Key words:

- Exponential – a function in the form $f(x) = ab^x$
- Logarithm – A logarithm answers the question "How many of this number do we multiply to get that number?

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Logarithms

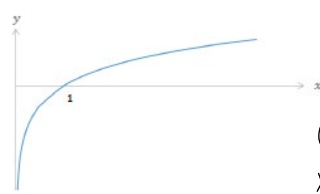
$\log_a n = x$ is equivalent to $a^x = n$

$$a \neq 1$$

Natural logarithms

Natural logarithms are logs in the base of e. Ln and e are the inverse of each other so they will cancel each other out

$$\ln e^x = x$$
$$e^{\ln x} = x$$



$y = \ln x$

Always crosses the x-axis at 1
The y-axis is an asymptote

Laws of logarithms

$$\log x + \log y = \log(xy)$$
$$\log x - \log y = \log\left(\frac{x}{y}\right)$$
$$\log(x^k) = k \log x$$
$$\log 0 = 1$$

Solving equations is the form $a^x = b$

1) Take logs of both sides
2) Use the power law to bring the power to the front
3) Solve the equation as normal

Y13 – Chapter 1 Algebraic Methods

Pure Maths Year 2

What do I need to be able to do?

By the end of this chapter you should be able to:

- Use proof by contradiction to prove true statements
- Add, subtract, multiply and divide two or more algebraic fractions
- Convert an expression with linear factors in the denominator into partial fractions
- Convert an expression with repeated linear fractions in the denominator into partial fractions
- Divide algebraic expressions
- Convert an improper fraction into partial fraction form

Key words:

- Contradiction – a disagreement between two statements which means that both cannot be true.
- Coefficient – A number used to multiply by a variable
- Improper algebraic fraction – One whose numerator has a degree equal to or larger than the denominator. It must be converted to a mixed fraction before you can express it in partial fractions

Partial Fractions

Sometimes it can be useful to split a single algebraic fraction into two or more partial fractions.

Eg: $\frac{7x-13}{(x-3)(x+1)} = \frac{2}{x-3} + \frac{5}{x+1}$

When solving partial fractions, you start by setting your function equal to the unknown fractions you are trying to find. There are 3 different layouts which depend on the starting function

1) All linear terms in the denominator

$$\frac{7x-13}{(x-3)(x+1)} = \frac{A}{(x-3)} + \frac{B}{(x+1)}$$

2) A repeated term in the denominator:

$$\frac{3x^2+7x-12}{(x-5)(x+2)^2} = \frac{A}{(x-5)} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$$

3) *Improper fractions:

$$\frac{3x^2-3x-2}{(x-1)(x-2)} = A + \frac{B}{(x-1)} + \frac{C}{(x-2)}$$

Steps to solve:

1) Sett your functions equal to the correct unknown fraction as above
2) Add the fractions using a common denominator (this should be the same as the original denominator)
3) Set the numerators as equal
4) Substitute values for x that will, in turn, make each bracket zero and/or equate coefficients to create enough equations to find the values of A, B, C etc

*NB: You can either use algebraic division or the relationship F(x)= Q(x) x divisor + remainder to convert an improper fraction into a mixed fraction

Proof by Contradiction

To prove by contradiction you start by assuming that the statement is false. You then use logical steps until you contradict yourself by leading to something that is impossible. You can then conclude that your assumption was incorrect and that the original statement was true.

Eg: Prove by contradiction that $\sqrt{2}$ is irrational

Assumption: $\sqrt{2}$ is rational, therefore $\sqrt{2}$ can be written as $\frac{a}{b}$ where a and b are in their lowest form and that $\frac{a}{b}$ is in its lowest terms

$$\therefore 2 = \left(\frac{a}{b}\right)^2$$
$$2 = \frac{a^2}{b^2}$$
$$\therefore 2b^2 = a^2$$

This means that a^2 is even which means that a is even.
If a is even then it can be expressed as 2k

$$\therefore a^2 = 2b^2$$
$$(2k)^2 = 2b^2$$
$$4k^2 = 2b^2$$
$$2k^2 = b^2$$

This means that b^2 is even which means that b is even.

Conclusion: If a and b are both even then the have a common factor of 2 so $\frac{a}{b}$ cannot be a fraction in its lowest terms which is a contradiction. This means that the original assumption is not correct and therefore $\sqrt{2}$ is irrational.

Y13 – Chapter 2 Functions and Graphs

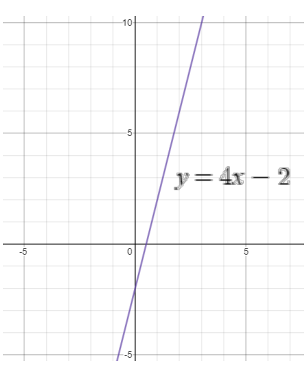
What do I need to be able to do?

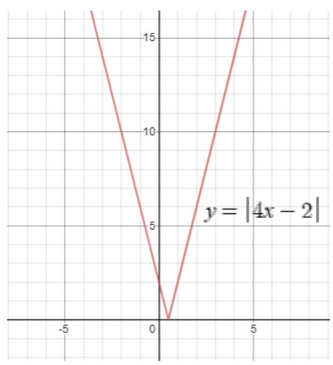
By the end of this chapter you should be able to:

- Understand and use the modulus function
- Understand mappings and functions, and use domain and range
- Combine two or more functions to get a composite function
- Know how to find the inverse of a function both graphically and algebraically
- Sketch the graphs of the modulus function
- Apply a combination of transformations to a curve
- Transform a modulus function

The Modulus Function

To sketch the graph of $y = |ax + b|$, sketch $y = ax + b$ and then reflect any section of the graph that is below the x-axis in the x-axis





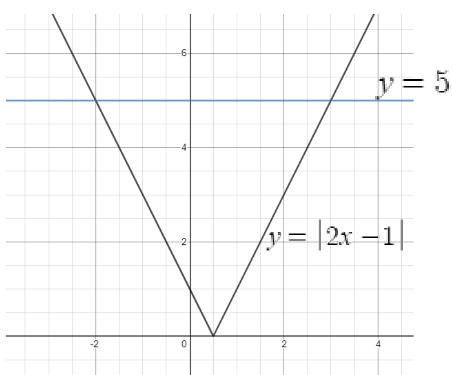
When solving modulus equations algebraically you consider the positive and negative argument (the function inside the modulus) separately

Eg:

Solve $|2x - 1| = 5$

$2x - 1 = 5$
$2x = 6$
$x = 3$

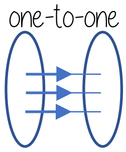
$-(2x - 1) = 5$
$-2x + 1 = 5$
$-2x = 4$
$x = -2$



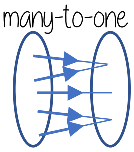
Functions and Mappings

A mapping is a function if each input has a distinct output. Functions can either be one-to-one or many-to-one

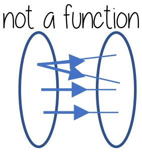
one-to-one



many-to-one



not a function



Key words:

- Modulus – the absolute value or modulus of a real number x, denoted $|x|$, is the non-negative value of x without regard to its sign. For example, the absolute value of 3 is 3, and the absolute value of –3 is also 3.
- Composite function – A function made of other functions, where the output of one is the input to the other
- Inverse function – An inverse function is a function that undoes the action of another function

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Composite Functions

Always apply the inside function first.

To find fg(x) do g(x) first then substitute your answer into f(x) to find the answer

Eg: $f(x) = x^2$ and $g(x) = x + 1$

a) Find fg(2)
$g(2) = 2 + 1 = 3$
$f(3) = 3^2 = 9$

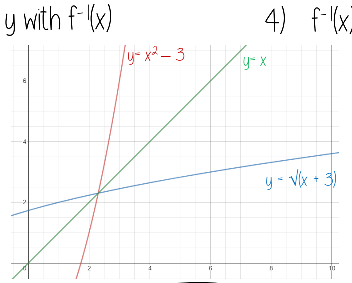
b) Find gf(x)
$f(x) = x^2$
$g(x^2) = x^2 + 1$

The Inverse Function

The inverse of a function performs the opposite operation to the original function. Inverse functions only exist for one-to-one functions.
The inverse of a function f(x) is written as $f^{-1}(x)$
The graphs of $y = f(x)$ and $y = f^{-1}(x)$ are reflections of each other in the line $y = x$
The domain of f(x) is the range of $f^{-1}(x)$
The range of f(x) is the domain of $f^{-1}(x)$

To find the inverse function:
1) Write it as y =
2) Swap x and y
3) Rearrange to make y the subject
4) Replace y with $f^{-1}(x)$

$f(x) = x^2 - 3$ find $f^{-1}(x)$
1) $y = x^2 - 3$
2) $x = y^2 - 3$
3) $\sqrt{(x + 3)} = y$
4) $f^{-1}(x) = \sqrt{(x + 3)}$



Y13 – Chapter 3 Sequences and Series

Pure Maths Year 2

What do I need to be able to do?

By the end of this chapter you should be able to:

- Find the nth term of an arithmetic sequence and a geometric sequence
- Prove and use the formula for the sum of the first n terms of an arithmetic series
- Prove and use the formula for the sum of a finite geometric series
- Prove and use the formula for the sum to infinity of a convergent geometric series
- Use sigma notation
- Generate sequences from recurrence relations
- Model real life situations

Key words:

- Sequence – A list of numbers or objects in a special order
- Series – The sum of terms in a sequence
- Arithmetic sequence – A sequence made by adding the same value each time
- Geometric sequence – A sequence made by multiplying by the same value each time. There is a common ratio between consecutive terms
- Arithmetic series– the sum of the terms of an arithmetic sequence
- Geometric series – the sum of the terms in a geometric sequence
- Common ratio – The amount we multiply by each time in a geometric sequence
- Converging sequence/series – A sequence/series converges when it keeps getting closer and closer to a certain value
- Divergent series – does not settle towards a certain value. When a series diverges it goes off to infinity, minus infinity, or up and down without settling towards some value.

Arithmetic Sequences and Series

The formula for the nth term of an arithmetic sequence is:

$$u_n = a + (n-1)d$$

u_n is the nth term

a is the first term

d is the common difference

The formula for the sum of the first n terms of an arithmetic series is:

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

It can also be written as:

$$S_n = \frac{n}{2}(a + l)$$

a is the first term

d is the common difference

l is the last term

Geometric Sequences and Series

The formula for the nth term of a geometric sequence is:

$$u_n = ar^{n-1}$$

u_n is the nth term

a is the first term

r is the common ratio

The formula for the sum of the first n terms of a geometric series is:

$$S_n = \frac{a(1-r^n)}{1-r}, r \neq 1$$

It can also be written as:

$$S_n = \frac{a(r^n-1)}{r-1}, r \neq 1$$

a is the first term

r is the common ratio

Sigma Notation

Σ means "the sum of". You write on the top and bottom to show which terms you are summing.

