Pearson

Topics 1 & 2
Algebra and Quadratics

Bronze, Silver, Gold and Platinum Worksheets for AS level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. We have, however, put these questions together with the intention that students can complete them without a calculator. It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculators may not be used

The total mark for this section is 32

Q1

(a) Find the value of $8^{\frac{4}{3}}$

(2)

(b) Simplify $\dfrac{15x^{\frac{4}{3}}}{3x}$

(2)

(Total for Question 1 is 4 marks)

Q2

(a) Write down the value of $32^{\frac{1}{5}}$

(1)

(b) Simplify fully $(32x^5)^{-\frac{2}{5}}$

(3)

(Total for Question 2 is 4 marks)

Q3

(a) Simplify

$$\sqrt{50} - \sqrt{18}$$

giving your answer in the form $a\sqrt{2}$, where a is an integer.

(2)

(b) Hence, or otherwise, simplify

$$\frac{12\sqrt{3}}{\sqrt{50} - \sqrt{18}}$$

giving your answer in the form $b\sqrt{c}$, where b and c are integers and $b \neq 1$

(3)

(Total for Question 3 is 5 marks)

Q4

Given that $32\sqrt{2} = 2^a$, find the value of a

(Total for Question 4 is 3 marks)

Q5

(a) Evaluate $32^{\frac{3}{5}}$, giving your answer as an integer.

(2)

(b) Simplify fully $\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}}$

(2)

(Total for Question 5 is 4 marks)

Q6

Express 8^{2x+3} in the form 2^y, stating y in terms of x

(Total for Question 6 is 2 marks)

Q7

Complete the table below. The first one has been done for you.

For each statement you must state if it is always true, sometimes true or never true, giving a reason in each case.

Statement	Always True	Sometimes True	Never True	Reason
The quadratic equation $ax^2 + bx + c = 0, \quad (a \neq 0)$ has 2 real roots.		✓		It only has 2 real roots when $b^2 - 4ac > 0$. When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots.
(i) When a real value of x is substituted into $x^2 - 6x + 10$ the result is positive. (2)				
(ii) If $ax > b$ then $x > \frac{b}{a}$ (2)				
(iii) The difference between consecutive square numbers is odd. (2)				

(Total for Question 7 is 6 marks)

Q8

The equation $x^2 + 3px + p = 0$, where p is a non-zero constant, has equal roots.

Find the value of p.

(Total for Question 8 is 4 marks)

End of Questions

Bronze Mark Scheme

Q1

Question number	Scheme	Marks
	(a) Attempt $\sqrt[3]{8}$ or $\sqrt[3]{(8^4)}$	M1
	$= \underline{16}$	A1 (2)
	(b) $\underline{5x^{\frac{1}{3}}}$ $\quad 5,\ x^{\frac{1}{3}}$	B1, B1 (2)
		4

(a)

M1 for: 2 (on its own) or $\left(2^3\right)^{\frac{4}{3}}$ or $\sqrt[3]{8}$ or $\left(\sqrt[3]{8}\right)^4$ or 2^4 or $\sqrt[3]{8^4}$ or $\sqrt[3]{4096}$

8^3 or 512 or $(4096)^{\frac{1}{3}}$ is M0

A1 for 16 only

(b)

1st B1 for 5 on its own or × something.

So e.g. $\frac{5x^{\frac{4}{3}}}{x}$ is B1 But $5^{\frac{1}{3}}$ is B0

An expression showing cancelling is not sufficient

(see first expression of QC0184500123945 the mark is scored for the second expression)

2nd B1 for $x^{\frac{1}{3}}$

Can use ISW (incorrect subsequent working)

e.g $5x^{\frac{4}{3}}$ scores B1B0 but it may lead to $\sqrt[3]{5x^4}$ which we ignore as ISW.

Correct answers only score full marks in both parts.

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Q2

Question Number	Scheme	Marks
	(a) $32^{\frac{1}{5}} = 2$	B1 (1)
	(b) For 2^{-2} or $\frac{1}{4}$ or $\left(\frac{1}{2}\right)^2$ or 0.25 as coefficient of x^k, for any value of k including $k = 0$	M1
	Correct index for x so Ax^{-2} or $\frac{A}{x^2}$ o.e. for any value of A	B1
	$= \frac{1}{4x^2}$ or $0.25x^{-2}$	A1 cao (3) **4 Marks**

Notes

(a) B1 Answer 2 must be in part (a) for this mark

(b) Look at their final answer

M1 For 2^{-2} or $\frac{1}{4}$ or $\left(\frac{1}{2}\right)^2$ or 0.25 in their answer as coefficient of x^k for numerical value of k (including $k = 0$) so final answer $\frac{1}{4}$ is M1 B0 A0

B1 Ax^{-2} or $\frac{A}{x^2}$ or equivalent e.g. $Ax^{-\frac{10}{5}}$ or $Ax^{\frac{50}{25}}$ i.e. correct power of x seen in final answer

May have a bracket provided it is $(Ax)^{-2}$ or $\left(\frac{A}{x}\right)^2$

A1 $\frac{1}{4x^2}$ or $\frac{1}{4}x^{-2}$ or $0.25x^{-2}$ oe but must be correct power **and** coefficient combined correctly and must not be followed by a different wrong answer.

Poor bracketing: $2x^{-2}$ earns M0 B1 A0 as correct power of x is seen in this solution (They can recover if they follow this with $\frac{1}{4x^2}$ etc)

Special case $(2x)^{-2}$ as a **final** answer and $\left(\frac{1}{2x}\right)^2$ can have M0 B1 A0 if the correct expanded answer is not seen

The correct answer $\frac{1}{4x^2}$ etc. followed by $\left(\frac{1}{2x}\right)^2$ or $(2x)^{-2}$, treat $\frac{1}{4x^2}$ as final answer so M1 B1 A1 isw

But the correct answer $\frac{1}{4x^2}$ etc clearly followed by the wrong $2x^{-2}$ or $4x^{-2}$, gets M1 B1 A0 do not ignore subsequent wrong work here

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Q3

Question Number	Scheme	Notes	Marks
(a)	$\sqrt{50}-\sqrt{18}=5\sqrt{2}-3\sqrt{2}$	$\sqrt{50}=5\sqrt{2}$ or $\sqrt{18}=3\sqrt{2}$ **and** the other term in the form $k\sqrt{2}$. This mark may be implied by the correct answer $2\sqrt{2}$	M1
	$=2\sqrt{2}$	Or $a=2$	A1
			[2]
(b) **WAY 1**	$\frac{12\sqrt{3}}{\sqrt{50}-\sqrt{18}}=\frac{12\sqrt{3}}{"2"\sqrt{2}}$	Uses part (a) by replacing denominator by their $a\sqrt{2}$ where a is numeric. This is all that is required for this mark.	M1
	$=\frac{12\sqrt{3}}{"2"\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{12\sqrt{6}}{4}$	Rationalises the denominator by a correct method e.g. multiplies numerator and denominator by $k\sqrt{2}$ to obtain a multiple of $\sqrt{6}$. Note that multiplying numerator and denominator by $2\sqrt{2}$ or $-2\sqrt{2}$ is quite common and is acceptable for this mark. May be implied by a correct answer. **This is dependent on the first M1.**	dM1
	$=3\sqrt{6}$ or $b=3, c=6$	Cao and cso	A1
			[3]
(b) **WAY 2**	$\frac{12\sqrt{3}}{\sqrt{50}-\sqrt{18}}\times\frac{\sqrt{50}+\sqrt{18}}{\sqrt{50}+\sqrt{18}}$ **or** $\frac{12\sqrt{3}}{5\sqrt{2}-3\sqrt{2}}\times\frac{5\sqrt{2}+3\sqrt{2}}{5\sqrt{2}+3\sqrt{2}}$	For rationalising the denominator by a correct method i.e. multiplying numerator and denominator by $k\left(\sqrt{50}+\sqrt{18}\right)$	M1
	$\frac{60\sqrt{6}+36\sqrt{6}}{50-18}$	For replacing numerator by $\alpha\sqrt{6}+\beta\sqrt{6}$. **This is dependent on the first M1 and there is no need to consider the denominator for this mark.**	dM1
	$=3\sqrt{6}$ or $b=3, c=6$	Cao and cso	A1
			[3]
(b) **WAY 3**	$\frac{12\sqrt{3}}{\sqrt{50}-\sqrt{18}}=\frac{12\sqrt{3}}{"2"\sqrt{2}}$	Uses part (a) by replacing denominator by their $a\sqrt{2}$ where a is numeric. This is all that is required for this mark.	M1
	$=\frac{12\sqrt{3}}{2\sqrt{2}}=\frac{6\sqrt{3}}{\sqrt{2}}=\frac{\sqrt{108}}{\sqrt{2}}=\sqrt{54}=\sqrt{9}\sqrt{6}$	Cancels to obtain a multiple of $\sqrt{6}$. **This is dependent on the first M1.**	dM1
	$=3\sqrt{6}$ Or $b=3, c=6$	Cao and cso	A1
			[3]
(b) **WAY 4**	$\frac{12\sqrt{3}}{\sqrt{50}-\sqrt{18}}=\frac{12\sqrt{3}}{"2"\sqrt{2}}$	Uses part (a) by replacing denominator by their $a\sqrt{2}$ where a is numeric. This is all that is required for this mark.	M1
	$\left(\frac{12\sqrt{3}}{"2"\sqrt{2}}\right)^2=\frac{432}{8}$		
	$\sqrt{54}=\sqrt{9}\sqrt{6}$	Obtains a multiple of $\sqrt{6}$. **This is dependent on the first M1.**	dM1
	$=3\sqrt{6}$ Or $b=3, c=6$	Cao and cso (do not allow $\pm 3\sqrt{6}$)	A1
			5 marks

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Q4

Question Number	Scheme	Marks
Q	$32=2^5$ or $2048=2^{11}$, $\sqrt{2}=2^{1/2}$ or $\sqrt{2048}=(2048)^{\frac{1}{2}}$ $a=\frac{11}{2}$ $\left(\text{or } 5\frac{1}{2} \text{ or } 5.5\right)$	B1, B1 B1 [3]
	1st B1 for $32=2^5$ or $2048=2^{11}$ This should be explicitly seen: $32\sqrt{2}=2^a$ followed by $2^5\sqrt{2}=2^a$ is OK Even writing $32\times2=2^5\times2\left(=2^6\right)$ is OK but simply writing $32\times2=2^6$ is NOT 2nd B1 for $2^{\frac{1}{2}}$ or $(2048)^{\frac{1}{2}}$ seen. This mark may be implied 3rd B1 for answer as written. **Need** $a=\ldots$ so $2^{\frac{11}{2}}$ is B0 $a=\frac{11}{2}$ $\left(\text{or } 5\frac{1}{2} \text{ or } 5.5\right)$ with no working scores full marks. If $a=5.5$ seen then award 3/3 unless it is clear that the value follows from totally incorrect work. Part solutions: e.g. $2^5\sqrt{2}$ scores the first B1. <u>Special case</u>: If $\sqrt{2}=2^{1/2}$ is not explicitly seen, but the final answer includes $\frac{1}{2}$, e.g. $a=2\frac{1}{2}$, $a=4\frac{1}{2}$, the second B1 is given by implication.	

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Q5

Question Number	Scheme	Marks
(a)	$\left\{(32)^{\frac{3}{5}}\right\} = \left(\sqrt[5]{32}\right)^3$ or $\sqrt[5]{(32)^3}$ or 2^3 or $\sqrt[5]{32768}$	M1
	$= 8$	A1
		[2]
(b)	$\left\{\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}}\right\} = \left(\frac{4}{25x^4}\right)^{\frac{1}{2}}$ or $\left(\frac{5x^2}{2}\right)^{-1}$ or $\frac{1}{\left(\frac{25x^4}{4}\right)^{\frac{1}{2}}}$ *See notes below*	M1
	$= \frac{2}{5x^2}$ or $\frac{2}{5}x^{-2}$ *See notes for other alternatives*	A1
		[2]
		4

Notes

(a)

M1: for a full correct interpretation of the fractional power. **Note:** $5\times(32)^3$ is M0.

A1: for 8 only.

Note: Award M1A1 for writing down 8.

(b)

M1: For use of $\frac{1}{2}$ OR use of -1

Use of $\frac{1}{2}$: Candidate needs to demonstrate the they have rooted all three elements in their bracket.

Use of -1: Either Candidate has $\frac{1}{\text{Bracket}}$ or $\left(\frac{Ax^C}{B}\right)$ becomes $\left(\frac{B}{Ax^C}\right)$.

Allow M1 for...

- $\left(\frac{4}{25x^4}\right)^{\frac{1}{2}}$ or $\left(\frac{5x^2}{2}\right)^{-1}$ or $\frac{1}{\left(\frac{25x^4}{4}\right)^{\frac{1}{2}}}$ or $\sqrt{\left(\frac{4}{25x^4}\right)}$ or $\frac{1}{\sqrt{\left(\frac{25x^4}{4}\right)}}$ or $\left(\frac{\frac{1}{25x^4}}{\frac{1}{4}}\right)^{\frac{1}{2}}$ or $\frac{\frac{1}{5x^2}}{\frac{1}{2}}$ or $\frac{\frac{1}{5}x^{-2}}{\frac{1}{2}}$

  or $-\left(\frac{5x^2}{2}\right)$ or $\left(\frac{-5x^{-2}}{-2}\right)$ or $-\left(\frac{5x^{-2}}{2}\right)$ or $\frac{5x^{-2}}{2}$
- $\left(\frac{4}{25x^4}\right)^K$ or $\left(\frac{5x^2}{2}\right)^C$ where K, C are any powers including 1.

A1: for either $\frac{2}{5x^2}$ or $\frac{2}{5}x^{-2}$ or $0.4x^{-2}$ or $\frac{0.4}{x^2}$.

Note: $\left(\sqrt{\left(\frac{25x^4}{4}\right)}\right)^{-1}$ is not enough work by itself for the method mark.

Note: A final answer of $\frac{1}{\frac{5}{2}x^2}$ or $\frac{1}{2\frac{1}{2}x^2}$ or $\frac{1}{2.5x^2}$ is A0.

Note: Also allow $\pm\frac{2}{5x^2}$ or $\pm\frac{2}{5}x^{-2}$ or $\pm 0.4x^{-2}$ or $\pm\frac{0.4}{x^2}$ for A1.

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Q6

Question Number	Scheme	Marks
	$(8^{2x+3} = (2^3)^{2x+3}) = 2^{3(2x+3)}$ or 2^{ax+b} with $a=6$ or $b=9$	M1
	$= 2^{6x+9}$ *or* $= 2^{3(2x+3)}$ as **final** answer with no errors or $(y=)6x+9$ or $3(2x+3)$	A1 [2]
		2 marks
	Notes	
	M1: Uses $8 = 2^3$, and **multiplies** powers $3(2x+3)$. Does **not** add powers. (Just $8 = 2^3$ or $8^{\frac{1}{3}} = 2$ is M0)	
	A1: Either 2^{6x+9} *or* $= 2^{3(2x+3)}$ or $(y=)6x+9$ or $3(2x+3)$	
	Note: Examples: 2^{6x+3} scores M1A0	
	: $8^{2x+3} = (2^3)^{2x+3} = 2^{3+2x+3}$ gets M0A0	
	Special case: : $= 2^{6x}2^9$ without seeing as single power M1A0	
	Alternative method using logs: $8^{2x+3} = 2^y \Rightarrow (2x+3)\log 8 = y\log 2 \Rightarrow y = \dfrac{(2x+3)\log 8}{\log 2}$	**M1**
	So $(y=)6x+9$ or $3(2x+3)$	**A1** [2]

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Q7

Question	Scheme	Marks	AOs
(i)	$x^2-6x+10=(x-3)^2+1$	M1	2.1
	Deduces "always true" as $(x-3)^2 \geqslant 0 \Rightarrow (x-3)^2+1 \geqslant 1$ and so is always positive	A1	2.2a
		(2)	
(ii)	For an explanation that it need not (always) be true This could be if $a<0$ then $ax>b \Rightarrow x<\frac{b}{a}$	M1	2.3
	States 'sometimes' and explains if $a>0$ then $ax>b \Rightarrow x>\frac{b}{a}$ if $a<0$ then $ax>b \Rightarrow x<\frac{b}{a}$	A1	2.4
		(2)	
(iii)	Difference $=(n+1)^2-n^2=2n+1$	M1	3.1a
	Deduces "Always true" as $2n+1$ = (even +1) = odd	A1	2.2a
		(2)	
			(6 marks)

Notes:

(i)

M1: Attempts to complete the square or any other valid reason. Allow for a graph of $y=x^2-6x+10$ or an attempt to find the minimum by differentiation

A1: States always true with a valid reason for their method

(ii)

M1: For an explanation that it need not be true (sometimes). This could be if $a<0$ then $ax>b \Rightarrow x<\frac{b}{a}$ or simply $-3x>6 \Rightarrow x<-2$

A1: Correct statement (sometimes true) and explanation

(iii)

M1: Sets up the proof algebraically.

For example by attempting $(n+1)^2-n^2=2n+1$ or $m^2-n^2=(m-n)(m+n)$ with $m=n+1$

A1: States always true with reason and proof

Accept a proof written in words. For example

If integers are consecutive, one is odd and one is even

When squared odd × odd = odd and even × even = even

The difference between odd and even is always odd, hence always true

Score M1 for two of these lines and A1 for a good proof with all three lines or equivalent.

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Q8

Question Number	Scheme	Marks
Q	b^2-4ac attempted, in terms of p. $(3p)^2-4p=0$ o.e. Attempt to solve for p e.g. $p(9p-4)=0$ Must potentially lead to $p=k,\ k\neq 0$ $p=\frac{4}{9}$ (Ignore $p=0$, if seen)	M1 A1 M1 A1cso [4]
	1st M1 for an attempt to substitute into b^2-4ac or $b^2=4ac$ with b or c correct Condone x's in one term only. This can be inside a square root as part of the quadratic formula for example. **Use of inequalities can score the M marks only** 1st A1 for any correct equation: $(3p)^2-4\times1\times p=0$ or better 2nd M1 for an attempt to factorize or solve their quadratic expression in p. Method must be sufficient to lead to their $p=\frac{4}{9}$. Accept factors or use of quadratic formula or $\left(p\pm\frac{2}{9}\right)^2=k^2$ (o.e. eg) $\left(3p\pm\frac{2}{3}\right)^2=k^2$ or equivalent work on <u>their</u> eqn. $9p^2=4p\Rightarrow\frac{9p^{\cancel{2}}}{\cancel{p}}=4$ which would lead to $9p=4$ is OK for this 2nd M1 ALT <u>Comparing coefficients</u> M1 for $(x+\alpha)^2=x^2+\alpha^2+2\alpha x$ and A1 for a correct equation eg $3p=2\sqrt{p}$ M1 for forming solving leading to $\sqrt{p}=\frac{2}{3}$ or better <u>Use of quadratic/discriminant formula (or any formula) Rule for awarding M mark</u> If the formula is quoted accept some correct substitution leading to a partially correct expression. If the formula is not quoted only award for a fully correct expression using their values.	

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Silver Questions

Calculators may not be used

The total mark for this section is 35

Q1

(a) Find the value of $16^{\frac{1}{4}}$

(2)

(b) Simplify $x(2x^{-\frac{1}{4}})^4$

(2)

(Total for Question 1 is 4 marks)

Q2

Show that $\frac{2}{\sqrt{(12)}-\sqrt{(8)}}$ can be written in the form $\sqrt{a}-\sqrt{b}$, where a and b are integers.

(Total for Question 2 is 5 marks)

Q3

Solve

(a) $2^y = 8$

(1)

(b) $2^x \times 4^{x+1} = 8$

(4)

(Total for Question 3 is 5 marks)

Q4

Given

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}$$

express y as a function of x

(Total for Question 4 is 3 marks)

Q5

Find, using algebra, all real solutions to the equation

(i) $16a^2 = 2\sqrt{a}$

(4)

(ii) $b^4 + 7b^2 - 18 = 0$

(4)

(Total for Question 5 is 8 marks)

Q6

The equation $kx^2 + 4kx + 3 = 0$, where k is a constant, has no real roots.

Prove that

$$0 \square k < \frac{3}{4}$$

(Total for Question 6 is 4 marks)

Q7

$$f(x) = x^2 + (k+3)x + k$$

where k is a real constant.

(a) Find the discriminant of f(x) in terms of k.

(2)

(b) Show that the discriminant of f(x) can be expressed in the form $(k + a)^2 + b$, where a and b are integers to be found.

(2)

(c) Show that, for all values of k, the equation $f(x) = 0$ has real roots.

(2)

(Total for Question 7 is 6 marks)

End of Questions

Silver Mark Scheme

Q1

Question Number	Scheme	Marks
(a)	$16^{\frac{1}{4}} = 2$ or $\frac{1}{16^{\frac{1}{4}}}$ or better $\left(16^{-\frac{1}{4}} =\right) \frac{1}{2}$ or 0.5 (ignore $\pm$)	M1 A1 **(2)**
(b)	$\left(2x^{-\frac{1}{4}}\right)^4 = 2^4 x^{-\frac{4}{4}}$ or $\frac{2^4}{x^{\frac{4}{4}}}$ or equivalent $x\left(2x^{-\frac{1}{4}}\right)^4 = 2^4$ or 16	M1 A1 cao **(2)** **4**
	Notes	
(a)	M1 for a correct statement dealing with the $\frac{1}{4}$ or the $-$ power This may be awarded if 2 is seen or for reciprocal of their $16^{\frac{1}{4}}$ s.c ¼ is M1 A0 , also 2^{-1} is M1 A0 $\pm\frac{1}{2}$ is not penalised so M1 A1	
(b)	M1 for correct use of the power 4 on both the 2 and the x terms A1 for cancelling the x and simplifying to one of these two forms. Correct answers with no working get full marks	

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Q2

Question Number	Scheme	Marks
	$\left\{\frac{2}{\sqrt{12}-\sqrt{8}}\right\} = \frac{2}{\left(\sqrt{12}-\sqrt{8}\right)} \times \frac{\left(\sqrt{12}+\sqrt{8}\right)}{\left(\sqrt{12}+\sqrt{8}\right)}$ Writing this is sufficient for M1.	M1
	$= \frac{\left\{2\left(\sqrt{12}+\sqrt{8}\right)\right\}}{12-8}$ For $12-8$. This mark can be implied.	A1
	$= \frac{2\left(2\sqrt{3}+2\sqrt{2}\right)}{12-8}$	B1 B1
	$= \sqrt{3}+\sqrt{2}$	A1 cso
		5

Notes

M1: for a correct method to rationalise the denominator.

1st A1: $\left(\sqrt{12}-\sqrt{8}\right)\left(\sqrt{12}+\sqrt{8}\right) \rightarrow 12-8$ or $\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right) \rightarrow 3-2$

1st B1: for $\sqrt{12}=2\sqrt{3}$ or $\sqrt{48}=4\sqrt{3}$ seen or implied in candidate's working.

2nd B1: for $\sqrt{8}=2\sqrt{2}$ or $\sqrt{32}=4\sqrt{2}$ seen or implied in candidate's working.

2nd A1: for $\sqrt{3}+\sqrt{2}$. Note: $\frac{\sqrt{3}+\sqrt{2}}{1}$ as a final answer is A0.

Note: The first accuracy mark is dependent on the first method mark being awarded.

Note: $\frac{1}{2}\sqrt{12}+\frac{1}{2}\sqrt{8}=\sqrt{3}+\sqrt{2}$ with no intermediate working implies the B1B1 marks.

Note: $\sqrt{12}=\sqrt{4}\sqrt{3}$ or $\sqrt{8}=\sqrt{4}\sqrt{2}$ are not sufficient for the B1 marks.

Note: A candidate who writes down (by misread) $\sqrt{18}$ for $\sqrt{8}$ can potentially obtain M1A0B1B1A0, where the 2nd B1 will be awarded for $\sqrt{18}=3\sqrt{2}$ or $\sqrt{72}=6\sqrt{2}$

Note: The final accuracy mark is for a correct solution only.

Alternative 1 solution

$\left\{\frac{2}{\sqrt{12}-\sqrt{8}}\right\} = \frac{2}{\left(2\sqrt{3}-2\sqrt{2}\right)}$ B1 B1

$= \frac{1}{\left(\sqrt{3}-\sqrt{2}\right)} \times \frac{\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}+\sqrt{2}\right)}$ M1

$= \frac{\left\{\left(\sqrt{3}+\sqrt{2}\right)\right\}}{3-2}$ A1 for $3-2$

$= \sqrt{3}+\sqrt{2}$ A1

Please record the marks in the relevant places on the mark grid.

Alternative 2 solution

$\left\{\frac{2}{\sqrt{12}-\sqrt{8}}\right\} = \frac{2}{\left(2\sqrt{3}-2\sqrt{2}\right)} = \frac{1}{\left(\sqrt{3}-\sqrt{2}\right)} = \sqrt{3}+\sqrt{2}$, or $\frac{2}{\left(2\sqrt{3}-2\sqrt{2}\right)} = \sqrt{3}+\sqrt{2}$

with no incorrect working seen is awarded M1A1B1B1A1.

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Q3

Question Number	Scheme	Notes	Marks
(a)	$2^y = 8 \Rightarrow y = 3$	Cao (Can be implied i.e. by 2^3)	B1
	(Alternative: Takes logs base 2: $\log_2 2^y = \log_2 8 \Rightarrow y\log_2 2 = 3\log_2 2 \Rightarrow y = 3$)		
			(1)
(b)	$8 = 2^3$	Replaces 8 by 2^3 (May be implied)	M1
	$4^{x+1} = (2^2)^{x+1}$ or $(2^{x+1})^2$	Replaces 4 by 2^2 **correctly.**	M1
	$2^{3x+2} = 2^3 \Rightarrow 3x+2 = 3 \Rightarrow x = \frac{1}{3}$	M1: Adds their powers of 2 on the lhs and puts this equal to 3 leading to a solution for x. A1: $x = \frac{1}{3}$ or $x = 0.\dot{3}$ or awrt 0.333	M1A1
			(4)
(b) Way 2	$4^{x+1} = 4 \times 4^x$	Obtains 4^{x+1} in terms of 4^x **correctly**	M1
	$2^x \times 4^x = 8^x$	Combines their 2^x and 4^x **correctly**	M1
	$4 \times 8^x = 8 \Rightarrow 8^x = 2 \Rightarrow x = \frac{1}{3}$	M1: Solves $8^x = k$ leading to a solution for x. A1: $x = \frac{1}{3}$ or $x = 0.\dot{3}$ or awrt 0.333	M1A1
			[5]

Q4

Part	Working or answer an examiner might expect to see	Mark	Notes
	$2^x \times (2^2)^y = 2^{-\frac{3}{2}} \Rightarrow 2^{x+2y} = 2^{-\frac{3}{2}}$	M1	This mark is given for writing all terms in the same base and applying an index law
	$x + 2y = -\frac{3}{2}$	M1	This mark is given for writing an equation to link x and y
	$y = -\frac{1}{2}x - \frac{3}{4}$	A1	This mark is given for rearranging to find a correct expression of y as a function of x
			(Total 3 marks)

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Q5

Question	Scheme		Marks	AOs
(i)	$16a^2 = 2\sqrt{a} \Rightarrow a^{\frac{3}{2}} = \frac{1}{8}$	$16a^2 - 2\sqrt{a} = 0$ $\Rightarrow 2a^{\frac{1}{2}}\left(8a^{\frac{3}{2}} - 1\right) = 0$ $\Rightarrow a^{\frac{3}{2}} = \frac{1}{8}$	M1	1.1b
	$\Rightarrow a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$	$\Rightarrow a = \left(\frac{1}{8}\right)^{\frac{2}{3}}$	M1	1.1b
	$\Rightarrow a = \frac{1}{4}$	$\Rightarrow a = \frac{1}{4}$	A1	1.1b
	Deduces that $a = 0$ is a solution		B1	2.2a
			(4)	
(ii)	$b^4 + 7b^2 - 18 = 0 \Rightarrow \left(b^2 + 9\right)\left(b^2 - 2\right) = 0$		M1	1.1b
	$b^2 = -9, 2$		A1	1.1b
	$b^2 = k \Rightarrow b = \sqrt{k}, k > 0$		dM1	2.3
	$b = \sqrt{2}$, $-\sqrt{2}$ only		A1	1.1b
			(4)	
				(8 marks)

Notes

(i)

M1: Combines the two algebraic terms to reach $a^{\pm\frac{3}{2}} = C$ or equivalent such as $\left(\sqrt{a}\right)^3 = C$ $\left(C \neq 0\right)$

An alternative is via squaring and combining the algebraic terms to reach $a^{\pm 3} = k, k > 0$

E.g. $...a^4 = ...a \Rightarrow a^{\pm 3} = k$ or $...a^4 = ...a \Rightarrow ...a^4 - ...a = 0 \Rightarrow ...a\left(a^3 - ...\right) = 0 \Rightarrow a^3 = ...$

Allow for slips on coefficients.

M1: Undoes the indices correctly for their $a^{\frac{m}{n}} = C$ (So M0 M1 A0 is possible)
You may even see logs used.

A1: $a = \frac{1}{4}$ and no other solutions apart from 0 Accept exact equivalents Eg 0.25

B1: Deduces that $a = 0$ is a solution.

(ii)

M1: Attempts to solve as a quadratic equation in b^2
Accept $\left(b^2 + m\right)\left(b^2 + n\right) = 0$ with $mn = \pm 18$ or solutions via the use of the quadratic formula Also allow candidates to substitute in another variable, say $u = b^2$ and solve for u

A1: Correct solution. Allow for $b^2 = 2$ or $u = 2$ with no incorrect solution given.
Candidates can choose to omit the solution $b^2 = -9$ or $u = -9$ and so may not be seen

dM1: Finds at least one solution from their $b^2 = k \Rightarrow b = \sqrt{k}, k > 0$. Allow $b = 1.414$

A1: $b = \sqrt{2}$, $-\sqrt{2}$ only. The solution asks for real values so if $3i$ is given then score A0

Pearson

Answers with minimal or no working:

In part **(i)**

- no working, just answer(s) with they can score the B1
- If they square and proceed to the quartic equation $256a^4 = 4a$ oe, and then write down the answers they can have access to all marks.

In part (ii)

- Accept for 4 marks $b^2 = 2 \Rightarrow b = \pm\sqrt{2}$
- No working, no marks.

Q6

Question	Scheme	Marks	AOs
	Realises that $k = 0$ will give no real roots as equation becomes $3 = 0$ (proof by contradiction)	B1	3.1a
	(For $k \neq 0$) quadratic has no real roots provided $b^2 < 4ac$ so $16k^2 < 12k$	M1	2.4
	$4k(4k-3) < 0$ with attempt at solution	M1	1.1b
	So $0 < k < \frac{3}{4}$, which together with $k = 0$ gives $0 \leqslant k < \frac{3}{4}$ *	A1*	2.1
			(4 marks)

Notes

B1 : Explains why $k = 0$ gives no real roots

M1 : Considers discriminant to give quadratic inequality – does not need the $k \neq 0$ for this mark

M1 : Attempts solution of quadratic inequality

A1*: Draws conclusion, which is a printed answer, with no errors (dependent on all three previous marks)

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Q7

Question Number	Scheme	Marks
(a)	Discriminant: $b^2 - 4ac = (k+3)^2 - 4k$ or equivalent	M1 A1 (2)
(b)	$(k+3)^2 - 4k = k^2 + 2k + 9 = (k+1)^2 + 8$	M1 A1 (2)
(c)	For real roots, $b^2 - 4ac \geq 0$ or $b^2 - 4ac > 0$ or $(k+1)^2 + 8 > 0$ $(k+1)^2 \geq 0$ for all k, so $b^2 - 4ac > 0$, so roots are real for all k (or equiv.)	M1 A1 cso (2) **6**
	Notes (a) M1: attempt to find discriminant – substitution is required If formula $b^2 - 4ac$ is seen at least 2 of a, b and c must be correct If formula $b^2 - 4ac$ is **not** seen all 3 of a, b and c must be correct Use of $b^2 + 4ac$ is M0 A1: correct unsimplified (b) M1: Attempt at completion of square (see earlier notes) A1: both correct (no ft for this mark) (c) M1: States condition as on scheme **or** attempts to explain that their $(k+1)^2 + 8$ is greater than 0 A1: The final mark (A1cso) requires $(k+1)^2 \geq 0$ and conclusion. We will allow $(k+1)^2 > 0$ (or word positive) also allow $b^2 - 4ac \geq 0$ and conclusion.	

Pearson

Gold Questions

Calculators may not be used

The total mark for this section is 33

Q1

Express 9^{3x+1} in the form 3^y, giving y in the form $ax + b$, where a and b are constants.

(Total for Question 1 is 2 marks)

Q2

The equation $x^2 + (k - 3)x + (3 - 2k) = 0$, where k is a constant, has two distinct real roots.

(a) Show that k satisfies

$$k^2 + 2k - 3 > 0$$

(3)

(b) Find the set of possible values of k.

(4)

(Total for Question 2 is 7 marks)

Q3

Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,

(a) show that $q^2 + 8q < 0$.

(2)

(b) Hence find the set of possible values of q.

(3)

(Total for Question 3 is 5 marks)

Q4

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(i) Solve the equation

$$x\sqrt{2} - \sqrt{18} = x$$

writing the answer as a surd in simplest form.

(3)

(ii) Solve the equation

$$4^{3x-2} = \frac{1}{2\sqrt{2}}$$

(3)

(Total for Question 4 is 6 marks)

Q5

Given that $y = 2^x$,

(a) express 4^x in terms of y.

(1)

(b) Hence, or otherwise, solve

$$8(4^x) - 9(2^x) + 1 = 0$$

(4)

(Total for Question 5 is 5 marks)

Pearson

Q6

$$f(x) = x^2 - 8x + 19$$

(a) Express $f(x)$ in the form $(x + a)^2 + b$, where a and b are constants.

(2)

The curve C with equation $y = f(x)$ crosses the y-axis at the point P and has a minimum point at the point Q.

(b) Sketch the graph of C showing the coordinates of point P and the coordinates of point Q.

(3)

(c) Find the distance PQ, writing your answer as a simplified surd.

(3)

(Total for Question 6 is 8 marks)

End of Questions

Gold Mark Scheme

Q1

Question Number	Scheme	Notes	Marks
	9^{3x+1} = for example $3^{2(3x+1)}$ or $\left(3^2\right)^{3x+1}$ or $\left(3^{(3x+1)}\right)^2$ or $3^{3x+1}\times 3^{3x+1}$ or $(3\times 3)^{3x+1}$ or $3^2\times\left(3^2\right)^{3x}$ or $\left(9^{\frac{1}{2}}\right)^y$ or $9^{\frac{1}{2}y}$ or $y=2(3x+1)$	Expresses 9^{3x+1} correctly as a power of 3 or expresses 3^y correctly as a power of 9 or expresses y correctly in terms of x **(This mark is <u>not</u> for just $3^2=9$)**	M1
	$=3^{6x+2}$ or $y=6x+2$ or $a=6$, $b=2$	Cao (isw if necessary)	A1
	Providing there is no incorrect work, allow sight of $6x+2$ to score both marks Correct answer only implies both marks **Special case: 3^{6x+1} only scores M1A0**		
			[2]
	Alternative using logs		
	$9^{3x+1}=3^y \Rightarrow \log 9^{3x+1}=\log 3^y$		
	$(3x+1)\log 9=y\log 3$	Use power law correctly on both sides	M1
	$y=\dfrac{\log 9}{\log 3}(3x+1)$		
	$y=6x+2$	cao	A1
			2 marks

Q2

Question	Scheme	Marks	AOs
	Realises that $k=0$ will give no real roots as equation becomes $3=0$ (proof by contradiction)	B1	3.1a
	(For $k\neq 0$) quadratic has no real roots provided $b^2<4ac$ so $16k^2<12k$	M1	2.4
	$4k(4k-3)<0$ with attempt at solution	M1	1.1b
	So $0<k<\frac{3}{4}$, which together with $k=0$ gives $0\leqslant k<\frac{3}{4}$ *	A1*	2.1
			(4 marks)

Notes

B1 : Explains why $k=0$ gives no real roots
M1 : Considers discriminant to give quadratic inequality – does not need the $k\neq 0$ for this mark
M1 : Attempts solution of quadratic inequality
A1*: Draws conclusion, which is a printed answer, with no errors (dependent on all three previous marks)

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Q3

Question Number	Scheme	Marks
Q	b^2-4ac attempted, in terms of p. $(3p)^2-4p=0$ o.e. Attempt to solve for p e.g. $p(9p-4)=0$ Must potentially lead to $p=k,\ k\neq 0$ $p=\frac{4}{9}$ (Ignore $p=0$, if seen)	M1 A1 M1 A1cso [4]
	1st M1 for an attempt to substitute into b^2-4ac or $b^2=4ac$ with b or c correct Condone x's in one term only. This can be inside a square root as part of the quadratic formula for example. **Use of inequalities can score the M marks only** 1st A1 for any correct equation: $(3p)^2-4\times1\times p=0$ or better 2nd M1 for an attempt to factorize or solve their quadratic expression in p. Method must be sufficient to lead to their $p=\frac{4}{9}$. Accept factors or use of quadratic formula or $\left(p\pm\frac{2}{9}\right)^2=k^2$ (o.e. eg) $\left(3p\pm\frac{2}{3}\right)^2=k^2$ or equivalent work on their eqn. $9p^2=4p\Rightarrow\frac{9p^{\cancel{2}}}{\cancel{p}}=4$ which would lead to $9p=4$ is OK for this 2nd M1 ALT Comparing coefficients M1 for $(x+\alpha)^2=x^2+\alpha^2+2\alpha x$ and A1 for a correct equation eg $3p=2\sqrt{p}$ M1 for forming solving leading to $\sqrt{p}=\frac{2}{3}$ or better Use of quadratic/discriminant formula (or any formula) Rule for awarding M mark If the formula is quoted accept some correct substitution leading to a partially correct expression. If the formula is not quoted only award for a fully correct expression using their values.	

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Q4

Question	Scheme	Marks	AOs
(i)	$x\sqrt{2}-\sqrt{18}=x \Rightarrow x\left(\sqrt{2}-1\right)=\sqrt{18} \Rightarrow x=\dfrac{\sqrt{18}}{\sqrt{2}-1}$	M1	1.1b
	$\Rightarrow x=\dfrac{\sqrt{18}}{\sqrt{2}-1}\times\dfrac{\sqrt{2}+1}{\sqrt{2}+1}$	dM1	3.1a
	$x=\dfrac{\sqrt{18}\left(\sqrt{2}+1\right)}{1}=6+3\sqrt{2}$	A1	1.1b
		(3)	
(ii)	$4^{3x-2}=\dfrac{1}{2\sqrt{2}} \Rightarrow 2^{6x-4}=2^{-\frac{3}{2}}$	M1	2.5
	$6x-4=-\dfrac{3}{2} \Rightarrow x=...$	dM1	1.1b
	$x=\dfrac{5}{12}$	A1	1.1b
		(3)	
			(6 marks)

Notes

(i)

M1: Combines the terms in x, factorises and divides to find x. Condone sign slips and ignore any attempts to simplify $\sqrt{18}$

Alternatively squares both sides $x\sqrt{2}-\sqrt{18}=x \Rightarrow 2x^2-12x+18=x^2$

dM1: Scored for a complete method to find x. In the main scheme it is for making x the subject and then multiplying both numerator and denominator by $\sqrt{2}+1$

In the alternative it is for squaring both sides to produce a 3TQ and then factorising their quadratic equation to find x. (usual rules apply for solving quadratics)

A1: $x=6+3\sqrt{2}$ only following a correct intermediate line. Allow $\dfrac{6+3\sqrt{2}}{1}$ as an intermediate line.

In the alternative method the $6-3\sqrt{2}$ must be discarded.

(ii)

M1: Uses correct mathematical notation and attempts to set both sides as powers of 2 or 4.

Eg $2^{ax+b}=2^c$ or $4^{dx+e}=4^f$ is sufficient for this mark.

Alternatively uses logs (base 2 or 4) to get a linear equation in x.

$4^{3x-2}=\dfrac{1}{2\sqrt{2}} \Rightarrow \log_2 4^{3x-2}=\log_2\dfrac{1}{2\sqrt{2}} \Rightarrow 2(3x-2)=\log_2\dfrac{1}{2\sqrt{2}}$.

Or $4^{3x-2}=\dfrac{1}{2\sqrt{2}} \Rightarrow 3x-2=\log_4\dfrac{1}{2\sqrt{2}}$

Or $4^{3x-2}=\dfrac{1}{2\sqrt{2}} \Rightarrow 4^{3x}=4\sqrt{2} \Rightarrow 3x=\log_4 4\sqrt{2}$

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dM1: Scored for a complete method to find x.
Scored for setting the indices of 2 or 4 equal to each other and then solving to find x.
There must be an attempt on both sides.

You can condone slips for this mark Eg bracketing errors $4^{3x-2}=2^{2\times 3x-2}$ or $\frac{1}{2\sqrt{2}}=2^{-1+\frac{1}{2}}$

In the alternative method candidates cannot just write down the answer to the rhs.

So expect some justification. E.g. $\log_2\frac{1}{2\sqrt{2}}=\log_2 2^{-\frac{3}{2}}=-\frac{3}{2}$

or $\log_4\frac{1}{2\sqrt{2}}=\log_4 2^{-\frac{3}{2}}=-\frac{3}{2}\times\frac{1}{2}$ condoning slips as per main scheme

or $3x=\log_4 4\sqrt{2}\Rightarrow 3x=1+\frac{1}{4}$

A1: $x=\frac{5}{12}$ with correct intermediate work

Q5

Question Number	Scheme		Marks
(a)	$\left(4^x=\right)y^2$	Allow y^2 or $y\times y$ or "y squared" "$4^x=$" not required	B1
	Must be seen in part (a)		
			(1)
(b)	$8y^2-9y+1=(8y-1)(y-1)=0\Rightarrow y=...$ or $(8(2^x)-1)((2^x)-1)=0\Rightarrow 2^x=...$	For attempting to solve the given equation as a **3 term quadratic** in y or as a **3 term quadratic** in 2^x leading to a value of y or 2^x (Apply usual rules for solving the quadratic – see general guidance) Allow x (or any other letter) instead of y for this mark e.g. an attempt to solve $8x^2-9x+1=0$	M1
	2^x(or y)$=\frac{1}{8}$, 1	Both correct answers of $\frac{1}{8}$ (oe) and 1 for 2^x or y or their letter but **not x** unless 2^x(or y) is implied later	A1
	$x=-3\quad x=0$	M1: A correct attempt to find one **numerical value** of x from their 2^x(or y) **which must have come from a 3 term quadratic equation**. If logs are used then they must be evaluated.	M1A1
		A1: Both $x=-3$ and/or $x=0$ May be implied by e.g. $2^{-3}=\frac{1}{8}$ and $2^0=1$ **and no extra values.**	
			(4)
			(5 marks)

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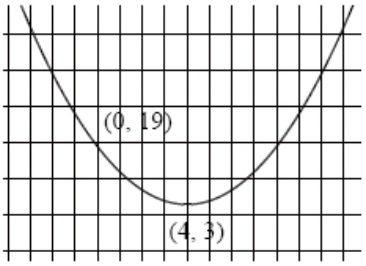
Q6

Question Number	Scheme		Marks
(a)	$f(x)=(x-4)^2+3$	M1: $f(x)=(x\pm4)^2\pm\alpha,\ \alpha\neq0$ (where α is a single number or a numerical expression $\neq 0$)	M1A1
		A1: Allow $(x+^-4)^2+3$ and ignore any spurious "= 0"	
	Allow a = -4, b = 3 to score both marks		
			(2)
(b)		B1: U shape anywhere even with no axes. Do not allow a "V" shape i.e. with an obvious vertex.	B1
		B1: $P(0, 19)$. Allow (0, 19) or just 19 marked in the correct place as long as the curve (or straight line) passes through or touches here and allow (19, 0) as long as it is marked in the correct place. Correct coordinates may be seen in the body of the script as long as the curve (or straight line) passes through or touches here. If there is any ambiguity, the sketch has precedence. **(There must be a sketch to score this mark)**	B1
		B1: $Q(4, 3)$. Correct coordinates that can be scored without a sketch but if a sketch is drawn then it must have a minimum in the first quadrant and no other turning points. May be seen in the body of the script. If there is any ambiguity, the sketch has precedence. Allow this mark if 4 is clearly marked on the x-axis below the minimum and 3 is marked clearly on the y-axis and corresponds to the minimum,	B1
			(3)

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(c)	$PQ^2 = (0-4)^2 + (19-3)^2$	**Correct** use of Pythagoras' Theorem on 2 points of the form $(0, p)$ and (q, r) where $q \neq 0$ and $p \neq r$ with p, q and r numeric.	M1
	$PQ = \sqrt{4^2 + 16^2}$	Correct un-simplified numerical expression for PQ including the square root. **This must come from a correct *P* and *Q*.** Allow e.g $PQ = \sqrt{(0-4)^2 + (19-3)^2}$. Allow $\pm\sqrt{(0-4)^2 + (19-3)^2}$	A1
	$PQ = 4\sqrt{17}$	Cao and cso i.e. **This must come from a correct *P* and *Q*.**	A1
	Note that it is possible to obtain the correct value for PQ from (-4,3) and (0, 19) and e.g. (0, 13) and (4, -3) but the A marks in (c) can only be awarded for the correct P and Q.		
			(3)
			(8 marks)

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Platinum Questions

Calculators may not be used

The total mark for this section is 18

1 A student was attempting to prove that $x = \frac{1}{2}$ is the only real root of

$$x^3 + \tfrac{3}{4}x - \tfrac{1}{2} = 0.$$

The attempted solution was as follows.

$$x^3 + \tfrac{3}{4}x = \tfrac{1}{2}$$

$\therefore$ $$x(x^2 + \tfrac{3}{4}) = \tfrac{1}{2}$$

$\therefore$ $$x = \tfrac{1}{2}$$

or $$x^2 + \tfrac{3}{4} = \tfrac{1}{2}$$

i.e. $$x^2 = -\tfrac{1}{4}$$ no solution

$\therefore$ only real root is $x = \frac{1}{2}$

(a) Explain clearly the error in the above attempt.

(2)

(b) Give a correct proof that $x = \frac{1}{2}$ is the only real root of $x^3 + \frac{3}{4}x - \frac{1}{2} = 0$.

(3)

The equation

$$x^3 + \beta x - \alpha = 0 \qquad \text{(I)}$$

where α, β are real, $\alpha \neq 0$, has a real root at $x = \alpha$.

(c) Find and simplify an expression for β in terms of α and prove that α is the only real root provided $|\alpha| < 2$.

(6)

An examiner chooses a positive number α so that α is the only real root of equation (I) but the incorrect method used by the student produces 3 distinct real "roots".

(d) Find the range of possible values for α.

(7)

(Total for Question 1 is 19 marks)

End of Questions

Platinum Mark Scheme

Question Number		Scheme		Marks
7.	(*a*)	$pq = \frac{1}{2} \not\Rightarrow p = \frac{1}{2}$ or $q = \frac{1}{2}$ (line 3)	identify; explain	B1; B1 (2)
	(*b*)	$x^3 + \frac{3}{4}x - \frac{1}{2} = 0 \Rightarrow (x - \frac{1}{2})(x^2 + \frac{1}{2}x + 1) = 0$	attempt to divide	M1
			correct quadratic	A1
		i.e. $x = \frac{1}{2}$ or $x^2 + \frac{1}{2}x + 1 = 0$, discriminant $= (\frac{1}{2})^2 - 4$		M1
		< 0 ∴ no real roots (so only root is $x = \frac{1}{2}$)		A1 cso (4)
	(*c*)	$x = \alpha$ is a root $\Rightarrow \alpha^3 + \beta\alpha - \alpha = 0$, i.e. $\beta = 1 - \alpha^2$ $(\alpha \neq 0)$		M1, A1
		$x^3 + \beta x - \alpha \equiv (x - \alpha)[x^2 + \alpha x + 1]$		M1 [A1]
		Discriminant of $x^2 + \alpha x + 1$ is $\alpha^2 - 4$		M1
		∴ $x = \alpha$ is the only real root if $\alpha^2 - 4 < 0$, i.e. $\lvert\alpha\rvert < 2$ (✱)		A1 cso (6)
	(*d*)	Student's method: $x(x^2 + \beta) = \alpha$		
		$\Rightarrow x = \alpha$ or $x^2 + \beta = \alpha$		M1
		require $\alpha - \beta > 0$		
		$\alpha^2 + \alpha - 1 > 0$		
		cvs $\alpha = \dfrac{-1 \pm \sqrt{5}}{2}$	attempt cvs	M1
			2 correct cvs	A1
		∴ $\dfrac{\sqrt{5}-1}{2} < \alpha < 2$ or $-2 < \alpha < -\dfrac{\sqrt{5}-1}{2}$		A1, A1 (7)
				(19 marks)

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(*a*)	STYLE INSIGHT & REASONING S marks For a novel or neat solution to any of questions 3—7. Apply once per question in up to 3 questions **S2** if solution is fully correct in principle and accuracy **S1** if principle is sound but includes a minor algebraic or numerical slip T mark **For a good and largely accurate attempt at the whole paper**	S6 (S2 × 3) **T1** **(7 marks)**

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Topic 3

Equations and Inequalities

Bronze, Silver, Gold and Platinum Worksheets for AS Level Mathematics

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Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculators may not be used

The total mark for this section is 29

Q1

Find the set of values of x for which

(a) $3(x - 2) < 8 - 2x$

(2)

(b) $(2x - 7)(1 + x) < 0$

(3)

(c) both $3(x - 2) < 8 - 2x$ **and** $(2x - 7)(1 + x) < 0$

(1)

(Total for Question 1 is 6 marks)

Q2

Find the set of values of x for which

(a) $2(3x + 4) > 1 - x$

(2)

(b) $3x^2 + 8x - 3 < 0$

(4)

(Total for Question 2 is 6 marks)

Q3

Solve the simultaneous equations

$$y - 3x + 2 = 0$$
$$y^2 - x - 6x^2 = 0$$

(Total for Question 3 is 7 marks)

Q4

A rectangular room has a width of x m.

The length of the room is 4 m longer than its width.

Given that the perimeter of the room is greater than 19.2 m,

(a) show that $x > 2.8$

(3)

Given also that the area of the room is less than 21 m^2,

(b) (i) write down an inequality, in terms of x, for the area of the room.

(ii) Solve this inequality.

(4)

(c) Hence find the range of possible values for x.

(1)

(Total for Question 4 is 8 marks)

End of Questions

Bronze Mark Scheme

Q1.

Question Number	Scheme	Marks
(a)	$3x-6<8-2x \rightarrow 5x<14$ (Accept $5x-14<0$ (o.e.)) $x<2.8$ or $\frac{14}{5}$ or $2\frac{4}{5}$ (condone $\leq$)	M1 A1 (2)
(b)	Critical values are $x=\frac{7}{2}$ and -1	B1
	Choosing "inside" $-1<x<\frac{7}{2}$	M1 A1 (3)
(c)	$-1<x<2.8$	B1ft (1)
	Accept any exact equivalents to -1, 2.8, 3.5	**6**

	Notes
(a)	M1 for attempt to rearrange to $kx<m$ (o.e.) Either $k=5$ or $m=14$ should be correct Allow $5x=14$ or even $5x>14$
(b)	B1 for both correct critical values. (May be implied by a correct inequality) M1 ft their values and choose the "inside" region A1 for fully correct inequality (Must be in part (b): do not give marks if only seen in (c)) Condone seeing $x<-1$ in working provided $-1<x$ is in the final answer. e.g. $x>-1$, $x<\frac{7}{2}$ or $x>-1$ "or" $x<\frac{7}{2}$ or $x>-1$ "blank space" $x<\frac{7}{2}$ score M1A0 BUT allow $x>-1$ and $x<\frac{7}{2}$ to score M1A1 (the "and" must be seen) Also $\left(-1,\frac{7}{2}\right)$ will score M1A1 NB $x<-1, x<\frac{7}{2}$ is of course M0A0 and a number line even with "open" ends is M0A0 Allow 3.5 instead of $\frac{7}{2}$
(c)	B1ft for $-1<x<2.8$ (ignoring their previous answers) or ft their answers to part (a) and part (b) provided both answers were regions and not single values. Allow use of "and" between inequalities as in part (b) If their set is empty allow a suitable description in words or the symbol $\varnothing$. Common error: If (a) is correct and in (b) they simply leave their answer as $x<-1$, $x<3.5$ then in (c) $x<-1$ would get B1ft as this is a correct follow through of these 3 inequalities. Penalise use of $\leq$ only on the A1 in part (b). [i.e. condone in part (a)]

Pearson

Q2.

Question Number	Scheme		Marks
(a)	$6x + x > 1 - 8$	Attempts to expand the bracket and collect x terms on one side and constant terms on the other. Condone sign errors and allow one error in expanding the bracket. Allow $<, \leq, \geq, =$ instead of $>$.	M1
	$x > -1$	Cao	A1
	Do not isw here, mark their final answer.		
			(2)
(b)	$(x+3)(3x-1)[=0]$ $\Rightarrow x = -3$ and $\frac{1}{3}$	M1: Attempt to solve the quadratic to obtain two critical values	M1A1
		A1: $x = -3$ **and** $\frac{1}{3}$ (may be implied by their inequality). Allow all equivalent fractions for -3 and 1/3. (Allow 0.333 for 1/3)	
	$-3 < x < \frac{1}{3}$	M1: Chooses "inside" region (The letter x does not need to be used here)	M1A1ft
		A1ft: Allow $x < \frac{1}{3}$ **and** $x > -3$ or $\left(-3, \frac{1}{3}\right)$ or $x < \frac{1}{3} \cap x > -3$. Follow through their critical values. (must be in terms of x here) Allow all equivalent fractions for -3 and 1/3. Both $(x < \frac{1}{3}$ **or** $x > -3)$ and $(x < \frac{1}{3}, x > -3)$ as a final answer score A0.	
			(4)
			[6]
	Note that use of $\leq$ or $\geq$ appearing in an otherwise correct answer in (a) or (b) should only be penalised once, the first time it occurs.		

Pearson

Q3.

Question number	Scheme	Marks
	$y = 3x - 2 \quad (3x-2)^2 - x - 6x^2 (= 0)$	M1
	$9x^2 - 12x + 4 - x - 6x^2 = 0$ $3x^2 - 13x + 4 = 0$ (or equiv., e.g. $3x^2 = 13x - 4$)	M1 A1cso
	$(3x-1)(x-4) = 0 \quad x = \ldots \quad x = \frac{1}{3}$ (or exact equivalent) $\quad x = 4$	M1 A1
	$y = -1 \quad y = 10$ (Solutions need not be "paired")	M1 A1
		[7]
	1st M: Obtaining an equation in x only (or y only). Condone missing "= 0" Condone sign slips, e.g. $(3x+2)^2 - x - 6x^2 = 0$, but not other algebraic mistakes (such as squaring individual terms... see bottom of page). 2nd M: Multiplying out their $(3x-2)^2$, which must lead to a 3 term quadratic, i.e. $ax^2 + bx + c$, where $a \neq 0$, $b \neq 0$, $c \neq 0$, and collecting terms. 3rd M: Solving a 3-term quadratic (see general principles at end of scheme). 2nd A: Both values. 4th M: Using an x value, found algebraically, to attempt at least one y value (or using a y value, found algebraically, to attempt at least one x value)... allow b.o.d. for this mark in cases where the value is wrong but working is not shown. 3rd A: Both values. If y solutions are given as x values, or vice-versa, penalise at the end, so that it is possible to score M1 M1A1 M1 A1 M0 A0. "Non-algebraic" solutions: No working, and only one correct solution pair found (e.g. $x = 4, y = 10$): M0 M0 A0 M0 A0 M1 A0 No working, and both correct solution pairs found, but not demonstrated: M0 M0 A0 M1 A1 M1 A1 Both correct solution pairs found, and demonstrated: Full marks Alternative: $x = \frac{y+2}{3} \quad y^2 - \frac{y+2}{3} - 6\left(\frac{y+2}{3}\right)^2 = 0$ M1 $y^2 - \frac{y+2}{3} - 6\left(\frac{y^2+4y+4}{9}\right) = 0 \quad y^2 - 9y - 10 = 0$ M1 A1 $(y+1)(y-10) = 0 \quad y = \ldots \quad y = -1 \quad y = 10$ M1 A1 $x = \frac{1}{3} \quad x = 4$ M1 A1 Squaring each term in the first equation, e.g. $y^2 - 9x^2 + 4 = 0$, and using this to obtain an equation in x only could score at most 2 marks: M0 M0 A0 M1 A0 M1 A0.	

Pearson

Q4.

Question Number	Scheme	Notes	Marks
	Ignore any references to the units in this question		
(a)	length is '$x+4$'	May be implied	B1
	$x+x+x+4+x+4>19.2 \Rightarrow x>..$	$2x+2(x\pm4)>19.2$ and proceeds to $x>$ ..... (Accept 'invisible' brackets) Attempts 2 widths + 2 lengths > 19.2 leading to $x>$	M1
	E.g. $x+x+4x+4x>19.2 \Rightarrow x>1.92$ scores B0M1A0		
	$x>2.8$ *	Achieves $x>2.8$ with no errors	A1(*)
			(3)
	Mark parts (b) and (c) together		
(b)(i)	$x(x+4)<21$	Cao	B1
b(ii)	$x^2+4x-21<0$ $(x+7)(x-3)<0 \Rightarrow x=...$	Multiply out lhs, produce 3TQ = 0 and attempt to solve leading to $x=...$ according to general guidelines	M1
	Either $-7<x<3$ **or** $0<x<3$	M1: Attempts the 'inside' for their critical values (may be from a 2TQ here) A1: Accept either $-7<x<3$ or $0<x<3$ or ($x>-7$ **and** $x<3$) or ($x>0$ **and** $x<3$) but not e.g. ($x>-7$, $x<3$) or ($x>-7$ **or** $x<3$) (There is no specific need for them to realise $x>0$)	M1A1
	Note that <u>many</u> candidates stop here		
			(4)
(c)	$2.8<x<3$	Follow through their answers to (a) and (b) Provided "their 3" > 2.8	B1ft
			(1)
			[8]
	Examples		
	$x(x-4)<21 \Rightarrow x^2-4x-21<0$ $(x-7)(x+3)<0,\ x=7, x=-3$ $-3<x<7$ *or* $0<x<7$ $2.8<x<7$ Scores B0M1M1A0B1ft	$x\times4x<21 \Rightarrow 4x^2-21<0$ $(2x-\sqrt{21})(2x+\sqrt{21})<0,\ x=\pm\frac{\sqrt{21}}{2}$ $-\frac{\sqrt{21}}{2}<x<\frac{\sqrt{21}}{2}$ *or* $0<x<\frac{\sqrt{21}}{2}$ $2.8<x<\frac{\sqrt{21}}{2}$ Scores B0M0M1A0B0	

Pearson

Silver Questions

Calculators may not be used

The total mark for this section is 34

Q1

Find the set of values of x for which

(a) $4x - 3 > 7 - x$

(2)

(b) $2x^2 - 5x - 12 < 0$

(4)

(c) **both** $4x - 3 > 7 - x$ **and** $2x^2 - 5x - 12 < 0$

(1)

Q2

Given the simultaneous equations

$$2x + y = 1$$
$$x^2 - 4ky + 5k = 0$$

where k is a non zero constant,

(a) show that

$$x^2 + 8kx + k = 0$$

(2)

Given that $x^2 + 8kx + k = 0$ has equal roots,

(b) find the value of k.

(3)

(c) For this value of k, find the solution of the simultaneous equations.

(3)

(Total for Question 2 is 8 marks)

Q3

Solve the simultaneous equations

$$x + y = 2$$
$$4y^2 - x^2 = 11$$

(Total for Question 3 is 7 marks)

Q4

The equation

$$(k + 3)\,x^2 + 6x + k = 5, \text{ where } k \text{ is a constant,}$$

has two distinct real solutions for x.

(a) Show that k satisfies

$$k^2 - 2k - 24$$

(4)

(b) Hence find the set of possible values of k.

(3)

(Total for Question 4 is 7 marks)

Q5

(i) Show that $x^2 - 8x + 17 > 0$ for all real values of x

(3)

(ii) "If I add 3 to a number and square the sum, the result is greater than the square of the original number."

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

(Total for Question 5 is 5 marks)

End of Questions

Pearson

Silver Mark Scheme

Q1

Question Number		Scheme	Marks
Q	(a)	$5x > 10$, $x > 2$ [Condone $x > \frac{10}{2} = 2$ for M1A1]	M1, A1 (2)
	(b)	$(2x+3)(x-4) = 0$, 'Critical values' are $-\frac{3}{2}$ and 4	M1, A1
		$-\frac{3}{2} < x < 4$	M1 A1ft (4)
	(c)	$2 < x < 4$	B1ft (1) [7]

	(a)	M1 for attempt to collect like terms on each side leading to $ax > b$, or $ax < b$, or $ax = b$ Must have a or b correct so eg $3x > 4$ scores M0
	(b)	1st M1 for an attempt to factorize or solve to find critical values. Method must potentially give 2 critical values 1st A1 for $-\frac{3}{2}$ and 4 seen. They may write $x < -\frac{3}{2}$, $x < 4$ and still get this A1 2nd M1 for choosing the "inside region" for their critical values 2nd A1ft follow through their 2 distinct critical values Allow $x > -\frac{3}{2}$ with "or" "," "$\cup$" "" $x < 4$ to score M1A0 but "and" or "$\cap$" score M1A1 $x \in (-\frac{3}{2}, 4)$ is M1A1 but $x \in [-\frac{3}{2}, 4]$ is M1A0. Score M0A0 for a number line or graph only

	(c)	B1ft Allow if **a correct answer is seen** or follow through their answer to (a) and their answer to (b) but their answers to (a) and (b) must be regions. Do not follow through single values. If their follow through answer is the empty set accept $\varnothing$ or {} or equivalent in words If (a) or (b) are not given then score this mark for cao NB You may see $x<4$ (with anything or nothing in-between) $x < -1.5$ in (b) and empty set in (c) for B1ft **Do not award marks for part (b) if only seen in part (c)** Use of $\leq$ instead of $<$ (or $\geq$ instead of $>$) loses one accuracy mark only, at first occurrence.

Pearson

Q2.

Question Number	Scheme	Marks	
(a)	$x^2 - 4k(1-2x) + 5k(=0)$	Makes y the subject from the first equation and substitutes into the second equation (= 0 not needed here) or eliminates y by a correct method.	M1
	So $x^2 + 8kx + k = 0$ *	Correct completion to printed answer. There must be no incorrect statements.	A1cso
			(2)
(b)	$(8k)^2 - 4k$	M1: Use of $b^2 - 4ac$ (Could be in the quadratic formula or an inequality, = 0 not needed yet). There must be some correct substitution but there must be no x's. No formula quoted followed by e.g. $8k^2 - 4k = 0$ is M0.	M1 A1
		A1: Correct expression. Do not condone missing brackets unless they are implied by later work but can be implied by $(8k)^2 > 4k$ etc.	
	$k = \frac{1}{16}$ (oe)	Cso (Ignore any reference to $k = 0$) but there must be no contradictory earlier statements. A fully correct solution with no errors.	A1
			(3)
(b) Way 2 Equal roots	$\Rightarrow x^2 + 8kx + k = (x + \sqrt{k})^2$ $\Rightarrow 8k = 2\sqrt{k}$	M1: Correct strategy for equal roots	M1A1
		A1: Correct equation	

Pearson

Q3.

Question Number	Scheme	Marks
	Either $y^2 = 4 - 4x + x^2$ $4(4 - 4x + x^2) - x^2 = 11$ or $4(2-x)^2 - x^2 = 11$ $3x^2 - 16x + 5 = 0$ $(3x-1)(x-5) = 0, \quad x =$ $x = \frac{1}{3} \quad x = 5$ $y = \frac{5}{3} \quad y = -3$ Or $x^2 = 4 - 4y + y^2$ $4y^2 - (4 - 4y + y^2) = 11$ or $4y^2 - (2-y)^2 = 11$ $3y^2 + 4y - 15 = 0$ Correct 3 terms $(3y-5)(y+3) = 0, \quad y = ...$ $y = \frac{5}{3} \quad y = -3$ $x = \frac{1}{3} \quad x = 5$	M1 M1 A1 M1 A1 M1 A1 (7) 7
	Notes 1st M: Squaring to give 3 or 4 terms (need a middle term) 2nd M: Substitute to give quadratic in one variable (may have just two terms) 3rd M: Attempt to solve a **3 term** quadratic. 4th M: Attempt to find at least one y value (or x value). (The second variable) This will be by substitution or by starting again. If y solutions are given as x values, or vice-versa, penalise accuracy, so that it is possible to score M1 M1A1 M1 A0 M1 A0. "Non-algebraic" solutions: No working, and only one correct solution pair found (e.g. $x = 5$, $y = -3$): M0 M0 A0 M1 A0 M1 A0 No working, and both correct solution pairs found, but not demonstrated: M0 M0 A0 M1 A1 M1 A1 Both correct solution pairs found, and demonstrated: Full marks are possible (send to review)	

Pearson

Q4.

Question Number	Scheme	Marks
(a)	**Method 1:** Attempts $b^2 - 4ac$ for $a = (k+3)$, $b = 6$ and their c. $c \neq k$ $b^2 - 4ac = 6^2 - 4(k+3)(k-5)$ $(b^2 - 4ac =) \quad -4k^2 + 8k + 96$ or $-(b^2 - 4ac =) \quad 4k^2 - 8k - 96$ (with no prior algebraic errors) As $b^2 - 4ac > 0$, then $-4k^2 + 8k + 96 > 0$ and so, $k^2 - 2k - 24 < 0$	M1 A1 B1 A1 *
	Method 2: Considers $b^2 > 4ac$ for $a = (k+3)$, $b = 6$ and their c. $c \neq k$ $6^2 > 4(k+3)(k-5)$ $4k^2 - 8k - 96 < 0$ or $-4k^2 + 8k + 96 > 0$ or $9 > (k+3)(k-5)$ (with no prior algebraic errors) and so, $k^2 - 2k - 24 < 0$ following correct work	M1 A1 B1 A1 * **[4]**
(b)	Attempts to solve $k^2 - 2k - 24 = 0$ to give $k =$ ($\Rightarrow$ Critical values, $k = 6, -4$.) $k^2 - 2k - 24 < 0$ gives $-4 < k < 6$	M1 M1 A1 **[3]** **7 marks**
	Notes	
(a)	**Method 1: M1:** Attempts $b^2 - 4ac$ for $a = (k+3)$, $b = 6$ and their c. $c \neq k$ or uses quadratic formula and has this expression under square root. (ignore > 0, < 0 or $= 0$ for first 3 marks) **A1:** Correct expression for $b^2 - 4ac$ - need not be simplified (may be under root sign) **B1:** Uses algebra to manipulate result **without error** into **one of these** three term quadratics. Again may be under root sign in quadratic formula. If inequality is used early in "proof" may see $4k^2 - 8k - 96 < 0$ and B1 would be given for $4k^2 - 8k - 96$ correctly stated. **A1:** Applies $b^2 - 4ac > 0$ correctly (**or writes** $b^2 - 4ac > 0$) to achieve the **result given** in the question. No errors should be seen. Any incorrect line of argument should be penalised here. There are several ways of reaching the answer; either multiplication of both sides of inequality by –1, or taking every term to other side of inequality. **Need conclusion i.e. printed answer.** **Method 2: M1:** Allow $b^2 > 4ac$, $b^2 < 4ac$ or $b^2 = 4ac$ for $a = (k+3)$, $b = 6$ and their c. $c \neq k$ **A1:** Correct expressions on either side (ignore $>$, $<$ or $=$). **B1:** Uses algebra to manipulate result into one of the two three term quadratics or divides both sides by 4 **again without error** **A1:** Produces result with no errors seen from initial consideration of $b^2 > 4ac$.
(b)	**M1:** Uses factorisation, formula, completion of square method to find two values for k, or finds two **correct** answers with no obvious method **M1:** Their Lower Limit $< k <$ Their Upper Limit Allow the M mark mark for $\leq$. (Allow $k <$ upper and $k >$ lower) **A1:** $-4 < k < 6$ Lose this mark for $\leq$ Allow (-4, 6) [not square brackets] or $k > -4$ **and** $k < 6$ (must be **and** not **or**) Can also use intersection symbol $\cap$ **NOT** $k > -4$, $k < 6$ (M1A0)
	Special case : In part (a) uses $c = k$ instead of k - 5 - scores 0 . Allow $k + 5$ for method marks
	Special Case: In part (b) Obtaining $-6 < k < 4$ This is a common wrong answer. Give M1 M1 A0 special case.
	Special Case: In part (b) Use of x instead of k – M1M1A0
	Special Case: $-4 < k < 6$ and $k < -4$, $k > 6$ both given is M0A0 for last two marks. Do not treat as isw.

Pearson

Q5.

Question	Scheme	Marks	AOs
(i)	$x^2-8x+17=(x-4)^2-16+17$	M1	3.1a
	$=(x-4)^2+1$ with comment (see notes)	A1	1.1b
	As $(x-4)^2\geqslant 0 \Rightarrow (x-4)^2+1\geqslant 1$ hence $x^2-8x+17>0$ for all x	A1	2.4
		(3)	
(ii)	For an explanation that it may not always be true Tests say $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$	M1	2.3
	States sometimes true and gives reasons Eg. when $x=5$ $(5+3)^2=64$ whereas $(5)^2=25$ True When $x=-5$ $(-5+3)^2=4$ whereas $(-5)^2=25$ Not true	A1	2.4
		(2)	
			(5 marks)

Pearson

Notes

(i) Method One: Completing the Square

M1: For an attempt to complete the square. Accept $(x-4)^2$ ...

A1: For $(x-4)^2+1$ with either $(x-4)^2 \geqslant 0, (x-4)^2+1 \geqslant 1$ or min at (4,1). Accept the inequality statements in words. Condone $(x-4)^2 > 0$ or a squared number is always positive for this mark.

A1: A fully written out solution, with correct statements and no incorrect statements. There must be a valid reason and a conclusion

---

$x^2-8x+17$

$=(x-4)^2+1 \geqslant 1$ as $(x-4)^2 \geqslant 0$

Hence $(x-4)^2+1>0$

scores M1 A1 A1

$x^2-8x+17>0$

$(x-4)^2+1>0$

This is true because $(x-4)^2 \geqslant 0$ and when you add 1 it is going to be positive

scores M1 A1 A1

$x^2-8x+17>0$

$(x-4)^2+1>0$

which is true because a squared number is positive

scores M1 A1 A0

incorrect and incomplete

$x^2-8x+17=(x-4)^2+1$

Minimum is (4,1) so $x^2-8x+17>0$

scores M1 A1 A0

correct but not explained

$x^2-8x+17=(x-4)^2+1$

Minimum is (4,1) so as $1>0 \Rightarrow x^2-8x+17>0$

scores M1 A1 A1

correct and explained

Pearson

Gold Questions

Calculators may not be used

The total mark for this section is 27

Q1

The equation

$$x^2 + kx + 8 = k$$

has no real solutions for x.

(a) Show that k satisfies $k^2 + 4k - 32 < 0$.

(3)

(b) Hence find the set of possible values of k.

(4)

(Total for Question 1 is 7 marks)

Q2

Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,

(a) show that $q^2 + 8q < 0$.

(2)

(b) Hence find the set of possible values of q.

(3)

(Total for Question 2 is 5 marks)

Q3

The equation $20x^2 = 4kx - 13kx^2 + 2$, where k is a constant, has no real roots.

(a) Show that k satisfies the inequality

$$2k^2 + 13k + 20 < 0$$

(4)

(b) Find the set of possible values for k.

(4)

(Total for Question 3 is 8 marks)

Q4

(a) By eliminating y from the equations

$$y = x - 4$$
$$2x^2 - xy = 8,$$

show that

$$x^2 + 4x - 8 = 0$$

(2)

(b) Hence, or otherwise, solve the simultaneous equations

$$y = x - 4,$$
$$2x^2 - xy = 8,$$

giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers.

(5)

(Total for Question 4 is 7 marks)

End of Questions

Gold Mark Scheme

Question number	Scheme	Marks
	(a) $x^2 + kx + (8-k) \quad (=0)$ $\quad 8-k$ need not be bracketed	M1
	$b^2 - 4ac = k^2 - 4(8-k)$	M1
	$b^2 - 4ac < 0 \Rightarrow k^2 + 4k - 32 < 0$ (*)	A1cso (3)
	(b) $(k+8)(k-4) = 0 \quad k = \ldots$	M1
	$k = -8 \quad k = 4$	A1
	Choosing 'inside' region (between the two k values)	M1
	$-8 < k < 4$ or $4 > k > -8$	A1 (4)
		7

(a) 1st M: Using the k from the right hand side to form 3-term quadratic in x
('= 0' can be implied), or…

attempting to complete the square $\left(x + \frac{k}{2}\right)^2 - \frac{k^2}{4} + 8 - k \ (=0)$ or equiv.,

using the k from the right hand side.
For either approach, condone sign errors.

1st M may be implied when candidate moves straight to the discriminant.

2nd M: Dependent on the 1st M.

Forming expressions in k (with no x's) by using b^2 and $4ac$. (Usually seen as the discriminant $b^2 - 4ac$, but separate expressions are fine, and also allow the use of $b^2 + 4ac$.
(For 'completing the square' approach, the expression must be clearly separated from the equation in x).
If b^2 and $4ac$ are used in the quadratic formula, they must be clearly separated from the formula to score this mark.
For any approach, condone sign errors.

If the wrong statement $\sqrt{b^2 - 4ac} < 0$ is seen, maximum score is M1 M1 A0.

(b) Condone the use of x (instead of k) in part (b).
1st M: Attempt to solve a 3-term quadratic equation in k.
It might be different from the given quadratic in part (a).

Ignore the use of $<$ in solving the equation. The 1st M1 A1 can be scored if -8 and 4 are achieved, even if stated as $k < -8$, $k < 4$.
Allow the first M1 A1 to be scored in part (a).

N.B. '$k > -8$, $k < 4$' scores 2nd M1 A0
'$k > -8$ or $k < 4$' scores 2nd M1 A0
'$k > -8$ and $k < 4$' scores 2nd M1 A1
'$k = -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3$' scores 2nd M0 A0

Use of $\leq$ (in the answer) loses the final mark.

Pearson

Q2.

Question Number	Scheme	Marks
(*a*)	[No real roots implies $b^2-4ac<0$.] $b^2-4ac=q^2-4\times2q\times(-1)$	M1
	So $q^2-4\times2q\times(-1)<0$ i.e. $q^2+8q<0$ (*)	A1 cso (2)
(*b*)	$q(q+8)=0$ or $(q\pm4)^2\pm16=0$	M1
	$(q)=0$ or -8 (2 cvs)	A1
	$-8<q<0$ or $q\in(-8,0)$ or $q<0$ and $q>-8$	A1 ft (3)
		(5 marks)

Q3.

(b)	$2k^2+13k+20=0\Rightarrow k=...$ e.g. $(2k+5)(k+4)=0\Rightarrow k=...$	Attempt to solve the **given** quadratic to find 2 values for k. See general guidance.	M1
	$\Rightarrow k=-\frac{5}{2},-4$	Both correct. May be implied by e.g. $k<-\frac{5}{2}$, $k<-4$ or seen on a sketch. If they use the quadratic formula allow $\frac{-13\pm3}{4}$ for this mark but not $\sqrt{9}$ for 3 and allow e.g. $-\frac{13}{4}\pm\frac{3}{4}$ if they complete the square.	A1
	$-4<k<-\frac{5}{2}$ Allow equivalent values e.g. $-\frac{10}{4}$ i.e. the critical values must be in the form $\frac{a}{b}$ where a and b are integers	M1: Chooses 'inside' region for **their critical values** i.e. Lower Limit $<k<$ Upper Limit or e.g. Lower Limit $\le k\le$ Upper Limit	M1A1
		A1: Allow $k\in(-4,-\frac{5}{2})$ or just $(-4,-\frac{5}{2})$ and allow $k>-4$ **and** $k<-2.5$ and $-\frac{5}{2}>k>-4$ but $k>-4$, $k<-\frac{5}{2}$ scores M1A0. $-\frac{5}{2}<k<-4$ is M0A0	
	Allow working in terms of x in (b) but the answer must be in terms of k for the final mark.		
			(4)
			(8 marks)

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Q4.

Question number	Scheme	Marks
	(a) $2x^2 - x(x-4) = 8$	M1
	$x^2 + 4x - 8 = 0$ (*)	A1cso (2)
	(b) $x = \dfrac{-4 \pm \sqrt{4^2 - (4\times1\times-8)}}{2}$ or $(x+2)^2 \pm 4 - 8 = 0$	M1
	$x = -2 \pm$ (any correct expression)	A1
	$\sqrt{48} = \sqrt{16}\sqrt{3} = 4\sqrt{3}$ or $\sqrt{12} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$	B1
	$y = (-2 \pm 2\sqrt{3}) - 4$ M: Attempt at least one y value	M1
	$\underline{x = -2 + 2\sqrt{3},\ \ y = -6 + 2\sqrt{3}}$ $\underline{x = -2 - 2\sqrt{3},\ \ y = -6 - 2\sqrt{3}}$	A1 (5)
		7

(a) M1 for correct attempt to form an equation in x only. Condone sign errors/slips but attempt at this line must be seen. E.g. $2x^2 - x^2 \pm 4x = 8$ is OK for M1.

A1cso for correctly simplifying to printed form. No incorrect working seen. The = 0 is required.

These two marks can be scored in part (b). For multiple attempts pick best.

(b) 1st M1 for use of correct formula. If formula is not quoted then a fully correct substitution is required. Condone missing $x =$ or just + or − instead of $\pm$ for M1.

For completing the square must have as printed or better.

If they have $x^2 - 4x - 8 = 0$ then M1 can be given for $(x-2)^2 \pm 4 - 8 = 0$.

1st A1 for -2 $\pm$ any correct expression. (The $\pm$ is required but $x =$ is not)

B1 for simplifying the surd e.g. $\sqrt{48} = 4\sqrt{3}$. Must reduce to $b\sqrt{3}$ so $\sqrt{16}\sqrt{3}$ or $\sqrt{4}\sqrt{3}$ are OK.

2nd M1 for attempting to find at least one y value. Substitution into one of the given equations and an attempt to solve for y.

2nd A1 for correct y answers. Pairings need not be explicit but they must say which is x and which y.

Mis-labelling x and y loses final A1 only.

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Platinum Questions

Calculators may not be used

The total mark for this section is 13

1 (*a*) Find the set of values of k for which the equation

$$\frac{x^2+3x+8}{x^2+x-2}=k$$

has no real roots.

(6)

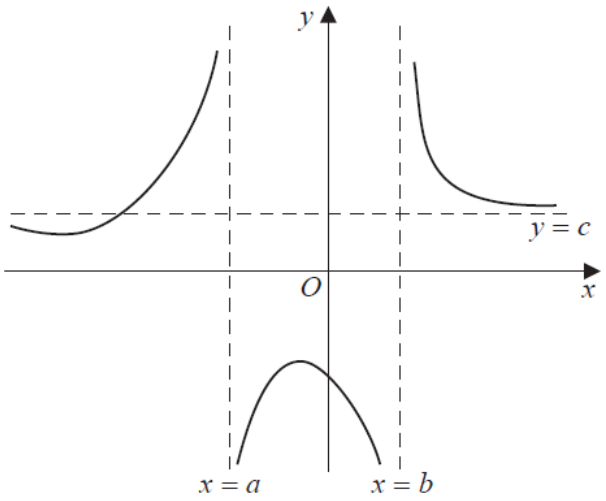

Figure 3

Figure 3 shows a sketch of the curve C_1 with equation $y = f(x)$ where $f(x) = \frac{x^2+3x+8}{x^2+x-2} = k$

The curve has asymptotes $x = a$, $x = b$ and $y = c$, where a, b and c are integers.

(*b*) Find the value of a, the value of b and the value of c.

(4)

(*c*) Find the coordinates of the points of intersection of C_1 with the line $y = 2$

(3)

(Total for Question 1 is 13 marks)

Platinum Mark Scheme

Qu	Scheme	Mark
7 (a)	$x^2+3x+8=kx^2+kx-2k \Rightarrow 0=(k-1)x^2+(k-3)x-(2k+8)$	M1
	No real roots so "$b^2-4ac<0$" $\Rightarrow (k-3)^2+4(k-1)(2k+8)\ [<0]$	M1
	So $9k^2+18k-23[<0]$	M1A1
	$(k+1)^2-1-\frac{23}{9}[<0]$	M1
	$k=-1\pm\frac{4\sqrt{2}}{3}$ so $-1-\frac{4\sqrt{2}}{3}<k<-1+\frac{4\sqrt{2}}{3}$ (o.e.)	A1cso
		(6)
(b)	$x^2+x-2=(x+2)(x-1)$	M1
	$x=-2,\ x=1$ or $a=-2$ and $b=1$	A1A1
	Division or limits of x $y=1$ or $c=1$	B1
		(4)
(c)	f(x) = 2 $\Rightarrow$ $x^2-x-12=0$	M1
	i.e. $(x-4)(x+3)=0$ so $x=4$ or -3	M1
	Coordinates are **(−3, 2)** and **(4, 2)**	A1
		(3)

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Topic 4

Graphs and transformations

Bronze, Silver, Gold
Worksheets for
AS Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculators may not be used

The total mark for this section is 29

Q1

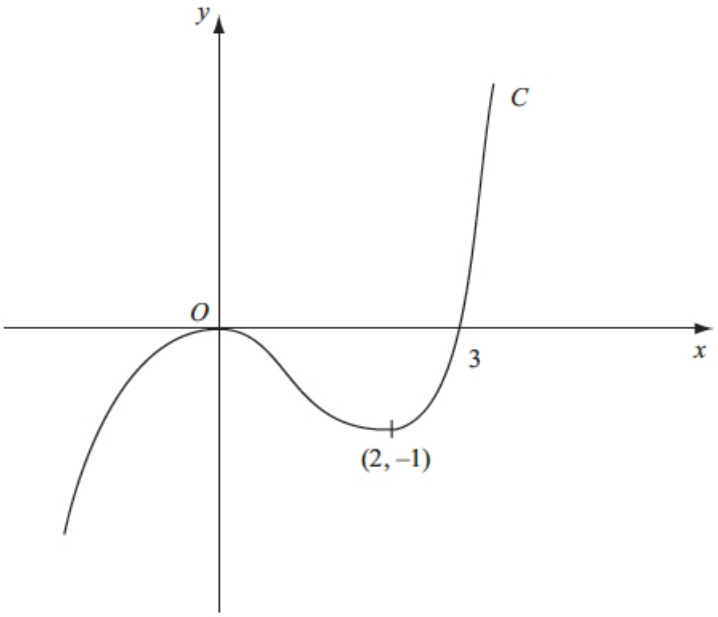

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$. There is a maximum at $(0, 0)$, a minimum at $(2, -1)$ and C passes through $(3, 0)$.

On separate diagrams sketch the curve with equation

(a) $y = f(x + 3)$,

(3)

(b) $y = f(-x)$.

(3)

On each diagram show clearly the coordinates of the maximum point, the minimum point and any points of intersection with the x-axis.

(Total for Question 1 is 6 marks)

Q2

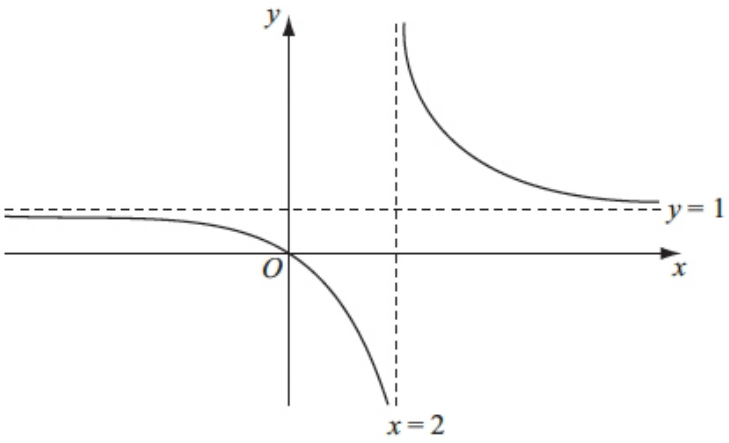

Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x)\frac{x}{x-2}, \quad x \neq 2$$

The curve passes through the origin and has two asymptotes, with equations $y = 1$ and $x = 2$, as shown in Figure 1.

(a) Sketch the curve with equation $y = f(x - 1)$ and state the equations of the asymptotes of this curve.

(3)

(b) Find the coordinates of the points where the curve with equation $y = f(x - 1)$ crosses the coordinate axes.

(4)

(Total for Question 2 is 7 marks)

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Q3

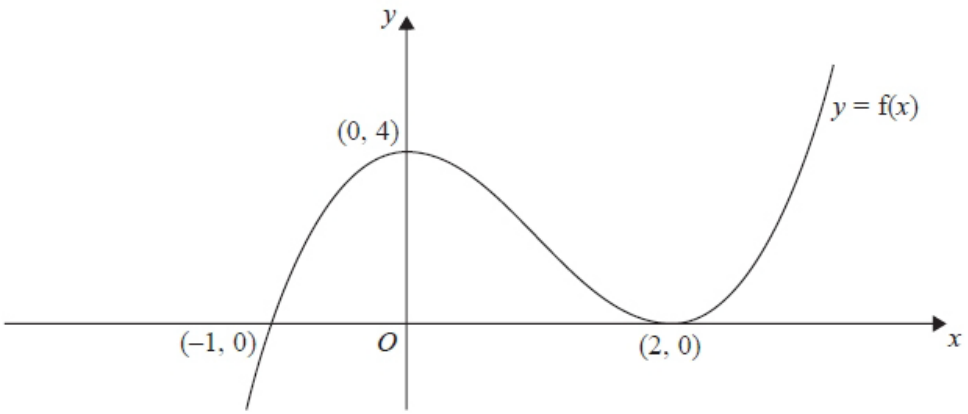

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$

The curve C passes through the point $(-1, 0)$ and touches the x-axis at the point $(2, 0)$

The curve C has a maximum at the point $(0, 4)$

(a) The equation of the curve C can be written in the form

$$y = x^3 + ax^2 + bx + c$$

where a, b and c are integers.

Calculate the values of a, b and c.

(5)

(b) Sketch the curve with equation $y = f\left(\frac{1}{2}\right)$

Show clearly the coordinates of all the points where the curve crosses or meets the coordinate axes.