Eg:

$$\sum_{r=1}^{5}(2r-3) = -1 + 1 + 3 + 5 + 7$$

This tells you that you are summing the expression in brackets with r = 1 up to r = 5

Substitute r = 1, r = 2, r = 3, r = 4 and r = 5 into the expression in brackets to find the 5 terms in this arithmetic series

Sum to Infinity

As n tends to infinity, the sum of a geometric series is called the sum to infinity.

A geometric series is convergent only when $|r| < 1$, where r is the common ratio

The formula for the sum to infinity of a convergent series is:

$$S_\infty = \frac{a}{1-r}$$

a is the first term

r is the common ratio

Recurrence Relations

The next term in the sequence is the function of the previous term

$$u_{n+1} = f(u_n)$$

Y13 – Chapter 4 Binomial Expansion

What do I need to be able to do?

By the end of this chapter you should be able to:

- Expand $(1+x)^n$ for any rational constant n and determine the range of values of x for which the expansion is valid
- Expand $(a+bx)^n$ for any rational constant n and determine the range of values of x for which the expansion is valid
- Use partial fractions to expand fractional expressions

Key words:

- Infinite series – The sum of infinite terms that follow a rule

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The Binomial Expansion

$(a+b)^n$ when n is a fraction or a negative number (ie. NOT a positive integer) there will be an infinite number of terms. This means that the binomial expansion can only be used when $-1 < x < 1$.

When n is a fraction or a negative number the following form of the binomial expansion should be used:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \binom{n}{r}x^r + \cdots, (|x| < 1, n \in \mathbb{R})$$

The expansion is valid when $|x| < 1$

The expansion of $(1+bx)^n$ is valid for $|bx| < 1$ or $|x| < \frac{1}{b}$

We can use the expansion of $(1+x)^n$ to expand $(a+bx)^n$ by taking out a factor of a^n out of the expression

$$(a+bx)^n = (a\left(1+\frac{b}{a}x\right))^n = a^n(1+\frac{b}{a}x)^n$$

The expansion of $(a+bx)^n$ is valid for $\left|\frac{b}{a}x\right| < 1$ or $|x| < \frac{a}{b}$

Partial Fractions

We can use partial fractions to simplify the expansions of more difficult expressions

E.g.

a) Express $\frac{4-5x}{(1+x)(2-x)}$ as partial fractions

b) Hence show that the cubic approximation of $\frac{4-5x}{(1+x)(2-x)}$ is $2 - \frac{7x}{2} + \frac{11}{4}x^2 - \frac{25}{8}x^3$

c) State the range of values of x for which the expansion is valid

a) $\frac{4-5x}{(1+x)(2-x)} \equiv \frac{A}{1+x} + \frac{B}{2-x}$

$\equiv \frac{A(2-x)+B(1+x)}{(1+x)(2-x)}$

$4 - 5x \equiv A(2-x) + B(1+x)$

Substitute x = 2:

$4 - 10 = A \times 0 + B \times 3$

$\mathrm{B} = -2$

Substitute x = -1:

$4 + 5 = A \times 3 + B \times 0$

$\mathrm{A} = 3$

b) $\frac{4-5x}{(1+x)(2-x)} = \frac{3}{1+x} - \frac{2}{2-x}$

$= 3(1+x)^{-1} - 2(2-x)^{-1}$

The expansion of $3(1+x)^{-1} = 3 - 3x + 3x^2 - 3x^3 + \cdots$

The expansion of $2(2-x)^{-1} = 1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} + \cdots$

Hence $\frac{4-5x}{(1+x)(2-x)} = (3 - 3x + 3x^2 - 3x^3) - \left(1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8}\right)$

$= 2 - \frac{7}{2}x + \frac{11}{4}x^2 - \frac{25}{8}x^3$

c) $\frac{3}{1+x}$ is valid if $|x| < 1$ $\qquad$ $\frac{2}{2-x}$ is valid if $|x| < 2$

So the expansion is valid when $|x| < 1$

Y13 – Chapter 5 Radians

What do I need to be able to do?

By the end of this chapter you should be able to:

- Convert between degrees and radians
- Know exact values of angles measured in radians
- Find arc length using radians
- Find areas of sectors and segments using radians
- Solve trigonometric equations
- Use small angle approximations

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Key words:

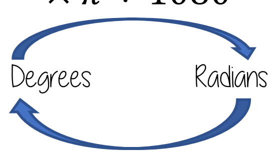
- Radian – The angle made by taking the radius and wrapping it round the circle
- Arc length – The distance along part of the circumference of a circle, or of any curve
- Sector – the area between two radiuses and the connecting arc of a circle
- Segment – The smallest part of a circle made when it is cut by a line

Converting between degrees and radians

2π radians = 360°

π radians = 180°

1 radian = $180/\pi$



Arc lengths, Sectors and Segments

When working in radians:

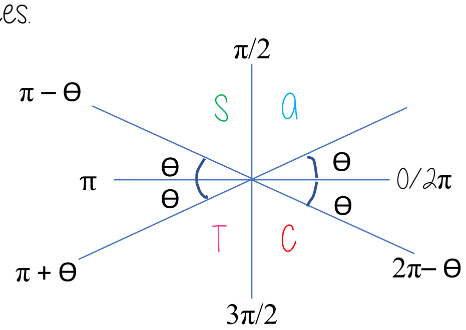
Arc length = $r\theta$

Area of sector = $\frac{1}{2}r^2\theta$

Area of a segment = $\frac{1}{2}r^2(\theta - \sin\theta)$

Solving Trigonometric Equations

This works the same way as solving trigonometric equations in degrees.

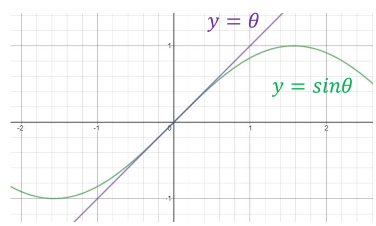


$$\sin\theta = \sin(\pi - \theta)$$
$$\cos\theta = \cos(2\pi - \theta)$$
$$\tan\theta = \tan(\pi + \theta)$$
$$-\sin\theta = \sin(\pi + \theta) = \sin(2\pi - \theta)$$
$$-\cos\theta = \cos(\pi - \theta) = \cos(\pi + \theta)$$
$$-\tan\theta = \tan(\pi - \theta) = \tan(2\pi - \theta)$$

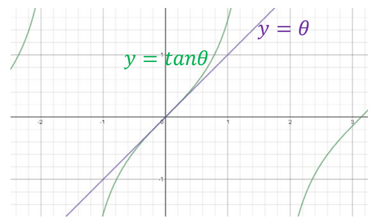
Small Angle Approximations

When ϴ is small and measured in radians:

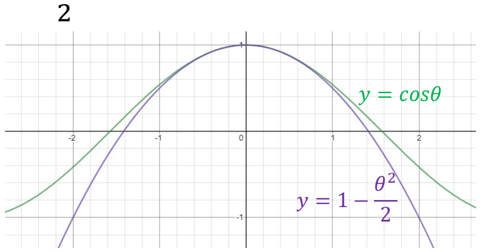
$\sin\theta \approx \theta$



$\tan\theta \approx \theta$



$\cos\theta \approx 1 - \frac{\theta^2}{2}$



Y13 – Chapter 6 Trigonometric Functions

What do I need to be able to do?

By the end of this chapter you should be able to:

- Understand secant, cosecant and cotangent and their relationship to cosine, sine and tangent
- Understand the graphs of secant, cosecant and cotangent
- Simplify expressions, prove identities and solve equations involving secant, cosecant and cotangent
- Understand and use inverse trigonometric functions

Key words:

- Cosecant – In a right angled triangle, the cosecant of an angle is: The length of the hypotenuse divided by the length of the side opposite the angle
- Secant– In a right angled triangle, the secant of an angle is: The length of the hypotenuse divided by the length of the adjacent side.
- Cotangent – In a right angled triangle, the cotangent of an angle is: The length of the adjacent side divided by the length of the side opposite the angle

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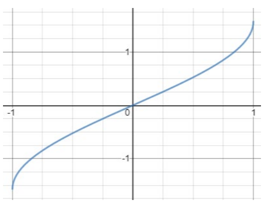
Trig Identities

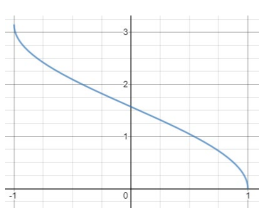
$$1 + tan^2 x \equiv sec^2 x$$
$$1 + cot^2 x \equiv cosec^2 x$$

Graphs of Inverse Functions

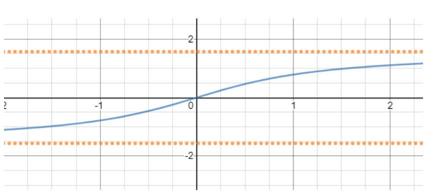
The inverse of sin(x) is arcsin(x)
The domain is $-1 \leq x \leq 1$
The range is $-\pi/2 \leq \arcsin(x) \leq \pi/2$



The inverse of cos(x) is arccos(x)
The domain is $-1 \leq x \leq 1$
The range is $0 \leq \arcsin(x) \leq \pi$



The inverse of tan(x) is arctan(x)
The domain is $x \in \mathbb{R}$
The range is $-\pi/2 \leq \arctan(x) \leq \pi/2$

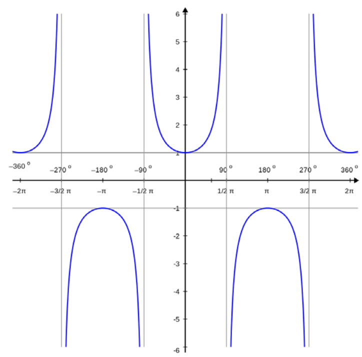


Secant, Cosecant and Cotangent

$$\sec x = \frac{1}{\cos x}$$

The graph has symmetry in the y-axis, a period of $360/2\pi$
It has vertical asymptotes at all of the values for which cos(x)=0

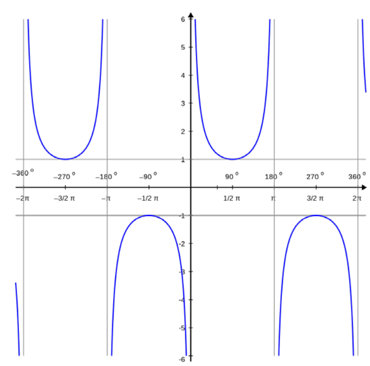
The domain of y=sec(x) is $x \in \mathbb{R}$
In degrees:
$x \neq$ 90, 270, 450… (any odd multiple of 90)
In radians:
$x \neq \pi/2, 3\pi/2, 5\pi/2$… (any odd multiple of $\pi/2$)
The range of y=sec(x) is $y \leq -1$ or $y \geq 1$



$$cosec\ x = \frac{1}{\sin x}$$

The graph has symmetry in the y-axis, a period of $360/2\pi$
It has vertical asymptotes at all of the values for which cos(x)=0