(3)

(Total for Question 3 is 8 marks)

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Q4

(a) Sketch the graphs of

$$y = x(x + 2)(3 - x)$$

$$y = -\frac{2}{x}$$

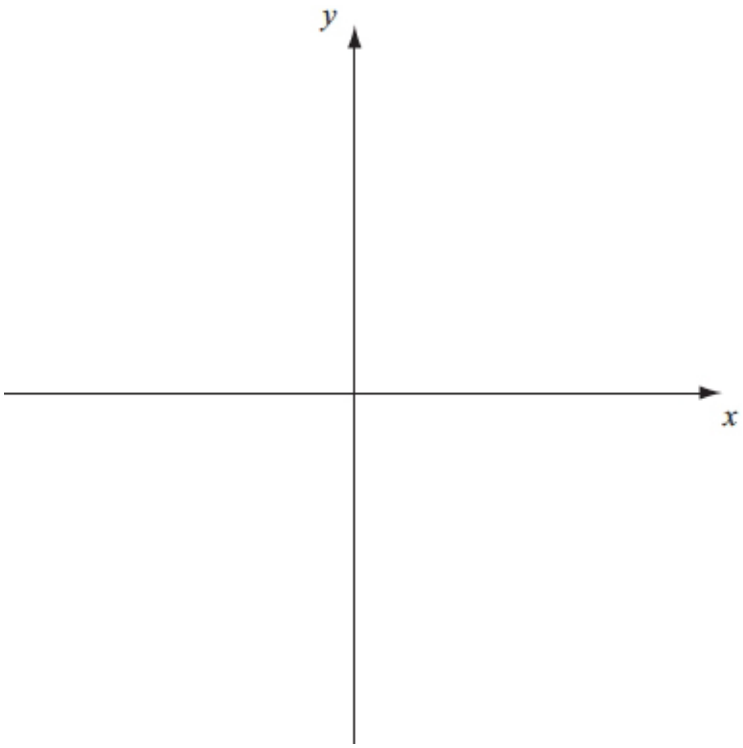
showing clearly the coordinates of all the points where the curves cross the coordinate axes.

(6)

(b) Using your sketch state, giving a reason, the number of real solutions to the equation

$$x(x+2)(3-x)+\frac{2}{x}=0$$

(2)



(Total for Question 4 is 8 marks)

End of Questions

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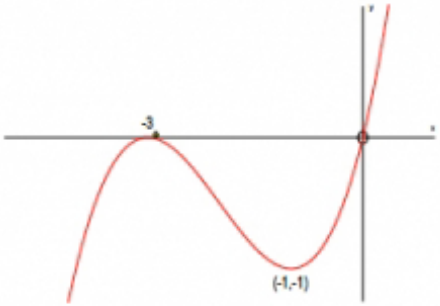
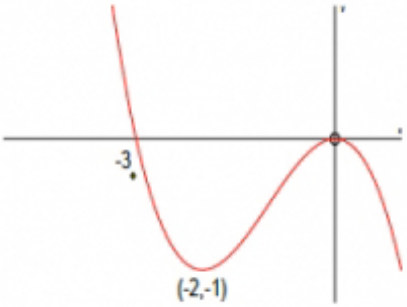
Bronze Mark Scheme

Q1

Question Number	Scheme	Marks
(a)	 Shape ∼ , touching the x-axis at its maximum. Through (0,0) & -3 marked on x-axis, or $(-3,0)$ seen. Allow $(0,-3)$ if marked on the x-axis. Marked in the correct place, but 3, is A0. Min at $(-1,-1)$	M1 A1 A1 (3)
(b)	 Correct shape ∧ (top left - bottom right) Through -3 and max at (0, 0). Marked in the correct place, but 3, is B0. Min at $(-2,-1)$	B1 B1 B1 (3) **[6]**
(a)	M1 as described above. Be generous, even when the curve seems to be composed of straight line segments, but there must be a discernible 'curve' at the max. and min. 1st A1 for curve passing through -3 and the origin. Max at $(-3,0)$ 2nd A1 for minimum at $(-1,-1)$. Can simply be indicated on sketch.	
(b)	1st B1 for the correct shape. A negative cubic passing from top left to bottom right. Shape: Be generous, even when the curve seems to be composed of straight line segments, but there must be a discernible 'curve' at the max. and min. 2nd B1 for curve passing through $(-3,0)$ having a max at (0, 0) and no other max. 3rd B1 for minimum at $(-2,-1)$ and no other minimum. If in correct quadrant but labelled, e.g. $(-2,1)$, this is B0. In each part the (0, 0) does <u>not</u> need to be written to score the second mark… having the curve pass through the origin is sufficient. The last mark (for the minimum) in each part is dependent on a sketch being attempted, and the sketch must show the minimum in approximately the correct place (not, for example, $(-2,-1)$ marked in the wrong quadrant). The mark for the minimum is <u>not</u> given for the coordinates just marked on the axes <u>unless</u> these are clearly linked to the minimum by vertical and horizontal lines.	

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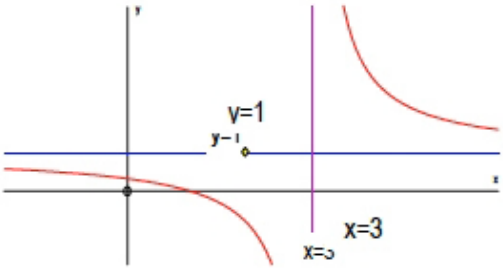
Q2

Question Number	Scheme	Marks
(a)	 Correct shape with a single crossing of each axis $y = 1$ labelled or stated $x = 3$ labelled or stated	B1 B1 B1 **(3)**
(b)	Horizontal translation so crosses the x-axis at (1, 0) New equation is $(y=)\dfrac{x\pm1}{(x\pm1)-2}$ When $x = 0$ $y =$ $=\dfrac{1}{3}$	B1 M1 M1 A1 **(4)** **7**
	Notes	
(b)	B1 for point (1,0) identified - this may be marked on the sketch as 1 on x axis. Accept $x = 1$. 1st M1 for attempt at new equation and either numerator or denominator correct 2nd M1 for setting $x = 0$ in their new equation and solving as far as $y =$… A1 for $\frac{1}{3}$ or exact equivalent. Must see $y = \frac{1}{3}$ or $(0, \frac{1}{3})$ or point marked on y-axis. **Alternative** $f(-1) = \dfrac{-1}{-1-2} = \dfrac{1}{3}$ scores M1M1A0 unless $x = 0$ is seen or they write the point as $(0, \frac{1}{3})$ or give $y = 1/3$ Answers only: $x = 1$, $y = 1/3$ is full marks as is (1,0) (0, 1/3) Just 1 and 1/3 is B0 M1 M1 A0 Special case : Translates 1 unit to left (a) B0, B1, B0 (b) Mark (b) as before May score B0 M1 M1 A0 so 3/7 or may ignore sketch and start again scoring full marks for this part.	

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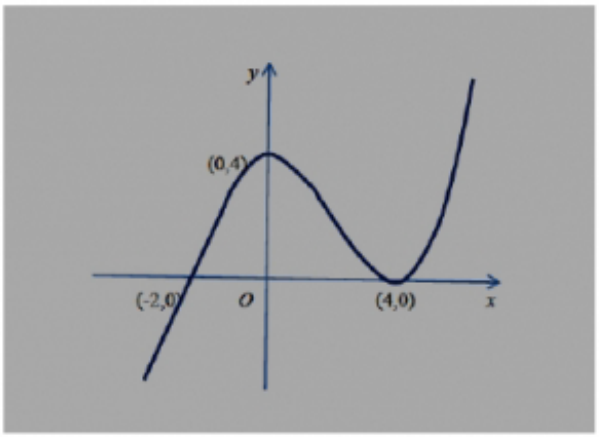
Q3

Question Number	Scheme	Notes	Marks
(a)	$f(x)=(x+1)(x-2)^2$	M1: Either stating or writing down that $(x \pm 1)$ **or** $(x \pm 2)$ is a factor – may be implied by their f(x) A1: **Both** $(x+1)$ **and** $(x-2)$ are factors - may be implied by their f(x) B1: y or f(x)= $(x+1)(x-2)^2$	M1A1B1
	$=(x+1)(x^2-4x+4)=x^3-3x^2+4$	M1: Multiplying out a quadratic to get 3 terms and then multiplying by the linear term to form a cubic. A1: x^3-3x^2+4 or $a=-3, b=0, c=4$	M1A1
			(5)
(b)		Same shape and position (ignore any coordinates) with the maximum on the y-axis	B1
		y intercept = 4 or their 'c'	B1ft
		x coordinates at -2 and 4 or marked as coordinates. Allow (0, -2) and (0, 4) if they are marked in the correct position. The curve must cross or at least stop at $x=-2$	B1
			(3)
			[8]

(a) Way 2	$x=0, y=4 \Rightarrow c=4$	Uses (0, 4) to obtain $c=4$ (can be just stated)	B1
	$x=-1, y=0 \Rightarrow -1+a-b+c=0$ $x=2, y=0 \Rightarrow 8+4a+2b+c=0$	Uses both (-1, 0) and (2, 0) in $y=x^3+ax^2+bx+c$ to form 2 simultaneous equations. Allow the equations to contain c here.	M1
	$a-b=-3$ $4a+2b=-12$ $\Rightarrow a=..$ or $b=...$	Solves simultaneously with a value for c to obtain a value for a or a value for b	M1
	Either $a=-3$ **or** $b=0$		A1
	Both $a=-3$ **and** $b=0$		A1

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(a) **Way 3**	$\frac{dy}{dx} = 3x^2 + 2ax + b$	M1: $x^n \rightarrow x^{n-1}$ at least once including c → 0	M1
	$x = 0 \Rightarrow \frac{dy}{dx} = 0 \Rightarrow b = 0$	Correct value for b	A1
	$x = 0, y = 4 \Rightarrow c = 4$	Uses (0, 4) to obtain $c = 4$ (can be just stated)	B1
	$3(2)^2 + 2a(2) + b = 0$ or $(-1)^3 + a(-1)^2 + b(-1) + 4 = 0$	Obtains an equation in a	M1
	$a = -3$	Correct value for a	A1
			(5)
	Special case: A common incorrect approach is to assume the cubic is of the form e.g. $f(x) = x(x \pm 1)(x \pm 2) + 4$ This scores B1 only for $c = 4$		
			[8]

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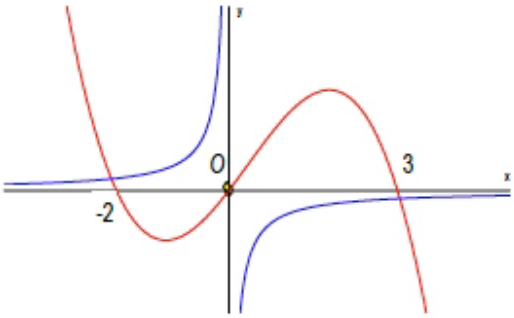
Q4

Question Number	Scheme	Marks
(a)	 (i) correct shape (-ve cubic) Crossing at (-2, 0) Through the origin Crossing at (3,0) (ii) 2 branches in correct quadrants not crossing axes One intersection with cubic on each branch	B1 B1 B1 B1 B1 B1 **(6)**
(b)	"2" solutions Since only "2" intersections	B1ft dB1ft **(2)** **8**
	Notes	
(b)	B1ft for a value that is compatible with their sketch dB1ft This mark is dependent on the value being compatible with their sketch. For a comment relating the number of solutions to the number of intersections. [Only allow 0, 2 or 4]	

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Silver Questions

Calculators may not be used

The total mark for this section is 30

Q1

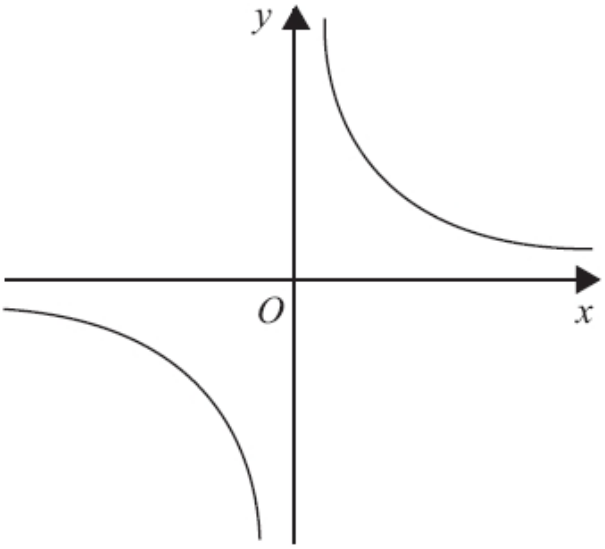

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \frac{3}{x}$, $x \neq 0$

(a) On a separate diagram, sketch the curve with equation $y = \frac{3}{x+2}$, $x \neq -2$,

showing the coordinates of any point at which the curve crosses a coordinate axis.

(3)

(b) Write down the equations of the asymptotes of the curve in part (a).

(2)

(Total for Question 1 is 5 marks)

Q2

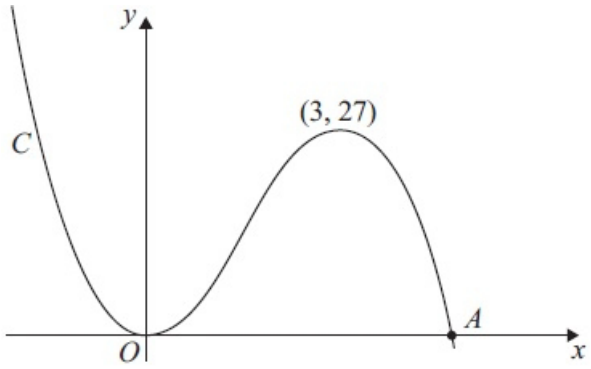

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = x^2(9 - 2x)$$

There is a minimum at the origin, a maximum at the point (3, 27) and C cuts the x-axis at the point A.

(a) Write down the coordinates of the point A.

(1)

(b) On separate diagrams sketch the curve with equation

(i) $y = \mathrm{f}(x+3)$

(ii) $y = \mathrm{f}(3x)$

On each sketch you should indicate clearly the coordinates of the maximum point and any points where the curves cross or meet the coordinate axes.

(6)

The curve with equation $y = \mathrm{f}(x) + k$, where k is a constant, has a maximum point at (3, 10).

(c) Write down the value of k.

(1)

(Total for Question 2 is 8 marks)

Q3

(a) Factorise completely $x^3 - 6x^2 + 9x$

(3)

(b) Sketch the curve with equation

$$y = x^3 - 6x^2 + 9x$$

showing the coordinates of the points at which the curve meets the x-axis.

(4)

Using your answer to part (b), or otherwise,

(c) sketch, on a separate diagram, the curve with equation

$$y = (x - 2)^3 - 6(x - 2)^2 + 9(x - 2)$$

showing the coordinates of the points at which the curve meets the x-axis.

(2)

(Total for Question 3 is 9 marks)

Q4

The curve C has equation

$$y = \frac{k^2}{x} + 1 \qquad x \in \mathbb{R},\ x \neq 0$$

where k is a constant.

(a) Sketch C stating the equation of the horizontal asymptote.

(3)

The line l has equation $y = -2x + 5$

(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

$$2x^2 - 4x + k^2 = 0$$

(2)

(c) Hence find the exact values of k for which l is a tangent to C.

(3)

(Total for Question 4 is 8 marks)

Silver Mark Scheme

Q1

Question number	Scheme	Marks
	(a) Translation parallel to x-axis Top branch intersects +ve y-axis Lower branch has no intersections No obvious overlap $\left(0,\frac{3}{2}\right)$ or $\frac{3}{2}$ marked on y- axis	M1 A1 B1 (3)
	(b) $x=-2,\quad y=0$	B1, B1 (2)
S.C.	[Allow ft on first B1 for $x=2$ when translated "the wrong way" but must be compatible with their sketch.]	
		5

(a)	M1 for a horizontal translation – two branches with one branch cutting $y-$ axis only. If one of the branches cuts both axes (translation up and across) this is M0. A1 for a horizontal translation to left. Ignore any figures on axes for this mark. B1 for correct intersection on positive y-axis. More than 1 intersection is B0. x=0 and $y=1.5$ in a table alone is insufficient unless intersection of their sketch is with +ve y-axis. A point marked on the graph overrides a point given elsewhere.
(b)	1st B1 for $x=-2$. NB $x\neq-2$ is B0. Can accept $x=+2$ if this is compatible with their sketch. Usually they will have M1A0 in part (a) (and usually B0 too) 2nd B1 for $y=0$.
S.C.	If $x=-2$ and $y=0$ and some other asymptotes are also given award B1B0 The asymptote equations should be clearly stated in part (b). Simply marking $x=-2$ or $y=0$ on the sketch is insufficient <u>unless</u> they are clearly marked "asymptote $x=-2$" etc.

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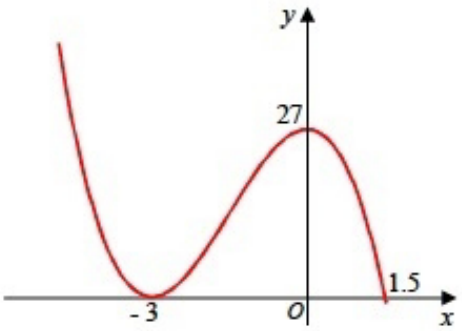
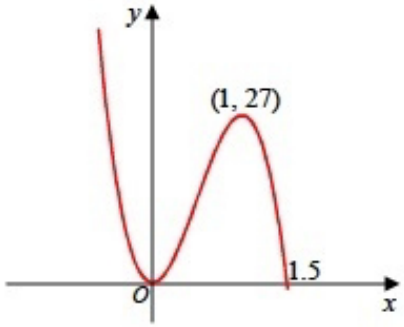
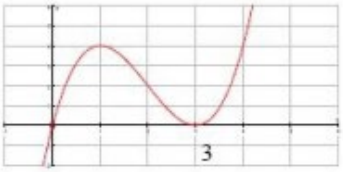
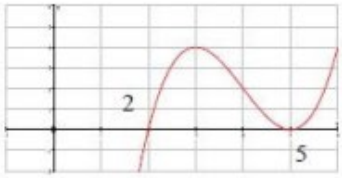
Q2

Question Number	Scheme	Marks
(a)	{Coordinates of A are} $(4.5, 0)$ See notes below	B1 [1]
(b)(i)	 Horizontal translation M1 -3 and their ft 1.5 on postitive x-axis A1 ft Maximum at 27 marked on the y-axis B1	[3]
(ii)	 Correct shape, minimum at $(0, 0)$ and a maximum within the first quadrant. M1 1.5 on x-axis A1 ft Maximum at $(1, 27)$ B1	[3]
(c)	$\{k =\} -17$	B1 [1]
		8

	Notes
(a)	**B1:** For stating either $x = 4.5$ or $\frac{9}{2}$ or $\frac{18}{4}$ etc. or $A = 4.5$ or $\frac{9}{2}$ or $(4.5, 0)$. Can be written on graph. Allow $(0, 4.5)$ marked on curve for B1. Otherwise $(0, 4.5)$ without reference to any of the above is B0.
(b)(i)	**M1:** for any horizontal (left-right) translation where minimum is still **on** x-axis not at $(0, 0)$. Ignore any values. **A1ft:** for -3 **(NOT 3)** and 1.5 (or their x in part (a) $-$ 3) *evaluated* and marked on the positive x-axis. Allow $(0, -3)$ and/or $(0,$ ft 1.5$)$ rather than $(-3, 0)$ and (ft 1.5, 0) if marked in the "correct" place on the x-axis. **Note:** Candidate *cannot* gain this mark if their x in part (a) is less than 3. **B1:** Maximum at 27 marked on the y-axis. **Note:** the maximum must be on the y-axis for this mark.
(ii)	**M1:** for correct shape with minimum still at $(0, 0)$ and a maximum within the first quadrant. Ignore values. **A1ft:** for $\frac{\text{their } x \text{ in part } (a)}{3}$; as intercept on x-axis eg: $\frac{4.5}{3}$ or 1.5 or $\frac{3}{2}$ or $\frac{9}{6}$ **Note:** a generalised $\frac{A}{3}$ is A0. Allow $(0,$ ft 1.5$)$ rather than (ft 1.5, 0) if marked in the "correct" place on the x-axis. **B1:** Maximum at $(1, 27)$ or allow 1 marked on the x-axis and the corresponding 27 marked on the y-axis. **Note: Be careful to look at the correct graph. The candidate may draw another graph to help them to answer part (c).** **Note:** You can recover (b)(i) $(-3, 0)$ and (ft 1.5, 0) or in (b)(ii) (ft 1.5, 0) as *correct coordinates only* in candidate's working if these are not marked on their sketch(es).
(c)	**B1:** for $(k =) -17$ only. **BEWARE**: This could be written in the middle or at the bottom of a page.

Pearson

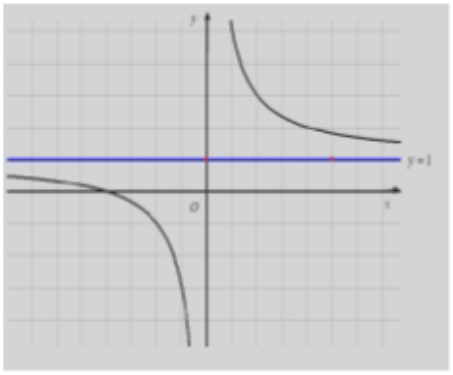
Q3

Question Number	Scheme	Marks
Q (a)	$x(x^2-6x+9)$ $=x(x-3)(x-3)$	B1 M1 A1 (3)
(b)	 Shape Through origin (not touching) Touching x-axis only once Touching at (3, 0), or 3 on x-axis [Must be on graph not in a table]	B1 B1 B1 B1ft (4)
(c)	 Moved horizontally (either way) (2, 0) and (5, 0), or 2 and 5 on x-axis	M1 A1 (2) **[9]**

(a)	B1 for correctly taking out a factor of x M1 for an attempt to factorize their 3TQ e.g. $(x+p)(x+q)$ where $\lvert pq\rvert=9$. So $(x-3)(x+3)$ will score M1 but A0 A1 for a fully correct factorized expression - accept $x(x-3)^2$ If they "solve" use ISW
S.C.	If the only correct linear factor is $(x - 3)$, perhaps from factor theorem, award B0M1A0 Do not award marks for factorising in part (b)
	For the graphs "Sharp points" will lose the 1st B1 in (b) but otherwise be generous on shape Condone (0, 3) in (b) and (0, 2), (0,5) in (c) if the points are marked in the correct places.
(b)	2nd B1 for a curve that starts or terminates at (0, 0) score B0 4th B1ft for a curve that touches (not crossing or terminating) at $(a, 0)$ where their $y=x(x-a)^2$
(c)	M1 for their graph moved horizontally (only) or a fully correct graph Condone a partial stretch if ignoring their values looks like a simple translation A1 for their graph translated 2 to the right and crossing or touching the axis at 2 and 5 only **Allow a fully correct graph (as shown above) to score M1A1 whatever they have in (b)**

Pearson

Q4

Question	Scheme	Marks	AOs
(a)	 $\frac{1}{x}$ shape in 1st quadrant	M1	1.1b
	Correct	A1	1.1b
	Asymptote $y=1$	B1	1.2
		(3)	
(b)	Combines equations $\Rightarrow \frac{k^2}{x}+1=-2x+5$	M1	1.1b
	$(\times x)\Rightarrow k^2+1x=-2x^2+5x\Rightarrow 2x^2-4x+k^2=0$ *	A1*	2.1
		(2)	
(c)	Attempts to set $b^2-4ac=0$	M1	3.1a
	$8k^2=16$	A1	1.1b
	$k=\pm\sqrt{2}$	A1	1.1b
		(3)	
			(8 marks)

Pearson

Notes

(a)

M1: For the shape of a $\frac{1}{x}$ type curve in Quadrant 1. It must not cross either axis and have acceptable curvature. Look for a negative gradient changing from $-\infty$ to 0 condoning "slips of the pencil". (See Practice and Qualification for clarification)

A1: Correct shape and position for both branches.
It must lie in Quadrants 1, 2 and 3 and have the correct curvature including asymptotic behaviour

B1: Asymptote given as $y=1$. This could appear on the diagram or within the text.
Note that the curve does not need to be asymptotic at $y=1$ but this must be the only horizontal asymptote offered by the candidate.

(b)

M1: Attempts to combine $y=\frac{k^2}{x}+1$ with $y=-2x+5$ to form an equation in just x

A1*: Multiplies by x (the processed line must be seen) and proceeds to given answer with no slips.
Condone if the order of the terms are different $2x^2+k^2-4x=0$

(c)

M1: Deduces that $b^2-4ac=0$ or equivalent for **the given equation.**
If a, b and c are stated only accept $a=2, b=\pm4, c=k^2$ so $4^2-4\times2\times k^2=0$
Alternatively completes the square $x^2-2x+\frac{1}{2}k^2=0\Rightarrow(x-1)^2=1-\frac{1}{2}k^2\Rightarrow "1-\frac{1}{2}k^2"=0$

A1: $8k^2=16$ or exact simplified equivalent. Eg $8k^2-16=0$

If a, b and c are stated they must be correct. Note that b^2 appearing as 4^2 is correct

A1: $k=\pm\sqrt{2}$ and following correct a, b and c if stated

A solution via differentiation would be awarded as follows

M1: Sets the gradient of the curve $=-2\Rightarrow-\frac{k^2}{x^2}=-2\Rightarrow x=(\pm)\frac{k}{\sqrt{2}}$ oe and attempts to substitute into $2x^2-4x+k^2=0$

A1: $2k^2=(\pm)2\sqrt{2}k$ oe

A1: $k=\pm\sqrt{2}$

Pearson

Gold Questions

Calculators may not be used

The total mark for this section is 35

Q1

The point $P\,(1, a)$ lies on the curve with equation $y = (x + 1)^2(2 - x)$.

(a) Find the value of a.

(1)

(b) Sketch the curves with the following equations:

(i) $y = (x + 1)^2\ (2 - x)$,

(ii) $y = \dfrac{2}{x}$

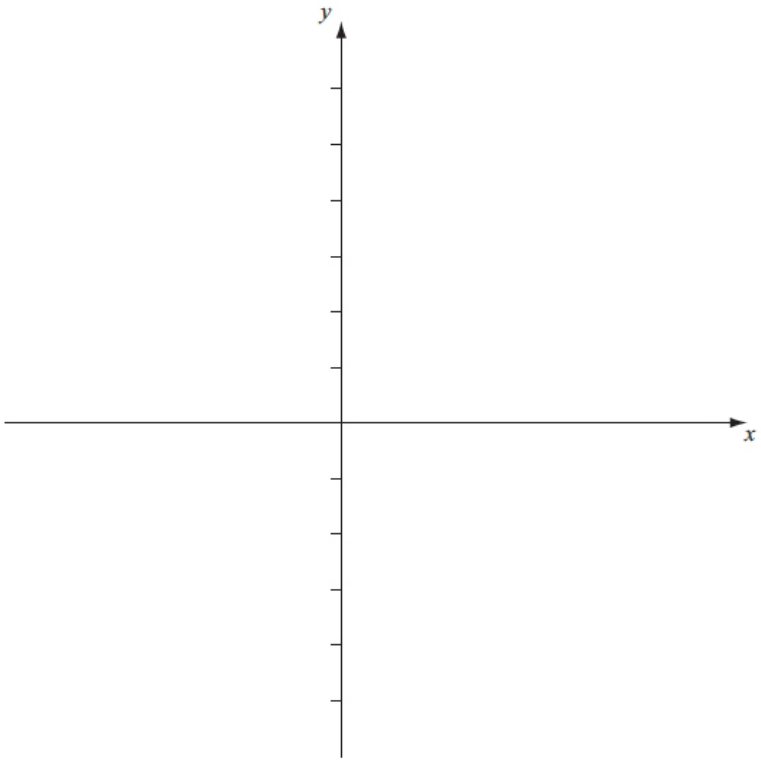
On your diagram show clearly the coordinates of any points at which the curves meet the axes.

(5)

(c) With reference to your diagram in part (b) state the number of real solutions to the equation

$$(x+1)^2\left(2-x\right)=\frac{2}{x}$$

(1)



(Total for Question 1 is 7 marks)

Q2

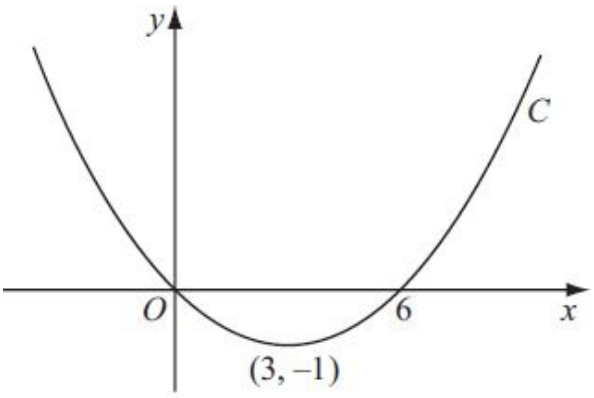


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$

The curve C passes through the origin and through $(6, 0)$

The curve C has a minimum at the point $(3, -1)$.

On separate diagrams, sketch the curve with equation

(a) $y = f(2x)$,

(3)

(b) $y = -f(x)$,

(3)

(c) $y = f(x + p)$, where p is a constant and $0 < p < 3$.

(4)

(Total for Question 1 is 10 marks)

Pearson

Q3

(a) Factorise completely $x^3 + 10x^2 + 25x$

(2)

(b) Sketch the curve with equation

$$y = x^3 + 10x^3 + 25x$$

showing the coordinates of the points at which the curve cuts or touches the x-axis.

(2)

The point with coordinates $(-3, 0)$ lies on the curve with equation

$$y = (x + a)^3 + 10(x + a)^2 + 25(x + a)$$

where a is a constant.

(c) Find the two possible values of a.

(3)

(Total for Question 3 is 7 marks)

Q4

(a) On separate axes sketch the graphs of

(i) $y = -3x + c$, where c is a positive constant,

(ii) $y = \frac{1}{x} + 5$

On each sketch show the coordinates of any point at which the graph crosses the y-axis and the equation of any horizontal asymptote.

(4)

Given that $y = -3x + c$, where c is a positive constant, meets the curve $y = \frac{1}{x} + 5$ at two distinct points,

(b) show that $(5 - c)^2 > 2$

(3)

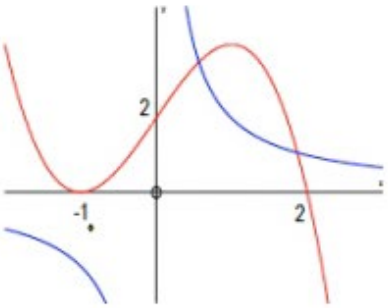
(c) Hence find the range of possible values for c

(4)

(Total for Question 4 is 11 marks)

Gold Mark Scheme

Q1

Question Number	Scheme	Marks
(a)	$(a=)\ (1+1)^2(2-1)=\underline{4}$ (1, 4) or $y=4$ is also acceptable	B1 (1)
(b)	 (i) Shape ∧ or ∼ anywhere	B1
	Min at $(-1,0)$ … can be -1 on x-axis. Allow $(0,-1)$ if marked on the x-axis. Marked in the correct place, but 1, is B0.	B1
	(2, 0) and (0, 2) can be 2 on axes	B1
	(ii) Top branch in 1st quadrant with 2 intersections	B1
	Bottom branch in 3rd quadrant (ignore any intersections)	B1 (5)
(c)	(2 intersections therefore) **2** (roots)	B1ft (1) **[7]**

(b)	1st B1 for shape ∧ or ∼ Can be anywhere, but there must be one max. and one min. and no further max. and min. turning points. Shape: Be generous, even when the curve seems to be composed of straight line segments, but there must be a discernible 'curve' at the max. and min. 2nd B1 for minimum at $(-1,0)$ (even if there is an additional minimum point shown) 3rd B1 for the sketch meeting axes at (2, 0) and (0, 2). They can simply mark 2 on the axes. The marks for minimum and intersections are dependent upon having a sketch. Answers on the diagram for min. and intersections take precedence over answers seen elsewhere. 4th B1 for the branch fully within 1st quadrant having 2 intersections with (not just 'touching') the other curve. The curve can 'touch' the axes. A curve of (roughly) the correct shape is required, but be very generous, even when the arc appears to turn 'inwards' rather than approaching the axes, and when the curve looks like two straight lines with a small curve at the join. Allow, for example, shapes like these: 5th B1 for a branch fully in the 3rd quadrant (ignore any intersections with the other curve for this branch). The curve can 'touch' the axes. A curve of (roughly) the correct shape is required, but be very generous, even when the arc appears to turn 'inwards' rather than approaching the axes.
(c)	B1ft for a statement about the number of roots - compatible with their sketch. No sketch is B0. The answer 2 incompatible with the sketch is B0 (ignore any algebra seen). If the sketch shows the 2 correct intersections and, for example, one other intersection, the answer here should be 3, not 2, to score the mark.

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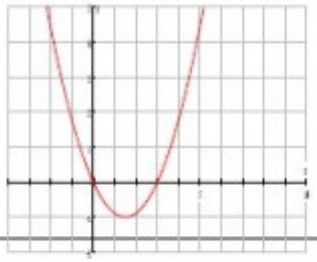
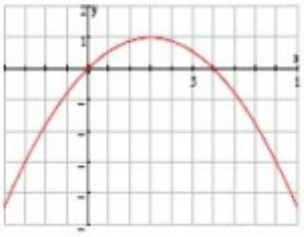
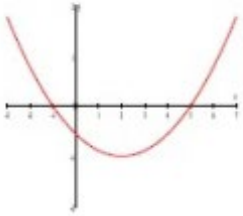
Q2

Question Number	Scheme	Marks
(a)	 Shape ∪ through (0, 0) (3, 0) (1.5, −1)	B1 B1 B1 (3)
(b)	 Shape ∩ (0, 0) and (6, 0) (3, 1)	B1 B1 B1 (3)
(c)	 Shape ∪, not through (0, 0) Minimum in 4th quadrant $(-p, 0)$ and $(6-p, 0)$ $(3-p, -1)$	M1 A1 B1 B1 (4) **10**

Notes

(a) B1: U shaped parabola through origin
B1: (3,0) stated or 3 labelled on x axis
B1: (1.5, -1) or equivalent e.g. (3/2, -1)
(b) B1: Cap shaped parabola in any position

B1: through origin (may not be labelled) and (6,0) stated or 6 labelled on x - axis
B1: (3,1) shown
(c) M1: U shaped parabola not through origin
A1: Minimum in 4th quadrant (depends on M mark having been given)
B1: Coordinates stated or shown on x axis
B1: Coordinates stated
Note: If values are taken for p, then it is possible to give M1A1B0B0 even if there are several attempts. (In this case all minima should be in fourth quadrant)

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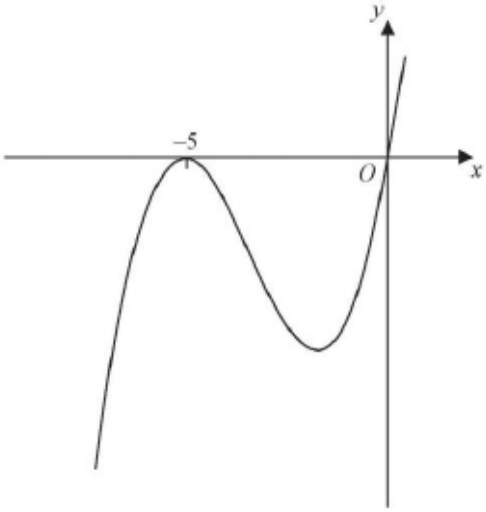
Q3

Question	Scheme	Marks	AOs
(a)	$x^3+10x^2+25x=x(x^2+10x+25)$	M1	1.1b
	$=x(x+5)^2$	A1	1.1b
		(2)	
(b)	 A cubic with correct orientation Curve passes through the origin (0, 0) and touches at $(-5, 0)$ (see note below for ft)	M1	1.1b
		A1ft	1.1b
		(2)	
(c)	Curve has been translated a to the left	M1	3.1a
	$a = -2$	A1ft	3.2a
	$a = 3$	A1ft	1.1b
		(3)	
			(7 marks)

Notes

(a) M1: Takes out factor x
A1: Correct factorisation - allow $x(x+5)(x+5)$

(b) M1: Correct shape
A1ft: Curve passes through the origin (0, 0) and touches at $(-5, 0)$ – allow follow through from incorrect factorisation

(c) M1: May be implied by one of the correct answers for a or by a statement
A1ft: ft from their cubic as long as it meets the x-axis only twice.
A1ft : ft from their cubic as long as it meets the x-axis only twice.

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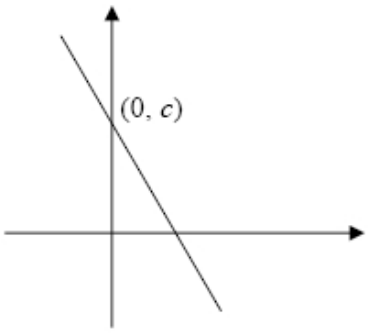
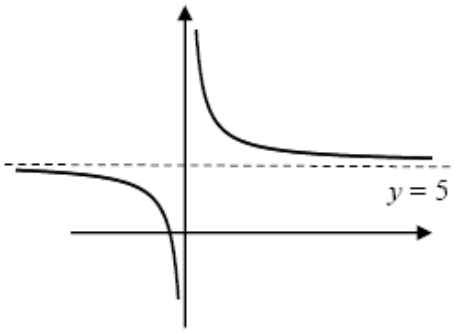
Q4

Question Number	Scheme		Marks
(a)(i)		B1: Straight line with negative gradient anywhere even with no axes.	B1
		B1: Straight line with an intercept at $(0, c)$ or just c marked on the positive y-axis provided the line passes through the positive y-axis. Allow $(c, 0)$ as long as it is marked in the correct place. Allow $(0, c)$ in the body of the script but in any ambiguity, the sketch has precedence. **Ignore any intercepts with the x-axis.**	B1
(a)(ii)		**Either:** For the shape of a $y = \frac{1}{x}$ curve in any position. It must have two branches and be asymptotic horizontally and vertically with no obvious "overlap" with the asymptotes, but otherwise be generous. The curve may bend away from the asymptote a little at the end. Sufficient curve must be seen to suggest the asymptotic behaviour, both vertically and horizontally and the branches must approach the same asymptote **Or** the **equation** $y = 5$ seen independently i.e. whether the sketch has an asymptote here or not. Do not allow $y \neq 5$ or $x = 5$.	B1
		B1: Fully correct graph **and** with a horizontal asymptote on the positive y-axis. The asymptote does not have to be drawn but the **equation** $y = 5$ must be seen. The shape needs to be reasonably accurate with the "ends" not bending away significantly from the asymptotes and the branches must approach the same asymptote. Ignore $x = 0$ given as an asymptote.	B1
	Allow sketches to be on the same axes.		
			(4)

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(b)	$\frac{1}{x}+5=-3x+c \Rightarrow 1+5x=-3x^2+cx$ $\Rightarrow 3x^2+5x-cx+1=0$	Sets $\frac{1}{x}+5=-3x+c$, attempts to multiply by x and collects terms (to one side). Allow e.g. "$>$" or "$<$" for "$=$". At least 3 of the terms must be multiplied by x, e.g. allow one slip. The '$=0$' may be implied by subsequent work and provided correct work follows, full marks are still possible in (b).	M1
	$b^2-4ac=(5-c)^2-4\times1\times3$	Attempts to use b^2-4ac with **their** a, b and c from **their equation** where $a=\pm3$, $b=\pm5\pm c$ and $c=\pm1$. This could be as part of the quadratic formula or as $b^2<4ac$ or as $b^2>4ac$ or as $\sqrt{b^2-4ac}$ etc. If it is part of the quadratic formula only look for use of b^2-4ac. There must be no x's.	M1
	$(5-c)^2>12$*	Completes proof with **no errors or incorrect statements** and with the "$>$" appearing correctly before the final answer, which could be from $b^2-4ac>0$. Note that the statement $3x^2+5x-cx+1>0$ or starting with e.g. $\frac{1}{x}+5>-3x+c$ would be an error.	A1*
	Note: A minimum for (b) could be, $\frac{1}{x}+5=-3x+c \Rightarrow 3x^2+5x-cx+1(=0)$ (M1) $b^2>4ac \Rightarrow (5-c)^2>12$ (M1A1) If $b^2>4ac$ is not seen then $4\times3\times1$ needs to be seen explicitly.		
			(3)

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(c)	$(5-c)^2=12\Rightarrow(c=)5\pm\sqrt{12}$ or $(5-c)^2=12\Rightarrow c^2-10c+13=0$ $\Rightarrow(c=)\dfrac{-^-10\pm\sqrt{(-10)^2-4\times13}}{2}$	M1: Attempts to find at least one critical value using the result in (b) or by expanding and solving a 3TQ (See General Principles) (the "= 0" may be implied) A1: Correct critical values in any form. Note that $\sqrt{12}$ may be seen as $2\sqrt{3}$.	M1A1
	$c<"5-\sqrt{12}",\ c>"5+\sqrt{12}"$	Chooses outside region. The '0 <' can be ignored for **this** mark. So look for $c <$ their $5-\sqrt{12}$, $c >$ their $5+\sqrt{12}$. This could be scored from $5+\sqrt{12}<c<5-\sqrt{12}$ or $5-\sqrt{12}>c>5+\sqrt{12}$. Evidence is to be taken from their answers not from a diagram.	M1
	$0<c<5-\sqrt{12},\ c>5+\sqrt{12}$	Correct ranges including the '0 <' e.g. answer as shown or each region written separately or e.g. $(0,5-\sqrt{12}),\ (5+\sqrt{12},\infty)$. The critical values may be un-simplified but must be at least $\dfrac{10+\sqrt{48}}{2},\ \dfrac{10-\sqrt{48}}{2}$. Note that $0<c<5-\sqrt{12}$ **and** $c>5+\sqrt{12}$ would score M1A0.	A1
	Allow the use of *x* rather than *c* in (c) but the final answer must be in terms of *c*.		
			(4)
			(11 marks)

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Topic 5

Straight Line Graphs

Bronze, Silver, Gold
Worksheets for
AS Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculators may not be used

The total mark for this section is 34

Q1

The line l passes through the points A (3, 1) and B (4, – 2).

Find an equation for l

(Total for Question 1 is 6 marks)

Q2

The line L_1 has equation $4x + 2y - 3 = 0$

(a) Find the gradient of L_1

(2)

The line L_2 is perpendicular to L_1 and passes through the point (2, 5)

(b) Find the equation of L_2 in the form $y = mx + c$, where m and c are constants.

(3)

(Total for Question 2 is 5 marks)

Q3

The line l_1 has equation $4y - 3x = 10$

The line l_2 passes through the points (5, –1) and (–1, 8)

Determine, giving full reasons for your answer, whether lines l_1 and l_2 are parallel, perpendicular or neither.

(Total for Question 3 is 4 marks)

Q4

The line l_1 has equation $y = -2x + 3$

The line l_2 is perpendicular to l_1 and passes through the point (5, 6).

(a) Find an equation for l_2 in the form $ax + by + c = 0$, where a , b and c are integers.

(3)

The line l_2 crosses the x-axis at the point A and the y-axis at the point B.

(b) Find the x-coordinate of A and the y-coordinate of B.

(2)

Given that O is the origin,

(c) find the area of the triangle OAB.

(2)

(Total for Question 4 is 7 marks)

Q5

The point A (–6, 4) and the point B (8, –3) lie on the line L.

(a) Find an equation for L in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

(b) Find the distance AB, giving your answer in the form $k\sqrt{5}$, where k is an integer.

(3)

(Total for Question 5 is 7 marks)

Q6

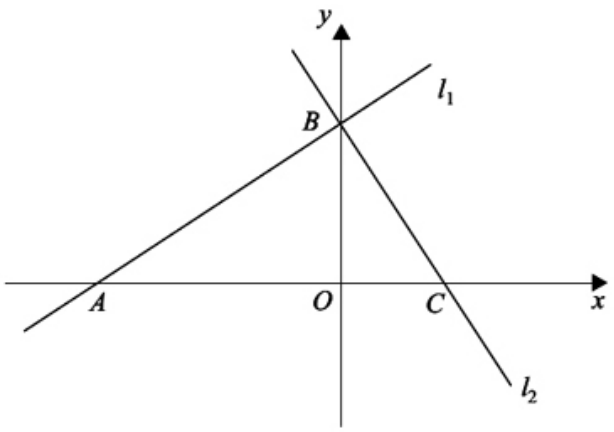

Figure 1

The line l_1 has equation $2x - 3y + 12 = 0$

(a) find the gradient of l_1.

(1)

The line l_1 crosses the x-axis at the point A and the y-axis at the point B, as shown in Figure 1.

The line l_2 is perpendicular to l_1 and passes through B.

(b) Find an equation of l_2.

(3)

The line l_2 crosses the x-axis at the point C.

(c) Find the area of triangle ABC.

(4)

(Total for Question 6 is 8 marks)

End of Questions

Pearson

Bronze Mark Scheme

Q1

Question	Scheme	Marks	AOs
(Way 1)	Uses $y = mx + c$ with both (3,1) and (4, -2) and attempt to find m or c	M1	1.1b
	$m = -3$	A1	1.1b
	$c = 10$ so $y = -3x + 10$ o.e.	A1	1.1b
		(3)	
Or (Way 2)	Uses $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$ with both (3,1) and (4, -2)	M1	1.1b
	Gradient simplified to -3 (may be implied)	A1	1.1b
	$y = -3x + 10$ o.e.	A1	1.1b
		(3)	
Or (Way 3)	Uses $ax + by + k = 0$ and substitutes both $x = 3$ when $y = 1$ and $x = 4$ when $y = -2$ with attempt to solve to find a, b or k in terms of one of them	M1	1.1b
	Obtains $a = 3b$, $k = -10b$ or $3k = -10a$	A1	1.1b
	Obtains $a = 3$, $b = 1$, $k = -10$ Or writes $3x + y - 10 = 0$ o.e.	A1	1.1b
		(3)	
			(3 marks)

Notes

M1: Need correct use of the given coordinates
A1: Need fractions simplified to -3 (in ways 1 and 2)
A1: Need constants combined accurately
N.B. Answer left in the form $(y - 1) = -3(x - 3)$ or $(y - (-2)) = -3(x - 4)$ is awarded M1A1A0 as answers should be simplified by constants being collected
Note that a correct answer implies all three marks in this question.

Pearson

Q2

Question Number	Scheme	Notes	Marks
(a)	$4x+2y-3=0 \Rightarrow y=-2x+\frac{3}{2}$	Attempt to write in the form $y=$	M1
	$\Rightarrow$ gradient $=-2$	Accept any un-simplified form and allow even with an incorrect value of "c"	A1
(a) Way 2	Alternative: $4+2\frac{dy}{dx}=0$	Attempt to differentiate Allow $p\pm q\frac{dy}{dx}=0,\ p,q\neq 0$	M1
	$\Rightarrow$ gradient $=-2$	Accept any un-simplified form	A1
	Answer only scores M1A1		
			[2]
(b)	Using $m_N=-\frac{1}{m_T}$	Attempt to use $m_N=$ $-\frac{1}{gradient\ from\ (a)}$	M1
	$y-5='\frac{1}{2}'(x-2)$ or Uses $y=mx+c$ in an attempt to find c	Correct straight line method using a 'changed' gradient and the point (2, 5)	M1
	$y=\frac{1}{2}x+4$	Cao (Isw)	A1
			(3)
			[5]

Pearson

Q3

Question	Scheme	Marks	AOs
	States gradient of $4y-3x=10$ is $\frac{3}{4}$ oe or rewrites as $y=\frac{3}{4}x+...$	B1	1.1b
	Attempts to find gradient of line joining $(5,-1)$ and $(-1,8)$	M1	1.1b
	$=\frac{-1-8}{5-(-1)}=-\frac{3}{2}$	A1	1.1b
	States neither with suitable reasons	A1	2.4
		(4)	
			(4 marks)

Notes

B1: States that the gradient of line l_1 is $\frac{3}{4}$ or writes l_1 in the form $y=\frac{3}{4}x+...$

M1: Attempts to find the gradient of line l_2 using $\frac{\Delta y}{\Delta x}$ Condone one sign error Eg allow $\frac{9}{6}$

A1: For the gradient of l_2 $=\frac{-1-8}{5-(-1)}=-\frac{3}{2}$ or the equation of l_2 $y=-\frac{3}{2}x+...$

Allow for any equivalent such as $-\frac{9}{6}$ or -1.5

A1: CSO (on gradients)

Explains that they are neither parallel as the gradients not equal nor perpendicular as $\frac{3}{4}\times-\frac{3}{2}\neq-1$ oe

Allow a statement in words "they are not negative reciprocals " for a reason for not perpendicular and "they are not equal" for a reason for not being parallel

Pearson

Q4

Question Number	Scheme	Marks
(a)	Gradient of l_2 is $\frac{1}{2}$ or 0.5 or $\frac{-1}{-2}$	B1
	Either $y-6="\frac{1}{2}"(x-5)$ or $y="\frac{1}{2}"x+c$ and $6="\frac{1}{2}"(5)+c \Rightarrow c=("\frac{7}{2}")$	M1
	$x-2y+7=0$ or $-x+2y-7=0$ or $k(x-2y+7)=0$ with k **an integer**	A1 [3]
(b)	Puts $x=0$, or $y=0$ in **their** equation and solves to find appropriate co-ordinate	M1
	x-coordinate of A is -7 and y-coordinate of B is $\frac{7}{2}$.	A1 cao [2]
(c)	Area $OAB=\frac{1}{2}(7)\left(\frac{7}{2}\right)=\frac{49}{4}$ (units)^2	Applies $\pm\frac{1}{2}$(base)(height) M1 $\frac{49}{4}$ A1cso [2]
		7 marks

	Notes
(a)	**B1**: Must have ½ or 0.5 or $\frac{-1}{-2}$ o.e. stated and stops, or used in their line equation
	M1: Full method to obtain an equation of the line through (5,6) with their "m". So $y-6=m(x-5)$ with their gradient or uses $y=mx+c$ with (5, 6) and their gradient to find c. Allow any numerical gradient here including –2 or –1 but **not zero** . (Allow (6,5) as a slip **if** $y-y_1=m(x-x_1)$ is quoted first)
	A1: Accept any multiple of the correct equation, provided that the coefficients are integers and equation = 0 e.g. $-x+2y-7=0$ or $k(x-2y+7)=0$ or even $2y-x-7=0$
(b)	**M1:** Either one of the x or y coordinates using their equation
	A1: Needs both correct values. Accept any correct equivalent.. Need not be written as co-ordinates. **Even** just –7 and 3.5 with **no** indication which is which may be awarded the A1.
(c)	**M1:** Any correct method for area of triangle AOB, with their values for co-ordinates of A and B (may include negatives) *Method usually half base times height but determinants could be used.*
	A1: Any exact equivalent to 49/4, e.g. 12.25. (**negative final** answer is A0 but replacing by **positive** is A1) Do not need units.
	c.s.o. implies if A0 is scored in (b) then A0 is scored in (c) as well. **However** if candidate has correct line equation in (a) of wrong form may score A0 in (a) and A1 in (b) and (c)
	Note: Special cases: $\frac{1}{2}(-7)\left(+\frac{7}{2}\right)=-\frac{49}{4}$ (units)^2 is M1 A0 but changing sign to area = $+\frac{49}{4}$ gets M1A1 (recovery)
	N.B. Candidates making sign errors in (b) and obtaining +7 and -$\frac{7}{2}$. may also get $\frac{49}{4}$ as their answer following previous errors. They should be awarded A0 as this answer is not ft and is for correct **solution** only
	Special Case: In (a) and (b): Produces parallel line instead of perpendicular line: So uses m = -2 This is not treated as a misread as it simplifies the question. The marks will usually be B0 M1 A0, M1 A0, M1 A0 i.e. maximum of 3/7

Pearson

Q5

Question number	Scheme	Marks
	(a) $m=\frac{4-(-3)}{-6-8}$ or $\frac{-3-4}{8-(-6)}$, $=\frac{7}{-14}$ or $\frac{-7}{14}$ $\left(=-\frac{1}{2}\right)$	M1, A1
	Equation: $y-4=-\frac{1}{2}(x-(-6))$ or $y-(-3)=-\frac{1}{2}(x-8)$	M1
	$x+2y-2=0$ (or equiv. with integer coefficients… must have '= 0')	A1 (4)
	(e.g. $14y+7x-14=0$ and $14-7x-14y=0$ are acceptable)	
	(b) $(-6-8)^2+(4-(-3))^2$	M1
	14^2+7^2 or $(-14)^2+7^2$ or $14^2+(-7)^2$ (M1 A1 may be implied by 245)	A1
	$AB=\sqrt{14^2+7^2}$ or $\sqrt{7^2(2^2+1^2)}$ or $\sqrt{245}$	
	$7\sqrt{5}$	A1cso (3)
		7
	(a) 1st M: Attempt to use $m=\frac{y_2-y_1}{x_2-x_1}$ (may be implicit in an equation of L).	
	2nd M: Attempting straight line equation in any form, e.g $y-y_1=m(x-x_1)$, $\frac{y-y_1}{x-x_1}=m$, with any value of m (except 0 or ∞) and either (–6, 4) or (8, –3).	
	N.B. It is also possible to use a different point which lies on the line, such as the midpoint of AB (1, 0.5).	
	Alternatively, the 2nd M may be scored by using $y=mx+c$ with a numerical gradient and substituting (–6, 4) or (8, –3) to find the value of c.	
	Having coords the wrong way round, e.g. $y-(-6)=-\frac{1}{2}(x-4)$, loses the 2nd M mark unless a correct general formula is seen, e.g. $y-y_1=m(x-x_1)$.	
	(b) M: Attempting to use $(x_2-x_1)^2+(y_2-y_1)^2$.	
	Missing bracket, e.g. -14^2+7^2 implies M1 if no earlier version is seen.	
	-14^2+7^2 with no further work would be M1 A0.	
	-14^2+7^2 followed by 'recovery' can score full marks.	

Pearson

Q6

Question	Scheme	Marks
(a)	$(m=)\frac{2}{3}$ (or exact equivalent)	B1 (1)
(b)	B: $(0, 4)$ [award when first seen – may be in (c)]	B1
	Gradient: $\frac{-1}{m}=-\frac{3}{2}$	M1
	$y-4=-\frac{3x}{2}$ or equiv. e.g. $\left(y=-\frac{3x}{2}+4,\quad 3x+2y-8=0\right)$	A1 (3)
(c)	A: $(-6,0)$ [award when first seen – may be in (b)]	B1
	C: $\frac{3x}{2}=4 \Rightarrow x=\frac{8}{3}$ [award when first seen – may be in (b)]	B1ft
	Area: Using $\frac{1}{2}(x_C-x_A)y_B$	M1
	$=\frac{1}{2}\left(\frac{8}{3}+6\right)4=\frac{52}{3}\left(=17\frac{1}{3}\right)$	A1 cso (4)
ALT	$BC=\frac{4}{6}\sqrt{52}$ (from similar triangles) (or possibly using C)	2nd B1ft
	Area: Using $\frac{1}{2}(AB\times BC)$ N.B. $AB=\sqrt{6^2+4^2}=\sqrt{52}$	M1
	$=\frac{1}{2}\times\sqrt{52}\times\left(\frac{2}{3}\sqrt{52}\right)=\frac{52}{3}\left(=17\frac{1}{3}\right)$	A1
		8 marks

	Notes
(a)	B1 for $\frac{2}{3}$ seen. Do not award for $\frac{2}{3}x$ and must be in part (a)
(b)	B1 for coordinates of B. Accept 4 marked on y-axis (clearly labelled) M1 for use of perpendicular gradient rule. Follow through their value for m A1 for a correct equation (any form, need not be simplified). Answer only 3/3
(c)	1st B1 for the coordinates of A (clearly labelled). Accept – 6 marked on x-axis 2nd B1ft for the coordinates of C (clearly labelled) or $AC=\frac{26}{3}$. Accept $x=\frac{8}{3}$ marked on x-axis. Follow through from l_2 if >0 M1 for an expression for the area of the triangle (all lengths > 0). Ft their 4, - 6 and $\frac{8}{3}$ A1 cso for $\frac{52}{3}$ or exact equivalent seen but must be a single fraction or $17\frac{1}{3}$ or $17\frac{2}{6}$ etc $17\frac{1}{3}$ on its own can only score full marks if A, B and C are all correct.
ALT	2nd B1ft If they use this approach award this mark for C (if seen) or BC
Use of Det	2nd M1 must get as far as: $\frac{1}{2}\lvert x_A\times y_B-x_C\times y_B\rvert$

Pearson

Silver Questions

Calculators may not be used

The total mark for this section is 31

Q1

The points P and Q have coordinates $(-1, 6)$ and $(9, 0)$ respectively.

The line l is perpendicular to PQ and passes through the mid-point of PQ.

Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

(Total for Question 1 is 5 marks)

Q2

The straight line L_1 passes through the points $(-1, 3)$ and $(11, 12)$.

(a) Find an equation for L_1 in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The line L_2 has equation $3y + 4x - 30 = 0$.

(b) Find the coordinates of the point of intersection of L_1 and L_2.

(3)

(Total for Question 2 is 7 marks)

Q3

The line L_1 has equation $2y - 3x - k = 0$, where k is a constant.

Given that the point A (1, 4) lies on L_1, find

(a) the value of k,

(1)

(b) the gradient of L_1.

(2)

The line L_2 passes through A and is perpendicular to L_1

(c) Find an equation of L_2 giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The line L_2 crosses the x-axis at the point B.

(d) Find the coordinates of B.

(2)

(e) Find the exact length of AB.

(2)

(Total for Question 3 is 11 marks)

Q4

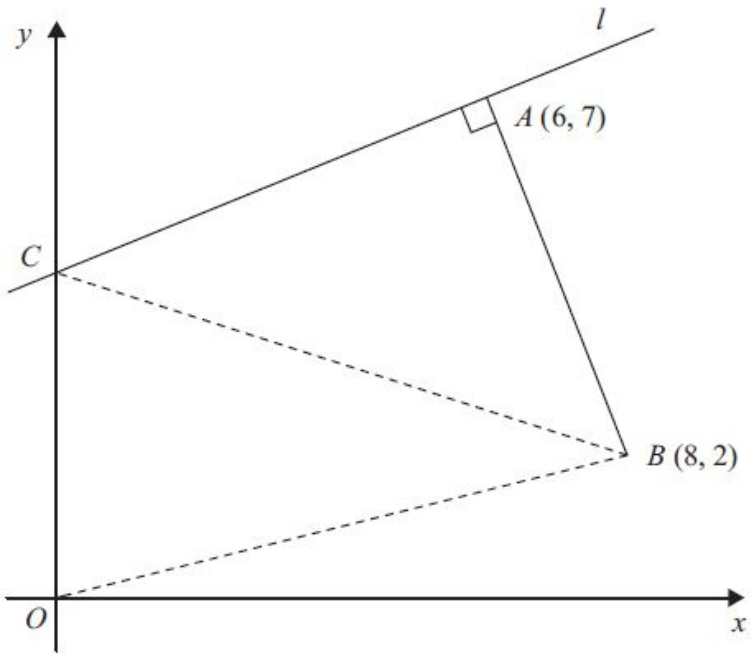

Figure 1

The points A and B have coordinates (6, 7) and (8, 2) respectively.

The line l passes through the point A and is perpendicular to the line AB, as shown in Figure 1.

(a) Find an equation for l in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Given that l intersects the y-axis at the point C, find

(b) the coordinates of C,

(2)

(c) the area of ΔOCB, where O is the origin.

(2)

(Total for Question 4 is 8 marks)

End of Questions

Pearson

Silver Mark Scheme

Q1

Question Number	Scheme	Marks
	Mid-point of PQ is (4, 3)	B1
	PQ: $m = \frac{0-6}{9-(-1)}$, $\left(=-\frac{3}{5}\right)$	B1
	Gradient perpendicular to PQ $=-\frac{1}{m}$ $\left(=\frac{5}{3}\right)$	M1
	$y-3=\frac{5}{3}(x-4)$	M1
	$5x-3y-11=0$ or $3y-5x+11=0$ or multiples e.g. $10x-6y-22=0$	A1 (5) **5**
	Notes B1: correct midpoint. B1: correct numerical expression for gradient – need not be simplified 1st M: Negative reciprocal of their numerical value for m 2nd M: Equation of a line through **their** (4, 3) with any gradient except 0 or ∞. If the 4 and 3 are the wrong way round the 2nd M mark can still be given if a correct formula (e.g. $y-y_1=m(x-x_1)$) is seen, otherwise M0. If (4, 3) is substituted into $y=mx+c$ to find c, the 2nd M mark is for attempting this. A1: Requires integer form with an = zero (see examples above)	

Pearson

Q2

Question Number	Scheme		Marks
	(-1, 3) , (11, 12)		
(a)	$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12-3}{11-(-1)}, = \frac{3}{4}$	M1:Correct method for the gradient A1: Any correct fraction or decimal	M1,A1
	$y-3 = ¾(x+1)$ or $y-12 = ¾(x-11)$ or $y = ¾x + c$ with attempt at substitution to find c	Correct straight line method using either of the given points and a numerical gradient.	M1
	$4y - 3x - 15 = 0$	Or equivalent with integer coefficients (= 0 **is** required)	A1
	This A1 should only be awarded in (a)		
			(4)
(a) Way 2	$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1} \Rightarrow \frac{y-3}{12-3} = \frac{x+1}{11+1}$	M1: Use of a correct formula for the straight line A1: Correct equation	M1A1
	$12(y-3) = 9(x+1)$	Eliminates fractions	M1
	$4y - 3x - 15 = 0$	Or equivalent with integer coefficients (= 0 **is** required)	A1
			(4)
(b)	Solves their equation from part (a) and L_2 simultaneously to eliminate one variable	Must reach as far as an equation in x only or in y only. (Allow slips in the algebra)	M1
	$x = 3$ or $y = 6$	One of $x = 3$ or $y = 6$	A1
	Both $x = 3$ **and** $y = 6$	Values can be un-simplified fractions.	A1
	Fully correct answers with no working can score 3/3 in (b)		
			(3)
(b) Way 2	$(-1,3) \rightarrow -a + 3b + c = 0$ $(11,12) \rightarrow 11a + 12b + c = 0$	Substitutes the coordinates to obtain two equations	M1
	$\therefore a = -\frac{3}{4}b,\ b = -\frac{4}{15}c$	Obtains sufficient equations to establish values for a, b and c	A1
	e.g. $c = 1 \Rightarrow b = -\frac{4}{15},\ a = \frac{3}{15}$	Obtains values for a, b and c	M1
	$\frac{3}{15}x - \frac{4}{15}y + 1 = 0 \Rightarrow 4y - 3x - 15 = 0$	Correct equation	A1
			(4)
			[7]

Pearson

Q3

Question Number	Scheme	Marks
(a)	$(8-3-k=0)$ so $\underline{k=5}$	B1 (1)
(b)	$2y=3x+k$ $y=\frac{3}{2}x+\ldots$ and so $m=\frac{3}{2}$ o.e.	M1 A1 (2)
(c)	Perpendicular gradient $=-\frac{2}{3}$ Equation of line is: $y-4=-\frac{2}{3}(x-1)$ $\underline{3y+2x-14=0}$ o.e.	B1ft M1A1ft A1 (4)
(d)	$y=0, \Rightarrow B(7,0)$ or $\underline{x=7}$ $x=7$ or $-\frac{c}{a}$	M1A1ft (2)
(e)	$AB^2=(7-1)^2+(4-0)^2$ $AB=\sqrt{52}$ or $2\sqrt{13}$	M1 A1 (2) 11
	Notes	
(b)	M1 for an attempt to rearrange to $y=\ldots$ A1 for clear statement that gradient is 1.5, can be $m=1.5$ o.e.	
(c)	B1ft for using the perpendicular gradient rule correctly on their "1.5" M1 for an attempt at finding the equation of the line through A using their gradient. Allow a sign slip 1st A1ft for a correct equation of the line follow through their changed gradient 2nd A1 as printed or equivalent with integer coefficients – allow $\underline{3y+2x=14}$ or $\underline{3y=14-2x}$	
(d)	M1 for use of $y=0$ to find $x=\ldots$ in their equation A1ft for $x=7$ or $-\frac{c}{a}$	
(e)	M1 for an attempt to find AB or AB^2 A1 for any correct surd form- need not be simplified	

Pearson

Q4

Question Number		Scheme	Marks
Q	(a)	AB: $m = \frac{2-7}{8-6}$, $\left(=-\frac{5}{2}\right)$	B1
		Using $m_1 m_2 = -1$: $m_2 = \frac{2}{5}$	M1
		$y-7=\frac{2}{5}(x-6)$, $2x-5y+23=0$ (o.e. with integer coefficients)	M1, A1 (4)
	(b)	Using $x=0$ in the answer to (a), $y=\frac{23}{5}$ or 4.6	M1, A1ft (2)
	(c)	Area of triangle $= \frac{1}{2}\times 8\times\frac{23}{5}=\frac{92}{5}$ (o.e) e.g. $\left(18\frac{2}{5},\ 18.4,\ \frac{184}{10}\right)$	M1 A1 (2)
			[8]

(a)
B1 for an expression for the gradient of AB. Does not need the $= -2.5$
1st M1 for use of the perpendicular gradient rule. Follow through their m
2nd M1 for the use of (6, 7) and their changed gradient to form an equation for l.
Can be awarded for $\frac{y-7}{x-6}=\frac{2}{5}$ o.e.
Alternative is to use (6, 7) in $y=mx+c$ to find a value for c. Score when $c=\ldots$ is reached.
A1 for a correct equation in the required form and must have "= 0" and integer coefficients

(b)
M1 for using $x=0$ in their answer to part (a) e.g. $-5y+23=0$
A1ft for $y=\frac{23}{5}$ provided that $x=0$ clearly seen or C (0, 4.6). Follow through their equation in (a)
If $x=0$, $y=4.6$ are clearly seen but C is given as (4.6,0) apply ISW and award the mark.
This A mark requires a simplified fraction or an exact decimal
Accept their 4.6 marked on diagram next to C for M1A1ft

(c)
M1 for $\frac{1}{2}\times 8\times y_C$ so can follow through their y coordinate of C.
A1 for 18.4 (o.e.) but their y coordinate of C must be positive

Use of 2 triangles or trapezium and triangle
Award M1 when an expression for area of OCB only is seen

Determinant approach
Award M1 when an expression containing $\frac{1}{2}\times 8\times y_C$ is seen

Pearson

Gold Questions

Calculators may not be used

The total mark for this section is 27

Q1

(a) Find an equation of the line joining $A(7, 4)$ and $B(2, 0)$, giving your answer in the form $ax+by+c = 0$, where a, b and c are integers.

(3)

(b) Find the length of AB, leaving your answer in surd form.

(2)

The point C has coordinates $(2, t)$, where $t > 0$, and $AC = AB$.

(c) Find the value of t.

(1)

(d) Find the area of triangle ABC.

(2)

(Total for Question 1 is 8 marks)

Q2

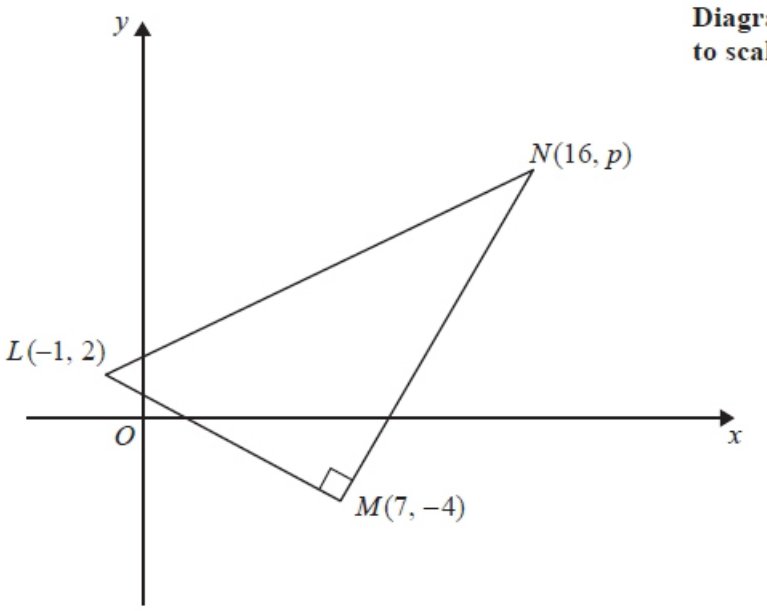

Figure 2

Figure 2 shows a right angled triangle *LMN*.

The points *L* and *M* have coordinates (−1, 2) and (7, −4) respectively.

(a) Find an equation for the straight line passing through the points *L* and *M*.

Give your answer in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers.

(4)

Given that the coordinates of point *N* are (16, *p*), where *p* is a constant, and angle LMN = 90°,

(b) find the value of *p*.

(3)

Given that there is a point *K* such that the points *L, M, N*, and *K* form a rectangle,

(c) find the *y* coordinate of *K*.

(2)

(Total for Question 2 is 9 marks)

Pearson

Q3

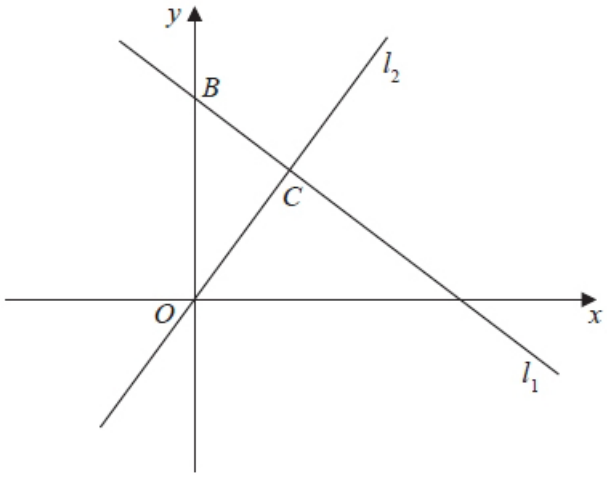

Figure 2

The line l_1, shown in Figure 2 has equation $2x + 3y = 26$

The line l_2 passes through the origin O and is perpendicular to l_1

(a) Find an equation for the line l_2

(4)

The line l_2 intersects the line l_1 at the point C

Line l_1 crosses the y-axis at the point B as shown in Figure 2.

(b) Find the area of triangle OBC

Give your answer in the form $\frac{a}{b}$, where a and b are integers to be determined.

(6)

(Total for Question 3 is 10 marks)

End of Questions

Pearson

Gold Mark Scheme

Q1

Question Number	Scheme	Marks
(a)	$m_{AB} = \frac{4-0}{7-2}$ $\left(=\frac{4}{5}\right)$	M1
	Equation of AB is: $y-0=\frac{4}{5}(x-2)$ or $y-4=\frac{4}{5}(x-7)$ (o.e.)	M1
	$4x - 5y - 8 = 0$ (o.e.)	A1 (3)
(b)	$(AB =)\sqrt{(7-2)^2+(4-0)^2}$	M1
	$=\sqrt{41}$	A1 (2)
(c)	Using isos triangle with $AB = AC$ then $t = 2 \times y_A = 2 \times 4 = 8$	B1 (1)
(d)	Area of triangle $= \frac{1}{2}t \times (7-2)$	M1
	$= \underline{20}$	A1 (2)
		8

	Notes
	Apply the usual rules for quoting formulae here.
	For a correctly quoted formula with some correct substitution award M1
	If no formula is quoted then a fully correct expression is needed for the M mark
(a)	1st M1 for attempt at gradient of AB. Some correct substitution in correct formula.
	2nd M1 for an attempt at equation of AB. Follow through their gradient, not e.g. $-\frac{1}{m}$
	Using $y = mx + c$ scores this mark when c is found.
	Use of $\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$ scores 1st M1 for denominator, 2nd M1 for use of a correct point
	A1 requires integer form but allow $5y + 8 = 4x$ etc. Must have an "=" or A0
(b)	M1 for an expression for AB or AB^2. Ignore what is "left" of the equals sign
(c)	B1 for $t = 8$. May be implied by correct coordinates (2, 8) or the value appearing in (d)
(d)	M1 for an expression for the area of the triangle, follow through their t ($\neq 0$) but must have the $(7-2)$ or 5 and the $\frac{1}{2}$.
DET	e.g. $\begin{matrix}2&7&2&2\\0&4&t&0\end{matrix}$ Area $= \frac{1}{2}\left[8+7t+0-(0+8+2t)\right]$ Must have the $\frac{1}{2}$ for M1

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Q2

Question Number	Scheme	Marks
(a)	Method 1: $gradient = \frac{y_1 - y_2}{x_1 - x_2} = \frac{2-(-4)}{-1-7}, = -\frac{3}{4}$ Method 2: $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$, so $\frac{y - y_1}{6} = \frac{x - x_1}{-8}$	M1, A1
	$y-2 = -\frac{3}{4}(x+1)$ or $y+4 = -\frac{3}{4}(x-7)$ or $y = their' -\frac{3}{4}' x + c$	M1
	$\Rightarrow \pm(4y+3x-5) = 0$	A1 (4)
	Method 3: Substitute $x = -1, y = 2$ and $x = 7, y = -4$ into $ax + by + c = 0$	M1
	$-a + 2b + c = 0$ and $7a - 4b + c = 0$	A1
	Solve to obtain $a = 3$, $b = 4$ and $c = -5$ or multiple of these numbers	M1 A1 (4)
(b)	Attempts $gradient\ LM \times gradient\ MN = -1$ so $-\frac{3}{4} \times \frac{p+4}{16-7} = -1$ or $\frac{p+4}{16-7} = \frac{4}{3}$ $p+4 = \frac{9 \times 4}{3} \Rightarrow p = \ldots$, $p = 8$ Or $(y+4) = \frac{4}{3}(x-7)$ equation with $x =$ 16 substituted So $y =$, $y = 8$	M1 M1, A1 (3)
Alternative for (b)	Attempt Pythagoras: $(p+4)^2 + 9^2 + (6^2 + 8^2) = (p-2)^2 + 17^2$	M1
	So $p^2 + 8p + 16 + 81 + 36 + 64 = p^2 - 4p + 4 + 289 \Rightarrow p = \ldots$	M1
	$p = 8$	A1 (3)
(c)	Either $(y=)\ p+6$ or $2+p+4$ $(y=)\ 14$ Or use 2 perpendicular line equations through L and N and solve for y	M1 A1 (2) (9 marks)

(a) M1 Uses the gradient formula with points L and M i.e. quote $gradient = \frac{y_1 - y_2}{x_1 - x_2}$ and attempt to substitute correct numbers. Formula may be implied by the correct $\frac{2-(-4)}{-1-7}$ or equivalent.

A1 Any correct single fraction gradient i.e $\frac{6}{-8}$ or equivalent

M1 Uses their gradient with either (-1, 2) or (7, -4) to form a linear equation

Eg $y-2 = their' -\frac{3}{4}'(x+1)$ or $y+4 = their' -\frac{3}{4}'(x-7)$ or $y = their' -\frac{3}{4}' x + c$ then find a value for c by substituting (-1,2) or (7, -4) in the correct way(not interchanging x and y)

A1 Accept $\pm k(4y+3x-5) = 0$ with k an integer (This implies previous M1)

(b) M1 Attempts to use $gradient\ LM \times gradient\ MN = -1$. ie. $-\frac{3}{4} \times \frac{p+4}{16-7} = -1$ (allow sign errors)

Or Attempts Pythagoras correct way round (allow sign errors)
M1 An attempt to solve their linear equation in 'p'. A1 cao $p = 8$

(c) M1 For using their numerical value of p and adding 6 . This may be done by any complete method (vectors, drawing, perpendicular straight line equations through L and N) or by no method. Assuming $x = 7$ is M0

A1 Accept 14 for both marks as long as no incorrect working seen (Ignore left hand side – allow k) . If there is wrong working resulting fortuitously in 14 give M0A0. Allow (8, 14) as the answer.

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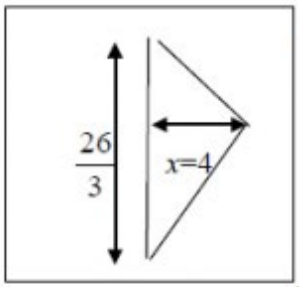
Q3

Question Number	Scheme	Marks
	(a) $2x+3y=26 \Rightarrow 3y=26\pm 2x$ and attempt to find m from $y=mx+c$	M1
	$(\Rightarrow y=\frac{26}{3}-\frac{2}{3}x)$ so gradient $=-\frac{2}{3}$	A1
	Gradient of perpendicular $=\frac{-1}{\text{their gradient}}$ $(=\frac{3}{2})$	M1
	Line goes through (0,0) so $y=\frac{3}{2}x$	A1
		(4)
	(b) Solves their $y=\frac{3}{2}x$ with their $2x+3y=26$ to form equation in x or in y	M1
	Solves their equation in x or in y to obtain $x=$ **or** $y=$	dM1
	$x=4$ or any equivalent e.g. 156/39 or $y=6$ o.a.e	A1
	$B=(0,\frac{26}{3})$ used or stated in (b)	B1
	 **Method 1** (see other methods in notes below)	
	Area $=\frac{1}{2}\times"4"\times\frac{"26"}{3}$	dM1
	$=\frac{52}{3}$ (oe with integer numerator and denominator)	A1
		(6) **(10 marks)**

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Notes

(a) M1 Complete method for finding gradient. (This may be implied by later correct answers.) e.g. Rearranges $2x + 3y = 26 \Rightarrow y = mx + c$ so $m =$

Or finds coordinates of two points on line and finds gradient e.g. (13, 0) *and* (1, 8) *so m* $= \frac{8-0}{1-13}$

A1 States or implies that gradient $= -\frac{2}{3}$ condone $-\frac{2}{3}x$ if they continue correctly. Ignore errors in constant term in straight line equation

M1 Uses $m_1 \times m_2 = -1$ to find the gradient of l_2. This can be implied by the use of $\frac{-1}{\text{their gradient}}$

A1 $y = \frac{3}{2}x$ or $2y - 3x = 0$ Allow $y = \frac{3}{2}x + 0$ Also accept $2y = 3x$, $y = 39/26x$ or even $y - 0 = \frac{3}{2}(x - 0)$ and isw

(b) M1 Eliminates variable between their $y = \frac{3}{2}x$ and their (possibly rearranged) $2x + 3y = 26$ to form an equation in x or y. (They may have made errors in their rearrangement)

dM1 (Depends on previous M mark) Attempts to solve their equation to find the value of x or y

A1 $x = 4$ or equivalent or $y = 6$ or equivalent

B1 y coordinate of B is $\frac{26}{3}$ (stated or implied) – isw if written as $(\frac{26}{3}, 0)$. **Must be used or stated in (b)**

dM1 (Depends on previous M mark) Complete method to find area of triangle OBC (using their values of x and/or y at point C and their 26/3)

A1 Cao $\frac{52}{3}$ or $\frac{104}{6}$ or $\frac{1352}{78}$ o.e.

Method 1:

Uses the area of a triangle formula $\frac{1}{2} \times OB \times$ (x coordinate of C)

Alternative methods:

Several Methods are shown below. The only mark which differs from Method 1 is the last M mark and its use in each case is described below:

Method 2 in (b) using $\frac{1}{2} \times BC \times OC$

dM1 Uses the area of a triangle formula $\frac{1}{2} \times BC \times OC$ Also finds OC ($=\sqrt{52}$) and $BC = (\frac{4}{3}\sqrt{13})$

Method 3 in (b) using $\frac{1}{2}\begin{vmatrix} 0 & 4 & 0 & 0 \\ 0 & 6 & \frac{26}{3} & 0 \end{vmatrix}$

dM1 States the area of a triangle formula $\frac{1}{2}\begin{vmatrix} 0 & 4 & 0 & 0 \\ 0 & 6 & \frac{26}{3} & 0 \end{vmatrix}$ or equivalent with their values

Method 4 in (b) using area of triangle OBX – area of triangle OCX where X is point (13, 0)

dM1 Uses the correct subtraction $\frac{1}{2} \times 13 \times "\frac{26}{3}" - \frac{1}{2} \times 13 \times "6"$

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Method 5 in (b) using area = ½ (6 × 4) + ½ (4 × 8/3) drawing a line from *C* parallel to the *x* axis and dividing triangle into two right angled triangles

dM1 for correct method area = ½ ("6" × "4") + ½ ("4" × ["26/3" − "6"])

Method 6 Uses calculus

dM1 $\int_0^4 \left("\frac{26}{3}" - \frac{2x}{3} - \frac{3x}{2}\right)dx = \left[\frac{26}{3}x - \frac{x^2}{3} - \frac{3x^2}{4}\right]_0^4$

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Topic 6

Circles

Bronze, Silver, Gold and Platinum Worksheets for AS Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculators may not be used

The total mark for this section is 29

Q1

A circle C has centre $(-1, 7)$ and passes through the point $(0, 0)$. Find an equation for C

(Total for Question 1 is 4 marks)

Q2

The circle C has equation $x^2 + y^2 + 4x - 2y - 11 = 0$

Find

(a) the coordinates of the centre of C,

(2)

(b) the radius of C,

(2)

(c) the coordinates of the points where C crosses the y-axis, giving your answers as simplified surds.

(4)

(Total for Question 2 is 8 marks)

Q3

The points A and B have coordinates $(-2, 11)$ and $(8, 1)$ respectively.

Given that AB is a diameter of the circle C,

(a) show that the centre of C has coordinates $(3, 6)$,

(1)

(b) find an equation for C.

(4)

(c) Verify that the point $(10, 7)$ lies on C.

(1)

(d) Find an equation of the tangent to C at the point $(10, 7)$, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

(Total for Question 3 is 10 marks)

Q4

The circle C, with centre A, passes through the point P with coordinates $(-9, 8)$ and the point Q with coordinates $(15, -10)$.

Given that PQ is a diameter of the circle C,

(a) find the coordinates of A

(2)

(b) find an equation for C

(3)

A point R also lies on the circle C.
Given that the length of the chord PR is 20 units,

(c) find the length of the shortest distance from A to the chord PR.
Give your answer as a surd in its simplest form.

(2)

(Total for Question 4 is 7 marks)

End of Questions

Bronze Mark Scheme

Q1

Question number	Scheme	Marks
	The equation of the circle is $(x+1)^2+(y-7)^2=(r^2)$	M1 A1
	The radius of the circle is $\sqrt{(-1)^2+7^2}=\sqrt{50}$ or $5\sqrt{2}$ or $r^2=50$	M1
	So $(x+1)^2+(y-7)^2=50$ or equivalent	A1
		(4)
		4
Notes	**M1** is for this expression on left hand side– allow *errors in sign* of 1 and 7. **A1** correct signs (just LHS) **M1** is for Pythagoras or substitution into equation of circle to give r or r^2 Giving this value as diameter is **M0** **A1**, cao for cartesian equation with numerical values but allow $(\sqrt{50})^2$ or $(5\sqrt{2})^2$ or any exact equivalent A correct answer implies a correct method – so answer given with no working earns all four marks for this question.	
Alternative method	Equation of circle is $x^2+y^2\pm 2x\pm 14y+c=0$	M1
	Equation of circle is $x^2+y^2+2x-14y+c=0$	A1
	Uses (0,0) to give $c=0$, **or** finds $r=\sqrt{(-1)^2+7^2}=\sqrt{50}$ or $5\sqrt{2}$ or $r^2=50$	M1
	So $x^2+y^2+2x-14y=0$ or equivalent	A1

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Q2

Question Number	Scheme		Marks
(a)	$x^2 + y^2 + 4x - 2y - 11 = 0$		
	$\{\underline{(x+2)^2 - 4} + \underline{\underline{(y-1)^2 - 1}} - 11 = 0\}$	$(\pm 2, \pm 1)$, see notes.	M1
	Centre is $(-2, 1)$.	$(-2, 1)$.	A1 cao [2]
(b)	$(x+2)^2 + (y-1)^2 = 11 + 1 + 4$	$r = \sqrt{11 \pm "1" \pm "4"}$	M1
	So $r = \sqrt{11+1+4} \Rightarrow r = 4$	4 or $\sqrt{16}$ (Award A0 for ± 4).	A1 [2]
(c)	When $x = 0$, $y^2 - 2y - 11 = 0$	Putting $x = 0$ in C or their C.	M1
		$y^2 - 2y - 11 = 0$ or $(y-1)^2 = 12$, etc	A1 aef
	$y = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2(1)} \left\{ = \frac{2 \pm \sqrt{48}}{2} \right\}$	Attempt to use formula or a method of completing the square in order to find $y = ...$	M1
			A1 cao cso
	So, $y = 1 \pm 2\sqrt{3}$	$1 \pm 2\sqrt{3}$	[4]
			8

Note: Please mark parts (a) and (b) together. Answers only in (a) and/or (b) get full marks.
Note in part (a) the marks are now M1A1 and not B1B1 as on ePEN.

(a) M1: for $(\pm 2, \pm 1)$. Otherwise, M1 for an attempt to complete the square eg. $\underline{(x \pm 2)^2 \pm \alpha}$, $\alpha \neq 0$ or $\underline{\underline{(y \pm 1)^2 \pm \beta}}$, $\beta \neq 0$. M1A1: Correct answer of $(-2, 1)$ stated from any working gets M1A1.

(b) M1: to find the radius using 11, "1" and "4", ie. $r = \sqrt{11 \pm "1" \pm "4"}$. By applying this method candidates will usually achieve $\sqrt{16}$, $\sqrt{6}$, $\sqrt{8}$ or $\sqrt{14}$ and not 16, 6, 8 or 14.

Note: $(x+2)^2 + (y-1)^2 = -11 - 5 = -16 \Rightarrow r = \sqrt{16} = 4$ should be awarded M0A0.

Alternative: M1 in part (a): For comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$ to write down centre $(-g, -f)$ directly. Condone sign errors for this M mark. M1 in part (b): For using $r = \sqrt{g^2 + f^2 - c}$. Condone sign errors for this method mark.

$(x+2)^2 + (y-1)^2 = 16 \Rightarrow r = 8$ scores M0A0, but $r = \sqrt{16} = 8$ scores M1A1 isw.

(c) 1st M1: Putting $x = 0$ in either $x^2 + y^2 + 4x - 2y - 11 = 0$ or their circle equation usually given in part (a) or part (b). 1st A1 for a correct equation in y **in any form** which can be implied by later working.

2nd M1: See rules for using the formula. Or completing the square on a 3TQ to give $y = a \pm \sqrt{b}$, where $\sqrt{b}$ is a surd, $b \neq$ their 11 and $b > 0$. This mark should not be given for an attempt to factorise.

2nd A1: Need exact pair in simplified surd form of $\{y =\} 1 \pm 2\sqrt{3}$. This mark is also cso.

Do not need to see $(0, 1 + 2\sqrt{3})$ and $(0, 1 - 2\sqrt{3})$. Allow 2nd A1 for bod $(1 + 2\sqrt{3}, 0)$ and $(1 - 2\sqrt{3}, 0)$.

Any incorrect working in (c) gets penalised the final accuracy mark. So, **beware:** incorrect $(x-2)^2 + (y-1)^2 = 16$ leading to $y^2 - 2y - 11 = 0$ and then $y = 1 \pm 2\sqrt{3}$ scores M1A1M1A0.

Special Case for setting $y = 0$: Award SC: M0A0M1A0 for an attempt at applying the formula $x = \frac{-4 \pm \sqrt{(-4)^2 - 4(1)(-11)}}{2(1)} \left\{ = \frac{-4 \pm \sqrt{60}}{2} = -2 \pm \sqrt{15} \right\}$

Award SC: M0A0M1A0 for completing the square to their equation in x which will usually be $x^2 + 4x - 11 = 0$ to give $a \pm \sqrt{b}$, where $\sqrt{b}$ is a surd, $b \neq$ their 11 and $b > 0$.

Special Case: For a candidate not using $\pm$ but achieving one of the correct answers then award SC: M1A1 M1A0 for one of either $y = 1 + 2\sqrt{3}$ or $y = 1 - 2\sqrt{3}$ or $y = 1 + \sqrt{12}$ or $y = 1 - \sqrt{12}$.