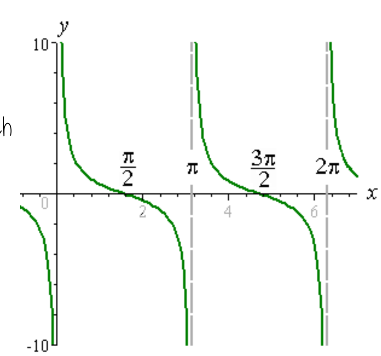
The domain of y=sec(x) is $x \in \mathbb{R}$
In degrees:
$x \neq$ 90, 270, 450… (any odd multiple of 90)
In radians:
$x \neq \pi/2, 3\pi/2, 5\pi/2$… (any odd multiple of $\pi/2$)
The range of y=sec(x) is $y \leq -1$ or $y \geq 1$



$$cot\ x = \frac{1}{tan\,x}$$

The graph has vertical asymptotes at all of the values for which tan(x)=0

The domain of y=cot(x) is $x \in \mathbb{R}$
In degrees:
$x \neq$ 0, 180, 360… (any odd multiple of 180)
In radians:
$x \neq 0, \pi, 2\pi$… (any multiple of π)
The range of y=sec(x) is $y \in \mathbb{R}$



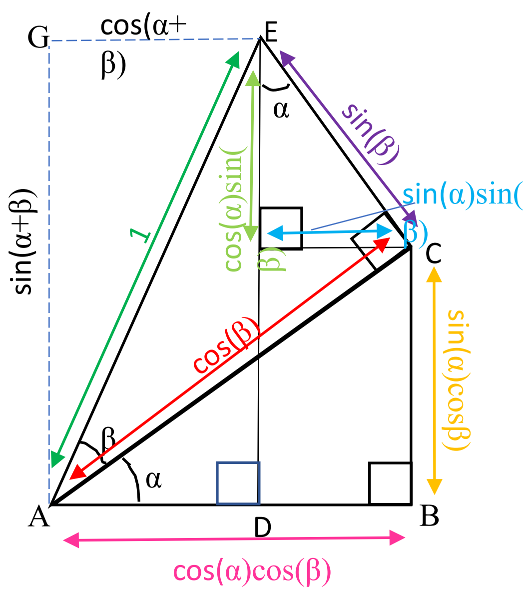
Y13 – Chapter 7 Trigonometry and Modelling

What do I need to be able to do?

By the end of this chapter you should be able to:

- Prove and use the addition formulae
- Understand and use the double angle formulae
- Solve trigonometric equations using addition and double angle formulae
- Write expressions in the form Rcos(θ±α) and Rsin(θ±α)
- Prove trigonometric identities
- Model real life situations

Proof of the Addition Formulae



Using the properties of sine and cosine we can label the diagram as above.

Using triangle ADE:
DE = sin(α+β)
AD = cos (α+β)

DE = DF + FE
∴ sin(α+β) = sin(α)cos(β) + cos(α)sin(β)

AD = AB – DB
∴ cos(α+β) = cos(α)cos(β) – sin(α)sin(β)

Addition Formulae

Sometimes are known as the compound angle formulae

$$sin(A+B) \equiv sinAcosB + cosAsinB$$
$$sin(A-B) \equiv sinAcosB - cosAsinB$$

$$cos(A+B) \equiv cosAcosB - sinAsinB$$
$$cos(A-B) \equiv cosAcosB + sinAsinB$$

$$tan(A+B) \equiv \frac{tanA + tanB}{1 - tanAtanB}$$

$$tan(A-B) \equiv \frac{tanA - tanB}{1 + tanAtanB}$$

Simplifying acos(x)± bcos(x)

Sometimes known as the harmonic form. You can write expressions in the form acos(x) ± bcos(x) as a function of sine or cosine only.

$acos\theta \pm bsin\theta$ can be written as either:

$Rsin(x \pm \alpha)$ where R>0 and 0< α<90

$Rcos(x \pm \beta)$ where R>0 and 0< β<90

Where $Rcos\alpha = a$ and $Rsin\alpha = b$ and $R = \sqrt{a^2 + b^2}$

Double Angle Formulae

You can use the addition formulae to derive the following double angle formulae:

$$sin(2A) \equiv 2sinAcosA$$

$$cos(2A) \equiv cos^2A - sin^2A \equiv 2cos^2A - 1 \equiv 1 - 2sin^2A$$

$$tan(2A) \equiv \frac{2tanA}{1 - tan^2A}$$

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Y13 – Chapter 8 Parametric Equations

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What do I need to be able to do?

By the end of this chapter you should be able to:

- Convert parametric equations into Cartesian form
- Understand and use parametric equations of curves and sketch parametric curves
- Solve problems involving parametric equations
- Use parametric equations in modelling

Key words:

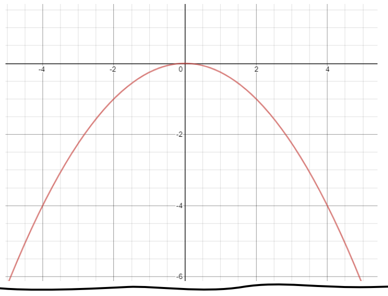
- Cartesian equations – Gives a direct relationship between x and y
- Parametric equations – Uses a third variable (usually t or θ) to define x and y

Sketching parametric equations of curves

When sketching a parametric equation, sub in values of t to find x and y values and then sketch as normal!

Sketch the curve defined by $x=2t$ and $y=-t^2$ between $t=-3$ **and** 3.

T	-3	-2	-1	0	1	2	3
X	-6	-4	-2	0	2	4	6
Y	-9	-4	-1	0	-1	-4	-9



Calculus with parametric equations*

* This section actually appears in your text book in chapters 9 and 11

Differentiation:

If x = f(t) and y = g(t)

Then:

$$\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$$

Integration:

If x = f(t) and y = g(t)

Then:

$$\int y\,dx=\int y\frac{dx}{dt}\,dt$$

Remember to adjust limits if you are using definite integration

Converting between parametric and cartesian equations

Combine the two equations by rearranging one of them to make t the subject and then substitute into the other equation.

OR

Rearrange both equations to make t the subject and then equate the two equations

Eg: Convert the following parametric equations into cartesian form

$x = t + 3 \quad y = 2t^2$

$x = t + 3 \rightarrow t = x - 3$

$y = 2(x - 3)^2$

OR

$x = t + 3 \quad y = 2t^2$

$x = t + 3 \rightarrow t = x - 3$

$y = 2t^2 \rightarrow t = \sqrt{(y/2)}$

$\sqrt{(y/2)} = x - 3$

$y/2 = (x - 3)^2$

$y = 2(x - 3)^2$

If your parametric equations contains trigonometric functions, first find an identity that connects then rearrange the parametric equations so that you can substitute into the identity

Domain and range

For parametric equations x = p(t) and y = q(t) with Cartesian equation y = f(x)

- The domain of f(x) is the range of p(t)
- The range of f(x) is the range of q(t)

Y13 – Chapter 9 Differentiation

Key words:

- Concave – Curves inwards
- Convex – Curves outwards

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What do I need to be able to do?

By the end of this chapter you should be able to:

- Differentiate trigonometric functions
- Differentiate exponentials and logarithms
- Use the chain, product and quotient rules
- Use the second derivative to describe a function's behaviour
- Solve problems involving connected rates of change
- Construct differential equations
- Differentiate parametric equations (see parametric equations sheet)

Differentiating Trigonometric functions

If $y = \sin kx$, then $\frac{dy}{dx} = k\cos kx$

If $y = \cos kx$, then $\frac{dy}{dx} = -k\sin kx$

If $y = \tan kx$, then $\frac{dy}{dx} = k\sec^2 kx$

If $y = \operatorname{cosec} kx$, then $\frac{dy}{dx} = -k\operatorname{cosec} kx \cot kx$

If $y = \sec kx$, then $\frac{dy}{dx} = -k\sec kx \tan kx$

If $y = \cot kx$, then $\frac{dy}{dx} = -k\operatorname{cosec}^2 kx$

If $y = \arcsin x$, then $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

If $y = \arccos x$, then $\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$

If $y = \arctan x$, then $\frac{dy}{dx} = \frac{1}{1+x^2}$

Implicit Differentiation

Use when equations are difficult to rearrange into the form $y = f(x)$

$$\frac{d}{dx}(y^n) = ny^{n-1}\frac{dy}{dx}$$

$$\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$$

Second Derivatives

The function f(x) is concave on a given interval if and only if $f''(x) \leq 0$ for every value of x in the value in that interval

The function f(x) is convex on a given interval if and only if $f''(x) \geq 0$ for every value of x in that interval

A point of inflection is a point at which $f''(x)$ changes sign

Chain, Product and Quotient Rules

Chain Rule:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Where y is a function of u and u is another function of x

In function notation:

If $y = (f(x))^n$ then $\frac{dy}{dx} = n(f(x))^{n-1}f'(x)$

If $y = f(g(x))$ then $\frac{dy}{dx} = f'(g(x))g'(x)$

Product Rule:

If y = uv where u and v are functions of x, then:

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

In function notation:

If $f(x) = g(x)h(x)$ then:

$$f'(x) = g(x)h'(x) + h(x)g'(x)$$

Quotient Rule:

If y = u/v where u and v are functions of x, then:

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

In function notation:

If $f(x) = g(x)/h(x)$ then:

$$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{(h(x))^2}$$

Differentiating Exponential and Logarithms

If $y = e^{kx}$, then $\frac{dy}{dx} = ke^{kx}$

If $y = \ln x$, then $\frac{dy}{dx} = \frac{1}{x}$

If $y = a^{kx}$, where k is a real constant and a>0, then $\frac{dy}{dx} = a^{kx}k\ln a$

What do I need to be able to do?

By the end of this chapter you should be able to:

- Locate roots of f(x)=0 by considering sign changes
- Use iteration to find an approximation to the equation f(x)=0
- Use the Newton Raphson method to find approximations to the solutions of equations in the form f(x)=0
- Use numerical methods to solve problems in context

Y13 – Chapter 10 Numerical Methods

Key words:

- Root – Where a function equals zero
- Continuous function – The function does not 'jump' from one value to another. If the graph of a function has a vertical asymptote between two points then the function is not continuous in the interval between the two points.

Locating Roots

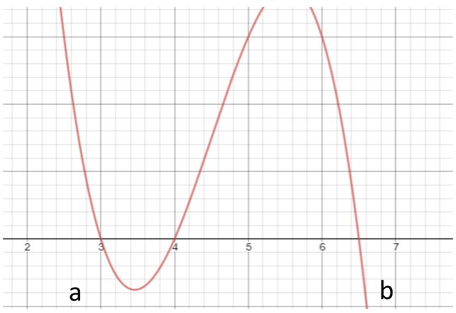
You can sometimes show the existence of a root within a given interval by showing that the function changes sign in that interval

Eg Show that $f(x) = x^3 - 4x^2 + 3x + 1$ has a root between x=1.4 and x=1.5

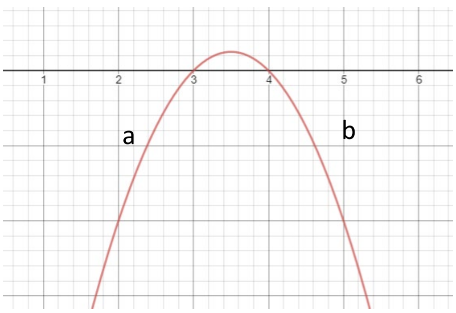
$f(1.4) = (1.4)^3 - 4(1.4)^2 + 3(1.4) + 1 = 0.104$
$f(1.5) = (1.5)^3 - 4(1.5)^2 + 3(1.5) + 1 = -0.125$

The function is continuous and there is a change of sign between 1.4 and 1.5 so there is at least one root in this interval

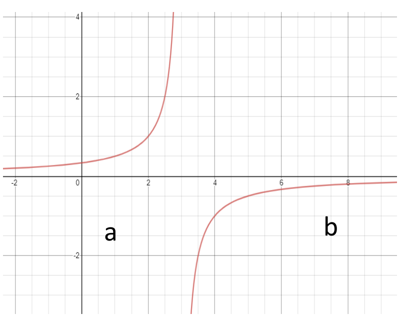
Watch out! The change of sign doesn't mean exactly one root and the absence of a sign change does not necessarily mean that a root does not exist



There are multiple roots within the interval [a, b] – odd number of roots



There are multiple roots within the interval [a, b] but a sign change does not occur – even number of roots



There is a vertical asymptote within the interval [a, b] – a sign change occurs but no roots

Iteration

To solve an equation in the form f(x) = 0 using iteration first rearrange f(x)=0 into the form x=g(x) and then use the iterative formula:

$$x_{n+1} = g(x_n)$$

Some iterations will converge and some will diverge
Successful iterations can be shown on staircase or cobweb diagrams

The Newton-Raphson Method

An alternative way to find roots using differentiation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Pure Maths Year 2

Y13 – Chapter 11 Integration

What do I need to be able to do?

By the end of this chapter you should be able to:

- Integrate standard mathematical function
- Use trigonometric identities in integration
- Use the reverse chain rule
- Integrate functions by substitution, by parts and using partial fractions
- Find the are under a curve using integration
- Use the trapezium rule
- Solve differential equations and model with differential equations

Reverse Chain Rule

Functions in the form f(ax+b):

$$\int f'(ax+b)\,dx = \frac{1}{a}f(ax+b)+c$$

To integrate expressions in the form $\int k\frac{f'(x)}{f(x)}dx$, try ln|f(x)| and differentiate to check and adjust the constant as necessary

To integrate functions in the form $\int kf'(x)(f(x))^n\,dx$, try $(f(x))^{n+1}$ and differentiate to check and adjust the constant as necessary

Partial Fractions and Integration by substitution

You can integrate by substitution by choosing a function ($u(x)$) (which can be differentiated) to help you to integrate a tricky function. You need to substitute all $x's$ (including dx) with terms involving u.

You can also use partial fractions in order to integrate algebraic fractions

Differential Equations

When $\frac{dy}{dx} = f(x)g(y)$ you can solve it by separating the variables as follows:

$$\int \frac{1}{g(y)}\,dy = \int f(x)\,dx$$

When you integrate to solve a differential equation you still need to include a constant of integration – this will give the general solution.

Key words:

- Differential equation – An equation with a function and one or more of its derivatives

Standard integrals

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}+c; \quad n\neq -1$$

$$\int e^x\,dx = e^x + c$$

$$\int \frac{1}{x}\,dx = \ln|x| + c$$

$$\int \cos x\,dx = \sin x + c$$

$$\int \sin x\,dx = -\cos x + c$$

$$\int \sec^2 x\,dx = \tan x + c$$

$$\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + c$$

$$\int \operatorname{cosec}^2 x\,dx = -\cot x + c$$

$$\int \sec x \tan x\,dx = \sec x + c$$

You can use trigonometric identities to replace expressions that cannot be integrated with one that can

Eg $\int \tan^2 x\,dx$

$$1+\tan^2 x \equiv \sec^2 x$$

So

$$\tan^2 x \equiv \sec^2 x - 1$$

So

$\int \tan^2 x\,dx = \int \sec^2 x - 1\,dx$ which we can integrate using a standard integral

Area Bounded by Two Curves

$$Area\ of\ R = \int_a^b f(x) - g(x)\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

Integration by Parts

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

The Trapezium Rule

$$\int_a^b y\,dx \approx \frac{1}{2}h(y_0 + 2(y_1 + y_2 \ldots + y_{n-1}) + y_n)$$

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Limit Notation

$$\int_a^b f(x)\,dx = \lim_{\delta x\to 0}\sum_a^b f(x)\delta x$$

Y13 – Chapter 12 Vectors

What do I need to be able to do?

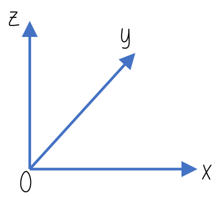
By the end of this chapter you should be able to:

- Understand 3D Cartesian coordinates
- Use vectors in three dimensions
- Use vectors to solve geometric problems
- Model 3D motion in mechanics with vectors

Key words:

- Coplanar vectors – Vectors in the same plane
- Magnitude– The size of the vector

3D Coordinates



When visualising 3D coordinates, think of the x and y axis drawn on a flat surface with the z axis sticking up from the flat surface.

The distance from the origin to the point (x, y, z) is:

$$\sqrt{x^2 + y^2 + z^2}$$

The distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is:

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

3D Vectors

Unit vectors along the x, y and z axes are denoted by I, j and k respectively

$$\boldsymbol{i} = \begin{pmatrix}1\\0\\0\end{pmatrix} \quad \boldsymbol{j} = \begin{pmatrix}0\\1\\0\end{pmatrix} \quad \boldsymbol{k} = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

For any 3D vector $\boldsymbol{pi} + \boldsymbol{qj} + \boldsymbol{rk} = \begin{pmatrix}p\\q\\r\end{pmatrix}$

Magnitude and Direction

Vector a = $\boldsymbol{pi} + \boldsymbol{qj} + \boldsymbol{rk}$

Magnitude of vector a:

$$|\boldsymbol{a}| = \sqrt{p^2 + q^2 + r^2}$$

Direction of vector a:

The angle with the x-axis: $\cos\theta_x = \frac{p}{|\boldsymbol{a}|}$

The angle with the y-axis: $\cos\theta_y = \frac{q}{|\boldsymbol{a}|}$

The angle with the z-axis: $\cos\theta_z = \frac{r}{|\boldsymbol{a}|}$

Parallel Vectors in 3D

If a, b, and c are 3D, non-coplanar vectors (not in the same plane) then you can compare coefficients on both sides of an equation:

Eg

If

$$\boldsymbol{pi} + \boldsymbol{qj} + \boldsymbol{rk} = \boldsymbol{ui} + \boldsymbol{vj} + \boldsymbol{wk}$$

Then: p=u, q=v and r=w

Pure Maths Year 2

THE LARGE DATA SET

KEY WORDS & DEFINITIONS

1. Daily Mean Temperature
The average of hourly temperature readings in a 24hour period, in Celsius.

2. Daily Total Rainfall
The depth of precipitation as a liquid. All precipitation is included, not just rainfall, but it is melted if necessary for the measurement. Heights less than 0.05mm are recorded as a "trace" or "tr".

3. Daily Total Sunshine
Recorded to the nearest 10th of an hour (6 minutes).

4. Daily Mean Wind Direction
Given as a bearing and/or in cardinal (compass) directions.

5. Daily Mean Windspeed
Averaged over 24 hours of a day (midnight to midnight), in Knots, nautical miles per hour where 1 Knot = 1.15mph. Can also be categorised by the Beaufort Scale.

6. Daily Maximum Gust
The highest instantaneous windspeed recorded, in Knots.

7. Daily Maximum Gust Direction
The direction of the maximum gust of wind recorded.

8. Daily Maximum Relative Humidity
A percentage of air saturation with water vapour. Relative humidities above 95% result in mist or fog.

9. Daily Mean Cloud Cover
Measured in eighths of the sky that is covered (Oktas).

10. Daily Mean Visibility
The greatest horizontal distance at which an object can be seen in daylight, measured in decametres (Dm).

11. Daily Mean Pressure
Measured in hectopascals (hPa)

WHAT DO I NEED TO KNOW?

1. What the Large Data Set is about
The Edexcel LDS has samples on weather data in different locations for certain time periods. The data is provided by the Met Office.
The LDS contains the weather data for 5 UK weather stations and 3 weather stations overseas.

2. How to clean the data
N/A should be removed before calculations
tr (trace) should be turned to 0

3. Locations
Learn maps and understand geographical significance of North, South, Coastal etc,

4. Dates
Remember the Large Data Set only has information from May–October 1987 and May–October 2015. Anything between November and April is outside the range of our data.

5. Understand OKTAS
A measure of the fraction of the celestial dome covered by cloud, measured in
eighths. 0 oktas represents a clear sky,
while a value of 8 indicates complete overcast.

6. How to convert units
1 knot = 1.151 mph

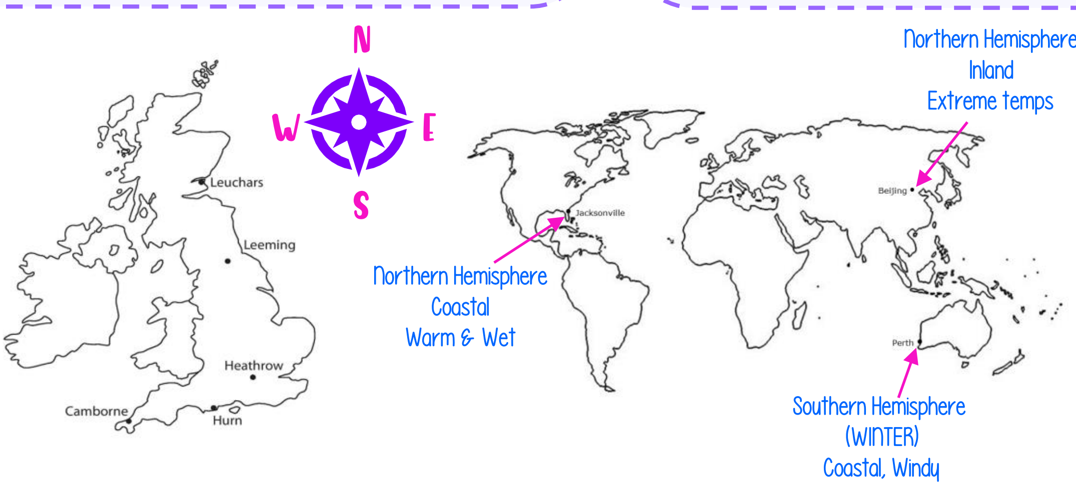
7. Limitations
These stations do not tell us about the whole UK

THE BEAUFORT SCALE

Beaufort Scale	Description	Av. Wind Speed 10m above ground
0	Calm	< 1 Knot
1-3	Light	1 – 10 Knots
4	Moderate	11 – 16 Knots
5	Fresh	17 – 21 Knots

UK DATA

Location (N to S)	Temp Range (°C)	Wind Speed Range (kn)
Leuchars	4 – 9	3 – 23
Leeming	4 – 23	3 – 17
Heathrow	8 - 29	3 – 19
Hurn	6 - 24	2 – 19
Camborne	10 - 20	3 – 18



DATA COLLECTION

KEY WORDS & DEFINITIONS

1. Population
Whole set of items that could be sampled.
2. Census
Observations taken from the entire population.
3. Sample
Observations taken from a subset of the population.
4. Sampling Unit
One individual observation set from the population.
5. Sampling Frame
A numbered (or named) list of individual sampling units.
6. Strata
A subset of the population.