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Q3

Question Number	Scheme	Marks
(a)	$C\left(\frac{-2+8}{2}, \frac{11+1}{2}\right) = C(3, 6)$ **AG** — Correct method (no errors) for finding the mid-point of AB giving $(3, 6)$	B1* (1)
(b)	$(8-3)^2 + (1-6)^2$ or $\sqrt{(8-3)^2 + (1-6)^2}$ or $(-2-3)^2 + (11-6)^2$ or $\sqrt{(-2-3)^2 + (11-6)^2}$ — Applies distance formula in order to find the radius.	M1
	Correct application of formula.	A1
	$(x-3)^2 + (y-6)^2 = 50$ (or $\left(\sqrt{50}\right)^2$ or $\left(5\sqrt{2}\right)^2$) — $(x \pm 3)^2 + (y \pm 6)^2 = k$, k is a positive value.	M1
	$(x-3)^2 + (y-6)^2 = 50$ (Not 7.07^2)	A1 (4)
(c)	{For $(10, 7)$,} $(10-3)^2 + (7-6)^2 = 50$, {so the point lies on C.}	B1 (1)
(d)	{Gradient of radius} $= \frac{7-6}{10-3}$ or $\frac{1}{7}$ — This must be seen in part (d).	B1
	Gradient of tangent $= \frac{-7}{1}$ — Using a perpendicular gradient method.	M1
	$y - 7 = -7(x-10)$ — $y - 7 =$ (their gradient)$(x-10)$	M1
	$y = -7x + 77$ — $y = -7x + 77$ or $y = 77 - 7x$	A1 cao (4) **[10]**

Notes

(a)	Alternative method: $C\left(-2 + \frac{8--2}{2}, 11 + \frac{1-11}{2}\right)$ or $C\left(8 + \frac{-2-8}{2}, 1 + \frac{11-1}{2}\right)$
(b)	You need to be convinced that the candidate is attempting to work out the radius and not the diameter of the circle to award the first M1. Therefore allow 1st M1 generously for $\frac{(-2-8)^2 + (11-1)^2}{2}$ Award 1st M1A1 for $\frac{(-2-8)^2 + (11-1)^2}{4}$ or $\frac{\sqrt{(-2-8)^2 + (11-1)^2}}{2}$. Correct answer in (b) with no working scores full marks.
(c)	B1 awarded for correct verification of $(10-3)^2 + (7-6)^2 = 50$ with no errors. Also to gain this mark candidates need to have the correct equation of the circle either from part (b) or re-attempted in part (c). They cannot verify $(10, 7)$ lies on C without a correct C. Also a candidate could either substitute $x = 10$ in C to find $y = 7$ or substitute $y = 7$ in C to find $x = 10$.

Question Number	Scheme	Marks
(d)	2nd M1 mark also for the complete method of applying $7 =$ (their gradient)$(10) + c$, finding c. **Note**: Award 2nd M0 in (d) if their numerical gradient is either 0 or ∞. **Alternative:** For first two marks (differentiation): $2(x-3) + 2(y-6)\frac{dy}{dx} = 0$ (or equivalent) scores B1. 1st M1 for substituting **both** $x = 10$ and $y = 7$ to find a value for $\frac{dy}{dx}$, which must contain both x and y. (This M mark can be awarded generously, even if the attempted "differentiation" is not "implicit".) **Alternative:** $(10-3)(x-3) + (7-6)(y-6) = 50$ scores B1M1M1 which leads to $y = -7x + 77$.	

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Q4

Question Number	Scheme		Marks
(a)	$A\left(\frac{-9+15}{2}, \frac{8-10}{2}\right) = A(3,-1)$	M1: A correct attempt to find the midpoint between P and Q. Can be implied by one of x or y-coordinates correctly evaluated.	M1A1
		A1: $(3,-1)$	
			[2]
(b)	$(-9-3)^2+(8+1)^2$ or $\sqrt{(-9-3)^2+(8+1)^2}$ or $(15-3)^2+(-10+1)^2$ or $\sqrt{(15-3)^2+(-10+1)^2}$ Uses Pythagoras correctly in order to find the **radius**. Must clearly be identified as the **radius** and may be implied by their circle equation. Or $(15+9)^2+(-10-8)^2$ or $\sqrt{(15+9)^2+(-10-8)^2}$ Uses Pythagoras correctly in order to find the **diameter**. Must clearly be identified as the **diameter** and may be implied by their circle equation. This mark can be implied by just 30 clearly seen as the **diameter** or 15 clearly seen as the **radius** (may be seen or implied in their circle equation) **Allow this mark if there is a correct statement involving the radius or the diameter but must be seen in (b)**		M1
	$(x-3)^2+(y+1)^2=225$ $\left(\text{or } (15)^2\right)$	$(x\pm\alpha)^2+(y\pm\beta)^2=k^2$ where $A(\alpha,\beta)$ and k is their radius.	M1
	$(x-3)^2+(y+1)^2=225$	Allow $(x-3)^2+(y+1)^2=15^2$	A1
	Accept correct answer only		
			[3]
	Alternative using $x^2+2ax+y^2+2by+c=0$		
	Uses $A(\pm\alpha,\pm\beta)$ and $x^2+2ax+y^2+2by+c=0$ e.g. $x^2+2(-3)x+y^2+2(1)y+c=0$		M1
	Uses P or Q and $x^2+2ax+y^2+2by+c=0$ e.g. $(-9)^2+2(-3)(-9)+(8)^2+2(1)(8)+c=0\Rightarrow c=-215$		M1
	$x^2-6x+y^2+2y-215=0$		A1
(c)	Distance $=\sqrt{15^2-10^2}$	$=\sqrt{(\text{their } r)^2-10^2}$ or a correct method for the distance e.g. their $r\times\cos\left[\sin^{-1}\left(\frac{10}{\text{their } r}\right)\right]$	M1
	$\left\{=\sqrt{125}\right\} = 5\sqrt{5}$	$5\sqrt{5}$	A1
			[2]

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Silver Questions

Calculators may not be used

The total mark for this section is 33

Q1

The circle C has equation

$$x^2 + y^2 - 20x - 24y + 195 = 0$$

The centre of C is at the point M

(a) Find

(i) the coordinates of the point M

(ii) the radius of the circle C

(5)

N is the point with coordinates (25, 32)

(b) Find the length of the line MN

(2)

The tangent to C at a point P on the circle passes through point N

(c) Find the length of the line NP

(2)

(Total for Question 1 is 9 marks)

Q2

The circle C has centre (3, 1) and passes through the point $P(8, 3)$.

(a) Find an equation for C.

(4)

(b) Find an equation for the tangent to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

(Total for Question 2 is 9 marks)

Q3

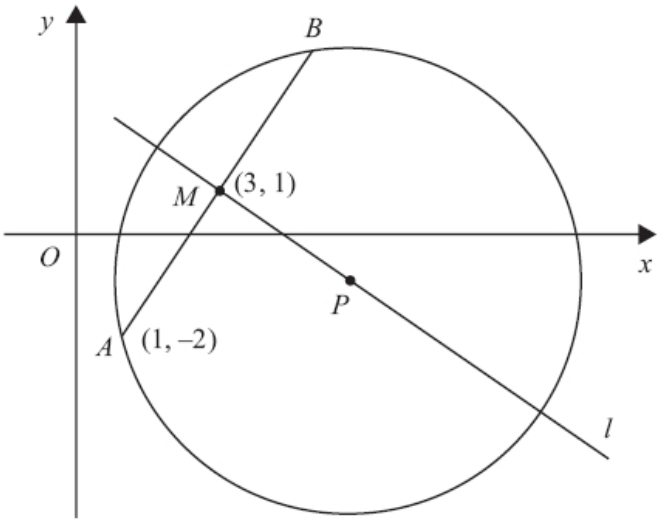

Figure 3

The points A and B lie on a circle with centre P, as shown in Figure 3.
The point A has coordinates $(1, -2)$ and the mid-point M of AB has coordinates $(3, 1)$.
The line l passes through the points M and P.

(a) Find an equation for l.

(4)

Given that the x-coordinate of P is 6,

(b) use your answer to part (a) to show that the y-coordinate of P is -1,

(1)

(c) find an equation for the circle.

(4)

(Total for Question 3 is 9 marks)

Pearson

Q4

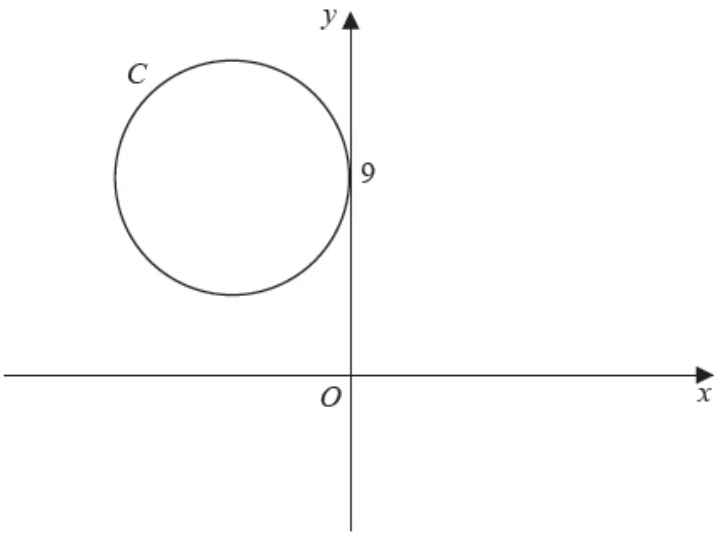

Figure 4

The circle C has radius 5 and touches the y-axis at the point (0, 9), as shown in Figure 4.

(a) Write down an equation for the circle C, that is shown in Figure 4.

(3)

A line through the point $P(8, -7)$ is a tangent to the circle C at the point T

(b) Find the length of PT

(3)

(Total for Question 4 is 6 marks)

End of Questions

Pearson

Silver Mark Scheme

Q1

Question Number	Scheme		Marks
(a)			
(i)	The centre is at (10, 12)	B1: $x = 10$ B1: $y = 12$	B1 B1
(ii)	Uses $(x-10)^2+(y-12)^2=-195+100+144 \Rightarrow r=...$		M1
	Completes the square for both x and y in an attempt to find r. $(x \pm "10")^2 \pm a$ and $(y \pm "12")^2 \pm b$ and $+195=0, (a,b \neq 0)$ Allow errors in obtaining their r^2 but must find square root		
	$r=\sqrt{10^2+12^2-195}$	A correct numerical expression for r including the square root and can implied by a correct value for r	A1
	$r=7$	Not $r=\pm 7$ unless -7 is rejected	A1
			(5)
(a) Way 2	Compares the given equation with $x^2+y^2+2gx+2fy+c=0$ to write down centre $(-g,-f)$ i.e. (10, 12)	B1: $x = 10$ B1: $y = 12$	B1B1
	Uses $r=\sqrt{(\pm"10")^2+(\pm"12")^2-c}$		M1
	$r=\sqrt{10^2+12^2-195}$	A correct numerical expression for r	A1
	$r=7$		A1
			(5)
(b)	$MN=\sqrt{(25-"10")^2+(32-"12")^2}$	Correct use of Pythagoras	M1
	$MN\left(=\sqrt{625}\right)=25$		A1
			(2)
(c)	$NP=\sqrt{("25"^2-"7"^2)}$	$NP=\sqrt{(MN^2-r^2)}$	M1
	$NP\left(=\sqrt{576}\right)=24$		A1
			(2)
(c) Way 2	$\cos(NMP)=\frac{7}{"25"} \Rightarrow NP="25"\sin(NMP)$	Correct strategy for finding NP	M1
	$NP=24$		A1
			(2)
			[9]

Pearson

Q2

Question Number	Scheme	Marks
(*a*)	$(8-3)^2+(3-1)^2$ or $\sqrt{(8-3)^2+(3-1)^2}$	M1 A1
	$(x\pm3)^2+(y\pm1)^2=k$ or $(x\pm1)^2+(y\pm3)^2=k$ (k a positive value)	M1
	$(x-3)^2+(y-1)^2=29$	A1 (4)
(*b*)	Gradient of radius $=\frac{2}{5}$ (or exact equivalent)	B1
	Gradient of tangent $=\frac{-5}{2}$	M1
	$y-3=\frac{-5}{2}(x-8)$	M1 A1 ft
	$5x+2y-46=0$ or equivalent	A1 (5)
		(9 marks)

Pearson

Q3

Question number	Scheme	Marks
	(a) Gradient of AM: $\frac{1-(-2)}{3-1}=\frac{3}{2}$ or $\frac{-3}{-2}$	B1
	Gradient of l: $=-\frac{2}{3}$ M: use of $m_1m_2=-1$, or equiv.	M1
	$y-1=-\frac{2}{3}(x-3)$ or $\frac{y-1}{x-3}=-\frac{2}{3}$ $[3y=-2x+9]$ (Any equiv. form)	M1 A1 (4)
	(b) $x=6$: $3y=-12+9=-3$ $y=-1$ (or show that for $y=-1$, $x=6$) (*) (A conclusion is not required).	B1 (1)
	(c) $\left(r^2=\right)$ $(6-1)^2+(-1-(-2))^2$ M: Attempt r^2 or r	M1 A1
	N.B. Simplification is not required to score M1 A1	
	$(x\pm6)^2+(y\pm1)^2=k$, $k\neq0$ (Value for k not needed, could be r^2 or r)	M1
	$(x-6)^2+(y+1)^2=26$ (or equiv.)	A1 (4)
	Allow $\left(\sqrt{26}\right)^2$ or other exact equivalents for 26.	
	(But… $(x-6)^2+(y--1)^2=26$ scores M1 A0)	
	(Correct answer with no working scores full marks)	
		9
	(a) 2nd M1: eqn. of a straight line through (3, 1) with any gradient except 0 or ∞.	
	Alternative: Using (3, 1) in $y=mx+c$ to find a value of c scores M1, but an equation (general or specific) must be seen.	
	Having coords the wrong way round, e.g. $y-3=-\frac{2}{3}(x-1)$, loses the 2nd M mark unless a correct general formula is seen, e.g. $y-y_1=m(x-x_1)$.	
	If the point $P(6,-1)$ is used to find the gradient of MP, maximum marks are (a) B0 M0 M1 A1 (b) B0.	
	(c) 1st M1: Condone one slip, numerical or sign, inside a bracket.	
	Must be attempting to use points $P(6,-1)$ and $A(1,-2)$, or perhaps P and B. (Correct coordinates for B are (5, 4)).	
	1st M alternative is to use a complete Pythag. method on triangle MAP, n.b. $MP=MA=\sqrt{13}$.	
	Special case: If candidate persists in using their value for the y-coordinate of P instead of the given −1, allow the M marks in part (c) if earned.	

Pearson

Q4

Question Number	Scheme	Marks
(a)	Equation of form $(x \pm 5)^2 + (y \pm 9)^2 = k$, $k>0$	M1
	Equation of form $(x-a)^2 + (y-b)^2 = 5^2$, with values for a and b	M1
	$(x+5)^2 + (y-9)^2 = 25 = 5^2$	A1 (3)
(b)	$P(8,-7)$. Let centre of circle $= X(-5,9)$	
	$PX^2 = (8-"-5")^2 + (-7-"9")^2$ or $PX = \sqrt{(8--5)^2 + (-7-9)^2}$	M1
	($PX = \sqrt{425}$ or $5\sqrt{17}$) $PT^2 = (PX)^2 - 5^2$ with numerical PX	dM1
	$PT\ \{=\sqrt{400}\} = 20$ (allow 20.0)	A1 cso (3) [6]
Alternative 2 for (a)	Equation of the form $x^2 + y^2 \pm 10x \pm 18y + c = 0$	M1
	Uses $a^2 + b^2 - 5^2 = c$ with their a and b or substitutes (0, 9) giving $+9^2 \pm 2b \times 9 + c = 0$	M1
	$x^2 + y^2 + 10x - 18y + 81 = 0$	A1 (3)
Alternative 2 for (b)	An attempt to find the point T may result in pages of algebra, but solution needs to reach (-8, 5) or $\left(\frac{-8}{17}, 11\frac{2}{17}\right)$ to get first M1 (even if gradient is found first)	M1
	M1: Use either of the correct points with P (8, -7) and distance between two points formula	dM1
	A1: 20	A1cso (3)
Alternative 3 for (b)	Substitutes (8, -7) into circle **equation** so $PT^2 = 8^2 + (-7)^2 + 10 \times 8 - 18 \times (-7) + 81$	M1
	Square roots to give $PT\ \{=\sqrt{400}\} = 20$	dM1A1 (3)
	Notes for Question	
(a)	**The three marks in (a) each require a circle equation – (see special cases which are not circles)**	
	M1: Uses coordinates of centre to obtain LHS of circle equation (RHS must be r^2 or $k>0$ or a positive value)	
	M1: Uses $r = 5$ to obtain RHS of circle equation as 25 or 5^2	
	A1: correct circle equation in any equivalent form	
	Special cases $(x \pm 5)^2 + (x \pm 9)^2 = (5^2)$ is **not a circle** equation so M0M0A0	
	Also $(x \pm 5)^2 + (y-9) = (5^2)$ **And** $(x \pm 5)^2 - (y \pm 9)^2 = (5^2)$ **are not circles** and gain M0M0A0	
	But $(x-0)^2 + (y-9)^2 = 5^2$ gains M0M1A0	
(b)	M1: Attempts to find distance from their **centre of circle** to P (or square of this value). If this is called PT and given as answer this is M0. Solution may use letter other than X, as centre was not labelled in the question.	
	N.B. Distance from (0, 9) to (8, -7) is incorrect method and is M0, followed by M0A0.	
	dM1: Applies the **subtraction** form of Pythagoras to find PT or PT^2 (depends on previous method mark for distance from **centre to P**) or uses appropriate complete method involving trigonometry	
	A1: 20 cso	

Pearson

Gold Questions

Calculators may not be used

The total mark for this section is 35

Q1

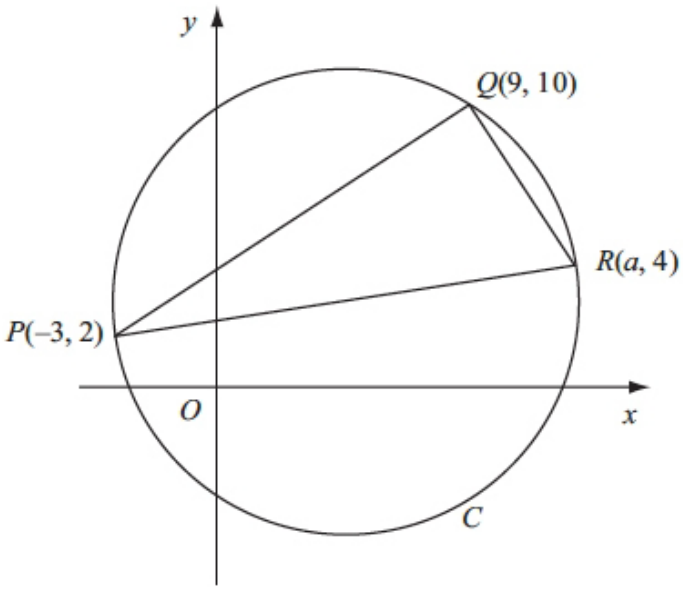

Figure 2

The points P(−3, 2), Q(9, 10) and R(a, 4) lie on the circle C, as shown in Figure 2. Given that PR is a diameter of C,

(a) show that $a = 13$

(3)

(b) find an equation for C

(5)

(2)

(Total for Question 1 is 10 marks)

Q2

The circle C has centre $A(2,1)$ and passes through the point $B(10, 7)$

(a) Find an equation for C

(4)

The line l_1 is the tangent to C at the point B

(b) Find an equation for l_1

(4)

The line l_2 is parallel to l_1 and passes through the mid-point of AB

Given that l_2 intersects C at the points P and Q,

(c) find the length of PQ, giving your answer in its simplest surd form.

(3)

(Total for Question 2 is 11 marks)

Q3

The circle C has equation

$$x^2 + y^2 - 6x + 10y + 9 = 0$$

(a) Find

(i) the coordinates of the centre of C
(ii) the radius of C

(3)

The line with equation $y = kx$, where k is a constant, cuts C at two distinct points.

(b) Find the range of values for k

(6)

(Total for Question 3 is 9 marks)

Q4

A circle C with centre at (– 2, 6) passes through the point (10, 11).
The circle C also passes through the point (10, 1) and has the equation
$(x + 2)^2 + (y - 6)^2 = 13^2$

The tangent to the circle C at the point (10, 11) meets the y axis at the point P
and the tangent to the circle C at the point (10, 1) meets the y axis at the point Q.
Show that the distance PQ is 58 explaining your method clearly.

(Total for Question 4 is 7 marks)

End of Questions

Gold Mark Scheme

Q1

Question Number	Scheme	Marks
(a)	PQ: $m_1 = \frac{10-2}{9-(-3)}$ $(=\frac{2}{3})$ **and** QR: $m_2 = \frac{10-4}{9-a}$	M1
(b)	$m_1 m_2 = -1$: $\frac{8}{12} \times \frac{6}{9-a} = -1$ $\quad a = 13$ $\quad$ (*)	M1 A1 (3)
Alt for (a)	(a) Alternative method (Pythagoras) Finds **all three** of the following	
	$(9-(-3))^2 + (10-2)^2$, (*i.e.*208), $(9-a)^2 + (10-4)^2$, $(a-(-3))^2 + (4-2)^2$	M1
	Using Pythagoras (correct way around) e.g. $a^2 + 6a + 9 = 240 + a^2 - 18a + 81$ to form equation	M1
	Solve (or verify) for a, $a = 13$ (*)	A1 (3)
	(b) Centre is at (5, 3)	B1
	$(r^2 =)\ (10-3)^2 + (9-5)^2$ or equiv., or $(d^2 =)\ (13-(-3))^2 + (4-2)^2$	M1 A1
	$(x-5)^2 + (y-3)^2 = 65$ or $x^2 + y^2 - 10x - 6y - 31 = 0$	M1 A1 (5)
Alt for (b)	Uses $(x-a)^2 + (y-b)^2 = r^2$ or $x^2 + y^2 + 2gx + 2fy + c = 0$ and substitutes (-3, 2), (9, 10) and (13, 4) then eliminates one unknown	M1
	Eliminates second unknown	M1
	Obtains $g = -5, f = -3, c = -31$ or $a = 5, b = 3, \quad r^2 = 65$	A1, A1, B1cao (5) **[8]**

Pearson

Notes	
(a)	**M1**-considers gradients of PQ and QR -must be y difference / x difference (or considers three lengths as in alternative method) **M1** Substitutes gradients into product = -1 (or lengths into Pythagoras' Theorem the correct way round) **A1** Obtains $a = 13$ with no errors by solution or verification. Verification can score 3/3.
(b)	Geometrical method: **B1** for coordinates of centre – can be implied by use in part (b) **M1** for attempt to find r^2, d^2, r or d (allow one slip in a bracket). **A1** cao. These two marks may be gained implicitly from circle equation **M1** for $(x \pm 5)^2 + (y \pm 3)^2 = k^2$ or $(x \pm 3)^2 + (y \pm 5)^2 = k^2$ ft their (5,3) Allow k^2 non numerical. **A1** cao for whole equation and rhs must be 65 or $\left(\sqrt{65}\right)^2$, (similarly B1 must be 65 or $\left(\sqrt{65}\right)^2$, in alternative method for (b))

Question Number	Scheme	Marks
Further alternatives	(i) A number of methods find gradient of PQ = 2/3 then give perpendicular gradient is −3/2 This is **M1**	M1
	They then proceed using equations of lines through point Q or by using gradient QR to obtain equation such as $\frac{4-10}{a-9} = -\frac{3}{2}$ **M1** (may still have x in this equation rather than a and there may be a small slip)	M1
	They then complete to give (a)= 13 **A1**	A1
	(ii) A long involved method has been seen finding the coordinates of the centre of the circle first. This can be done by a variety of methods Giving centre as (c, 3) and using an equation such as $(c-9)^2 + 7^2 = (c+3)^2 + 1^2$ (equal radii) or $\frac{3-6}{c-3} = -\frac{3}{2}$ **M1** (perpendicular from centre to chord bisects chord)	M1
	Then using c (= 5) to find a is **M1**	M1
	Finally $a = 13$ **A1**	A1
	(iii) Vector Method: States **PQ**. **QR** = 0, with vectors stated 12i +8j and $(9 - a)$**i** + **6j** is **M1**	M1
	Evaluates scalar product so $108 - 12a + 48 = 0$ **(M1)**	M1
	solves to give $a = 13$ **(A1)**	A1

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Q2

Question Number	Scheme	Marks
	(a) $(10-2)^2+(7-1)^2$ or $\sqrt{(10-2)^2+(7-1)^2}$	M1 A1
	$(x\pm 2)^2+(y\pm 1)^2=k$ (k a positive value)	M1
	$(x-2)^2+(y-1)^2=100$ (Accept 10^2 for 100)	A1
	(Answer only scores full marks)	(4)
	(b) (Gradient of radius =) $\frac{7-1}{10-2}=\frac{6}{8}$ (or equiv.) Must be seen in part (b)	B1
	Gradient of tangent $=\frac{-4}{3}$ (Using perpendicular gradient method)	M1
	$y-7=m(x-10)$ Eqn., in any form, of a line through (10, 7) with any numerical gradient (except 0 or ∞)	M1
	$y-7=\frac{-4}{3}(x-10)$ or equiv (ft gradient of radius, dep. on both M marks)	A1ft
	$\{3y=-4x+61\}$ (N.B. The A1 is only available as ft after B0)	
	The unsimplified version scores the A mark (isw if necessary... subsequent mistakes in simplification are not penalised here. The equation must at some stage be exact, not, e.g. $y=-1.3x+20.3$	(4)
	(c) $\sqrt{r^2-\left(\frac{r}{2}\right)^2}$ Condone sign slip if there is evidence of correct use of Pythag.	M1
	$=\sqrt{10^2-5^2}$ or numerically exact equivalent	A1
	$PQ\left(=2\sqrt{75}\right)=10\sqrt{3}$ Simplest surd form $10\sqrt{3}$ required for final mark	A1
		(3) 11
	(b) 2nd M: Using (10, 7) to find the equation, in any form, of a straight line through (10, 7), with any numerical gradient (except 0 or ∞).	
	Alternative: 2nd M: Using (10, 7) and an m value in $y=mx+c$ to find a value of c.	
	(b) Alternative for first 2 marks (differentiation):	
	$2(x-2)+2(y-1)\frac{dy}{dx}=0$ or equiv. B1	
	Substitute $x=10$ and $y=7$ to find a value for $\frac{dy}{dx}$ M1	
	(This M mark can be awarded generously, even if the attempted 'differentiation' is not 'implicit').	
	(c) Alternatives:	
	To score M1, must be a fully correct method to obtain $\frac{1}{2}PQ$ or PQ.	
	1st A1: For alternative methods that find PQ directly, this mark is for an exact numerically correct version of PQ.	

Pearson

Q3

Question	Scheme	Marks	AOs
(a)	Attempts to complete the square $(x\pm3)^2+(y\pm5)^2=...$	M1	1.1b
	(i) Centre $(3,-5)$	A1	1.1b
	(ii) Radius 5	A1	1.1b
		(3)	
(b)	Uses a sketch or otherwise to deduce $k=0$ is a critical value	B1	2.2a
	Substitute $y=kx$ in $x^2+y^2-6x+10y+9=0$	M1	3.1a
	Collects terms to form correct 3TQ $(1+k^2)x^2+(10k-6)x+9=0$	A1	1.1b
	Attempts $b^2-4ac...0$ for their a, b and c leading to values for k $"(10k-6)^2-36(1+k^2)...0"\rightarrow k=...,...$ $\left(0 \text{ and } \frac{15}{8}\right)$	M1	1.1b
	Uses $b^2-4ac>0$ and chooses the outside region (see note) for **their** critical values (Both a and b must have been expressions in k)	dM1	3.1a
	Deduces $k<0, k>\frac{15}{8}$ oe	A1	2.2a
		(6)	
			(9 marks)

Pearson

Notes

(a)

M1: Attempts $(x\pm3)^2+(y\pm5)^2=..$

This mark may be implied by candidates writing down a centre of $(\pm3,\pm5)$ or $r^2=25$

(i) A1: Centre $(3,-5)$

(ii) A1: Radius 5. Do not accept $\sqrt{25}$

Answers only (no working) scores all three marks

(b)

B1: Uses a sketch or their subsequent quadratic to deduce that $k=0$ is a critical value.
You may award for the correct $k<0$ but award if $k\leqslant 0$ or even with greater than symbols

M1: Substitutes $y=kx$ in $x^2+y^2-6x+10y+9=0$ or their $(x\pm3)^2+(y\pm5)^2=...$ to form an equation in just x and k. It is possible to substitute $x=\frac{y}{k}$ into their circle equation to form an equation in just y and k.

A1: Correct 3TQ $(1+k^2)x^2+(10k-6)x+9=0$ with the terms in x collected. The "= 0" can be implied by subsequent work. This may be awarded from an equation such as $x^2+k^2x^2+(10k-6)x+9=0$ so long as the correct values of a, b and c are used in $b^2-4ac...0$.

FYI The equation in y and k is $(1+k^2)y^2+(10k^2-6k)y+9k^2=0$ oe

M1: Attempts to find two critical values for k using $b^2-4ac...0$ or $b^2...4ac$ where ... could be "=" or any inequality.

dM1: Finds the outside region using their critical values. Allow the boundary to be included. It is dependent upon all previous M marks and both a and b must have been expressions in k.
Note that it is possible that the correct region could be the inside region if the coefficient of k^2 in $4ac$ is larger than the coefficient of k^2 in b^2 Eg

$b^2-4ac=(k-6)^2-4\times(1+k^2)\times9>0\Rightarrow-35k^2-12k>0\Rightarrow k(35k+12)<0$

Pearson

Q4

Question	Scheme		Marks	AOs
	Finds radius gradient $\frac{11-6}{10-(-2)}$ or $\frac{1-6}{10-(-2)}$ (m)		M1	3.1a
	Finds gradient perpendicular to their radius using $-\frac{1}{m}$		M1	1.1b
	Finds (equation and) y intercept of tangent (see note below)		M1	1.1b
	Obtains a correct value for y intercept of their tangent i.e.35 or –23		A1	1.1b
	Way 1: Deduces gradient of second tangent	Way 2: Deduces midpoint of PQ from symmetry ((0,6))	M1	1.1b
	Finds (equation and) y intercept of second tangent	Uses this to find other intercept	M1	1.1b
	So obtains distance PQ=35+23=58*		A1*	1.1b
			(7)	
				(7 marks)

Notes

M1: Calculates $\frac{11-6}{10-(-2)}$ or $\frac{1-6}{10-(-2)}$ (m)

M1: Finds $-\frac{1}{m}$ (correct answer is $-\frac{12}{5}$ or $\frac{12}{5}$) This is referred to as m' in the next note.

M1: Attempts $y-11=their\left(-\frac{12}{5}\right)(x-10)$ or $y-1=their\left(\frac{12}{5}\right)(x-10)$ and puts $x=0$, or uses vectors to find intercept e.g. $\frac{y-11}{10}=-m'$

A1: One correct intercept 35 or – 23

Pearson

Qu 17 continued

Way 1:

M1: Uses the negative of their previous tangent gradient or uses a correct $-\frac{12}{5}$ or $\frac{12}{5}$

M1: Attempts the second tangent equation and puts $x = 0$ or uses vectors to find intercept

e.g. $\frac{11-y}{10} = m'$

Way 2:

M1: Finds midpoint of PQ from symmetry. (This is at (0,6))

M1: Uses this midpoint to find second intercept or to find difference between midpoint and first intercept. e.g. $35 - 6 = 29$ then $6 - 29 = -23$ so second intercept is at $(-23, 0)$

Ways 1 and 2:

A1*: Obtain 58 correctly from a valid method.

Pearson

Platinum Questions

Calculators may not be used

The total mark for this section is 20

1 A point P lies on the curve with equation

$$x^2 + y^2 - 6x + 8y = 24.$$

Find the greatest and least possible values of the length OP, where O is the origin.

(6)

(Total for Question 1 is 6 marks)

---

2 The line with equation $y = mx$ is a tangent to the circle C_1 with equation

$$(x + 4)^2 + (y - 7)^2 = 13.$$

(a) Show that m satisfies the equation

$$3m^2 + 56m + 36 = 0.$$

(4)

The tangents from the origin O to C_1 touch C_1 at the points A and B.

(b) Find the coordinates of the points A and B.

(8)

Another circle C_2 has equation $x^2 + y^2 = 13$. The tangents from the point $(4, -7)$ to C_2 touch it at the points P and Q.

(c) Find the coordinates of either the point P or the point Q.

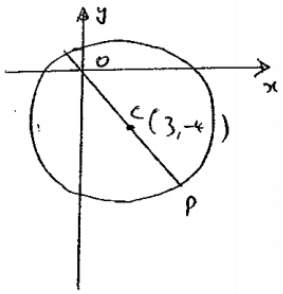
(2)

(Total for Question 2 is 20 marks)

End of Questions

Platinum Mark Scheme

1.

Question Number	Scheme	Marks
1	$(x-3)^2+(y+4)^2 = 24+9+16 = 49$	M1
	Curve is circle, centre $(3,-4)$, radius 7	A1
	 $OC = \sqrt{3^2+4^2} = 5$	M1
	Greatest length of $= 5+r$ (or least) $= \underline{\underline{12}}$	M1 A1
	Least length $= r-5 = \underline{\underline{2}}$	A1 (6)

Pearson

2.

Q4 (a)

$(x+4)^2 + (y-7)^2 = 13$ and $y = mx$

$\therefore (x^2+8x+16) + (m^2x^2 - 14mx + 49) = 13$ — M1

$(1+m^2)x^2 + (8-14m)x + 52 = 0$ (3 t-terms) — A1

Touches, so "$b^2 = 4ac$" $(8-14m)^2 = 4 \times 52 \times (1+m^2)$ — M1 ($b^2=4ac$)

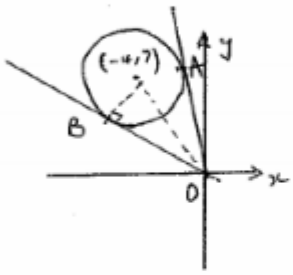
$(4-7m)^2 = 52 + 52m^2$

$\therefore \underline{3m^2 + 56m + 36 = 0}$ * — A1 cso (4)

(b)

$(3m+2)(m+18) = 0$ — M1

$\therefore m = -2/3$ or -18 (both m) — A1



Let A or B be (x, y)

Then $(x^2+y^2) + 13 = 4^2 + 7^2 = 65$

$x^2 + y^2 = 52$ — M1, A1

$y = -\frac{2}{3}x \Rightarrow \frac{13}{9}x^2 = 52 \Rightarrow x = \pm 6$ — M1, A1

From the configuration $x_B = -6 \therefore y_B = +4$: $\underline{B \equiv (-6, 4)}$

$y = -18x \Rightarrow 325x^2 = 52 \Rightarrow x^2 = \frac{4}{25}$

Again $x < 0$ for A $\therefore x_A = -\frac{2}{5}$; $y_A = \frac{36}{5}$ — M1, A1

$\underline{A = (-\frac{2}{5}, \frac{36}{5})}$ (8)

(c)

Situation is a translation of problem in (b) by $\begin{pmatrix} 4 \\ -7 \end{pmatrix}$ — M1

So P, Q are $\begin{pmatrix} -6 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ -7 \end{pmatrix}$ and $\left(-\frac{2}{5}, \frac{36}{5}\right) + (4, -7)$

$= \underline{(-2, -3)}$ and $\underline{\left(\frac{18}{5}, \frac{1}{5}\right)}$ — A1 (either) (2)

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Topic 7

Algebraic Methods

Bronze, Silver, Gold and Platinum Worksheets for AS Level Mathematics

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Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculators may not be used

The total mark for this section is 26

Q1

$$f(x) = 2x^3 - 7x^2 - 5x + 4$$

(a) Find the remainder when f(x) is divided by ($x - 1$)

(2)

(b) Use the factor theorem to show that (x+1) is a factor of f(x)

(2)

(c) Factorise f(x) completely.

(4)

(Total for Question 1 is 9 marks)

Q2

(a) Find the remainder when

$$x^3 - 2x^2 - 4x + 8$$

is divided by

(i) $x - 3$

(ii) $x + 2$

(3)

(b) Hence, or otherwise, find all the solutions to the equation

$$x^3 - 2x^2 - 4x + 8 = 0$$

(4)

(Total for Question 2 is 7 marks)

Q3

$$f(x) = x^4 + x^3 + 2x^2 + ax + b$$

where a and b are constants.

When $f(x)$ is divided by $(x - 1)$, the remainder is 7

(a) Show that $a + b = 3$

(2)

When $f(x)$ is divided by $(x + 2)$, the remainder is -8

(b) Find the value of a and the value of b

(5)

(Total for Question 3 is 7 marks)

Q4

Given $n \in \mathbb{Z}$, prove that $n^3 + 2$ is not divisible by 8

(Total for Question 4 is 4 marks)

End of Questions

Bronze Mark Scheme

Q1

Question Number	Scheme	Marks
(a)	$f(x)=2x^3-7x^2-5x+4$ Remainder $=f(1)=2-7-5+4=-6$ Attempts $f(1)$ or $f(-1)$. $=-6$ -6	M1 A1 [2]
(b)	$f(-1)=2(-1)^3-7(-1)^2-5(-1)+4$ Attempts $f(-1)$. and so $(x+1)$ is a factor. $f(-1)=0$ with no sign or substitution errors **and for conclusion.**	M1 A1 [2]
(c)	$f(x)=\{(x+1)\}(2x^2-9x+4)$ $=(x+1)(2x-1)(x-4)$ (**Note**: *Ignore the ePEN notation of (b) (should be (c)) for the final three marks in this part*).	M1 A1 dM1 A1 [4] 8

(a) M1 for ***attempting*** either $f(1)$ or $f(-1)$. Can be implied. Only one slip permitted.
M1 can also be given for an attempt (at least two "subtracting" processes) at long division to give a remainder which is independent of x. A1 can be given also for -6 seen at the bottom of long division working. Award A0 for a candidate who finds -6 but then states that the remainder is 6.
Award M1A1 for -6 without any working.

(b) M1: attempting only $f(-1)$. A1: must correctly show $f(-1)=0$ and give a conclusion ***in part (b) only***.
Note: Stating "hence factor" or "it is a factor" or a "tick" or "QED" is fine for the conclusion.
Note also that a conclusion can be implied from a preamble, eg: "If $f(-1)=0$, $(x+1)$ is a factor…."
Note: Long division scores no marks in part (b). The factor theorem is required.

(c) 1st M1: Attempts long division or other method, to obtain $(2x^2\pm ax\pm b)$, $a\neq 0$, even with a remainder.
Working need not be seen as this could be done "by inspection." $(2x^2\pm ax\pm b)$ must be seen ***in part (c) only***. Award 1st M0 if the quadratic factor is clearly found from dividing $f(x)$ by $(x-1)$. Eg. Some candidates use their $(2x^2-5x-10)$ in part (c) found from applying a long division method in part (a).
1st A1: For seeing $(2x^2-9x+4)$.
2nd dM1: Factorises a 3 term quadratic. (see rule for factorising a quadratic). This is dependent on the previous method mark being awarded. This mark can also be awarded if the candidate applies the quadratic formula correctly.
2nd A1: is cao and needs all three factors on one line. Ignore following work (such as a solution to a quadratic equation.)
Note: Some candidates will go from $\{(x+1)\}(2x^2-9x+4)$ to $\{x=-1\}$, $x=\frac{1}{2}$, 4, and not list all three factors. Award these responses M1A1M1A0.
Alternative: 1st M1: For finding either $f(4)=0$ or $f\left(\frac{1}{2}\right)=0$.
1st A1: A second correct factor of usually $(x-4)$ or $(2x-1)$ found. Note that any one of the other correct factors found would imply the 1st M1 mark.
2nd dM1: For using two known factors to find the third factor, usually $(2x\pm 1)$.
2nd A1 for correct answer of $(x+1)(2x-1)(x-4)$.
Alternative: (for the first two marks)
1st M1: Expands $(x+1)(2x^2+ax+b)$ {giving $2x^3+(a+2)x^2+(b+a)x+b$ } then compare coefficients to find values for a and b. 1st A1: $a=-9$, $b=4$
Not dealing with a factor of 2: $(x+1)(x-\frac{1}{2})(x-4)$ or $(x+1)(x-\frac{1}{2})(2x-8)$ scores M1A1M1A0.
Answer only, with one sign error: eg. $(x+1)(2x+1)(x-4)$ or $(x+1)(2x-1)(x+4)$ scores M1A1M1A0. **(c) Award M1A1M1A1 for Listing all three correct factors with no working.**

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Q2

Question Number	Scheme	Marks
a)i)	$f(3) = 3^3 - 2 \times 3^2 - 4 \times 3 + 8$; $= 5$	M1; A1
ii)	$f(-2) = (-8 - 8 + 8 + 8) = 0$ (B1 on Epen, but A1 in fact)	B1 (3)
	M1 is for attempt at **either** f(3) or f(–3) in (i) **or** f(–2) or f(2) in (ii).	
(b)	$[(x + 2)](x^2 - 4x + 4)$ (= 0 not required) [must be seen or used in (b)]	M1 A1
	$(x + 2)(x - 2)^2$ (= 0) (can imply previous 2 marks)	M1
	Solutions: $x = 2$ or -2 (both) or (–2, 2, 2) [**no wrong working seen**]	A1 (4) **[7]**

Notes: (a) **No** working seen: Both answers correct scores full marks

One correct ;M1 then A1B0 or A0B1, whichever appropriate.

Alternative (Long division)

Divide by (x - 3) OR $(x + 2)$ to get $x^2 + ax + b$, a may be zero [M1]

$x^2 + x - 1$ and $+5$ seen i.s.w. (or "remainder = 5") [A1]

$x^2 - 4x + 4$ and 0 seen (or "no remainder") [B1]

(b) First M1 requires division by a **found factor** ; e.g $(x + 2)$, $(x - 2)$ or

what candidate thinks is a **factor** to get $(x^2 + ax + b)$, a may be zero .

First A1 for $[(x + 2)](x^2 - 4x + 4)$ or $(x - 2)(x^2 - 4)$

Second M1:attempt to factorise **their** found quadratic. (or use formula correctly)

[Usual rule: $x^2 + ax + b = (x + c)(x + d)$, *where* $|cd| = |b|$.]

N.B. Second A1 is for solutions, not factors

SC: (i) Answers only: Both correct, and no wrong, award M0A1M0A1 (as if B1,B1)

One correct, (even if 3 different answers) award M0A1M0A0 (as if B1)

(ii) Factor theorem used to find two correct factors, award M1A1,

then M0, A1 if both correct solutions given. (-2,2,2 would earn all marks)

(iii) If in (a) candidate has $(x+2)(x^2-4)$ B0, but then repeats in (b),

can score M1A0M1(if goes on to factorise)A0 (answers fortuitous)

Alternative (first two marks)

$(x+2)(x^2 + bx + c) = x^3 + (2+b)x^2 + (2b+c)x + 2c = 0$ and then compare

with $x^3 - 2x^2 - 4x + 8 = 0$ to find b and c. [M1]

$b = -4$, $c = 4$ [A1]

Method of grouping

$x^3 - 2x^2 - 4x + 8 = x^2(x - 2), 4(x \pm 2)$ M1; $= x^2(x - 2) - 4(x - 2)$ A1

$[= (x^2 - 4)(x - 2)] = (x+2)(x-2)^2$ M1

Solutions: $x = 2$, $x = -2$ both A1

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Q3

Question Number	Scheme	Marks
(a)	$f(x)=x^4+x^3+2x^2+ax+b$ Attempting $f(1)$ or $f(-1)$. $f(1)=1+1+2+a+b=7$ or $4+a+b=7 \Rightarrow a+b=3$ (as required) **AG**	 M1 A1 * **cso** **(2)**
(b)	Attempting $f(-2)$ or $f(2)$. $f(-2)=\underline{16-8+8-2a+b=-8}$ $\{\Rightarrow -2a+b=-24\}$ Solving both equations simultaneously to get as far as $a=\ldots$ or $b=\ldots$ Any one of $a=9$ or $b=-6$ Both $a=9$ and $b=-6$	M1 A1 dM1 A1 A1 **cso** **(5)** **[7]**
	Notes	
(a)	M1 for attempting either $f(1)$ or $f(-1)$. A1 for applying $f(1)$, setting the result equal to 7, and manipulating this correctly to give the result given on the paper as $a+b=3$. Note that the answer is given in part (a).	
(b)	M1: attempting either $f(-2)$ or $f(2)$. A1: correct underlined equation in a and b; eg $\underline{16-8+8-2a+b=-8}$ or equivalent, eg $-2a+b=-24$. dM1: an attempt to eliminate one variable from 2 linear simultaneous equations in a and b. Note that this mark is dependent upon the award of the first method mark. A1: any one of $a=9$ or $b=-6$. A1: both $a=9$ and $b=-6$ and a correct solution only.	
	Alternative Method of Long Division: (a) M1 for long division by $(x-1)$ to give a remainder in a and b which is independent of x. A1 for $\{\text{Remainder}=\}\ b+a+4=7$ leading to the correct result of $a+b=3$ (answer given.) (b) M1 for long division by $(x+2)$ to give a remainder in a and b which is independent of x. A1 for $\{\text{Remainder}=\}\ \underline{b-2(a-8)=-8}$ $\{\Rightarrow -2a+b=-24\}$. Then dM1A1A1 are applied in the same way as before.	