TYPES OF SAMPLING

1. Simple Random Sampling
Every sample of a specified size has an equal chance of being selected from a sampling frame.
2. Systematic Sampling
Items are chosen at regular intervals from a sampling frame.
3. Stratified Sampling
Random samples are taken proportionally from mutually exclusive groups or strata.
4. Quota Sampling
Non-random sample is taken to fulfil predetermined quotas for different categories.
5. Opportunity Sampling
Non-random sample is selected from available sampling units.

TYPES OF DATA

1. Quantitative Data
Variables or data associated with a numerical value.
2. Qualitative Data
Variables or data associated with a non-numerical value.
3. Continuous
Variables that can take any value.. **Measured**.
4. Discrete
Variables that can only take specific values.. **Counted**.

CENSUS VS SAMPLE

	Census	Sample
Advantages	Includes every member of the population to give a fully representative set of data	Less time consuming to collect and process data. Fewer people needed therefore cheaper to conduct.
Disadvantages	Time consuming & expensive. Cannot be used when testing process destroys the item being tested	May not be fully representative of population. Outliers or whole subgroups possibly excluded

WHAT DO I NEED TO KNOW?

1. Advantages & Disadvantages
Why is one type of sampling more appropriate than another. Consider time, cost, bias, ease, accuracy of population representation.
2. How to work with Grouped Data
Understand inequalities. Find maximum, minimum & midpoint of each group.
3. How to use the Large Data Set
Be able to clean data, take samples and comment on findings.

MEASURES OF LOCATION & SPREAD

KEY WORDS & DEFINITIONS

1. Measure of Location
A single value which describes a position in a data set.
2. Measure of Central Tendency
A measure of location which describes the central position in a data set.
3. Measure of Spread or Dispersion
A value which describes how spread out the data is.
4. Mean
The sum of all the data divided by how many pieces of data there are. Includes all pieces of data. Affected by outliers.
5. Median Q_2
The middle value when the data values are put in order. Does not include all pieces of data. Not affected by outliers.
6. Mode
The value that occurs most often in the data. Good for non-numerical data.
7. Modal class
The class that has the highest frequency in grouped data.
8. Lower Quartile Q_1
A measure of location that is one quarter of the way through the data set.
9. Upper Quartile Q_3
A measure of location that is three-quarters of the way through the data set.
10. Percentile
A measure of location that is the specified percentage of the way through the data set.
11. Range
The difference between the largest and smallest values in a data set. Affected by outliers.
12. Inter-quartile Range
The difference between the upper and lower quartiles in a data set. $Q_3 - Q_1$
Not affected by outliers.

IMPORTANT FORMULAE

Mean: $\bar{x} = \frac{\Sigma x}{n}$

Mean from Frequency Table: $\bar{x} = \frac{\Sigma f x}{\sum f}$

Variance σ^2:

$$\frac{\sum (x - \bar{x})^2}{n} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

Standard Deviation $\sigma = \sqrt{\text{Variance}}$

CODING

If data is coded using $y = \frac{x - a}{b}$

Mean of coded data = $\bar{y} = \frac{\bar{x} - a}{b}$

s.d. of coded data = $\sigma_y = \frac{\sigma_x}{b}$

To find mean & s.d. of original data use:

$$\bar{x} = b\bar{y} + a$$

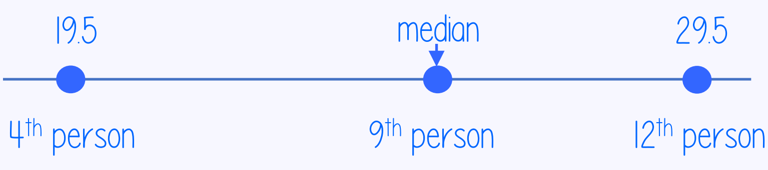
$$\sigma_x = b\sigma_y$$

INTERPOLATION

Assume data values are evenly distributed within each class then estimate median or percentile values using proportional reasoning.

Age	10 – 19	20 – 29	30 - 39
Frequency	4	8	5
Cumulative Freq	4	12	17

17 people ∴ median is 9^{th} person
9^{th} person is in 20 – 29 group
Take boundaries to be 19.5 & 29.5



$$\frac{m - 19.5}{29.5 - 19.5} = \frac{9 - 4}{12 - 4}$$

$$m = 25.75$$

REPRESENTATIONS OF DATA

KEY WORDS & DEFINITIONS

1. Outlier

A data value that lies beyond expected extremities. These are usually calculated as a multiple of the interquartile range above the upper quartile or below the lower quartile.

i.e. either greater than $Q_3 + k(Q_3 - Q_1)$

or less than $Q_1 - k(Q_3 - Q_1)$

2. Cleaning

The process of removing anomalies from the data set.

WHAT DO I NEED TO KNOW

Comparing 2 sets of data:

Calculate & compare the measures of location

Calculate & compare the measures of spread

Compare outliers if applicable

Mean & s.d go together

Median & IQR go together.

Ensure all comparisons are done IN CONTEXT

Histograms

Area of bar $\propto$ Frequency so

Area of bar = k x Frequency

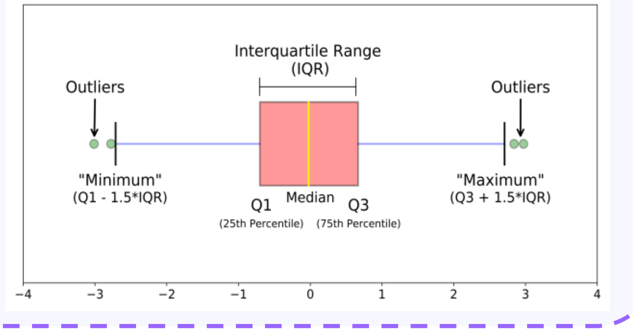
Area does NOT always = Frequency

BOX PLOTS

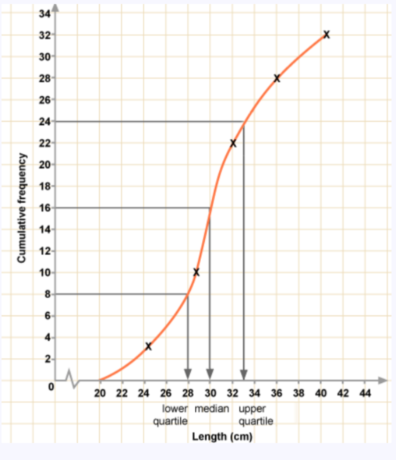
Box plots are rarely symmetrical

25% of the data lies within each section

Always use the same scale when comparing box plots



CUMULATIVE FREQUENCY



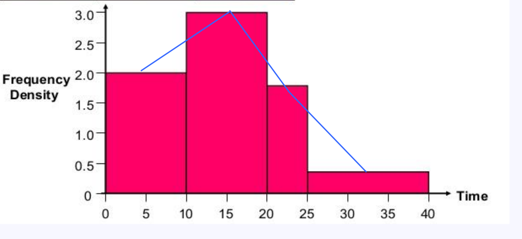
Plot points at the upper limits of group boundaries

Ensure it makes sense to extrapolate the curve at the beginning

Be careful of questions that ask "How many are **more** than..."

HISTOGRAMS

Time	Frequency	Frequency Density	Frequency Density = Frequency / Class width
$0 < t \leq 10$	20	2	20 ÷ 10
$10 < t \leq 15$	15	3	15 ÷ 5
$15 < t \leq 20$	10	2	10 ÷ 5
$20 < t \leq 25$	9	1.8	9 ÷ 5
$25 < t \leq 40$	6	0.4	6 ÷ 15



Histograms are used to represent grouped continuous data

Area of bar = k x frequency

If k = 1, then frequency density = $\frac{\text{frequency}}{\text{class width}}$

You may need to find the areas of parts of bars if questions don't use the class boundaries.

Joining the middle of the tops of each bar in a histogram forms a frequency polygon

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CORRELATION & REGRESSION

KEY WORDS & DEFINITIONS

1. **Correlation** A description of the linear relationship between two variables.
2. **Bivariate data** Pairs of values for two variables
3. **Causal relationship** Where a change in a variable causes a change in another. Not always true.
4. **Least squares regression line**
   A type of line of best fit which is a straight line in the form $y = a + bx$
5. **'b' of a regression line**
   The gradient of the line; indicating positive correlation if it is positive and negative correlation if it is negative..
6. **Independent or Explanatory variable**
   The variable which occurs regardless of the other variable (e.g. time passing). Plotted on the x axis.
7. **Dependent or Response variable**
   The variable whose value depends on the independent variable's data points.
8. **Interpolation** Estimating a value within the range of the data. Reliable.
9. **Extrapolation** Estimating a value outside of the range of the data. NOT reliable.
10. **Product Moment Correlation Coefficient**
    A measure of the strength and type of correlation.

WHAT DO I NEED TO KNOW

Interpreting 'b' of a regression line:
Refer to the change in the variable y for each unit change of the variable x IN CONTEXT

PMCC, r is the PMCC for a population sample

PMCC, ρ is the PMCC for the entire population

Range of PMCC, r: $-1 \leq r \leq 1$

Hypotheses for one tailed test on PMCC:
$H_0: \rho = 0$
$H_1: \rho > 0$ or $H_1: \rho < 0$

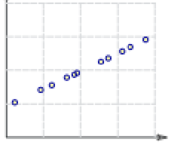
Hypotheses for two tailed test on PMCC:
$H_0: \rho = 0$
$H_1: \rho \neq 0$

Check sample size is big enough to draw a valid conclusion and comment on it if not.

A regression line is only a valid model when the data shows linear correlation.

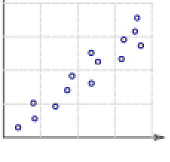
Only make predictions for the dependent variable using the regression line of y on x within the range of the original data

Perfect positive correlation



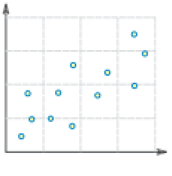
$r = 1$

Strong positive correlation



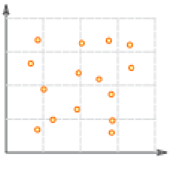
$r = 0.8$

Weak positive correlation



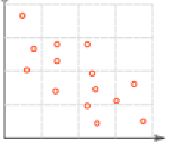
$r = 0.3$

No correlation



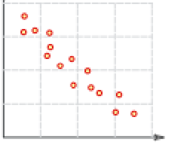
$r = 0$

Weak negative correlation



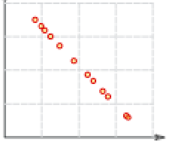
$r = -0.3$

Strong negative correlation



$r = -0.8$

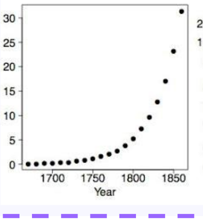
Perfect negative correlation

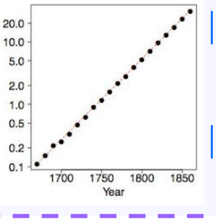


$r = -1$

EXPONENTIAL MODELS

You can use logarithms and coding to transform graphs and examine trends in non-linear data





If $y = ax^n$ then $\log y = \log a + n \log x$

If $y = kb^x$ then $\log y = \log k + x \log b$

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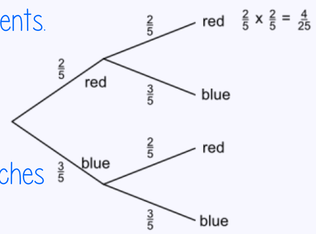
PROBABILITY

KEY WORDS & DEFINITIONS

1. **Experiment** A repeatable process that results in a number of outcomes.
2. **Event** A collection of one or more outcomes.
3. **Sample Space** The set of all possible outcomes. ξ is the universal set.
4. **Mutually Exclusive** Events that have no outcomes in common.
5. **Independent** When events have no effect on another.
6. **Intersection** When two or more events all happen.
7. **Union** When one or both events happen.
8. **Complement** When an event does not happen.

TREE DIAGRAMS

You can use tree diagrams to show the outcome of 2 or more successive events.



Multiply ALONG the branches

Add all the favourable final probabilities.

WHAT DO I NEED TO KNOW

Probabilities of all possible outcomes add to 1

Probability values must be between 0 and 1

Intersection $A \cap B$ $\Rightarrow$ A AND B happen

Union $A \cup B$ $\Rightarrow$ A OR B OR BOTH happen

Complement of A is A′ $\Rightarrow$ NOT A

$P(A') = 1 - P(A)$

Mutually Exclusive events:

$P(A \cup B) = P(A) + P(B)$

Independent Events:

$P(A \cap B) = P(A) \times P(B)$

Probability of B, given A has occurred:

$P(B \mid A)$

For independent events:

$P(A \mid B) = P(A \mid B') = P(A)$

In formulae book:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

VENN DIAGRAMS

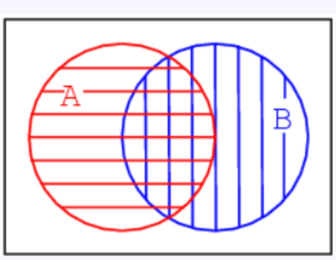
Venn diagrams can be used to show either probabilities or the number of outcomes.

n(A) is the number of outcomes while P(A) is the probability of an outcome

e.g. n(Aces) = 4   P(Ace) = 4/52

Use cross hatch shading to help you work out probabilities.

Focus on one condition at a time, ignoring the other condition completely when you shade.



If P(A) = // and P(B) = \\

$P(A \cap B)$ = #

$P(A \cup B)$ = // + \\ + #

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STATISTICAL DISTRIBUTIONS

KEY WORDS & DEFINITIONS

1 **Random variable** A variable whose outcome depends on a random event.
2 **Sample space** The range of values a variable can take.
3 **Discrete variable** A variable that can only take specific values.
4 **Probability Distribution** A full description of the probability of all possible outcomes in a sample space.
5 **Uniform distribution** When the probabilities in a distribution are all equal.
6 **Binomial Distribution** A distribution where the random variable, X, represents the number of successful trials in an experiment.
7 **Cumulative probability distribution** The sum of probabilities up to and including the given value..

WHAT DO I NEED TO KNOW

Probabilities of all possible outcomes add to 1
$\sum P(X = x) = 1$ for all x

Probability distributions can be described in different ways. E.g. if X = the score when a fair die is rolled

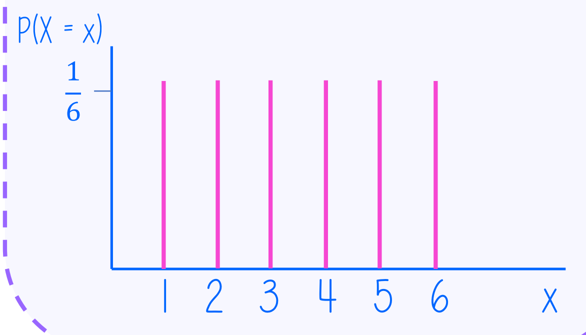
Table:

x	1	2	3	4	5	6
P(X=x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Probability Mass Function:

$$P(X = x) = \begin{cases} \frac{1}{6}, & x = 1,2,3,4,5,6 \\ 0 & \text{otherwise} \end{cases}$$

Diagram:



BINOMIAL DISTRIBUTION

Conditions for a binomial distribution B(n , p)

- Only two possible outcomes (success/failure)
- Fixed number of trials, n
- Fixed probability of success, p
- Trials are independent of each other .

Probability mass function of a Binomial distribution

$$p(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$$

Binomial Cumulative Probability Function

The sum of all the individual probabilities up to and including the given value of x in the calculation for $P(X \leq x)$

These values can be found in the tables or on a calculator.

Phrase	Means	Calculation
Greater than 5	$X > 5$	$1 - P(X \leq 5)$
No more than 3	$X \leq 3$	$P(X \leq 3)$
At least 7	$X \geq 7$	$1 - P(X \leq 6)$
Fewer than 10	$X < 10$	$P(X \leq 9)$
At most 8	$X \leq 8$	$P(X \leq 8)$

CALCULATORS FOR BINOMIAL

Casio fx-991EX:
Menu 7 – Binomial CD or Binomial PD

Casio CG50:
Menu 2 - F5 Dist – F5 Binomial – Bpd or Bcd

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HYPOTHESIS TESTING

KEY WORDS & DEFINITIONS

1 Hypothesis Test
A process that considers the probability of an observed (or calculated) value occurring.

2 Null Hypothesis, H_0
The hypothesis about the parameter that is assumed to be correct.

3 Alternative Hypothesis, H_1
The hypothesis about the parameter if the assumption is not correct.

4 Test Statistic
The result of an experiment, or the value calculated from a sample.

5 One-tailed Test
A hypothesis test that involves the alternative hypothesis describing the parameter as being less than or greater than the null hypothesis value.

6 Two-tailed test
A hypothesis test that involves the alternative hypothesis describing the parameter as taking any value that is not the null hypothesis value.

7 Critical Region
The region of the probability distribution where the test statistic value would result in the null hypothesis being rejected.

8 Critical value
The first value of the test statistic that could fall in the critical region.

9 Significance Level
The total probability of incorrectly rejecting the null hypothesis.

WHAT DO I NEED TO KNOW

To carry out a Hypothesis Test, assume H_0 is true, then consider how likely the observed value of the test statistic was to occur. Remember we need it to be **even more unlikely** than the significance level in order to be 'significant' and to reject H_0.

If the test is two-tailed there are two critical regions, one at each end of the distribution. We therefore need to halve the significance level at the end we are testing.

If the test statistic is $X \sim B(n, p)$ then the **expected** outcome is **np**

If the observed value lies in critical region we say there is sufficient evidence to reject H_0 and conclude that H_1 is correct.

If observed value is not in critical region we say there is insufficient evidence to reject H_0.

ALWAYS add a final line in your conclusion in the **context** of the question

Beware of questions that say 'The probability in the tail should be as close as possible to the significance level'. In these cases we may choose a value that is actually *slightly* more likely than the significance level.

THE NORMAL DISTRIBUTION

KEY WORDS & DEFINITIONS

The Normal Distribution

A continuous probability distribution that can be used to model variables that are more likely to be grouped around a central value than at extremities.

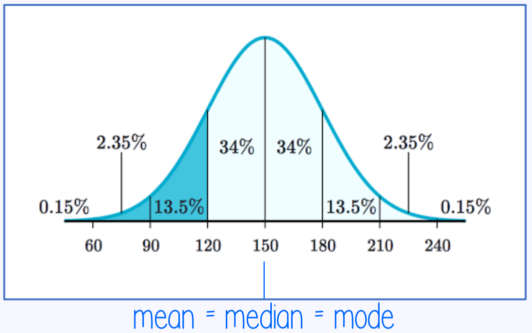
THE NORMAL DISTRIBUTION CURVE

Symmetrically bell-shaped, with asymptotes at each end.

68% percent of data is within one s.d. of μ

95% percent of data is within two s.d. of μ

99.7% percent of data is within three s.d. of μ



mean = median = mode

THE NORMAL DISTRIBUTION TABLE

To find z-values that correspond to given probabilities, i.e. $P(Z > z) = p$ use this table:

p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0364	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905

CALCULATORS FOR NORMAL DISTRIBUTION

Casio fx-991EX:

Menu 7 – Normal PD, Normal CD or Inverse Normal

Casio CG50:

Menu 2 - F5 Dist – F1 Normal – Npd, Ncd or InvN

Choose extremely large or small values for upper or lower limits as appropriate

WHAT DO I NEED TO KNOW

1. The area under a continuous probability distribution curve = 1

2. If X is a normally distributed random variable, with population mean, μ, and population variance, σ^2 we say $X \sim N(\mu, \sigma^2)$

3. To find an unknown value that is a limit for a given probability value, use the inverse normal distribution function on the calculator.

4. The notation of the standard normal variable Z is $Z \sim N(0, 1^2)$

5. The formula to standardise X is $z = \frac{x-\mu}{\sigma}$

6. The notation for the probability $P(Z < a)$ is $\phi(a)$

7. To find an unknown mean or standard deviation use coding and the standard normal variable, Z.

8. Conditions for a Binomial distribution to be approximated by a Normal distribution:
n must be large
p must be close to 0.5

9. The mean calculated from an approximated Binomial distribution is $\mu = np$

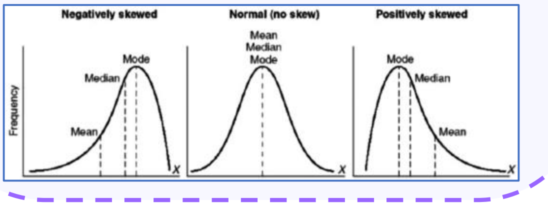
10. The variance calculated from an approximated Binomial distribution is $\sigma^2 = np(1-p)$

11. Apply a continuity correction when calculating probabilities from an approximated Binomial distribution using limits so that the integers are completely included or excluded, as required.