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Score as below so M0 A0 M1 A1 or M1 A0 M1 A1 are not possible

Generally the marks are awarded for
M1: Suitable approach to answer the question for n being even **OR** odd
A1: Acceptable proof for n being even **OR** odd
M1: Suitable approach to answer the question for n being even **AND** odd
A1: Acceptable proof for n being even **AND** odd **WITH** concluding statement.

There is no merit in a

- student taking values, or multiple values, of n and then drawing conclusions.
  So $n = 5 \Rightarrow n^3 + 2 = 127$ which is not a multiple of 8 scores no marks.
- student using divided when they mean divisible. Eg. "Odd numbers cannot be divided by 8" is incorrect. We need to see either "odd numbers are not divisible by 8" or "odd numbers cannot be divided by 8 **exactly**"
- stating $\frac{n^3+2}{8} = \frac{1}{8}n^3 + \frac{1}{4}$ which is not a whole number
- stating $\frac{(n+1)^3+2}{8} = \frac{1}{8}n^3 + \frac{3}{8}n^2 + \frac{3}{8}n + \frac{3}{8}$ which is not a whole number

Logical approach	States that if n is odd, n^3 is odd	M1	2.1
	so $n^3 + 2$ is odd and therefore cannot be divisible by 8	A1	2.2a
	States that if n is even, n^3 is a multiple of 8	M1	2.1
	so $n^3 + 2$ cannot be a multiple of 8 So (Given $n \in \mathbb{N}$), $n^3 + 2$ is not divisible by 8	A1	2.2a
		(4)	
			4 marks

First M1: States the result of cubing an odd or an even number
First A1: Followed by the result of adding two and gives a valid reason why it is not divisible by 8.
So for odd numbers accept for example
"odd number + 2 is still odd and odd numbers are not divisible by 8"
"$n^3 + 2$ is odd and cannot be divided by 8 **exactly**"
and for even numbers accept
"a multiple of 8 add 2 is not a multiple of 8, so $n^3 + 2$ is not divisible by 8"
"if n^3 is a multiple of 8 then $n^3 + 2$ cannot be divisible by 8"
Second M1: States the result of cubing an odd and an even number
Second A1: Both valid reasons must be given followed by a concluding statement.

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Q4

Question	Scheme	Marks	AOs
Algebraic approach	(If n is even,) $n=2k$ and $n^3+2=(2k)^3+2=8k^3+2$	M1	2.1
	Eg.'This is 2 more than a multiple of 8, hence not divisible by 8' Or 'as $8k^3$ is divisible by 8 , $8k^3+2$ isn't'	A1	2.2a
	(If n is odd,) $n=2k+1$ and $n^3+2=(2k+1)^3+2$	M1	2.1
	$=8k^3+12k^2+6k+3$ which is an even number add 3, therefore odd. Hence it is not divisible by 8 So (given $n\in\mathrm{N}$,) n^3+2 is not divisible by 8	A1	2.2a
		(4)	
Alt algebraic approach	(If n is even,) $n=2k$ and $\frac{n^3+2}{8}=\frac{(2k)^3+2}{8}=\frac{8k^3+2}{8}$	M1	2.1
	$=k^3+\frac{1}{4}$ oe which is not a whole number and hence not divisible by 8	A1	2.2a
	(If n is odd,) $n=2k+1$ and $\frac{n^3+2}{8}=\frac{(2k+1)^3+2}{8}$	M1	2.1
	$=\frac{8k^3+12k^2+6k+3}{8}$ ** The numerator is odd as $8k^3+12k^2+6k+3$ is an even number +3 hence not divisible by 8 So (Given $n\in\mathrm{N}$,) n^3+2 is not divisible by 8	A1	2.2a
		(4)	

Notes
Correct expressions are required for the M's. There is no need to state "**If *n* is even,**" $n=2k$ and "**If *n* is odd,** $n=2k+1$" for the two M's as the expressions encompass all numbers. However the concluding statement must attempt to show that it has been proven for all $n\in\mathrm{N}$ Some students will use $2k-1$ for odd numbers There is no requirement to change the variable. They may use $2n$ and $2n\pm1$ Reasons must be correct. Don't accept $8k^3+2$ cannot be divided by 8 for example. (It can!) Also $**"=\frac{8k^3+12k^2+6k+3}{8}=k^3+\frac{3}{2}k^2+\frac{3}{4}k+\frac{3}{8}$ which is not whole number" is too vague so A0

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Silver Questions

Calculators may not be used

The total mark for this section is 29

Q1

$$f(x) = 4x^3 - 12x^2 + 2x - 6$$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$

(2)

(b) Hence show that 3 is the only real root of the equation $f(x) = 0$

(4)

(Total for Question 1 is 6 marks)

Q2

$$f(x) = 2x^3 - 7x^2 - 10x + 24$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$

(2)

(b) Factorise $f(x)$ completely.

(4)

(Total for Question 2 is 6 marks)

Q3

$$f(x) = (3x - 2)(x - k) - 8$$

where k is a constant.

(a) Write down the value of f(k)

(1)

When f(x) is divided by $(x - 2)$ the remainder is 4

(b) Find the value of k

(2)

(c) Factorise f(x) completely.

(3)

(Total for Question 3 is 6 marks)

Q4

$$f(x) = x^3 + ax^2 + bx + 3$$

where a and b are constants.

Given that when f (x) is divided by $(x + 2)$ the remainder is 7,

(a) show that $2a - b = 6$

(2)

Given also that when f (x) is divided by $(x - 1)$ the remainder is 4,

(b) find the value of a and the value of b.

(4)

(Total for Question 4 is 6 marks)

Q5

(a) Prove that for all positive values of a and b

$$\frac{4a}{b} + \frac{b}{a} \geq 4$$

(4)

(b) Prove, by counter example, that this is not true for all values of a and b

(1)

(Total for Question 5 is 5 marks)

End of Questions

Silver Mark Scheme

Q1

Question	Scheme	Marks	AOs
(a)	States or uses $f(+3)=0$	M1	1.1b
	$4(3)^3-12(3)^2+2(3)-6=108-108+6-6=0$ and so $(x-3)$ is a factor	A1	1.1b
		(2)	
(b)	Begins division or factorisation so $4x^3-12x^2+2x-6=(x-3)(4x^2+...)$	M1	2.1
	$4x^3-12x^2+2x-6=(x-3)(4x^2+2)$	A1	1.1b
	Considers the roots of their quadratic function using completion of square or discriminant	M1	2.1
	$(4x^2+2)=0$ has no real roots with a reason (e.g. negative number does not have a real square root, or $4x^2+2>0$ for all x) So $x=3$ is the only real root of $f(x)=0$ *	A1*	2.4
		(4)	
			(6 marks)

Notes

(a) M1: States or uses $f(+3)=0$
A1: See correct work evaluating and achieving zero, together with correct conclusion

(b) M1: Needs to have $(x-3)$ and first term of quadratic correct
A1: Must be correct – may further factorise to $2(x-3)(2x^2+1)$
M1: Considers **their** quadratic for no real roots by use of completion of the square or consideration of discriminant then
A1*: a correct explanation.

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Q2

Question number	Scheme	Marks
(a)	$f(-2)=2.(-2)^3-7.(-2)^2-10.(-2)+24$ $=0$ so $(x+2)$ is a factor	M1 A1 (2)
(b)	$f(x)=(x+2)(2x^2-11x+12)$ $f(x)=(x+2)(2x-3)(x-4)$	M1 A1 dM1 A1 (4)
		6 marks

Notes (a)

M1 : Attempts f(±2) (Long division is M0)
A1 : is for =0 **and conclusion**
Note: Stating "hence factor" or "it is a factor" or a "√" (tick) or "QED" is fine for the conclusion.
Note also that a conclusion can be implied from a preamble, eg: "If $f(-2)=0$, $(x+2)$ is a factor…." (Not just f(-2)=0)

(b)

1st M1: Attempts long division by correct factor or other method leading to obtaining $(2x^2 \pm ax \pm b)$, $a \neq 0$, $b \neq 0$, even with a remainder. Working need not be seen as could be done "by inspection."
Or *Alternative Method*: **1st M1**: Use $(x+2)(ax^2+bx+c)=2x^3-7x^2-10x+24$ with expansion and comparison of coefficients to obtain $a=2$ and to obtain values for b and c
1st A1: For seeing $(2x^2-11x+12)$. [Can be seen here in (b) after work done in (a)]
2nd M1: Factorises quadratic. (see rule for factorising a quadratic). This is dependent on the previous method mark being awarded and **needs factors**
2nd A1: is cao and needs all three factors together. Ignore subsequent work (such as a solution to a quadratic equation.)

Note: Some candidates will go from $\{(x+2)\}(2x^2-11x+12)$ to $\{x=-2\}$, $x=\frac{3}{2}$, 4, and not list all three factors. Award these responses M1A1M0A0.

Finds $x=4$ and $x=1.5$ by factor theorem, formula or calculator and produces factors M1
$f(x)=(x+2)(2x-3)(x-4)$ **or** $f(x)=2(x+2)(x-1.5)(x-4)$ **o.e. is full marks**
$f(x)=(x+2)(x-1.5)(x-4)$ **loses last A1**

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Q3

Question Number		Scheme	Marks
Q	(a)	$f(k) = -8$	B1 (1)
	(b)	$f(2) = 4 \Rightarrow \quad 4 = (6-2)(2-k) - 8$	M1
		So $\qquad k = -1$	A1 (2)
	(c)	$f(x) = 3x^2 - (2+3k)x + (2k-8) \quad = 3x^2 + x - 10$	M1
		$= (3x - 5)(x + 2)$	M1A1 (3)
			[6]

(b)

M1 for substituting $x = 2$ (not $x = -2$) and equating to 4 to form an equation in k.
If the expression is expanded in this part, condone 'slips' for this M mark.
Treat the omission of the -8 here as a 'slip' and allow the M mark.

Beware:
Substituting $x = -2$ and equating to 0 (M0 A0) also gives $k = -1$.

Alternative:
M1 for dividing by $(x-2)$, to get $3x + (\text{function of } k)$, with remainder as a function of k, and equating the remainder to 4. [Should be $3x + (4 - 3k)$, remainder $-4k$].

No working:
$k = -1$ with no working scores M0 A0.

(c)

1st M1 for multiplying out and substituting their (constant) value of k (in either order).
The multiplying-out may occur earlier.
Condone, for example, sign slips, but if the 4 (from part (b)) is included in the $f(x)$ expression, this is M0. The 2nd M1 is still available.

2nd M1 for an attempt to factorise their three term quadratic (3TQ).

A1 The correct answer, as a product of factors, is required.
Allow $3\left(x - \frac{5}{3}\right)(x+2)$

Ignore following work (such as a solution to a quadratic equation).
If the 'equation' is solved but factors are never seen, the 2nd M is not scored.

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Q4

Question number	Scheme	Marks
(a)	$f(-2) = -8 + 4a - 2b + 3 = 7$ so $2a - b = 6$ *	M1 A1 (2)
(b)	$f(1) = 1 + a + b + 3 = 4$ Solve two linear equations to give $a = 2$ and $b = -2$	M1 A1 M1 A1 (4) **6**
Notes	(a) **M1** : Attempts $f(\pm 2) = 7$ or attempts long division as far as putting remainder equal to 7 (There may be sign slips) **A1** is for correct equation with remainder = 7 and for the printed answer with no errors and no wrong working between the two (b) **M1** : Attempts $f(\pm 1) = 4$ or attempts long division as far as putting remainder equal to 4 **A1** is for correct equation with remainder = 4 and powers calculated correctly **M1** : Solving simultaneous equations (may be implied by correct answers). This mark may be awarded for attempts at elimination or substitution leading to values for both a and b. Errors are penalised in the accuracy mark. **A1** is cao for values of a and b and explicit values are needed. Special case: Misreads and puts remainder as 7 again in (b). This may earn M1A0M1A0 in part (b) and will result in a maximum mark of 4/6	
Long Divisions	$(x+2)\overline{)\,x^3 + ax^2 + bx + 3}$ $\quad x^2 + (a-2)x + (b-2a+4)$ $x^3 + 2x^2$ ..... $(x-1)\overline{)\,x^3 + ax^2 + bx + 3}$ $\quad x^2 + (a+1)x + (b+a+1)$ $x^3 - x^2$ ..... A marks as before	and reach their "3 - 2b + 4a - 8" = 7 M1 and reach their "3 + b + a + 1" = 4 M1

Pearson

Q5

Question	Scheme	Marks	AOs
(a)	States $(2a-b)^2 \dots 0$	M1	2.1
	$4a^2+b^2 \dots 4ab$	A1	1.1b
	(As $a>0, b>0$) $\quad \frac{4a^2}{ab}+\frac{b^2}{ab} \dots \frac{4ab}{ab}$	M1	2.2a
	Hence $\quad \frac{4a}{b}+\frac{b}{a} \dots 4 \quad *$ CSO	A1*	1.1b
		(4)	
(b)	$a=5, b=-1 \Rightarrow \frac{4a}{b}+\frac{b}{a}=-20-\frac{1}{5}$ which is less than 4	B1	2.4
		(1)	
			(5 marks)

Notes

(a) (condone the use of $>$ for the first three marks)

M1: For the key step in stating that $(2a-b)^2 \dots 0$

A1: Reaches $4a^2+b^2 \dots 4ab$

M1: Divides each term by $ab \Rightarrow \frac{4a^2}{ab}+\frac{b^2}{ab} \dots \frac{4ab}{ab}$

A1*: Fully correct proof with steps in the correct order and gives the reasons why this is true:

- when you square any (real) number it is always greater than or equal to zero
- dividing by ab does not change the inequality as $a>0$ and $b>0$

(b)

B1: Provides a counter example and shows it is not true.
This requires values, a calculation or embedded values(see scheme) and a conclusion. The conclusion must be in words eg the result does not hold or not true
Allow 0 to be used as long as they explain or show that it is undefined so the statement is not true.

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Gold Questions

Calculators may not be used

The total mark for this section is 27

Q1

$$f(x) = -6x^3 - 7x^2 + 40x + 21$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$

(2)

(b) Factorise $f(x)$ completely.

(4)

(Total for Question 1 is 6 marks)

Q2

$$f(x) = x^4 + 5x^3 + ax + b,$$

where a and b are constants.

The remainder when $f(x)$ is divided by $(x - 2)$ is equal to the remainder when $f(x)$ is divided by $(x + 1)$.

(a) Find the value of a.

(5)

Given that $(x + 3)$ is a factor of $f(x)$,

(b) find the value of b.

(3)

(Total for Question 2 is 8 marks)

Q3

$$f(x) = 2x^3 - 13x^2 + 8x + 48$$

(a) Prove that $(x - 4)$ is a factor of $f(x)$

(2)

(b) Hence, using algebra, show that the equation $f(x) = 0$ has only two distinct roots.

(4)

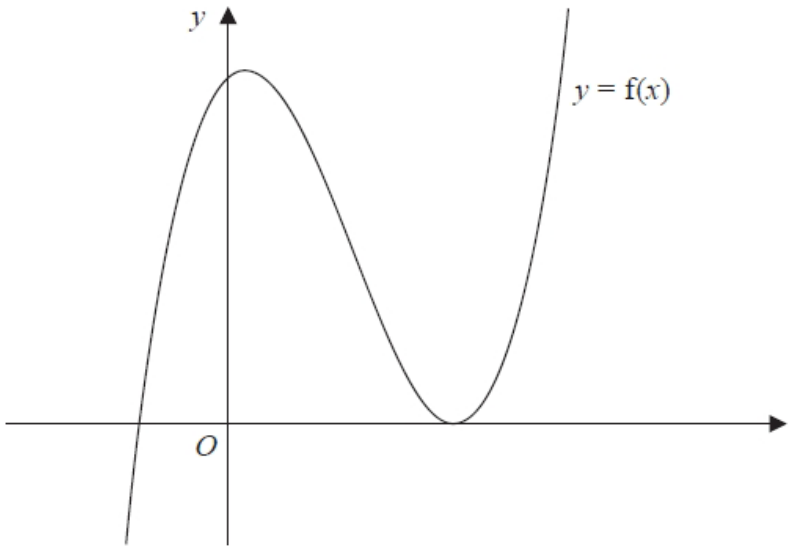

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$.

(c) Deduce, giving reasons for your answer, the number of real roots of the equation

$$2x^3 - 13x^2 + 8x + 46 = 0$$

(2)

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(d) find the two possible values of k.

(2)

(Total for Question 3 is 10 marks)

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Q4

(a) Prove that for all positive values of x and y

$$\sqrt{xy} \leqslant \frac{x+y}{2}$$

(2)

(b) Prove by counter example that this is not true when x and y are both negative.

(1)

(Total for Question 4 is 3 marks)

End of Questions

Gold Mark Scheme

Q1

Question Number	Scheme	Marks
(a)	Attempt $f(3)$ or $f(-3)$ Use of long division is M0A0 as factor theorem was required. $f(-3)=162-63-120+21=0$ so $(x+3)$ is a factor	M1 A1 **(2)**
(b)	Either (Way 1): $f(x)=(x+3)(-6x^2+11x+7)$ $=(x+3)(-3x+7)(2x+1)$ or $-(x+3)(3x-7)(2x+1)$	M1A1 M1A1 **(4)**
	Or (Way 2) Uses trial or factor theorem to obtain $x=-1/2$ **or** $x=7/3$ Uses trial or factor theorem to obtain both $x=-1/2$ **and** $x=7/3$ Puts three factors together (see notes below) Correct factorisation : $(x+3)(7-3x)(2x+1)$ or $-(x+3)(3x-7)(2x+1)$ oe	M1 A1 M1 A1 **(4)**
	Or (Way 3) No working three factors $(x+3)(-3x+7)(2x+1)$ otherwise need working	M1A1M1A1 (4)

	Notes
(a)	M1 for attempting either $f(3)$ or $f(-3)$ – with **numbers substituted into expression** A1 for calculating $f(-3)$ **correctly to 0**, and they must state $(x+3)$ **is a factor** for A1 (or equivalent ie. QED, □ or a tick). A conclusion may be implied by a preamble, "if f(−3) = 0, (x+3) is a factor". $-6(-3)^3-7(-3)^2+40(-3)+21=0$ so $(x+3)$ is a factor of $f(x)$ is M1A1 providing bracketing is correct.
(b)	1st M1: attempting to divide by $(x+3)$ leading to a 3TQ beginning with the correct term, usually $-6x^2$. This may be done by a variety of methods including long division, comparison of coefficients, inspection etc. Allow for work in part (a) if the result is used in (b). 1st A1: usually for $(-6x^2+11x+7)$… Credit when seen and use isw if miscopied 2nd M1: for a ***valid**** attempt to factorise their quadratic (* see notes on page 6 - General Principles for Core Mathematics Marking section 1) 2nd A1 is cao and needs all three factors together fully factorised. Accept e.g. $-3(x+3)(x-\frac{7}{3})(2x+1)$ but $(x+3)(x-\frac{7}{3})(-6x-3)$ and $(x+3)(3x-7)(-2x-1)$ are A0 as not fully factorised. Ignore subsequent work (such as a solution to a quadratic equation.) Way 2: The second M mark needs three roots together so $\pm 6(x-\alpha)(x-\beta)(x+3)$ or equivalent where they obtained α and β by trial, so if correct roots identified, then $(x+3)(3x-7)(2x+1)$ can gain M1A1M1A0. N.B. Replacing $(-6x^2+11x+7)$ (already awarded M1A1) by $(6x^2-11x-7)$ giving $(x+3)(3x-7)(2x+1)$ can have M1A0 for factorization so M1A1M1A0

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Q2

Question Number	Scheme	Marks
(a)	$f(2)=16+40+2a+b$ **or** $f(-1)=1-5-a+b$	M1 A1
	Finds 2nd remainder and equates to 1st $\Rightarrow$ $16+40+2a+b=1-5-a+b$	M1 A1
	$a=-20$	A1cso (5)
(b)	$f(-3)=(-3)^4+5(-3)^3-3a+b=0$	M1 A1ft
	$81-135+60+b=0$ gives $b=-6$	A1 cso (3) **[8]**
Alternative for (a)	(a) Uses long division, to get remainders as $b+2a+56$ or $b-a-4$ or correct equivalent	M1 A1
	Uses second long division as far as remainder term, to get $b+2a+56=b-a-4$ or correct equivalent	M1 A1
	$a=-20$	A1cso (5)
Alternative for (b)	(b) Uses long division of $x^4+5x^3-20x+b$ by $(x+3)$ to obtain $x^3+2x^2-6x+a+18$ (with their value for a)	M1 A1ft
	Giving remainder $b+6=0$ and so $b=-6$	A1 cso (3) **[8]**
Notes (a)	**M1** : Attempts $f(\pm2)$ or $f(\pm1)$ **A1** is for the answer shown (or simplified with terms collected) for one remainder **M1:** Attempts other remainder and puts one equal to the other **A1:** for correct equation in a (and b) then **A1 for** $a=-20$ **cso**	
(b)	**M1** : Puts $f(\pm3)=0$ **A1** is for f(-3) = 0, (where f is original function), with no sign or substitution errors (follow through on 'a' and could still be in terms of a) **A1:** $b=-6$ is cso.	
Alternatives	(a) **M1**: Uses long division of x^4+5x^3+ax+b by $(x\pm2)$ **or** by $(x\pm1)$ as far as three term quotient **A1**: Obtains at least one correct remainder **M1:** Obtains second remainder and puts two remainders (no x terms) equal **A1:** correct equation **A1**: correct answer $a=-20$ following correct work. (b) **M1**: complete long division as far as constant (ignore remainder) **A1ft:** needs correct answer for their a **A1:** correct answer	
Beware: It is possible to get **correct answers with wrong working**. If remainders are equated to 0 in part (a) both correct answers are obtained fortuitously. This could score M1A1M0A0A0M1A1A0		

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Q3

Question	Scheme	Marks	AOs
(a)	Attempts $f(4)=2\times4^3-13\times4^2+8\times4+48$	M1	1.1b
	$f(4)=0\Rightarrow(x-4)$ is a factor	A1	1.1b
		(2)	
(b)	$2x^3-13x^2+8x+48=(x-4)(2x^2...x-12)$	M1	2.1
	$=(x-4)(2x^2-5x-12)$	A1	1.1b
	Attempts to factorise quadratic factor or solve quadratic eqn	dM1	1.1b
	$f(x)=(x-4)^2(2x+3)\Rightarrow f(x)=0$ has only two roots, 4 and -1.5	A1	2.4
		(4)	
(c)	Deduces either three roots or deduces that $f(x)$ is moved down two units	M1	2.2a
	States three roots, as when $f(x)$ is moved down two units there will be three points of intersection (with the x - axis)	A1	2.4
		(2)	
(d)	For sight of $k=\pm4,\pm\frac{3}{2}$	M1	1.1b
	$k=4,-\frac{3}{2}$	A1ft	1.1b
		(2)	
			(10 marks)

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Notes
(a) **M1:** Attempts to calculate $f(4)$. Do not accept $f(4)=0$ without sight of embedded values or calculations. If values are not embedded look for two correct terms from $f(4)=128-208+32+48$ Alternatively attempts to divide by $(x-4)$. Accept via long division or inspection. See below for awarding these marks. **A1:** Correct reason with conclusion. Accept $f(4)=0$, hence factor as long as M1 has been scored. This should really be stated on one line after having performed a correct calculation. It could appear as a preamble if the candidate states "If $f(4)=0$, then $(x-4)$ is a factor before doing the calculation and then writing hence proven or ✓ oe. If division/inspection is attempted it must be correct and there must be some attempt to explain why they have shown that $(x-4)$ is a factor. Eg Via division they must state that there is no remainder, hence factor **(b)** **M1:** Attempts to find the quadratic factor by inspection (correct first and last terms) or by division (correct first two terms)

So for inspection award for $2x^3-13x^2+8x+48=(x-4)(2x^2...x\pm12)$

$$x-4\overline{)\,2x^3-13x^2+8x+48}\qquad 2x^2-5x$$

For division look for $2x^3-8x^2$ / $-5x^2$

A1: Correct quadratic factor $(2x^2-5x-12)$ For division award for sight of this "in the correct place" You don't have to see it paired with the $(x-4)$ for this mark.

If a student has used division in part (a) they can score the M1 A1 in (b) as soon as they start attempting to factorise their $(2x^2-5x-12)$.

dM1: Correct attempt to solve or factorise their $(2x^2-5x-12)$ including use of formula

Apply the usual rules $(2x^2-5x-12)=(ax+b)(cx+d)$ where $ac=\pm2$ and $bd=\pm12$

Allow the candidate to move from $(x-4)(2x^2-5x-12)$ to $(x-4)^2(2x+3)$ for this mark.

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dM1: Correct attempt to solve or factorise their $\left(2x^2-5x-12\right)$ including use of formula

Apply the usual rules $\left(2x^2-5x-12\right)=(ax+b)(cx+d)$ where $ac=\pm 2$ and $bd=\pm 12$

Allow the candidate to move from $(x-4)\left(2x^2-5x-12\right)$ to $(x-4)^2(2x+3)$ for this mark.

A1: Via factorisation

Factorises twice to $f(x)=(x-4)(2x+3)(x-4)$ or $f(x)=(x-4)^2(2x+3)$ or

$f(x)=2(x-4)^2\left(x+\frac{3}{2}\right)$ followed by a valid explanation why there are only two roots.

The explanation can be as simple as

- hence $x=4$ and $-\frac{3}{2}$ (only). The roots must be correct
- only two distinct roots as 4 is a repeated root

There must be some understanding between roots and factors.

E.g. $f(x)=(x-4)^2(2x+3)$

only two distinct roots is insufficient.

This would require two distinct factors, so there are two distinct roots.

Via solving.

Factorsises to $(x-4)\left(2x^2-5x-12\right)$ and solves $2x^2-5x-12=0 \Rightarrow x=4,-\frac{3}{2}$ followed by an explanation that the roots are $4,4,-\frac{3}{2}$ so only two distinct roots.

Note that this question asks the candidate to use algebra so you cannot accept any attempt to use their calculators to produce the answers.

(c)

M1: For a valid **deduction.**

Accept **either** there are 3 roots **or** state that it is a solution of $f(x)=2$ or $f(x)-2=0$

A1: Fully explains:

Eg. States three roots, as $f(x)$ is moved down by **two** units (giving three points of intersection with the x - axis)

Eg. States three roots, as it is where $f(x)=2$ (You may see $y=2$ drawn on the diagram)

(d)

M1: For sight of ± 4 **and** $\pm\frac{3}{2}$ Follow through on ± their roots.

A1ft: $k=4,-\frac{3}{2}$ Follow through on their roots. Accept $4,-\frac{3}{2}$ but not $x=4,-\frac{3}{2}$

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Q4

Question	Scheme	Marks	AOs
(a)	States $(2a-b)^2 \geq 0$	M1	2.1
	$4a^2+b^2 \geq 4ab$	A1	1.1b
	(As $a>0, b>0$) $\frac{4a^2}{ab}+\frac{b^2}{ab} \geq \frac{4ab}{ab}$	M1	2.2a
	Hence $\frac{4a}{b}+\frac{b}{a} \geq 4$ * CSO	A1*	1.1b
		(4)	
(b)	$a=5, b=-1 \Rightarrow \frac{4a}{b}+\frac{b}{a}=-20-\frac{1}{5}$ which is less than 4	B1	2.4
		(1)	
			(5 marks)

Notes

(a) (condone the use of $>$ for the first three marks)

M1: For the key step in stating that $(2a-b)^2 \geq 0$

A1: Reaches $4a^2+b^2 \geq 4ab$

M1: Divides each term by $ab \Rightarrow \frac{4a^2}{ab}+\frac{b^2}{ab} \geq \frac{4ab}{ab}$

A1*: Fully correct proof with steps in the correct order and gives the reasons why this is true:

- when you square any (real) number it is always greater than or equal to zero
- dividing by ab does not change the inequality as $a>0$ and $b>0$

(b)

B1: Provides a counter example and shows it is not true.
This requires values, a calculation or embedded values(see scheme) and a conclusion. The conclusion must be in words eg the result does not hold or not true
Allow 0 to be used as long as they explain or show that it is undefined so the statement is not true.

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Platinum Questions

Calculators may not be used

The total mark for this section is 9

1 (*a*) Show that $(x + 1)$ is a factor of $2x^3 + 3x^2 - 1$

(1)

(*b*) Solve the equation

$$\sqrt{x^2 + 2x + 5} = x + \sqrt{2x + 3}$$

(8)

(Total for Question 1 is 9 marks)

End of Questions

Platinum Mark Scheme

1

(a)	$-2+3-1=0$ so $(x+1)$ is a factor	B1cso (1)	
(b)	$x^2+2x+5=\underline{x^2}+\underline{2x\sqrt{2x+3}}+\underline{2x+3}$	M1	Attempt to square. 3 terms on RHS
	$1=x\sqrt{2x+3}$	M1	Prepare for final sq
	$0=2x^3+3x^2-1$ (Accept $2x^3+3x^2=1$ o.e.)	A1cso	
	$0=(x+1)(2x^2+x-1)$	M1	Div attempt. At least 2 correct terms of quadratic
	$0=(x+1)(2x-1)(x+1)$		
	$\underline{x}=-1$ or $\frac{1}{2}$	A1	Correct factors and both roots
	Check -1: LHS $=2$ RHS $=0$ **so -1 is not a solution**	B1	Must reject -1
	Check $\frac{1}{2}$: LHS $=\sqrt{\frac{25}{4}}=\frac{5}{2}$ RHS$=\frac{1}{2}+\sqrt{4}=2.5$	M1	Attempts 0.5 in original or line 2
	(Only) solution is 0.5	A1 (8)	Only award if check is in <u>original</u>
	[S- for treating $\sqrt{4}$ as ± 2 etc]		
		[9]	

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Topic 8

Binomial Expansion

Bronze, Silver, Gold and Platinum Worksheets for AS Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculators may not be used

The total mark for this section is 30

Q1

Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(2 - 3x)^5$ giving each term in its simplest form.

(Total for Question 1 is 4 marks)

Q2

(a) Find the first 4 terms, in ascending powers of x, in the binomial expansion of

$$(1 + kx)^{10}$$

where k is a positive constant. Give each term in its simplest form.

(3)

Given that, in this expansion, the coefficients of x and x^3 are equal,

(b) find the exact value of k,

(3)

(c) find the coefficient of x^2

(1)

(Total for Question 2 is 7 marks)

Q3

(a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(2 + kx)^7$$

where k is a non-zero constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^3 in this expansion is 1890

(b) find the value of k.

(3)

(Total for Question 3 is 7 marks)

Q4

Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{x}{4}\right)^{10}$$

giving each term in its simplest form.

(Total for Question 4 is 4 marks)

Q5

Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(3 - 2x)^5$, giving each term in its simplest form.

(Total for Question 5 is 4 marks)

Q6

Given that $\binom{40}{4} = \frac{40!}{4!b!}$,

(a) write down the value of b.

(1)

In the binomial expansion of $(1 + x)^{40}$, the coefficients of x^4 and x^5 are p and q respectively.

(b) Find the value of $\frac{q}{p}$

(3)

(Total for Question 6 is 4 marks)

End of Questions

Pearson

Bronze Mark Scheme

Q1

Question number	Scheme	Marks
	$\left[(2-3x)^5\right] = \ldots + \binom{5}{1}2^4(-3x) + \binom{5}{2}2^3(-3x)^2 + \ldots, \ \ldots\ldots$	M1
	$= 32, \ -240x, \ +720x^2$	B1, A1 , A1
		Total 4
Notes	**M1**: The **method** mark is awarded for an attempt at Binomial to get the second and/or third term – need **correct** binomial coefficient combined **with correct power of *x***. Ignore errors (or omissions) in powers of 2 or 3 or sign or bracket errors. Accept any notation for ${}^5C_1$ and ${}^5C_2$, e.g. $\binom{5}{1}$ and $\binom{5}{2}$ (unsimplified) or 5 and 10 from Pascal's triangle This mark may be given if no working is shown, but either or both of the terms including x is correct. **B1**: must be simplified to 32 (writing just 2^5 is **B0**). **32** must be the only constant term in the final answer- so $32 + 80 - 3x + 80 + 9x^2$ is B0 but may be eligible for M1A0A0 . **A1: is cao and is for** $-240x$. (not +-240x) The x is required for this mark **A1**: is c.a.o and is for $720x^2$ (can follow omission of negative sign in working) A list of correct terms may be given credit i.e. series appearing on different lines Ignore extra terms in x^3 and/or x^4 (isw)	
Special Case	Special Case: *Descending powers* of x would be $(-3x)^5 + 2\times5\times(-3x)^4 + 2^2\times\binom{5}{3}\times(-3x)^3 + \ldots$ i.e. $-243x^5 + 810x^4 - 1080x^3 + \ldots$ This is a misread but award as s.c. M1B1A0A0 if completely "correct" or M1 B0A0A0 for correct binomial coefficient in any form with the correct power of x	
Alternative Method	**Method 1**: $\left[(2-3x)^5\right] = 2^5(1 + \binom{5}{1}(-\frac{3x}{2}) + \binom{5}{2}(\frac{-3x}{2})^2 + \ldots)$ is **M1B0A0A0** { The M1 is for the expression **in the bracket** and as in first method– need **correct** binomial coefficient combined with correct power of x. Ignore bracket errors or errors (or omissions) in powers of 2 or 3 or sign or bracket errors} – answers must be simplified to $= 32, -240x, +720x^2$ for full marks (awarded as before) $\left[(2-3x)^5\right] = 2(1 + \binom{5}{1}(-\frac{3x}{2}) + \binom{5}{2}(\frac{-3x}{2})^2 + \ldots)$ would also be awarded **M1B0A0A0** **Method 2: Multiplying out** : B1 for 32 and M1A1A1 for other terms with M1 awarded if x or x^2 term is correct. Completely correct is 4/4	

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Q2

Question Number	Scheme	Marks
(a)	$(1+kx)^{10}$ $1+{}^{10}C_1(kx)+{}^{10}C_2(kx)^2+{}^{10}C_3(kx)^3\ldots$ $1+\left({}^{10}C_1\times\ldots\times x\right)+\left({}^{10}C_2\times\ldots\times x^2\right)+\left({}^{10}C_3\times\ldots\times x^3\right)\ldots$ $=1+10kx,+45k^2x^2+120k^3x^3\ldots$	M1 B1, A1 **(3)**
(b)	$120k^3=10k$ $k^2=\frac{1}{12}$ *so* $k=\ldots$ $k=\frac{\sqrt{3}}{6}$ o.e	M1 M1 A1 **(3)**
(c)	$\frac{15}{4}$ o.e.	B1 **(1)**
		(7 marks)

Notes

(a)

M1: All three binomial coefficients must be correct and must be with the correct power of x. (Ignore k) Accept ${}^{10}C_1$ or $\binom{10}{1}$ or 10 **as a coefficient, and** ${}^{10}C_2$ or $\binom{10}{2}$ or 45 **as another and** ${}^{10}C_3$ or $\binom{10}{3}$ or 120 **as another……..**

Pascal's triangle may be used to establish coefficients.

B1: The first two terms correct (i.e. $=1+10kx$)

A1: The third and fourth terms are correct – allow with brackets (kx) (i.e. $45k^2x^2+120k^3x^3$ or $45(kx)^2+120(kx)^3$)

(Accept answers without + signs, can be listed with commas or appear on separate lines)

If extra terms are given then isw

(b)

M1: Sets their **Coefficient** of x equal to their **Coefficient** of x^3but must have differing powers of k

M1: Divides then takes a square root to give a value for k (May use difference of two squares to find k which is fine)

A1: $k=\frac{1}{\sqrt{12}}$ or $\frac{\sqrt{12}}{12}$ or $\frac{\sqrt{3}}{6}$ o.e. (needs to have just the one positive answer – if negative square root is also given, this is A0)

If there are x terms present e.g. $120k^3x^3=10kx$ then this is M0M0A0

If both powers of k are the same this is also M0M0A0

(c)

B1: $\frac{45}{12}$ or $\frac{15}{4}$ or 3.75 or equivalent Allow $\frac{15}{4}x^2$ (can follow negative value for k)

Pearson

Q3

Question Number	Scheme	Marks
(a)	$(2+kx)^7$ $2^7 + {}^7C_1 2^6(kx) + {}^7C_2 2^5(kx)^2 + {}^7C_3 2^4(kx)^3 \ldots$ First term of 128 $\left({}^7C_1 \times \ldots \times x\right) + \left({}^7C_2 \times \ldots \times x^2\right) + \left({}^7C_3 \times \ldots \times x^3\right) \ldots$ $= (128 \ldots) + 448kx + 672k^2x^2 + 560k^3x^3 \ldots$	B1 M1 A1, A1 **(4)**
(b)	$560k^3 = 1890$ $k^3 = \frac{1890}{560}$ *so* $k =$ $k = 1.5$ o.e.	M1 dM1 A1 **(3)** **(7marks)**
Alternative method For (a)	$(2+kx)^7 = 2^7(1+\frac{kx}{2})^7$ $2^7(1 + {}^7C_1(\frac{k}{2}x) + {}^7C_2(\frac{k}{2}x)^2 + {}^7C_3(\frac{k}{2}x)^3 \ldots)$ Scheme is applied exactly as before	

Notes

(a)

B1: The constant term should be 128 in their expansion (should not be followed by other constant terms)

M1: Two of the three binomial coefficients must be correct and must be with the correct power of x. Accept ${}^7C_1$ or $\binom{7}{1}$ or 7 **as a coefficient, and** ${}^7C_2$ or $\binom{7}{2}$ or 21 **as another and** ${}^7C_3$ or $\binom{7}{3}$ or 35 **as another……..**

Pascal's triangle may be used to establish coefficients.

A1: Two of the final three terms correct (i.e. two of $448kx + 672k^2x^2 + 560k^3x^3 \ldots$).

A1: All three final terms correct. (Accept answers without + signs, can be listed with commas or appear on separate lines)

e.g. The common error $= (128 \ldots) + 448kx + 672kx^2 + 560kx^3 \ldots$ would earn B1, M1, A0, A0, so 2/4 Then would gain a maximum of 1/3 in part (b)

If extra terms are given then isw

If the **final** answer is given as $= (128 \ldots) + 448kx + 672(kx)^2 + 560(kx)^3 \ldots$ with correct brackets and no errors are seen, this may be given full marks. If they continue and remove the brackets wrongly then they lose the accuracy marks.

Special case using Alternative Method: Uses $2\ (1+\frac{kx}{2})^7$ is likely to result in a maximum mark of B0M1A0A0 then M1M1A0

If the correct expansion is seen award the marks and isw

(b)

M1: Sets their **Coefficient** of x^3 equal to 1890. They should have an equation which does not include a power of x. This mark may be recovered if they continue on to get $k = 1.5$

dM1: This mark depends upon the previous M mark. Divides then attempts a cube root of their answer to give k – the intention must be clear. (You may need to check on a calculator) The correct answer implies this mark.

A1: Any equivalent to 1.5 If they give – 1.5 as a second answer this is A0

Pearson

Q4

Question Number	Scheme	Marks
Way 1	$\left(2-\frac{x}{4}\right)^{10}$ $2^{10}+\underline{\underline{\binom{10}{1}}}2^9\left(-\frac{1}{4}x\right)+\underline{\underline{\binom{10}{2}}}2^8\left(-\frac{1}{4}x\right)^2+\ldots$ For **either** the x term **or** the x^2 term including a correct binomial coefficient with a correct power of x First term of 1024 $=\underline{1024}-1280x+720x^2$ **Either** $-1280x$ **or** $720x^2$ (Allow +-1280x here) Both $-1280x$ and $720x^2$ (Do not allow +-1280x here)	M1 B1 A1 A1 [4]
Way 2	$\left(2-\frac{x}{4}\right)^{10}=2^k\left(1-\underline{\underline{10}}\times\frac{x}{8}+\frac{10\times9}{\underline{\underline{2}}}\left(-\frac{x}{8}\right)^2\right)$ $1024(1\pm.....)$ $=\underline{1024}-1280x+720x^2$	M1 B1A1 A1 [4]

Notes

M1: For either the x term or the x^2 term having correct structure i.e. a correct binomial coefficient in any form with the correct power of x. Condone sign errors and condone missing brackets and allow alternative forms for binomial coefficients e.g. ${}^{10}C_1$ or $\binom{10}{1}$ or even $\left(\frac{10}{1}\right)$ or 10. The powers of 2 or of ¼ may be wrong or missing.

B1: Award this for 1024 when first seen as a distinct constant term (not $1024x^0$) and not 1 + 1024

A1: For one correct term in x with coefficient simplified. Either -1280x or $720x^2$ (allow +-1280x here)

Allow $720x^2$ to come from $\left(\frac{x}{4}\right)^2$ with no negative sign. So use of + sign throughout could give M1 B1 A1 A0

A1: For both correct simplified terms i.e. -1280x and $720x^2$ (**Do not** allow +-1280x here)

Allow terms to be listed for full marks e.g. $\underline{1024}$, $-1280x$, $+720x^2$

N.B. If they follow a correct answer by a factor such as $512-640x+360x^2$ then isw

Terms may be listed. Ignore any extra terms.

Notes for Way 2

M1: Correct structure for at least one of the underlined terms. i.e. a correct binomial coefficient in any form with the correct power of x. Condone sign errors and condone missing brackets and allow alternative forms for binomial coefficients e.g. ${}^{10}C_1$ or $\binom{10}{1}$ or even $\left(\frac{10}{1}\right)$ or 10. k may even be 0 or 2^k may not be seen. Just consider the bracket for this mark.