12. The mean of a sample from normally distributed population, is distributed as:

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right) \quad \text{then} \quad z = \frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}}$$

13. Skewed data is NOT 'Normal'



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MODELLING IN MECHANICS

KEY WORDS & DEFINITIONS

1. **Model** – A mathematical system which enables a problem to be solved
2. **Light** – Has negligible mass
3. **Static** – Not moving
4. **Rigid** – Doesn't bend
5. **Thin** – Has negligible thickness
6. **Smooth** – Has a surface that results in no friction between itself and an object
7. **Rough** – Has a surface that requires frictional forces between itself and an object to be considered
8. **Particle** – Dimensions are negligible, so mass or object is at a point. Rotational forces and air resistance can be ignored.
9. **Rod** – A long, thin, straight object. Mass is along a line that is rigid.
10. **Lamina** – A thin 2-dimensional surface with mass distributed evenly across its flat surface.
11. **Uniform Body** – Mass is distributed evenly, so acts at the centre of mass.
12. **Light string** – Has negligible mass and equal tension at both ends.
13. **Inextensible string** – A string that does not stretch so that connected objects can move with the same acceleration if the string is taut.
14. **Wire** – A rigid, thin length of metal.
15. **Smooth and Light Pulley** – A pulley that has no mass and results in tension being equal on either side.
16. **Bead** – A particle with a hole in it which can freely move along a wire or string, resulting in equal tension either side of the bead.
17. **Peg** – A supporting object that is dimensionless and fixed but may be rough or smooth.
18. **Air Resistance** – The resistance force as experienced as on object moves through the air, which is often modelled as negligible.
19. **Gravity** – The force of attraction between objects.
20. **Earth's Gravity** – Assumed to apply to all objects with mass. Acts uniformly and vertically downwards with a value of $9.8 m/s^2$
21. **Scalar** – A quantity which has magnitude only – distance, speed, time, mass. Always positive.
22. **Vector** – A quantity which has magnitude and direction – displacement, velocity, acceleration, force, weight. Can be described using column or i j notation. Can be positive or negative.

Distance is the magnitude of the displacement vector

Speed is the magnitude of the velocity vector

SI BASE UNITS

Quantity	Mass	Length/ Displacement	Time	Speed/ Velocity	Acceleration	Weight/ Force
Symbol	kg	m	s	ms^{-1}	ms^{-2}	N (= $kgms^{-2}$)

CONSTANT ACCELERATION

KEY WORDS & DEFINITIONS

1. **Velocity**
   The rate of change of displacement
2. **Acceleration**
   The rate of change of velocity

SUVAT EQUATIONS

For motion in a straight line with constant acceleration:

$$v = u + at$$

$$v^2 = u^2 + 2as$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

s – displacement
u – initial velocity
v – final velocity
a – acceleration
t - time

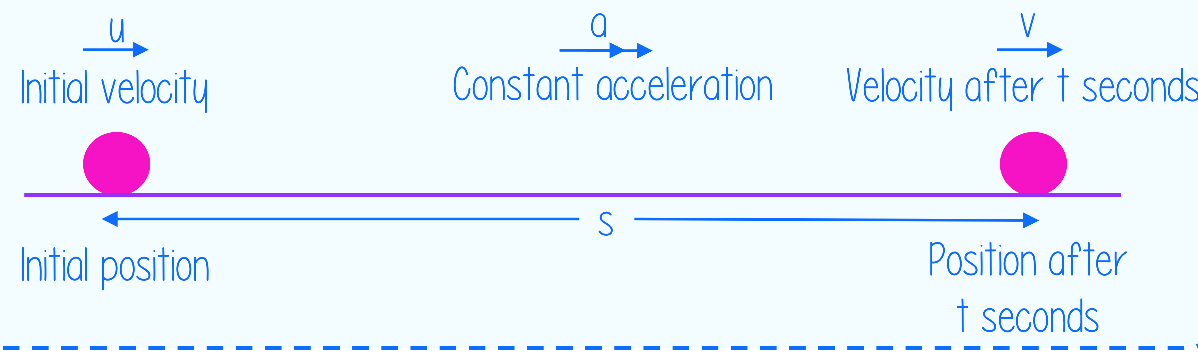
To derive the SUVAT equations:

- Find the gradient of a velocity time graph labelled with u, v, t
- Find the area underneath the velocity-time graph
- Use these two equations to replace each variable at a time to derive the other three equations.

WHAT DO I NEED TO KNOW

1. The gradient on a displacement-time graph = velocity
2. If a displacement-time graph is a straight line then the velocity is constant.
3. The gradient on a velocity-time graph = acceleration
4. If a velocity-time graph is a straight line then the acceleration is constant.
5. The area between a velocity-time graph and the time axis = Distance travelled
6. Average Speed = $\frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$
7. Average velocity = $\frac{\text{Displacement From Start Point}}{\text{Total Time Taken}}$
8. Acceleration due to gravity = $9.8 m/s^2$
9. Acceleration due to gravity does not depend on the mass of the object.
10. The degree of accuracy in your answers must be consistent with the values given in the question. I.e. if $g = 10 m/s^2$ in the question, your answer should also be given to 1 sig. fig.

ALWAYS DRAW A SKETCH!



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FORCES & MOTION

KEY WORDS & DEFINITIONS

1. Resultant Force

The result of resolving forces on an object in a particular direction.

2. Weight

The force due to gravity acting on an object

NEWTON'S LAWS OF MOTION

Newton's First Law of Motion

Objects in equilibrium will not accelerate. An object will only accelerate (or decelerate) if an unbalanced force acts on the object.

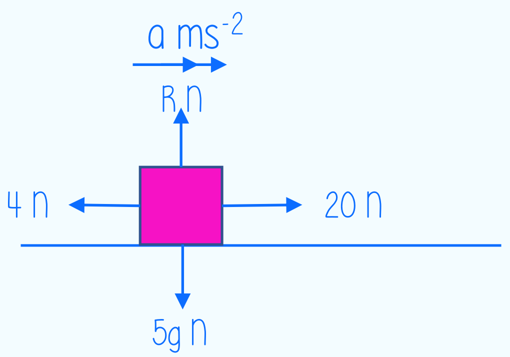
Newton's Second Law of Motion

The acceleration of an object depends on the overall net force acting on the object and the object's mass.

Newton's Third Law of Motion

For every action there is an equal and opposite reaction.

ALWAYS DRAW A DIAGRAM!



FORMULAE

Formula of Newton's Second Law of Motion

$$F = ma$$

Formula to calculate the weight of an object

$$W = mg$$

WHAT DO I NEED TO KNOW

1. To resolve forces given as vectors add the vectors

   If 2 forces ($p\mathbf{i} + q\mathbf{j}$) N and ($r\mathbf{i} + s\mathbf{j}$)N are acting on a particle, the resultant force will b $((p + r)\,\mathbf{i} + (q + s)\,\mathbf{j}\,)$N

2. To solve problems involving connected particles moving in the same straight line consider the particles as a single unit, moving as one.

   Particles need to remain in contact or be connected by an inextensible rod or string to be considered a single particle

3. To solve problems involving connected particles that are not moving in the same straight line consider the particles, and the forces acting on them, separately.

   Particles need to be considered separately in order to find the tension in any string between them

4. The tension in an inextensible string passing over a smooth pulley is the same on both sides

   You cannot treat a system involving a pulley as a single particle as the particles are moving in different directions

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VARIABLE ACCELERATION

KEY WORDS & DEFINITIONS

1. **Velocity**
   The rate of change of displacement

2. **Acceleration**
   The rate of change of velocity

FORMULAE

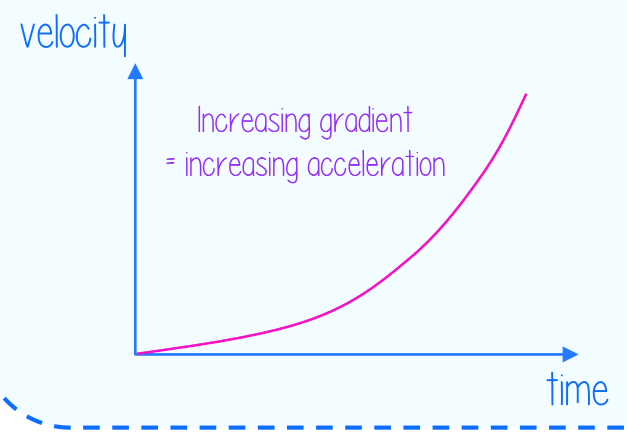
Velocity, if displacement is a function of time:

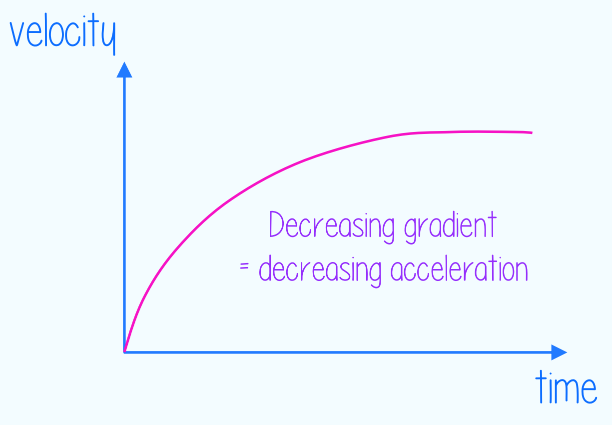
$$v = \frac{\mathrm{d}s}{\mathrm{d}t}$$

Acceleration, if velocity is a function of time

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$$

VELOCITY TIME GRAPHS





WHAT DO I NEED TO KNOW

1. The area under a velocity time graph represents the displacement
2. Integration is the reverse process to differentiation
3. Differentiate displacement with respect to time to get velocity
4. Differentiate velocity with respect to time to get acceleration
5. Integrate acceleration with respect to time to get velocity

$$\int (a)\,\mathrm{d}t = v$$

6. Integrate velocity with respect to time to get displacement

$$\int (v)\,\mathrm{d}t = s$$

7. The suvat equations can only be used when the acceleration is constant

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MOMENTS

KEY WORDS & DEFINITIONS

1. Moment
The turning effect of a force on a rigid body.

2. Resultant Moment
The sum of all moments acting on a rigid body.

3. The Point of Tilting
The instantaneous situation where the reaction at any support or the tension in any supporting string or wire, other than at the pivot, will be zero.