B1: Needs 1024(1…. To become 1024

A1, A1: as before

Pearson

Q5

Question Number	Scheme	Marks
	$(3-2x)^5 = 243$, $\quad \ldots\ldots + 5\times(3)^4(-2x) = -810x \quad \ldots\ldots$ $+\frac{5\times 4}{2}(3)^3(-2x)^2 = \quad +1080x^2$	B1, B1 M1 A1 (4) **[4]**
Notes	First term must be 243 for **B1**, writing just 3^5 is B0 (Mark their final answers except in second line of special cases below). Term must be simplified to $-810x$ for **B1** The x is required for this mark. The **method mark (M1) is generous** and is awarded for an attempt at Binomial to get the third term. There must be an x^2 (or no x- i.e. not wrong power) and attempt at Binomial Coefficient and at dealing with powers of 3 and 2. The power of 3 should not be one, but the power of 2 may be one (regarded as bracketing slip). So allow $\binom{5}{2}$ or $\binom{5}{3}$ or ${}^5C_2$ or ${}^5C_3$ or even $\left(\frac{5}{2}\right)$ or $\left(\frac{5}{3}\right)$ or use of '10' (maybe from Pascal's triangle) May see ${}^5C_2(3)^3(-2x)^2$ or ${}^5C_2(3)^3(-2x^2)$ or ${}^5C_2(3)^5(-\frac{2}{3}x^2)$ or $10(3)^3(2x)^2$ which would each score the **M1** **A1**is c.a.o and needs $1080x^2$ (if $1080x^2$ is written with no working this is awarded both marks i.e. **M1 A1.)**	
Special cases	$243+810x+1080x^2$ is **B1B0M1A1** (condone no negative signs) Follows correct answer with $27-90x+120x^2$ can isw here (sp case)– full marks for correct answer Misreads *ascending* and gives $-32x^5+240x^4-720x^3$ is marked as **B1B0M1A0** special case and must be completely correct. *(If any slips could get B0B0M1A0)* Ignores 3 and expands $(1\pm 2x)^5$ is **0/4** 243, -810x, $1080x^2$ is full marks but 243, -810, 1080 is **B1,B0,M1,A0** NB Alternative method $3^5(1-\frac{2}{3}x)^5 = 3^5 - 5\times 3^5\times(\frac{2}{3}x)+\binom{5}{3}3^5(-\frac{2}{3}x)^2+..$ is **B0B0M1A0** – answers must be simplified to $243-810x+1080x^2$ for full marks (awarded as before) Special case $3(1-\frac{2}{3}x)^5 = 3-5\times 3\times(\frac{2}{3}x)+\binom{5}{3}3(-\frac{2}{3}x)^2+..$ is **B0, B0, M1, A0** Or $\quad 3(1-2x)^5$ is **B0B0M0A0**	

Pearson

Q6

Question Number	Scheme	Marks
(a)	$\binom{40}{4}=\frac{40!}{4!b!}$; $(1+x)^n$ coefficients of x^4 and x^5 are p and q respectively. $b=36$ Candidates should usually "identify" two terms as their p and q respectively.	B1 **(1)**
(b)	Term 1: $\binom{40}{4}$ or ${}^{40}C_4$ or $\frac{40!}{4!36!}$ or $\frac{40(39)(38)(37)}{4!}$ or 91390 Term 2: $\binom{40}{5}$ or ${}^{40}C_5$ or $\frac{40!}{5!35!}$ or $\frac{40(39)(38)(37)(36)}{5!}$ or 658008 Hence, $\frac{q}{p}=\frac{658008}{91390}\left\{=\frac{36}{5}=7.2\right\}$ Any one of Term 1 or Term 2 correct. (Ignore the label of p and/or q.) Both of them correct. (Ignore the label of p and/or q.) for $\frac{658008}{91390}$ oe	M1 A1 A1 oe **cso** **(3)** **[4]**
	Notes	
(a)	B1: for only $b=36$.	
(b)	The candidate may expand out their binomial series. At this stage no marks should be awarded until they start to identify either one or both of the terms that they want to focus on. Once they identify their terms then if one out of two of them (ignoring which one is p and which one is q) is correct then award M1. If both of the terms are identified correctly (ignoring which one is p and which one is q) then award the first A1. Term 1 $=\binom{40}{4}x^4$ or ${}^{40}C_4(x^4)$ or $\frac{40!}{4!36!}x^4$ or $\frac{40(39)(38)(37)}{4!}x^4$ or $91390x^4$, Term 2 $=\binom{40}{5}x^5$ or ${}^{40}C_5(x^5)$ or $\frac{40!}{5!35!}x^5$ or $\frac{40(39)(38)(37)(36)}{5!}x^5$ or $658008x^5$ are fine for any (or both) of the first two marks in part (b). 2nd A1 for stating $\frac{q}{p}$ as $\frac{658008}{91390}$ or equivalent. Note that $\frac{q}{p}$ must be independent of x. Also note that $\frac{36}{5}$ or 7.2 or any equivalent fraction is fine for the 2nd A1 mark. SC: If candidate states $\frac{p}{q}=\frac{5}{36}$, then award M1A1A0. Note that either $\frac{4!36!}{5!35!}$ or $\frac{5!35!}{4!36!}$ would be awarded M1A1.	

Pearson

Silver Questions

Calculators may not be used

The total mark for this section is 25

Q1

Find the first 3 terms, in ascending powers of x, in the binomial expansion of

$$(2 - 5x)^6$$

Give each term in its simplest form.

(Total for Question 1 is 4 marks)

Q2

(a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1 + ax)^7$, where a is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^2 in this expansion is 525,

(b) find the possible values of a.

(2)

(Total for Question 2 is 6 marks)

Q3

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 + kx)^7$$

where k is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^2 is 6 times the coefficient of x,

(b) find the value of k.

(2)

(Total for Question 3 is 6 marks)

Q4

(a) Use the binomial theorem to find all the terms of the expansion of

$$(2 + 3x)^4$$

Give each term in its simplest form.

(4)

(b) Write down the expansion of

$$(2 - 3x)^4$$

in ascending powers of x, giving each term in its simplest form.

(1)

(Total for Question 4 is 5 marks)

Q5

Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{1}{2}x\right)^8$$

giving each term in its simplest form.

(Total for Question 5 is 4 marks)

End of Questions

Silver Mark Scheme

Q1

Question Number	Scheme		Marks
	$(2-5x)^6$		
	$(2^6 =)\ 64$	Award this when first seen (not $64x^0$)	B1
	$+6\times(2)^5(-5x)+\frac{6\times5}{2}(2)^4(-5x)^2$	Attempt binomial expansion with correct structure for at least one of these terms. E.g. a term of the form: $\binom{6}{p}\times(2)^{6-p}(-5x)^p$ with $p=1$ or $p=2$ consistently. Condone sign errors. Condone missing brackets if later work implies correct structure and allow alternative forms for binomial coefficients e.g. ${}^6C_1$ or $\binom{6}{1}$ or even $\left(\frac{6}{1}\right)$	M1
	$-960x$	**Not** $+-960x$	A1 (first)
	$(+)6000x^2$		A1 (Second)
			(4)
Way 2	$64(1\pm.....)$	64 **and** $(1\pm.....$ – Award when first seen.	B1
	$\left(1-\frac{5x}{2}\right)^6=1\ \underline{-6\times\frac{5x}{2}}+\underline{\frac{6\times5}{2}\left(-\frac{5x}{2}\right)^2}$	Correct structure for at least one of the underlined terms. E.g. a term of the form: $\binom{6}{p}\times(kx)^p$ with $p=1$ or $p=2$ consistently and $k\neq\pm5$ Condone sign errors. Condoned missing brackets if later work implies correct structure but it must be an expansion of $(1-kx)^6$ where $k\neq\pm5$	M1
	$-960x$	**Not** $+-960x$	A1
	$(+)6000x^2$		A1
			(4)

Pearson

Q2

Question Number	Scheme	Marks
	(a) $(1+ax)^7 = 1+7ax....$ or $1+7(ax)....$ (Not unsimplified versions)	B1
	$+\frac{7\times6}{2}(ax)^2+\frac{7\times6\times5}{6}(ax)^3$ Evidence from one of these terms is enough	M1
	$+21a^2x^2$ or $+21(ax)^2$ or $+21(a^2x^2)$	A1
	$+35a^3x^3$ or $+35(ax)^3$ or $+35(a^3x^3)$	A1
		(4)
	(b) $21a^2 = 525$	M1
	$a = \pm5$ (Both values are required)	A1
	(The answer $a = 5$ with no working scores M1 A0)	(2)
		6
	(a) The terms can be 'listed' rather than added.	
	M1: Requires correct structure: a correct binomial coefficient in any form (perhaps from Pascal's triangle) with the correct power of x. Allow missing a's and wrong powers of a, e.g. $\frac{7\times6}{2}ax^2$, $\frac{7\times6\times5}{3\times2}x^3$	
	However, $21+a^2x^2+35+a^3x^3$ or similar is M0.	
	$1+7ax+21+a^2x^2+35+a^3x^3 = 57+.....$ scores the B1 (isw).	
	$\binom{7}{2}$ and $\binom{7}{3}$ or equivalent such as ${}^7C_2$ and ${}^7C_3$ are acceptable,	
	but not $\left(\frac{7}{2}\right)$ or $\left(\frac{7}{3}\right)$ (unless subsequently corrected).	
	1st A1: Correct x^2 term. 2nd A1: Correct x^3 term (The binomial coefficients must be simplified).	
	Special case: If $(ax)^2$ and $(ax)^3$ are seen within the working, but then lost… … A1 A0 can be given if $21ax^2$ and $35ax^3$ are both achieved.	
	a's omitted throughout: Note that only the M mark is available in this case.	
	(b) M: Equating their coefficent of x^2 to 525. An equation in a or a^2 alone is required for this M mark, but allow 'recovery' that shows the required coefficient, e.g. $21a^2x^2 = 525 \Rightarrow 21a^2 = 525$ is acceptable, but $21a^2x^2 = 525 \Rightarrow a^2 = 25$ is not acceptable.	
	After $21ax^2$ in the answer for (a), allow 'recovery' of a^2 in (b) so that full marks are available for (b) (but not retrospectively for (a)).	

Pearson

Q3

Question Number		Scheme	Marks
Q	(a)	$(7\times\ldots\times x)$ or $(21\times\ldots\times x^2)$ The 7 or 21 can be in 'unsimplified' form.	M1
		$(2+kx)^7 = 2^7 + 2^6\times 7\times kx + 2^5\times\binom{7}{2}k^2x^2$	
		$= 128;\quad +448kx,\quad +672k^2x^2$ [or $672(kx)^2$]	B1; A1, A1 (4)
		(If $672kx^2$ follows $672(kx)^2$, isw and allow A1)	
	(b)	$6\times 448k = 672k^2$	M1
		$k = 4$ (Ignore $k = 0$, if seen)	A1 (2) [6]

(a)

The terms can be 'listed' rather than added. Ignore any extra terms.

M1 for either the x term or the x^2 term. Requires correct binomial coefficient in any form with the correct power of x, but the other part of the coefficient (perhaps including powers of 2 and/or k) may be wrong or missing.

Allow binomial coefficients such as $\binom{7}{1}, \left(\frac{7}{1}\right), \binom{7}{2}, {}^7C_1, {}^7C_2$.

However, $448 + kx$ or similar is M0.

B1, A1, A1 for the simplified versions seen above.

Alternative:

Note that a factor 2^7 can be taken out first: $2^7\left(1+\frac{kx}{2}\right)^7$, but the mark scheme still applies.

Ignoring subsequent working (isw):

Isw if necessary after correct working:

e.g. $128 + 448kx + 672k^2x^2$ M1 B1 A1 A1

$= 4 + 14kx + 21k^2x^2$ isw

(Full marks are still available in part (b)).

(b)

M1 for equating their coefficient of x^2 to 6 times that of x... to get an equation in k,

... or equating their coefficient of x to 6 times that of x^2, to get an equation in k.

Allow this M mark even if the equation is trivial, providing their coefficients from part (a) have been used, e.g. $6\times 448k = 672k$, but beware $k = 4$ following from this, which is A0.

An equation in k alone is required for this M mark, so...

e.g. $6\times 448kx = 672k^2x^2 \Rightarrow k = 4$ or similar is M0 A0 (equation in coefficients only is never seen), but ...

e.g. $6\times 448kx = 672k^2x^2 \Rightarrow 6\times 448k = 672k^2 \Rightarrow k = 4$ will get M1 A1 (as coefficients rather than terms have now been considered).

The mistake $2\left(1+\frac{kx}{2}\right)^7$ would give a maximum of 3 marks: M1B0A0A0, M1A1

Pearson

Q4

Question Number	Scheme	Marks
(a)	$(2+3x)^4$ - Mark (a) and (b) together $2^4 + {}^4C_1 2^3(3x) + {}^4C_2 2^2(3x)^2 + {}^4C_3 2^1(3x)^3 + (3x)^4$ First term of 16 $\left({}^4C_1 \times ... \times x\right) + \left({}^4C_2 \times ... \times x^2\right) + \left({}^4C_3 \times ... \times x^3\right) + \left({}^4C_4 \times ... \times x^4\right)$ $=(16+)\ 96x + 216x^2 + 216x^3 + 81x^4$ Must use Binomial – otherwise A0, A0	B1 M1 A1 A1 (4)
(b)	$(2-3x)^4 = 16 - 96x + 216x^2 - 216x^3 + 81x^4$	B1ft (1) 5
Alternative method (a)	$(2+3x)^4 = 2^4(1+\frac{3x}{2})^4$ $2^4(1+{}^4C_1(\frac{3x}{2}) + {}^4C_2(\frac{3x}{2})^2 + {}^4C_3(\frac{3x}{2})^3 + (\frac{3x}{2})^4)$ Scheme is applied exactly as before	
	Notes for Question	
(a)	B1: The constant term should be 16 in their expansion M1: Two binomial coefficients must be correct and must be with the correct power of x. Accept ${}^4C_1$ or $\binom{4}{1}$ or 4 **as a coefficient, and** ${}^4C_2$ or $\binom{4}{2}$ or 6 **as another**…….. Pascal's triangle may be used to establish coefficients. A1: Any two of the final four terms correct (i.e. two of $96x + 216x^2 + 216x^3 + 81x^4$) in expansion following Binomial Method. A1: All four of the final four terms correct in expansion. (Accept answers without + signs, can be listed with commas or appear on separate lines)	
(b)	B1ft: Award for correct answer as printed above or **ft their previous answer** provided it has **five terms** ft and must be **subtracting the** x **and** x^3 **terms** Allow terms in (b) to be in descending order and allow +-96x and +-216x^3 in the series. (Accept answers without + signs, can be listed with commas or appear on separate lines)	
	e.g. The common error $2^4 + {}^4C_1 2^3 3x + {}^4C_2 2^2 3x^2 + {}^4C_3 2^1 3x^3 + 3x^4 = (16) + 96x + 72x^2 + 24x^3 + 3x^4$ would earn B1, M1, A0, A0, and if followed by $=(16) - 96x + 72x^2 - 24x^3 + 3x^4$ gets B1ft so 3/5 **Fully correct answer with no working can score B1 in part (a) and B1 in part (b).** The question stated use the Binomial theorem and if there is no evidence of its use then M mark and hence A marks cannot be earned. **Squaring the bracket and squaring again** may also earn **B1 M0 A0 A0 B1 if correct** **Omitting the final term** but otherwise correct is B1 M1 A1 A0 B0ft so 3/5 If the series is divided through by 2 or a power of 2 at the final stage after an error or omission resulting in all even coefficients then apply scheme to series before this division and ignore subsequent work (isw)	

Pearson

Q5

Question Number	Scheme	Marks
Way 1	$\left(2-\frac{1}{2}x\right)^8 = 2^8+\binom{8}{1}.2^7\left(-\frac{1}{2}x\right)+\binom{8}{2}2^6\left(-\frac{1}{2}x\right)^2+\binom{8}{3}2^5\left(-\frac{1}{2}x\right)^3$	
	First term of 256	B1
	$\left({}^8C_1\times\ldots\times x\right)+\left({}^8C_2\times\ldots\times x^2\right)+\left({}^8C_3\times\ldots\times x^3\right)$	M1
	$=(256)-512x+448x^2-224x^3$	A1, A1
		(4)
		Total 4
Way 2	$\left(2-\frac{1}{2}x\right)^8 = 2^8\left(1-\frac{1}{4}x\right)^8 = 2^8\left(1+\binom{8}{1}.\left(-\frac{1}{4}x\right)+\binom{8}{2}\left(-\frac{1}{4}x\right)^2+\binom{8}{3}\left(-\frac{1}{4}x\right)^3\right)$ Scheme is applied exactly as before except in special case below*	
	Notes for Question	
	B1: The first term should be 256 in their expansion M1: Two binomial coefficients must be correct and must be with the correct power of x. Accept ${}^8C_1$ or $\binom{8}{1}$ or 8 **as a coefficient, and** ${}^8C_2$ or $\binom{8}{2}$ or 28 **as another**…….. Pascal's triangle may be used to establish coefficients. A1: Any two of the final three terms correct (but allow +- instead of -) A1: All three of the final three terms correct and simplified. (Deduct last mark for $+-512x$ *and* $+-224x^3$ in the series). Also deduct last mark for the three terms correct but unsimplified. (Accept answers without + signs, can be listed with commas or appear on separate lines) The common error $\left(2-\frac{1}{2}x\right)^8 = 256+\binom{8}{1}.2^7\left(-\frac{1}{2}x\right)+\binom{8}{2}2^6\left(-\frac{1}{2}x^2\right)+\binom{8}{3}2^5\left(-\frac{1}{2}x^3\right)$ would earn B1, M1, A0, A0 **Ignore extra terms** involving higher powers. Condone terms in reverse order i.e. $=-224x^3+448x^2-512x+(256)$ ***In Way 2 the error** $=2\left(1+\binom{8}{1}.\left(-\frac{1}{4}x\right)+\binom{8}{2}\left(-\frac{1}{4}x\right)^2+\binom{8}{3}\left(-\frac{1}{4}x\right)^3\right)$ giving $=2-4x+\frac{7}{2}x^2-\frac{7}{4}x^3$ is a **special case** B0, M1, A1, A0 i.e. 2/4	

Pearson

Gold Questions

Calculators may not be used

The total mark for this section is 33

Q1

Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(1+\frac{3x}{2}\right)^8$$

giving each term in its simplest form.

(Total for Question 1 is 4 marks)

Q2

(a) Find the first 4 terms of the expansion of $\left(1+\frac{x}{2}\right)^{10}$ in ascending powers of x, giving each term in its simplest form.

(4)

(b) Use your expansion to estimate the value of $(1.005)^{10}$, giving your answer to 5 decimal places.

(3)

(Total for Question 2 is 7 marks)

Q3

(a) Find the first four terms, in ascending powers of x, in the binomial expansion of $(1 + kx)^6$, where k is a non-zero constant.

(3)

Given that, in this expansion, the coefficients of x and x^2 are equal, find

(b) the value of k,

(2)

(c) the coefficient of x^3 .

(1)

(Total for Question 3 is 6 marks)

Q4

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 9x)^4$$

giving each term in its simplest form.

(4)

$$f(x) = (1 + kx)(2 - 9x)^4, \text{ where } k \text{ is a constant}$$

The expansion, in ascending powers of x, of $f(x)$ up to and including the term in x^2 is

$$A - 232x + Bx^2$$

where A and B are constants.

(b) Write down the value of A.

(1)

(c) Find the value of k.

(2)

(d) Hence find the value of B.

(2)

(Total for Question 4 is 9 marks)

Q5

(a) Find the first 4 terms of the binomial expansion, in ascending powers of x, of

$$\left(1+\frac{x}{4}\right)^8$$

giving each term in its simplest form.

(4)

(b) Use your expansion to estimate the value of $(1.025)^8$, giving your answer to 4 decimal places.

(3)

(Total for Question 5 is 7 marks)

End of Questions

Gold Mark Scheme

Q1

Question Number	Scheme		Marks
	$\left(1+\frac{3x}{2}\right)^8$		
	$1+12x$	Both terms correct as printed (allow $12x^1$ but not 1^8)	B1
	$\ldots+\frac{8(7)}{2!}\left(\frac{3x}{2}\right)^2+\frac{8(7)(6)}{3!}\left(\frac{3x}{2}\right)^3+\ldots$ $\ldots+{}^8C_2\left(\frac{3x}{2}\right)^2+{}^8C_3\left(\frac{3x}{2}\right)^3+\ldots$	$\left(\frac{8(7)}{2!}\times\ldots\times x^2\right)$ or $\left(\frac{8(7)(6)}{3!}\times\ldots\times x^3\right)$ or $\left({}^8C_2\times\ldots\times x^2\right)$ or $\left({}^8C_3\times\ldots\times x^3\right)$ M1: For either the x^2 term or the x^3 term. Requires correct binomial coefficient in any form with the correct power of x, but the other part of the coefficient (perhaps including powers of 2 and/or 3 or signs) may be wrong or missing.	M1
	Special Case: Allow this M1 only for an attempt at a descending expansion provided the equivalent conditions are met for any term other than the first e.g. $\ldots+8\left(\frac{3x}{2}\right)^7(1)+\frac{8(7)}{2!}\left(\frac{3x}{2}\right)^6(1)^2+\ldots$ $\ldots+{}^8C_1\left(\frac{3x}{2}\right)^7+{}^8C_2\left(\frac{3x}{2}\right)^6+\ldots$		
	$\ldots+63x^2+189x^3+\ldots$	A1: **Either** $63x^2$ **or** $189x^3$ A1: **Both** $63x^2$ **and** $189x^3$	A1A1
	Terms may be listed but must be positive		
			[4]
			Total 4
	Note it is common not to square the 2 in the denominator of $\left(\frac{3x}{2}\right)$ and this gives $1+12x+126x^2+756x^3$. This could score B1M1A0A0.		
	Note $\ldots+{}^8C_2\left(1^4+\frac{3x}{2}\right)^2+{}^8C_3\left(1^3+\frac{3x}{2}\right)^3+\ldots$ would score M0 unless a correct method was implied by later work		

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Q2

Question Number	Scheme	Marks
(a)	$\left(1+\frac{1}{2}x\right)^{10} = 1 + \underline{\binom{10}{1}\left(\frac{1}{2}x\right)+\binom{10}{2}\left(\frac{1}{2}x\right)^2+\binom{10}{3}\left(\frac{1}{2}x\right)^3}$	M1 A1
	$= 1 + 5x\ ; + \frac{45}{4}(\text{or } 11.25)x^2 + 15x^3$ (coeffs need to be these, i.e, simplified)	A1; A1 (4)
	[Allow A1A0, if totally correct with unsimplified, single fraction coefficients)	
(b)	$(1 + \tfrac{1}{2}\times 0.01)^{10} = 1 + 5(0.01) + (\frac{45}{4} \text{ or } 11.25)(0.01)^2 + 15(0.01)^3$	M1 A1√
	$= 1 + 0.05 + 0.001125 + 0.000015$	
	$= 1.05114$ cao	A1 (3) **[7]**
Notes:	(a) For M1 first A1: Consider underlined expression only. M1 Requires correct structure for at least two of the three terms: (i) Must be attempt at binomial coefficients. [Be generous :allow all notations e.g. $^{10}C_2$, even $\binom{10}{2}$; allow "slips".] (ii) Must have increasing powers of x, (iii) May be listed, need not be added; *this applies for all marks*. First A1: Requires **all three** correct terms but need not be simplified, allow 1^{10} etc, $^{10}C_2$ etc, and condone omission of brackets around powers of $\frac{1}{2}x$ Second A1: Consider as B1: **$1 + 5x$ can score A1 on Epen, even after M0** (b) For M1: Substituting **their** (0.01) into their (a) result [0.1, 0.001, 0.25, 0.025,0.0025 acceptable but **not** 0.005 or 1.005] First A1 (f.t.): Substitution of (**0.01**) into **their 4 termed** expression in (a) **Answer with no working** scores **no marks** (calculator gives this answer)	

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Q3

Question number	Scheme	Marks
	(a) $1+6kx$ [Allow unsimplified versions, e.g. $1^6+6(1^5)kx$, ${}^6C_0+{}^6C_1kx$]	B1
	$+\frac{6\times5}{2}(kx)^2+\frac{6\times5\times4}{3\times2}(kx)^3$ [See below for acceptable versions]	M1 A1 (3)
	N.B. THIS NEED NOT BE SIMPLIFIED FOR THE A1 (isw is applied)	
	(b) $6k=15k^2$ $k=\frac{2}{5}$ (or equiv. fraction, or 0.4) (Ignore $k=0$, if seen)	M1 A1cso (2)
	(c) $c=\frac{6\times5\times4}{3\times2}\left(\frac{2}{5}\right)^3=\frac{32}{25}$ (or equiv. fraction, or 1.28)	A1cso (1)
	(Ignore x^3, so $\frac{32}{25}x^3$ is fine)	
		6

(a) The terms can be 'listed' rather than added.

M1: Requires correct structure: 'binomial coefficients' (perhaps from Pascal's triangle), increasing powers of x. Allow a 'slip' or 'slips' such as:

$+\frac{6\times5}{2}kx^2+\frac{6\times5\times4}{3\times2}kx^3$, $+\frac{6\times5}{2}(kx)^2+\frac{6\times5}{3\times2}(kx)^3$

$+\frac{5\times4}{2}kx^2+\frac{5\times4\times3}{3\times2}kx^3$, $+\frac{6\times5}{2}x^2+\frac{6\times5\times4}{3\times2}x^3$

But: $15+k^2x^2+20+k^3x^3$ or similar is M0.

Both x^2 and x^3 terms must be seen.

$\binom{6}{2}$ and $\binom{6}{3}$ or equivalent such as ${}^6C_2$ and ${}^6C_3$ are acceptable, and

even $\left(\frac{6}{2}\right)$ and $\left(\frac{6}{3}\right)$ are acceptable for the method mark.

A1: Any correct (possibly unsimplified) version of these 2 terms.

$\binom{6}{2}$ and $\binom{6}{3}$ or equivalent such as ${}^6C_2$ and ${}^6C_3$ are acceptable.

Descending powers of x:
Can score the M mark if the required first 4 terms are not seen.

Multiplying out $(1+kx)(1+kx)(1+kx)(1+kx)(1+kx)(1+kx)$:
M1: A full attempt to multiply out (power 6)
B1 and A1 as on the main scheme.

(b) M: Equating the coefficients of x and x^2 (even if trivial, e.g. $6k=15k$).
Allow this mark also for the 'misread': equating the coefficients of x^2 and x^3.
An equation in k alone is required for this M mark, although…

…condone $6kx=15k^2x^2 \Rightarrow \left(6k=15k^2 \Rightarrow\right) k=\frac{2}{5}$.

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Q4.

Question Number	Scheme		Marks
	(a) $(2-9x)^4 = 2^4 + {}^4C_1 2^3(-9x) + {}^4C_2 2^2(-9x)^2$, (b) $f(x) = (1+kx)(2-9x)^4 = A - 232x + Bx^2$		
(a) **Way 1**	First term of 16 in their final series		B1
	At least one of $\left({}^4C_1 \times ... \times x\right)$ or $\left({}^4C_2 \times ... \times x^2\right)$		M1
	$= (16) - 288x + 1944x^2$	At least one of $-288x$ or $+1944x^2$	A1
		Both $-288x$ **and** $+1944x^2$	A1
			[4]
(a) **Way 2**	$(2-9x)^4 = (4 - 36x + 81x^2)(4 - 36x + 81x^2)$		
		First term of 16 in their final series	B1
	$= 16 - 144x + 324x^2 - 144x + 1296x^2 + 324x^2$	Attempts to multiply a 3 term quadratic by the same 3 term quadratic to achieve either 2 terms in x or at least 2 terms in x^2.	M1
	$= (16) - 288x + 1944x^2$	At least one of $-288x$ or $+1944x^2$	A1
		Both $-288x$ **and** $+1944x^2$	A1
			[4]
(a) **Way 3**	$\{(2-9x)^4 =\}\ 2^4\left(1 - \frac{9}{2}x\right)^4$	First term of 16 in final series	B1
	$= 2^4\left(1 + 4\left(-\frac{9}{2}x\right) + \frac{4(3)}{2}\left(-\frac{9}{2}x\right)^2 + ...\right)$	At least one of $(4 \times ... \times x)$ or $\left(\frac{4(3)}{2} \times ... \times x^2\right)$	M1
	$= (16) - 288x + 1944x^2$	At least one of $-288x$ or $+1944x^2$	A1
		Both $-288x$ **and** $+1944x^2$	A1
			[4]
	Parts (b), (c) and (d) may be marked together		
(b)	$A =$ "16"	Follow through their value from (a)	B1ft
			[1]
(c)	$\left\{(1+kx)(2-9x)^4\right\} = (1+kx)\left(16 - 288x + \left\{1944x^2 + ...\right\}\right)$	*May be seen in part (b) or (d) and can be implied by work in parts (c) or (d).*	M1
	x terms: $-288x + 16kx = -232x$		
	giving, $16k = 56 \Rightarrow k = \frac{7}{2}$	$k = \frac{7}{2}$	A1
			[2]
(d)	x^2 terms: $1944x^2 - 288kx^2$		
	So, $B = 1944 - 288\left(\frac{7}{2}\right); = 1944 - 1008 = 936$	See notes	M1
		936	A1
			[2]
			9

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	Question	Notes
(a) Ways 1 and 3	**B1 cao**	16
	M1	Correct binomial coefficient associated with correct power of x *i.e* $\left({}^4C_1\times\ldots\times x\right)$ or $\left({}^4C_2\times\ldots\times x^2\right)$ They may have 4 and 6 or 4 and $\frac{4(3)}{2}$ or even $\binom{4}{1}$ and $\binom{4}{2}$ as their coefficients. Allow missing signs and brackets for the M marks.
	1st A1	At least one of $-288x$ or $+1944x^2$ (allow +- $288x$)
	2nd A1	Both $-288x$ **and** $+1944x^2$ (May list terms separated by commas) Also full marks for correct answer with no working here. Again allow +- $288x$
	Note	If the candidate then divides their final correct answer through by 8 or any other common factor then isw and mark correct series when first seen. So (a) B1M1A1A1 .It is likely that this approach will be followed by (b) B0, (c) M1A0, (d) M1A0 if they continue with their new series e.g. $2-36x+283x^2+\ldots$ (Do not ft the value 2 as a mark was awarded for 16)
Way 2b	**Special Case**	**Slight Variation on the solution given in the scheme** $(2-9x)^4=(2-9x)(2-9x)(4-36x+81x^2)$ $=(2-9x)(8-108x+486x^2+\ldots)$ $=16-216x+972x^2-72x+972x^2$ $=(16)-288x+1944x^2+\ldots$ First term of 16 — **B1** Multiplies out to give either 2 terms in x or 2 terms in x^2. — **M1** At least one of $-288x$ or $+1944x^2$ — **A1** Both $-288x$ **and** $+1944x^2$ — **A1**
(b)	**B1ft**	**Parts (b), (c) and (d) may be marked together.** Must **identify** $A=16$ or $A=$ *their* constant term found in part (a). Or may write just 16 if this is clearly their answer to part (b). If they expand their series and have 16 as first term of a series it is not sufficient for this mark.
(c)	**M1**	Candidate shows intention to multiply $(1+kx)$ by part of their series from (a) e.g. Just $(1+kx)(16-288x+\ldots)$ or $(1+kx)(16-288x+1944x^2+\ldots)$ are fine for M1.
	Note	This mark can also be implied by candidate multiplying out to find **two terms** (or coefficients) in x. i.e. f.t. their $-288x+16kx$ N.B. $-288kx=-232x$ with no evidence of brackets is M0 – allow copying slips, or use of factored series, as this is a method mark
	A1	$k=\frac{7}{2}$ o.e. so 3.5 is acceptable
(d)	**M1**	Multiplies out their $(1+kx)(16-288x+1944x^2+\ldots)$ to give **exactly** two terms (or coefficients) in x^2 and attempts to find B using **these two** terms and a numerical value of k.
	A1	936
	Note	Award A0 for $B=936x^2$ But allow A1 for $B=936x^2$ **followed by** $B=936$ and treat this as a correction Correct answers in parts (c) and (d) with no method shown may be awarded full credit.

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Q5

Question number	Scheme	Marks
(a).	$(1+\frac{x}{4})^8 = 1+2x+..,$	B1
	$+\frac{8\times7}{2}(\frac{x}{4})^2+\frac{8\times7\times6}{2\times3}(\frac{x}{4})^3,$	M1 A1
	$= +\frac{7}{4}x^2+\frac{7}{8}x^3$ or $= +1.75x^2+0.875x^3$	A1 (4)
(b)	States or implies that $x = 0.1$	B1
	Substitutes their value of x (provided it is <1) into series obtained in (a)	M1
	i.e. 1 + 0.2 + 0.0175 + 0.000875, = 1.2184	A1 cao (3) 7
Alternative for (b) Special case	Starts again and expands $(1+0.025)^8$ to $1+ 8\times0.025 +\frac{8\times7}{2}(0.025)^2+\frac{8\times7\times6}{2\times3}(0.025)^3, = 1.2184$ (Or 1 + 1/5 + 7/400 + 7/8000 = 1.2184)	B1,M1,A1
Notes	**(a) B1** must be simplified The **method** mark **(M1)** is awarded for an attempt at Binomial to get the third and/or fourth term – need correct binomial coefficient combined with correct power of x. Ignore bracket errors or errors in powers of 4. Accept any notation for ${}^8C_2$ and ${}^8C_3$, e.g. $\binom{8}{2}$ and $\binom{8}{3}$ (unsimplified) or 28 and 56 from Pascal's triangle. (The terms may be listed without + signs) First **A1** is for two completely correct unsimplified terms **A1** needs the fully simplified $\frac{7}{4}x^2$ *and* $\frac{7}{8}x^3$. **(b) B1** – states or uses x =0.1 or $\frac{x}{4}=\frac{1}{40}$ **M1** for substituting their value of x (0 < x <1) into expansion (e.g. 0.1 (correct) or 0.01, 0.00625 or even 0.025 but **not** 1 nor 1.025 which would earn **M0**) **A1** Should be answer printed cao (not answers which round to) and should follow correct work. **Answer with no working at all** is B0, M0, A0 States 0.1 then just writes down answer is B1 M0A0	

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Platinum Questions

Calculators may not be used

The total mark for this section is 15

1 In the binomial expansion of

$$\left(1+\frac{12n}{5}x\right)^n$$

the coefficients of x^2 and x^3 are equal and non-zero.

(*a*) Find the possible values of n.

(4)

(*b*) State, giving a reason, which value of n gives a valid expansion when $x = \frac{1}{2}$

(2)

(Total for Question 1 is 6 marks)

---

2 In the binomial expansion of

$$(1-4x)^p, \quad |x| < \tfrac{1}{4},$$

the coefficient of x^2 is equal to the coefficient of x^4 and the coefficient of x^3 is positive.

Find the value of p.

(Total for Question 2 is 9 marks)

End of Questions

Platinum Mark Scheme

Question	Scheme	Marks
1. (a)	$\frac{n(n-1)}{2!}\left(\frac{12n}{5}\right)^2 = \frac{n(n-1)(n-2)}{3!}\left(\frac{12n}{5}\right)^3$	M1
	$3\times5 = n(n-2)\times12$ or $4n^2-8n-5=0$ (o.e.)	A1
	$(2n+1)(2n-5)=0$	dM1
	$\underline{n=-\frac{1}{2},\frac{5}{2}}$	A1 (4)
(b)	$n=-\frac{1}{2}$ in $\left\|\frac{12nx}{5}\right\|<1$ gives $\|x\|<\frac{5}{6}$ and $n=\frac{5}{2}$ in $\left\|\frac{12nx}{5}\right\|$ gives $\|x\|<\frac{1}{6}$	M1
	So should choose $n=-\frac{1}{2}$	A1 (2)
		[6]

2.	$(1-4x)^p =$	
	$\frac{p(p-1)}{2!}(-4x)^2 + \frac{p(p-1)(p-2)}{3!}(-4x)^3 + \frac{p(p-1)(p-2)(p-3)}{4!}(-4x)^4\ldots$	M1 at least x^2 term
	Equation: $\frac{p(p-1)}{2!}\times4^2 = \frac{p(p-1)(p-2)(p-3)}{4!}\times4^4$ attempt equation	M1 (ignore x's)
	$1=\frac{(p-2)(p-3)\times16}{12}$ cancel or factor $p(p-1)$	M1
	i.e. $0=4p^2-20p+21$	A1
	i.e. $0=(2p-3)(2p-7)$	M1
	solving	
	i.e. $p=\frac{3}{2}$ or $\frac{7}{2}$	A1
	both	
	coefficient of $x^3>0 \Rightarrow p(p-1)(p-2)<0$ x^3 coefficient examined	M1
	so $p\neq0$ and $p\neq1$	A1
	$p\neq\frac{7}{2}$ $\therefore p=\frac{3}{2}$	A1
		(9 marks)

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Topic 9
Trigonometric Ratios

Bronze, Silver, Gold
Worksheets for
AS Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculators may not be used

The total mark for this section is 25

Q1

Find the exact value of tan 30° × sin 60°
Give your answer in its simplest form.

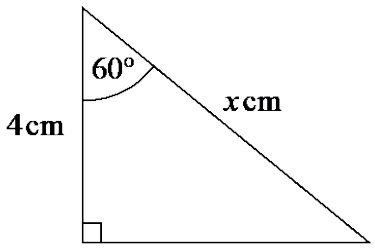
(Total for Question 1 is 2 marks)

Q2

(a) Write down the exact value of tan 45°

(1)

Here is a right-angled triangle.



cos 60° = 0.5

(b) Work out the value of x.

(2)

(Total for Question 2 is 3 marks)

Q3

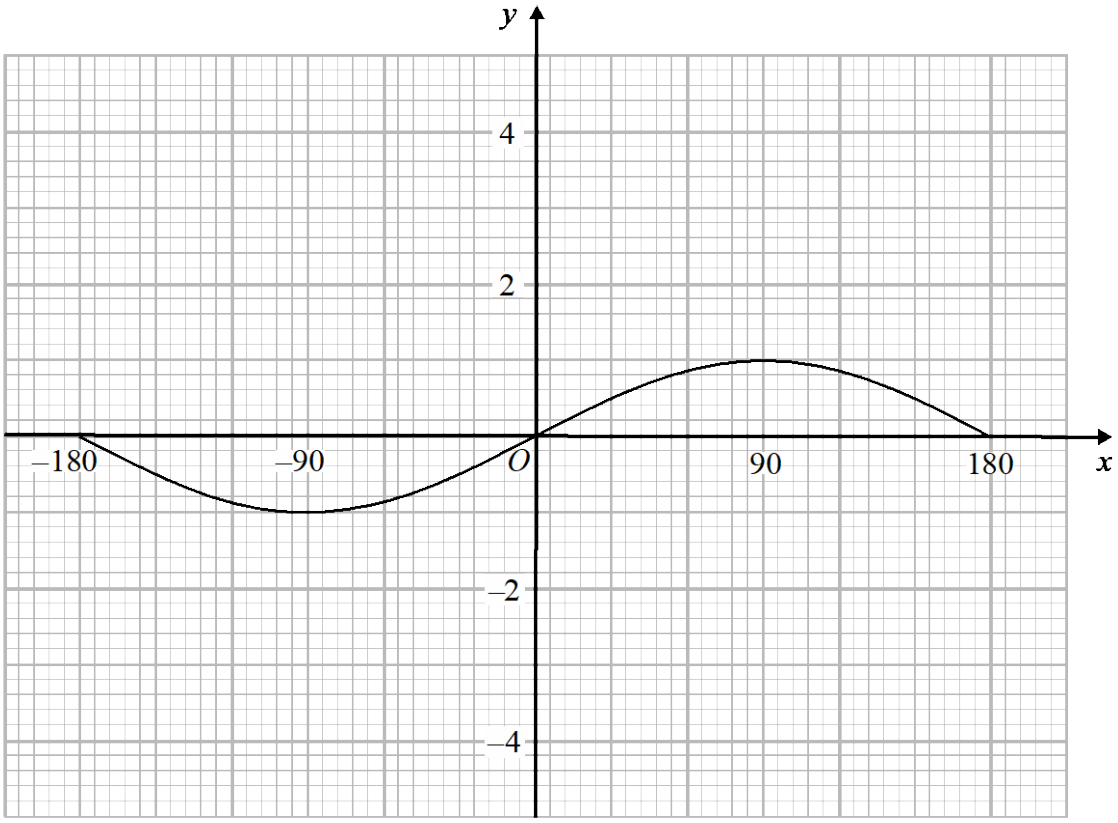
In the triangle ABC, $AB = 1\text{m}$, $AC = \sqrt{3}\text{m}$, angle $ABC = 60°$ and angle $BCA = x°$

Find the two possible values for x.

(Total for Question 3 is 4 marks)

Q4

Here is the graph of $y = \sin x°$ for $-180 \leqslant x \leqslant 180$



On the grid, sketch the graph of $y = \sin x° - 2$ for $-180 \leqslant x \leqslant 180$

(Total for Question 4 is 2 marks)

Q5

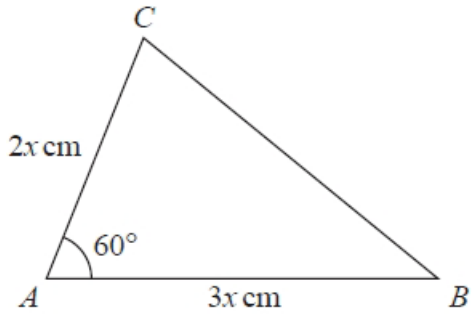


Figure 1

Figure 1 shows a sketch of a triangle ABC with $AB = 3x$ cm, $AC = 2x$ cm and angle $CAB = 60°$

Given that the area of triangle ABC is $18\sqrt{3}$ cm^2

(a) Show that $x = 2\sqrt{3}$

(3)

(b) Hence find the exact length of BC, giving your answer as a simplified surd.

(3)

(Total for Question 5 is 6 marks)

Q6

In triangle RPQ,

$RP = \sqrt{3}$ cm
$PQ = 1$ cm
Angle $PRQ = 30°$

(a) Assuming that angle PQR is an acute angle,
calculate the area of triangle RPQ.
Give your answer in exact form.

(4)

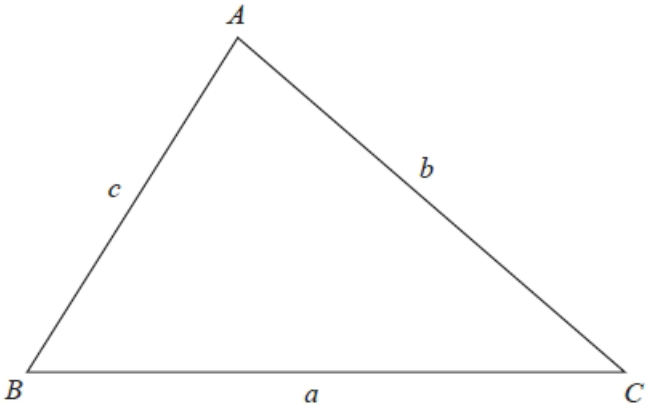
(b) If you did not know that angle PQR is an acute angle, what effect would this have on your calculation of the area of triangle RPQ?

(1)

(Total for Question 6 is 5 marks)

Q7

The diagram shows an acute-angled triangle ABC.



Prove that area of triangle $ABC = \frac{1}{2}ab\sin C$

(Total for Question 7 is 3 marks)

Bronze Mark Scheme

Q1.

	$\frac{1}{2}$	M1 A1	for $\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$ **or** $\frac{\sqrt{3}}{3} \times \frac{\sqrt{3}}{2}$ **or** $(\frac{1}{2} \div \frac{\sqrt{3}}{2}) \times \frac{\sqrt{3}}{2}$ **OR** $\tan 30 = \frac{1}{\sqrt{3}}$ oe **or** $\sin 60 = \frac{\sqrt{3}}{2}$ for $\frac{1}{2}$ or 0.5

Q2.

(a)	1	B1	cao	
(b)	8	M1 A1	starts process, eg $\cos(60) = \frac{4}{x}$ or $0.5 = \frac{4}{x}$ oe or $\sin 30 = \frac{4}{x}$ or $\frac{\sin 30}{4} = \frac{\sin 90}{x}$ oe cao	All three elements of cos, 4, x must be present in an equation. eg $\cos = 4/x$ is acceptable but $\cos(4/x)$ is insufficient

Q3.

Question Number	Scheme	Marks
	$\frac{\sin x}{1} = \frac{\sin 60°}{\sqrt{3}}$ $(\sin x) = \frac{1 \times \sin 60°}{\sqrt{3}}$ $\left(= \frac{1}{2}\right)$ $x =$ awrt 30 **and** 150	M1 A1 dM1 A1 **(4)** **[4]**

Q4.

18	Graph drawn	C2 (C1	for graph translated by −2 in the y direction for a graph translated in the y direction **OR** for a correct graph through four of the five key points)	Key points: (–180, –2), (–90, –3), (0, –2), (90, –1), (180, –2)

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Q5.