4. Coplanar Forces
Forces that act in the same plane.

5. Lamina
A 2D object whose thickness can be ignored.

MODELLING ASSUMPTIONS & IMPLICATIONS

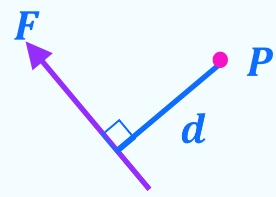
A plank is uniform ⇒ Weight acts at the centre of the plank

A plank is a rod ⇒ The plank remains straight

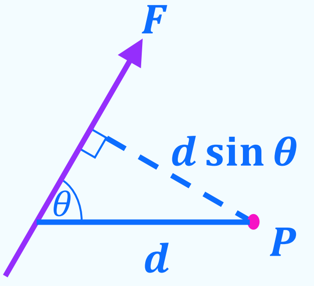
Any people/objects are particles ⇒ Their weight acts at the end of any rod

FORMULAE

Moment about P = magnitude of force x perpendicular distance of the force from P



Moment of **F** about P = | **F** | x d **N**m clockwise



Moment of **F** about P = FdSinθ **N**m clockwise

WHAT DO I NEED TO KNOW

1. The **units** of Moments are **Newton metres Nm**
2. The **direction** of the Moment (clockwise or anticlockwise) must be included with a moment's value.
3. When a rigid body is in **equilibrium**, the **resultant force** in any direction is 0**N** and the **resultant moment** about any point is 0**N**m
4. The centre of mass of a **non-uniform rod** is **not** necessarily at the **midpoint** of the rod.

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FORCES & FRICTION

KEY WORDS & DEFINITIONS

1. Friction
A force which opposes motion.

2. Coefficient of Friction μ
A measure of how resistant to motion two surfaces are

3. Limiting Equilibrium
The point at which there is equilibrium, but friction is at its maximum

FORMULAE

To calculate Maximum Friction:

$$F_{max} = \mu R$$

Where:

F is the frictional force

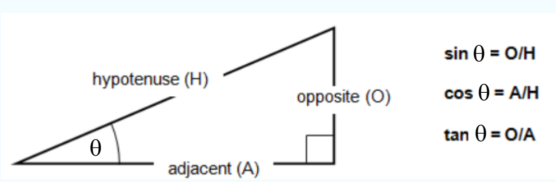
μ is the coefficient of friction

R is the normal reaction between the surfaces.

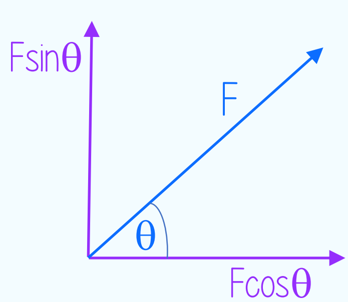
RESOLVING FORCES

"Cos through, Sine away"

Using SOH CAH TOA



when resolving forces gives the following result:



Component $F_x = F\cos\theta$ (through the angle)

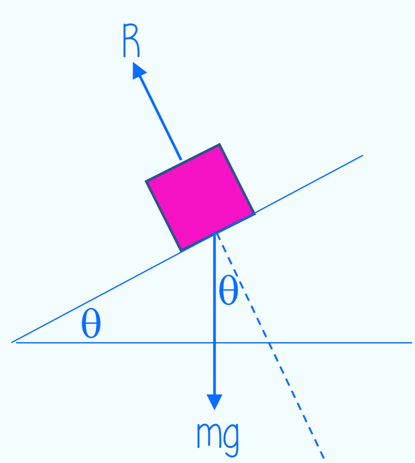
Component $F_y = F\sin\theta$ (away from the angle)

(or $= F\cos(90 - \theta)$)

WHAT DO I NEED TO KNOW

1. If a force is applied at an angle to the direction of motion, resolve it in two perpendicular directions to find the component of force that acts in the direction of motion OR use the triangle law for vector addition.

2. To solve problems on inclined planes, resolve parallel and perpendicular to the plane. REMEMBER, the normal reaction force acts at right angles to the plane, not vertically.



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PROJECTILES

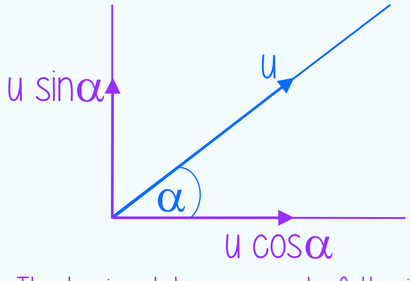
KEY WORDS & DEFINITIONS

1. **Projectile**
   A particle moving in a vertical plane under the action of gravity.
2. **Angle of Projection**
   The initial angle the projectile makes with the horizontal direction.
3. **Speed**
   The magnitude of the velocity, or the resultant velocities.
4. **Range**
   The horizontal distance that the particle travels.
5. **Time of Flight**
   The time taken for the projectile to hit the ground, or other horizontal surface, after being projected.

HORIZONTAL & VERTICAL COMPONENTS OF INITIAL VELOCITY

If a particle is projected with an initial velocity u, at an angle α above the horizontal, α is called 'The angle of projection'.

The velocity can be resolved into components that act horizontally and vertically.



The horizontal component of the initial velocity = $u \cos \alpha$
The vertical component of the initial velocity = $u \sin \alpha$

WHAT DO I NEED TO KNOW

1. The horizontal acceleration of a particle = 0
2. The horizontal velocity of a particle is constant. Therefore s = vt
3. The vertical acceleration a of a partical = g (constant)
4. To find the horizontal & vertical components of the initial velocity, resolve horizontally & vertically
5. When a projectile reaches its maximum height, the vertical component of velocity = 0
6. Acceleration due to gravity = $9.8 m/s^2$
   This does not depend on the mass of the object.
7. The degree of accuracy in your answers must be consistent with the values given in the question. I.e. if g = $10 m/s^2$ in the question, your answer should also be given to 1 sig. fig. Do not leave exact surd answers.
8. Many projectile problems can be solved by first using the vertical motion to find the total time taken.

POSSIBLE EQUATIONS TO DERIVE

For a particle projected with initial velocity U at angle α above horizontal and moving freely under gravity:

- Time of flight $= \dfrac{2U \sin \alpha}{g}$
- Time to reach greatest height $= \dfrac{U \sin \alpha}{g}$
- Range on horizontal plane $= \dfrac{U^2 \sin 2\alpha}{g}$
- Equation of trajectory:

$$y = x \tan \alpha - \frac{gx^2}{2U^2}(1 + \tan^2 \alpha)$$

where y is the vertical height of particle and x is the horizontal distance from the point of projection.

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APPLICATIONS OF FORCES

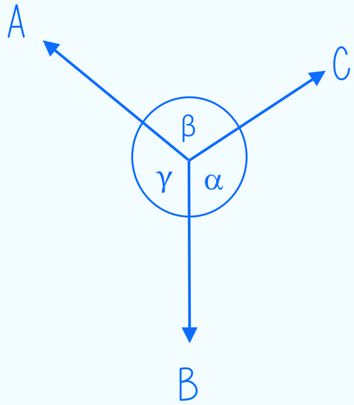
KEY WORDS & DEFINITIONS

Static Equilibrium

A particle is in static equilibrium if it is at rest and the resultant force acting upon it = 0

A rigid body is in static equilibrium if the body is stationary, the resultant force in any direction = 0 and the resultant moment = 0

LAMI'S THEOREM



$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

MODELLING

If a particle is attached separately to two strings, the tension can be different in each string.

If a smooth bead is threaded on a string, the tension in the string will be the same on both sides.

Unless connected particles are moving in the same direction, they must be considered separately.

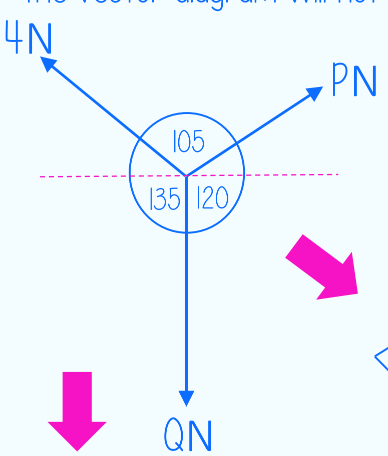
WHAT DO I NEED TO KNOW

1. The maximum value of the frictional force $F_{max} = \mu R$ is reached when the body being considered is on the point of moving. The body is then said to be in 'limiting equilibrium'.

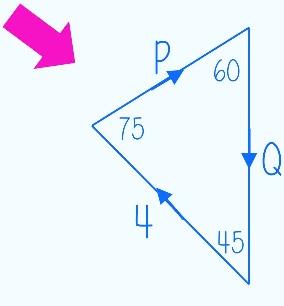
2. In general, the force of friction F is such that $F \leq \mu R$ and the direction of the frictional force is opposite to the direction in which the body would move, if the frictional force were absent.

3. To solve equilibrium problems, draw both a force diagram and a vector diagram.

4. If the angle between forces on a force diagram is θ, the angle between those forces in a triangle of forces is 180° - θ. The length of each side of the triangle is the magnitude of the force. (If the particle is not in equilibrium, the vector diagram will not be a closed triangle).



Method 2:
Use Sine Rule



Method 1:
Resolve horizontally & vertically

→ $P\cos 30 - 4\cos 45 = 0$

↑ $P\sin 30 + 4\sin 45 - Q = 0$

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FURTHER KINEMATICS

WHAT DO I NEED TO KNOW

1. To solve problems involving constant acceleration in 2 dimensions, use the SUVAT equations with vector components where
u is the initial velocity
a is the acceleration
v is the velocity at time t (t is a scalar)
r is the displacement at time t

2. To solve problems involving variable acceleration in 2 dimensions, use calculus with vectors by considering each function of time (the vector component) separately.

3. When integrating a vector for a variable acceleration problem, the constant of integration, c, will also be a vector.

4. To find constants of integration, look for initial conditions or boundary conditions.

5. Displacement, velocity & acceleration can be given using ***i-j*** notation, or as column vectors.

FORMULAE

The formula to find **the position vector, r,** of a particle starting at position $\mathbf{r}_0$ that is moving with constant velocity **v** is

$$\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{v}t$$

Constant acceleration vector equations:

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$

$$\boldsymbol{r} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$$

Calculus for variable acceleration:

Velocity, if displacement is a function of time:

$$\boldsymbol{v} = \frac{\mathrm{d}\boldsymbol{s}}{\mathrm{d}t}$$

$$\int (\boldsymbol{v})\,\mathrm{d}t = \boldsymbol{s}$$

Acceleration, if velocity is a function of time

$$\boldsymbol{a} = \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\boldsymbol{s}}{\mathrm{d}t^2}$$

$$\int (\boldsymbol{a})\,\mathrm{d}t = \boldsymbol{v}$$

DOT NOTATION & DIFFERENTIATING VECTORS

Dot notation is a shorthand for differentiation with respect to time.

$$\dot{x} = \frac{\mathrm{d}x}{\mathrm{d}t} \qquad \dot{y} = \frac{\mathrm{d}y}{\mathrm{d}t} \qquad \ddot{x} = \frac{\mathrm{d}^2x}{\mathrm{d}t^2} \qquad \ddot{y} = \frac{d^2y}{dt^2}$$

To differentiate a vector quantitiy in the form f(t)I + g(t)j, differentiate each function of time separately.

If $r = x\mathrm{i} + yj$, then $v = \frac{\mathrm{d}r}{\mathrm{d}t} = \dot{r} = \dot{x}\mathrm{i} + \dot{y}j$ and $a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2r}{\mathrm{d}t^2} = \ddot{r} = \ddot{x}\mathrm{i} + \ddot{y}j$