Question	Scheme	Marks	AOs
(a)	Uses $18\sqrt{3} = \frac{1}{2} \times 2x \times 3x \times \sin 60°$	M1	1.1a
	Sight of $\sin 60° = \frac{\sqrt{3}}{2}$ and proceeds to $x^2 = k$ oe	M1	1.1b
	$x = \sqrt{12} = 2\sqrt{3}$ *	A1*	2.1
		(3)	
(b)	Uses $BC^2 = \left(6\sqrt{3}\right)^2 + \left(4\sqrt{3}\right)^2 - 2 \times 6\sqrt{3} \times 4\sqrt{3} \times \cos 60°$	M1	1.1b
	$BC^2 = 84$	A1	1.1b
	$BC = 2\sqrt{21}$ (cm)	A1	1.1b
		(3)	
			(6 marks)

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Notes
(a) **M1:** Attempts to use the formula $A=\frac{1}{2}ab\sin C$. If the candidate writes $18\sqrt{3}=\frac{1}{2}\times 5x\times \sin 60°$ **without** sight of a previous correct line then this would be M0 **M1:** Sight of $\sin 60°=\frac{\sqrt{3}}{2}$ or awrt 0.866 and proceeds to $x^2=k$ oe such as $px^2=q$ This may be awarded from the correct formula or $A=ab\sin C$ **A1*:** Look for $x^2=12\Rightarrow x=2\sqrt{3}$, $x^2=4\times 3\Rightarrow x=2\sqrt{3}$ or $x=\sqrt{12}=2\sqrt{3}$ This is a given answer and all aspects must be correct including one of the above intermediate lines. It cannot be scored by using decimal equivalents to $\sqrt{3}$ Alternative using the given answer of $x=2\sqrt{3}$ **M1:** Attempts to use the formula $A=\frac{1}{2}\times 4\sqrt{3}\times 6\sqrt{3}\sin 60°$ oe **M1:** Sight of $\sin 60°=\frac{\sqrt{3}}{2}$ and proceeds to $A=18\sqrt{3}$ **A1*:** Concludes that $x=2\sqrt{3}$ **(b)** **M1:** Attempts the cosine rule with the sides in the correct position. This can be scored from $BC^2=(3x)^2+(2x)^2-2\times 3x\times 2x\times\cos 60°$ as long as there is some attempt to substitute x in later. Condone slips on the squaring **A1:** $BC^2=84$ Accept $BC^2=7\times 12$, $BC=\sqrt{84}$ or $BC=2\sqrt{21}$ If they replace the surds with decimals they can score the A1 for $BC^2=$ awrt 84.0 **A1:** $BC=2\sqrt{21}$ Condone other variables, say $x=2\sqrt{21}$, but it cannot be scored via decimals.

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Q6.

Paper 1MA1: 2H			
Question	**Working**	**Answer**	**Notes**
(a)		$\frac{\sqrt{3}}{2}$	P1 start to process eg draw a labelled triangle or use of sine rule $\frac{\sin Q}{\sqrt{3}} = \frac{\sin 30}{1.0}$
			P1 process to find of Q eg. $Q = \sin^{-1}\left[\frac{\sin 30}{1.0} \times \sqrt{3}\right]$
			P1 process to find area of triangle PRQ.
			A1 $\frac{\sqrt{3}}{2}$
(b)			C1 angle PRQ is obtuse so need to find area of two triangles.

Q7.

Question	Working	Answer	Mark	Notes
		Shown	M1	for use of sine to find height, e.g. $\sin C = \frac{h}{b}$
			M1	for use of expression for the height of the triangle, e.g. area $= \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2}ab\sin C$
			C1	for complete proof

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Silver Questions

Calculators may not be used

The total mark for this section is 29

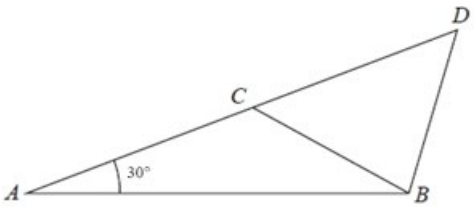
Q1

In the triangle ABC, $AB = 5\sqrt{6}$ cm, $AC = 4$ cm, angle $ABC = 45°$ and angle $BCA = x°$

Find the two possible values for x, giving your answers in exact form.

(Total for Question 1 is 4 marks)

Q2



Not to scale

Figure 1

Figure 1 shows the design for a structure used to support a roof.

The structure consists of four steel beams, AB, BD, BC and AD.

Given $AB = \sqrt{2}$m, $BC = BD = 1$m and angle $BAC = 30°$

Find, the size of angle ACB.

(Total for Question 2 is 3 marks)

Q3

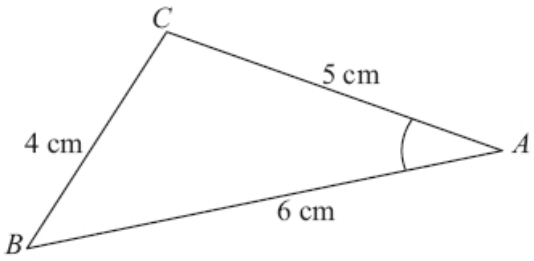


Figure 1

Figure 1 shows the triangle ABC, with $AB = 6$ cm, $BC = 4$ cm and $CA = 5$ cm.

(a) Show that $\cos A = \frac{3}{4}$.

(3)

(b) Hence, or otherwise, find the exact value of $\sin A$.

(2)

(Total for Question 3 is 5 marks)

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Q4

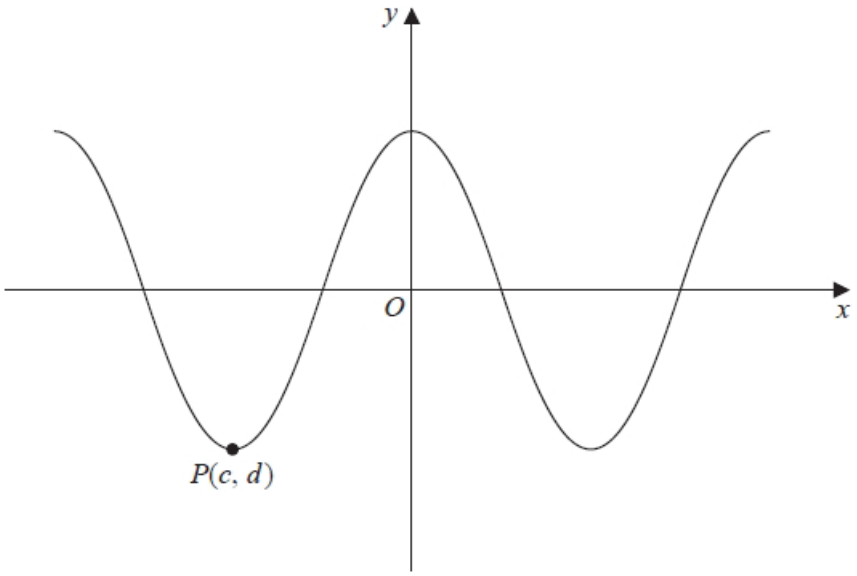

Figure 3

Figure 3 shows part of the curve with equation $y = 3 \cos x°$.

The point $P(c, d)$ is a minimum point on the curve with c being the smallest negative value of x at which
a minimum occurs.

(a) State the value of c and the value of d.

(1)

(b) State the coordinates of the point to which P is mapped by the transformation which transforms the curve with equation $y = 3 \cos x°$ to the curve with equation

(i) $y = 3\cos\left(\frac{x°}{4}\right)$

(ii) $y = 3 \cos (x - 36)°$

(2)

(Total for Question 4 is 3 marks)

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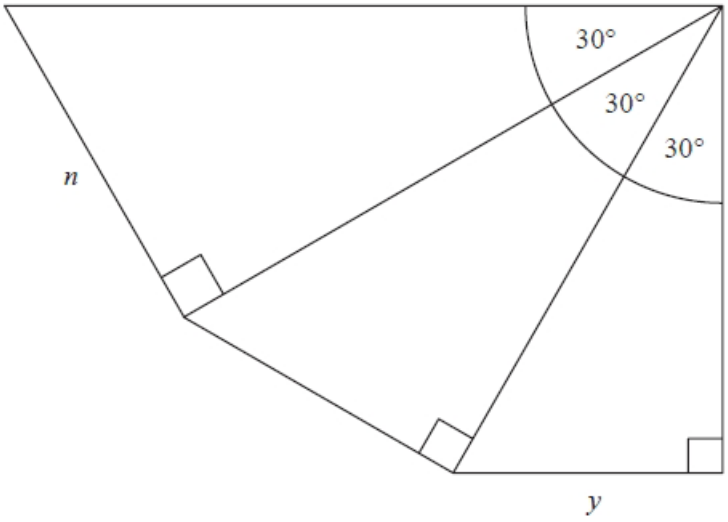
Q5



The diagram shows three right-angled triangles.

Prove that $y = \frac{3}{4}n$

(Total for Question 5 is 4 marks)

Q6

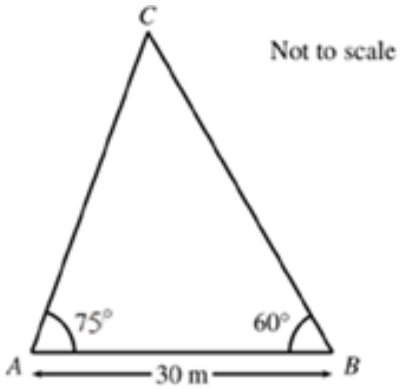


Figure 1

A triangular lawn is modelled by the triangle *ABC*, shown in Figure 1. The length *AB* is to be 30 m long.

Given that angle *BAC* = 75° and angle *ABC* = 60°,

(a) Calculate the length AC **(2)**

Given that BC=$15+15\sqrt{3}$

(b) Calculate the area of the lawn in exact form.

(2)

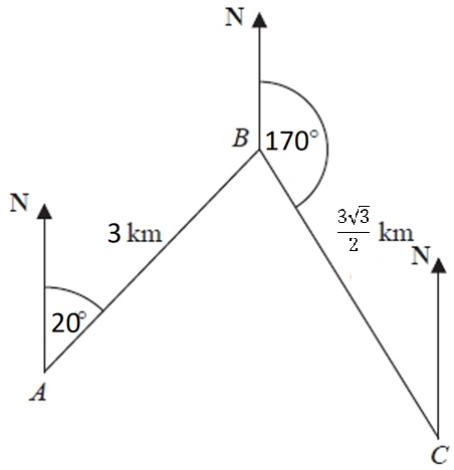
(c) Why is your answer unlikely to be accurate to the nearest square metre?

(1)

(Total for Question 6 is 5 marks)

Q7

The diagram shows the positions of three towns, Acton (A), Barston (B) and Chorlton (C).



Barston is 3 km from Acton on a bearing of 020°

Chorlton is $\frac{3\sqrt{3}}{2}$ km from Barston on a bearing of 170°

Find the bearing of Chorlton from Acton.
You must show all your working.

(Total for Question 7 is 5 marks)

Silver Mark Scheme

Q1.

Question Number	Scheme	Marks
	$\frac{sinx}{5\sqrt{6}} = \frac{sin45}{4}$ $(sinx) = \frac{5\sqrt{6} \times sin45}{4}$ $x = 60$ and 120	M1 A1 dM1 A1 (4) [4]

Q2.

Question	Scheme	Marks	AOs
	States $\frac{\sin\theta}{\sqrt{2}} = \frac{\sin 30°}{1}$	M1	1.1b
	Finds $\theta =$ awrt $45°$ or awrt $135°$	A1	1.1b
	$=$ awrt $135°$	A1	1.1b
		(3)	

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Q3.

Question number	Scheme	Marks
	(a) $4^2 = 5^2 + 6^2 - (2\times5\times6\cos\theta)$	M1
	$\cos\theta = \dfrac{5^2+6^2-4^2}{2\times5\times6}$	A1
	$\left(=\dfrac{45}{60}\right)=\dfrac{3}{4}$ (*)	A1cso (3)
	(b) $\sin^2 A + \left(\dfrac{3}{4}\right)^2 = 1$ (or equiv. Pythag. method)	M1
	$\left(\sin^2 A = \dfrac{7}{16}\right)$ $\sin A = \dfrac{1}{4}\sqrt{7}$ or equivalent exact form, e.g. $\sqrt{\dfrac{7}{16}}$, $\sqrt{0.4375}$	A1 (2)
		5
	(a) M: Is also scored for $5^2 = 4^2 + 6^2 - (2\times4\times6\cos\theta)$ or $6^2 = 5^2 + 4^2 - (2\times5\times4\cos\theta)$ or $\cos\theta = \dfrac{4^2+6^2-5^2}{2\times4\times6}$ or $\cos\theta = \dfrac{5^2+4^2-6^2}{2\times5\times4}$. 1st A: Rearranged correctly and numerically correct (possibly unsimplified), in the form $\cos\theta = \ldots$ or $60\cos\theta = 45$ (or equiv. in the form $p\cos\theta = q$). Alternative (verification): $4^2 = 5^2 + 6^2 - \left(2\times5\times6\times\dfrac{3}{4}\right)$ [M1] Evaluate correctly, at least to $16 = 25 + 36 - 45$ [A1] Conclusion (perhaps as simple as a tick). [A1cso] (Just achieving 16 = 16 is insufficient without at least a tick). (b) M: Using a correct method to find an equation in $\sin^2 A$ or $\sin A$ which would give an exact value. Correct answer without working (or with unclear working or decimals): Still scores both marks.	

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Q4.

Question	Scheme	Marks	AOs
(a)	$(-180°, -3)$	B1	1.1b
		(1)	
(b)	(i) $(-720°, -3)$	B1ft	2.2a
	(ii) $(-144°, -3)$	B1 ft	2.2a
		(2)	
			(3 marks)

(a)

B1: Deduces that $P(-180°, -3)$ or $c = -180^{(°)}, d = -3$

(b)(i)

B1ft: Deduces that $P'(-720°, -3)$ Follow through on their $(c, d) \rightarrow (4c, d)$ where d is negative

(b)(ii)

B1ft: Deduces that $P'(-144°, -3)$ Follow through on their $(c, d) \rightarrow (c + 36°, d)$ where d is negative

Q5.

Question	Working	Answer	Mark	Notes
		Proof	B1	for using any correct trig value for 30°, e.g. $\sin 30 = 0.5$, $\cos 30 = \frac{\sqrt{3}}{2}$ or $\tan 30 = \frac{1}{\sqrt{3}}$
			M1	for hypotenuse of small triangle $= 2y$ or hypotenuse of large triangle $= 2n$
			A1	for method to find the hypotenuse of middle triangle, e.g. $\sqrt{(2n)^2 - n^2}$ $(= \sqrt{3}n)$
			A1	for a correct equation linking y and n and correct working leading to the given result

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Q6.

Question	Scheme	Marks	AOs
(a)	Finds third angle of triangle and uses or states $\frac{x}{\sin 60^\circ} = \frac{30}{\sin 45^\circ}$	M1	2.1
	So $x = \frac{30\sin 60^\circ}{\sin 45^\circ}$ $(= 15\sqrt{6})$	A1	1.1b
(b)	Area $= \frac{1}{2} \times 30 \times (15 + 15\sqrt{3}) \times \sin 60$	M1	3.1a
	$= 675 + 225\sqrt{3}$ m^2	A1ft	1.1b
		(4)	
(c)	Plausible reason e.g. Because the angles and the side length are not given to four significant figures Or e.g. The lawn may not be flat	B1	3.2b
		(1)	
			(5 marks)

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Q7.

Question	Answer	Mark	Mark scheme	Additional guidance
	80	P1	for using bearings to determine ABC as 30°	Accept 67 written on the diagram.
		P1	for using the cosine rule to find AC eg $(AC^2=)\ 3^2 + (\frac{3\sqrt{3}}{2})^2 - 2 \times 3 \times \frac{3\sqrt{3}}{2} \cos(30)$ $AC=\frac{3}{2}$	Accept correct substitution into RHS of equation Accept AC in the range 9.41 to 9.42
		P1	(dep P1) for using the sine rule to find angle BAC $\frac{\sin(30)}{\frac{3}{2}} = \frac{\sin(BAC)}{\frac{3\sqrt{3}}{2}}$	
		P1	for rearranging $\frac{\sin(30)}{\frac{3}{2}} \times \frac{3\sqrt{3}}{2} = \sin(BAC)$ for angle $BAC = 60$	Accept any equivalent form with values substituted
		A1	for angle 080°	If the correct answer is given without supportive evidence award 0 marks. Condone missing "0" at the front. If an answer within the range is seen in working and rounded incorrectly award full marks.

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Gold Questions

Calculators may not be used

The total mark for this section is 29

Q1

Figure 1

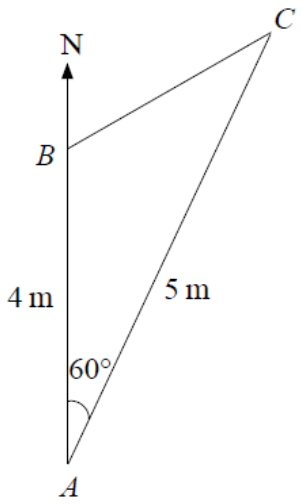


Figure 1 shows 3 yachts A, B and C which are assumed to be in the same horizontal plane. Yacht B is 4 m due north of yacht A and yacht C is 5 m from A. The bearing of C from A is 060°.

Calculate the distance between yacht B and yacht C, in exact form.

(Total for Question 1 is 3 marks)

Q2

In a triangle ABC, side AB has length 10 cm, side AC has length 5 cm, and angle $BAC = \theta$ where θ is measured in degrees. The area of triangle ABC is 15cm^2

(a) Find the two possible values of $\cos\theta$

(4)

Given that BC is the longest side of the triangle,

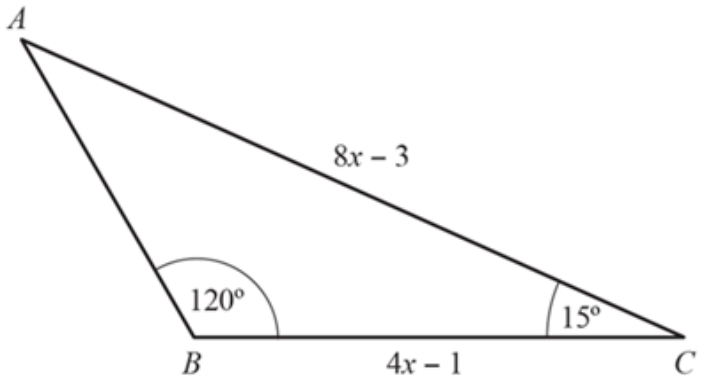
(b) find the exact length of BC.

(2)

(Total for Question 2 is 6 marks)

Q3

The diagram shows $\triangle ABC$ with $AC = 8x - 3$, $BC = 4x - 1$, $\angle ABC = 120°$ and $\angle ACB = 15°$.



(*a*) Show that the exact value of x is $\frac{9+\sqrt{6}}{20}$.

(7)

(*b*) Find the area of $\triangle ABC$, giving your answer in exact form

(3)

(Total for Question 3 is 10 marks)

Q4

A buoy is a device which floats on the surface of the sea and moves up and down as waves pass.

For a certain buoy, its height, above its position in still water, y in metres, is modelled by a sine function of the form $y = \frac{1}{2} \sin 180t°$, where t is the time in seconds.

(*a*) Sketch a graph showing the height of the buoy above its still water level for $0 \leqslant t \leqslant 10$ showing the coordinates of points of intersection with the t-axis.

(3)

(*b*) Write down the number of times the buoy is 0.4 m above its still water position during the first 10 seconds.

(1)

(*c*) Give one reason why this model might not be realistic.

(1)

(Total for Question 4 is 5 marks)

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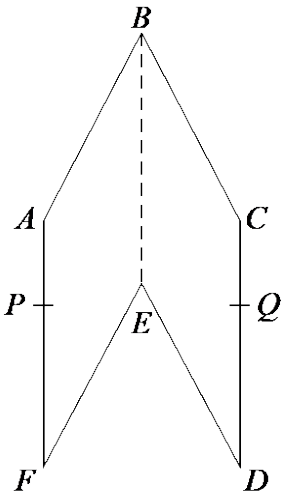
Q5 The diagram shows a hexagon $ABCDEF$.



$ABEF$ and $CBED$ are congruent parallelograms where $AB = BC = x$ cm.
P is the point on AF and Q is the point on CD such that $BP = BQ = 10$ cm.

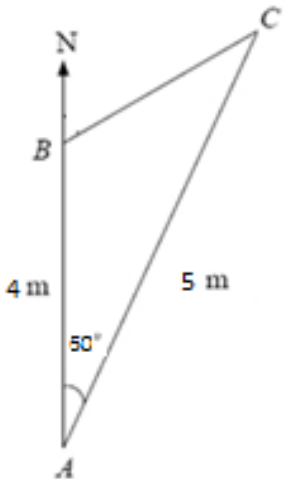
Given that angle $ABC = 30°$,

prove that $\cos PBQ = 1 - \dfrac{(2-\sqrt{3})}{200}x^2$

(Total for Question 5 is 5 marks)

Gold Mark Scheme

Q1.

Question Number	Scheme	Marks
	Figure 1  $BC^2 = 4^2 + 5^2 - 2 \times 4 \times 5 \cos 60^\circ$ $BC = 4m$	M1 A1 A1 (3) [3]

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Q2.

Question	Scheme	Marks	AOs
(a)	Uses $15=\frac{1}{2}\times5\times10\times\sin\theta$	M1	1.1b
	$\sin\theta=\frac{3}{5}$ oe	A1	1.1b
	Uses $\cos^2\theta=1-\sin^2\theta$	M1	2.1
	$\cos\theta=\pm\frac{4}{5}$	A1	1.1b
		(4)	
(b)	Uses $BC^2=10^2+5^2-2\times10\times5\times"-\frac{4}{5}"$	M1	3.1a
	$BC=\sqrt{205}$	A1	1.1b
		(2)	
			(6 marks)

Notes

(a)

M1: Uses the formula Area $=\frac{1}{2}ab\sin C$ in an attempt to find the value of $\sin\theta$ or θ

A1: $\sin\theta=\frac{3}{5}$ oe This may be implied by $\theta=$ awrt $36.9°$ or awrt 0.644 (radians)

M1: Uses their value of $\sin\theta$ to find two values of $\cos\theta$ This may be scored via the formula $\cos^2\theta=1-\sin^2\theta$ or by a triangle method. Also allow the use of a graphical calculator or candidates may just write down the **two values**. The values must be symmetrical $\pm k$

A1: $\cos\theta=\pm\frac{4}{5}$ or ±0.8 Condone these values appearing from $\pm0.79...$

(b)

M1: Uses a suitable method of finding the longest side. For example chooses the negative value (or the obtuse angle) and proceeds to find BC using the cosine rule. Alternatively works out BC using both values and chooses the larger value. If stated the cosine rule should be correct (with a minus sign). Note if the sign is +ve and the acute angle is chosen the correct value will be seen. It is however M0 A0

A1: $BC=\sqrt{205}$

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Q3.

3a	$\angle A = 45^\circ$ seen or implied in later working.	**B1**
	Makes an attempt to use the sine rule, for example, writing $\frac{\sin 120^\circ}{8x-3} = \frac{\sin 45^\circ}{4x-1}$	**M1**
	States or implies that $\sin 120^\circ = \frac{\sqrt{3}}{2}$ and $\sin 45^\circ = \frac{\sqrt{2}}{2}$ **NOTE:** Award ft marks for correct work following incorrect values for sin 120° and sin 45°	**A1**
	Makes an attempt to solve the equation for x. Possible steps could include: $\frac{\sqrt{3}}{16x-6} = \frac{\sqrt{2}}{8x-2}$ or $\frac{\sqrt{6}}{16x-6} = \frac{1}{4x-1}$ or $\frac{3}{16x-6} = \frac{\sqrt{6}}{8x-2}$ $(8\sqrt{3})x - 2\sqrt{3} = (16\sqrt{2})x - 6\sqrt{2}$ or $(4\sqrt{6})x - \sqrt{6} = 16x - 6$ or $24x - 6 = (16\sqrt{6})x - 6\sqrt{6}$ $6\sqrt{2} - 2\sqrt{3} = x(16\sqrt{2} - 8\sqrt{3})$ or $(4\sqrt{6})x - \sqrt{6} = 16x - 6$ or $12x - 3 = (8\sqrt{6})x - 3\sqrt{6}$	**M1ft**
	$x = \frac{6\sqrt{2} - 2\sqrt{3}}{16\sqrt{2} - 8\sqrt{3}}$ or $x = \frac{6 - \sqrt{6}}{16 - 4\sqrt{6}}$ or $x = \frac{3\sqrt{6} - 3}{8\sqrt{6} - 12}$ o.e.	**A1ft**
	Makes an attempt to rationalise the denominator by multiplying top and bottom by the conjugate. Possible steps could include: $x = \frac{(3\sqrt{2} - \sqrt{3})}{(8\sqrt{2} - 4\sqrt{3})} \times \frac{(8\sqrt{2} + 4\sqrt{3})}{(8\sqrt{2} + 4\sqrt{3})}$ $x = \frac{48 + 12\sqrt{6} - 8\sqrt{6} - 12}{128 - 48}$ $x = \frac{36 + 4\sqrt{6}}{80}$	**M1ft**
	States the fully correct simplifed version for x. $x = \frac{9 + \sqrt{6}}{20}$ *	**A1***
	NOTE: Award ft marks for correct work following incorrect values for sin 120° and sin 45°	**(7 marks)**
3b	States or implies that the formula for the area of a triangle is $\frac{1}{2}ab\sin C$ or $\frac{1}{2}ac\sin B$ or $\frac{1}{2}bc\sin A$	**M1**
	$\frac{1}{2}\left(4\left(\frac{9+\sqrt{6}}{20}\right) - 1\right)\left(8\left(\frac{9+\sqrt{6}}{20}\right) - 3\right)(\sin 15\ )$ or $\frac{1}{2}\ (\sin 15\ )$.	**M1**
	Finds the correct area is $\frac{1}{200}(24 + 11\sqrt{6})(\sqrt{6} - \sqrt{2})$.	**A1**
		(3 marks) **Total** **10 marks**

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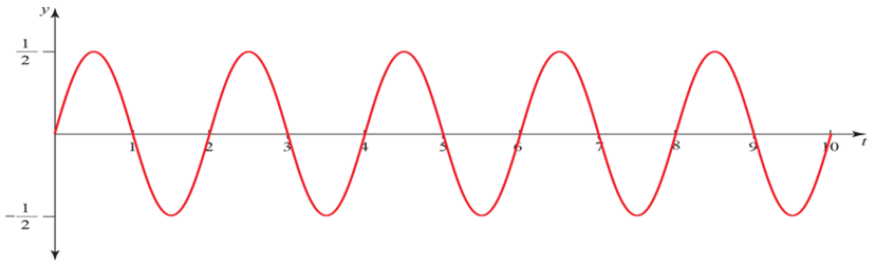
Q4.

4a	 Correct shape of sine curve through (0, 0). Sine curve has max value of $\frac{1}{2}$ and min value of $-\frac{1}{2}$ Sine curve has a period of 2 (can be implied by 5 complete cycles) and passes through (1,0), (2,0),..., (10,0).	**B1** **B1** **B1**
		(3 marks)
4b	Student states that the buoy will be 0.4 m above the still water level 10 times.	**B1**
		(1 mark)
4c	Sensible and correct reason. For example: A buoy would not move up and down at exactly the same rate during each oscillation. The period of oscillation is likely to change each oscillation. The maximum (or minimum) height is likely to change with time. Waves in the sea are not uniform.	**B1**
	Award the mark for a different explanation that is mathematically correct. For example, stating that the buoy would not move exactly vertically each time.	**(1 mark)**
		total 5 marks

Q5.

	cos PBQ= $\frac{10^2+10^2-x^2(2-\sqrt{3})}{200}$ $= \frac{200-x^2(2-\sqrt{3})}{200}$	Proof	B1 M1 M1 M1 A1	(indep) for stating cos 30 = $\frac{\sqrt{3}}{2}$ for $PQ^2 = 10^2 + 10^2 - 2 \times 10 \times 10 \times \cos PBQ$ **or** $AC^2 = x^2 + x^2 - 2 \times x \times x \times \cos 30$ ($=x^2(2-\sqrt{3})$) oe for $\cos PBQ = \frac{10^2+10^2-PQ^2}{2\times10\times10}$ (implies previous M1) for $\cos PBQ = \frac{10^2+10^2-(x^2+x^2-2\times x\times x\times\cos 30)}{2\times10\times10}$ conclusion of proof with all working seen

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Topic 10
Trigonometric Identities and Equations

Bronze, Silver, Gold and Platinum Worksheets for AS Level Mathematics

ALWAYS LEARNING

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Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculators may not be used

The total mark for this section is 26

Q1

(a) Show that the equation

$$5 \sin x = 1 + 2 \cos^2 x$$

can be written in the form

$$2 \sin^2 x + 5 \sin x - 3 = 0$$

(2)

(b) Solve, for $0 \leqslant x < 360°$,

$$2 \sin^2 x + 5 \sin x - 3 = 0$$

(4)

(Total for Question 1 is 6 marks)

Q2

Show that the equation

$$\cos^2 x = 8\sin^2 x - 6\sin x$$

can be written in the form

$$(3\sin x - 1)^2 = 2$$

(Total for Question 2 is 3 marks)

Q3

(a) Show that

$$\frac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta} = 4 - 5\cos\theta$$

(4)

(b) Hence, or otherwise, solve, for $0 \leq x \leq 360°$, the equation

$$\frac{10\sin^2 x - 7\cos x + 2}{3 + 2\cos x} = 4 + 5\sin x$$

(3)

(Total for Question 3 is 7 marks)

Q4

Solve, for $0 \leq x < 360°$,

(a) $\sin(x - 20^\circ) = \frac{1}{\sqrt{2}}$,

(4)

(b) $\cos 3x = -\frac{1}{2}$.

(6)

(Total for Question 4 is 10 marks)

End of Questions

Bronze Mark Scheme

Q1.

Question Number	Scheme	Marks
(a)	$5\sin x = 1 + 2\left(1 - \sin^2 x\right)$	M1
	$2\sin^2 x + 5\sin x - 3 = 0$ (*)	A1cso (2)
(b)	$(2s-1)(s+3) = 0$ **giving** s =	M1
	$[\sin x = -3$ has no solution$]$ so $\sin x = \frac{1}{2}$	A1
	$\therefore \quad x = 30,\ 150$	B1, B1ft (4) **[6]**
(a)	M1 for a correct method to change $\cos^2 x$ into $\sin^2 x$ (must use $\cos^2 x = 1 - \sin^2 x$) A1 need 3 term quadratic printed in any order with =0 included	
(b)	M1 for attempt to solve given quadratic (usual rules for solving quadratics) (can use any variable here, s, y, x, or $\sin x$) A1 requires no incorrect work seen and is for $\sin x = \frac{1}{2}$ or $x = \sin^{-1}\frac{1}{2}$ $y = \frac{1}{2}$ is A0 (unless followed by $x = 30$) B1 for 30 (α) not dependent on method 2nd B1 for 180 - α provided in required range (otherwise 540 - α) Extra solutions outside required range: Ignore Extra solutions inside required range: Lose final B1 Answers in radians: Lose final B1 S.C. Merely writes down two correct answers is M0A0B1B1 Or $\sin x = \frac{1}{2} \quad \therefore \quad x = 30,\ 150$ is **M1A1B1B1** Just gives one answer : 30 only is M0A0B1B0 or 150 only is M0A0B0B1 NB Common error is to factorise wrongly giving $(2\sin x + 1)(\sin x - 3) = 0$ $[\sin x = 3$ gives no solution$]$ $\sin x = -\frac{1}{2} \quad \Rightarrow \quad x = 210,\ 330$ This earns M1 A0 B0 B1ft Another common error is to factorise correctly $(2\sin x - 1)(\sin x + 3) = 0$ and follow this with $\sin x = \frac{1}{2}$, $\sin x = 3$ then $x = 30°, 150°$ This would be M1 A0 B1 B1	

Pearson

Q2.

Question Number	Scheme		Marks
	Way 1 $1-\sin^2 x = 8\sin^2 x - 6\sin x$ E.g. $9\sin^2 x - 6\sin x = 1$ or $9\sin^2 x - 6\sin x - 1 = 0$ or $9\sin^2 x - 6\sin x + 1 = 2$ So $9\sin^2 x - 6\sin x + 1 = 2$ or $(3\sin x - 1)^2 - 2 = 0$ so $(3\sin x - 1)^2 = 2$ or $2 = (3\sin x - 1)^2$ *	Way 2 $2 = (3\sin x - 1)^2$ gives $9\sin^2 x - 6\sin x + 1 = 2$ so $\sin^2 x + 8\sin^2 x - 6\sin x + 1 = 2$ so $8\sin^2 x - 6\sin x = 1 - \sin^2 x$ $8\sin^2 x - 6\sin x = \cos^2 x$ *	B1 M1 A1cso* (3) 3

	Notes
	Way 1 B1: Uses $\cos^2 x = 1 - \sin^2 x$ M1: Collects $\sin^2 x$ terms to form a three term quadratic or into a suitable completed square format. May be sign slips in the collection of terms. A1*: cso This needs an intermediate step from 3 term quadratic and no errors in answer and printed answer stated but allow $2 = (3\sin x - 1)^2$. If sin is used throughout instead of sinx it is A0. **Way 2** B1: **Needs correct expansion and split** M1: Collects $1 - \sin^2 x$ together A1*: Conclusion and no errors seen

Pearson

Q3.

Question	Scheme	Marks	AOs
(a)	$\frac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta} \equiv \frac{10(1-\cos^2\theta) - 7\cos\theta + 2}{3 + 2\cos\theta}$	M1	1.1b
	$\equiv \frac{12 - 7\cos\theta - 10\cos^2\theta}{3 + 2\cos\theta}$	A1	1.1b
	$\equiv \frac{(3 + 2\cos\theta)(4 - 5\cos\theta)}{3 + 2\cos\theta}$	M1	1.1b
	$\equiv 4 - 5\cos\theta$ *	A1*	2.1
		(4)	
(b)	$4 + 5\sin x = 4 - 5\cos x \Rightarrow \tan x = -1$	M1	2.1
	$x = 135°, 215°$	A1 A1	1.1b 1.1b
		(3)	
			(7 marks)

Q4.

Question Number	Scheme	Marks
(a)	45 (α)	B1
	$180 - \alpha$, Add 20 (for at least one angle)	M1 M1
	65 155	A1 (4)
(b)	120 or 240 (β):	B1
	$360 - \beta$, $360 + \beta$	M1 M1
	Dividing by 3 (for at least one angle)	M1
	40 80 160 200 280 320	A1 A1 (6)
		(10 marks)

Pearson

Silver Questions

Calculators may not be used

The total mark for this section is 34

Q1

(i) Solve, for $-180° \leq \theta < 180°$,

$$(1+\tan\theta)(2\sin\theta-\sqrt{3})=0$$

(4)

(ii) Solve, for $0 \leq x < 360°$,

$$2\sin x = \sqrt{2}\tan x.$$

(6)

(Total for Question 1 is 10 marks)

Q2

(a) Show that the equation

$$\tan 2x = 2\sin 2x$$

can be written in the form

$$(1-2\cos 2x)\sin 2x = 0$$

(2)

(b) Hence solve, for $0 \leq x \leq 180°$,

$$\tan 2x = 2\sin 2x$$

You must show clearly how you obtained your answers.

(5)

(Total for Question 2 is 7 marks)

Q3

(a) Show that the equation

$$8\sin^2\theta - 2\cos^2\theta = 3$$

can be written as

$$10\sin^2\theta = 5.$$

(2)

(b) Hence solve, for $0° \leq \theta < 360°$, the equation

$$8\sin^2\theta - 2\cos^2\theta = 3,$$

(7)

(Total for Question 3 is 10 marks)

Q4

(a) Show that the equation

$$\sin\theta\tan\theta = \cos\theta + 1$$

can be written in the form

$$2\cos^2\theta + \cos\theta - 1 = 0$$

(3)

(b) Hence solve, for $0 \leq \theta < 360°$,

$$\sin\theta\tan\theta = \cos\theta + 1$$

showing each stage of your working.

(5)

(Total for Question 4 is 8 marks)

End of Questions

Silver Mark Scheme

Q1.

Question Number		Scheme	Marks
Q	(i)	$\tan\theta = -1 \Rightarrow \quad \theta = -45, \quad 135$	B1, B1ft
		$\sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 60, 120$	B1, B1ft (4)
	(ii)	$2\sin x = \frac{\sqrt{2}\sin x}{\cos x}$	M1
		$2\sin x\cos x = \sqrt{2}\sin x \Rightarrow \quad \sin x(2\cos x - \sqrt{2}) = 0$	M1
		$x = 0, \ 180$ seen	B1, B1
		$x = 45, 135$	B1, B1ft (6)
			[10]

Q2.

Question number	Scheme		Marks
(a)	States or uses $\tan 2x = \frac{\sin 2x}{\cos 2x}$		M1
	$\frac{\sin 2x}{\cos 2x} = 2\sin 2x \Rightarrow \sin 2x - 2\sin 2x\cos 2x = 0 \Rightarrow \sin 2x(1 - 2\cos 2x) = 0$ *		A1 (2)
(b)	$\sin 2x = 0$ gives $2x = 0, 180, 360$ so $x = 0, 90, 180$	B1 for two correct answers, second B1 for all three correct. Excess in range – lose last B1	B1, B1
	$\cos 2x = \frac{1}{2}$ gives $2x = 60$ or $2x = 300$		M1
	$x = 30, 150$		A1, A1 (5)
			7 marks

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Q3.

Question Number	Scheme	Marks
(a)	$8\sin^2\theta - 2\cos^2\theta = 3$	
here	$8\sin^2\theta - 2(1-\sin^2\theta) = 3$ (M1: Use of $\sin^2\theta + \cos^2\theta = 1$)	M1
	$8\sin^2\theta - 2 + 2\sin^2\theta = 3$	
	$10\sin^2\theta = 5$ cso AG	A1 (2)
(b)	$\sin^2\theta = \frac{1}{2}$, so $\sin\theta = (\pm)\frac{1}{\sqrt{2}}$	M1
	Attempt to solve both $\sin\theta$ = +.. and $\sin\theta$ = - ...(may be implied by later work)	M1
	$\theta = 45°$ (dependent on first M1 only)	A1
	θ (= 180° – 45°); = 135 °	M1; A1 √
	[f.t. dependent on first M and 3rd M]	
	$\sin\theta = -\frac{1}{\sqrt{2}}$	
	$\theta = 225°$ and 315 °	M1A1 (7)
		[9]

Q4.

Question Number	Scheme	Marks
(a)	$\sin\theta\left(\frac{\sin\theta}{\cos\theta}\right) = \cos\theta + 1$	M1
	$\left(\frac{1-\cos^2\theta}{\cos\theta}\right) = \cos\theta + 1$	dM1
	$1-\cos^2\theta = \cos^2\theta + \cos\theta \Rightarrow 0 = 2\cos^2\theta + \cos\theta - 1$	A1 cso (3)
(b)	$(\cos\theta + 1)(2\cos\theta - 1) = 0$	M1
	$\cos\theta = -1$ $\cos\theta = \frac{1}{2}$	
	One solution is 60° or 300°, Two solutions are 60° and 300°	A1, A1
	$\theta = \{60, 180, 300\}$	M1 A1 (5)

Pearson

Gold Questions

Calculators may not be used

The total mark for this section is 32

Q1

(i) Solve, for $0 \leq \theta < 360°$, the equation

$$90\sin(\theta + 60°) = 45$$

You must show each step of your working.

(4)

(ii) Solve, for $-180 \leq x < 180$, the equation

$$\tan x - \sqrt{2}\sin x = 0$$

(5)

(Total for Question 1 is 9 marks)

Q2

(i) Solve, for $0 \leqslant \theta < 180°$, the equation

$$\sin 3\theta - \sqrt{3}\cos 3\theta = 0$$

(3)

(ii) Given that

$$4\sin^2 x + \cos x = 4 - k, \quad 0 \text{ „ } k \text{ „ } 3\,3\,3\,;$$

3

(a) Find cos x in terms of k.

(3)

(Total for Question 2 is 6 marks)

Q3

Solve, for $0 \leq x < 180^\circ$,

$$\cos(3x - 10^\circ) = \frac{1}{\sqrt{2}}$$

You should show each step in your working.

(7)

(Total for Question 3 is 7 marks)

Q4

(i) Find the solutions of the equation $\sin(3x - 15°) = \frac{1}{2}$, for which $0 \leqslant x \leqslant 180°$

(6)

(ii)

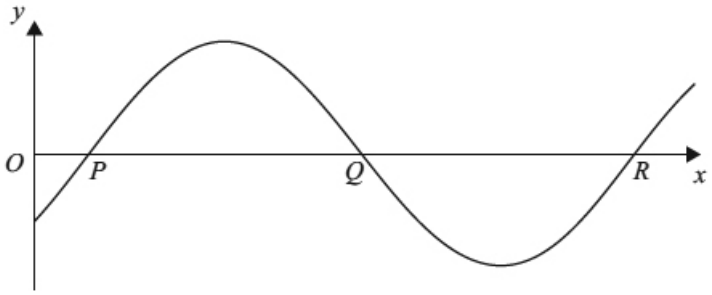

Figure 4

Figure 4 shows part of the curve with equation

$y = \sin(ax - b)$, where $a > 0$, $0 < b < 180$

The curve cuts the x-axis at the points P, Q and R as shown.

Given that the coordinates of P, Q and R are (11, 0), (108, 0) and 198, 0) respectively, find the values of a and b.

(4)

(Total for Question 4 is 10 marks)

End of Questions

Gold Mark Scheme

Pearson

Q1.

Question Number	Scheme		Marks
	(i) $90\sin(\theta+60)=45$; $0\le\theta<360^\circ$ (ii) $\tan x-\sqrt{2}\sin x=0$; $-180\le x<180$		
(i)	$\sin(\theta+60^\circ)=\frac{1}{2}$ so $(\theta+60^\circ)=30^\circ$ $(\alpha=30^\circ)$	Sight of $\sin^{-1}\left(\frac{1}{2}\right)$	M1
	So, $\theta+60^\circ=\{150, 390\}$	$\theta+60^\circ$ = either "180 – their α" or "360° + their α" and not for θ = either "180 – their α" or "360° + their α". This can be implied by later working. The candidate's α could also be in radians but do not allow mixing of degrees and radians.	M1
	and $\theta=\{90, 330\}$	A1: At least one of awrt 90° or awrt 330° A1: Both 90, and 330	A1 A1
	Both answers are cso and must come from correct work		
	Ignore extra solutions outside the range. In an otherwise fully correct solution deduct the final A1for any extra solutions in range		
			[4]
(ii)	$\left(\frac{\sin x}{\cos x}\right)-\sqrt{2}\sin x=0$	Applies $\tan x=\frac{\sin x}{\cos x}$	M1
	Note: Applies $\tan x=\frac{\sin x}{\cos x}$ can be implied by $\tan x-\sqrt{2}\sin x=0\Rightarrow\tan x(1-\sqrt{2}\cos x)$		
	$\sin x-\sqrt{2}\sin x\cos x=0$		
	$\sin x(2-\sqrt{2}\cos x)=0$		
	$\cos x=\frac{1}{\sqrt{2}}$	$\cos x=\frac{1}{\sqrt{2}}$	A1
	$x=\{45, -45\}$	A1: One of either 45, or −45. A1ft: You can apply ft for $x=\pm\alpha$, where $\alpha=\cos^{-1}k$ and $-1\le k\le 1$	A1A1ft
	In this part of the solution, if there are any extra answers in range in an otherwise correct solution withhold the A1ft.		
	$\{\sin x=0\Rightarrow\}\ x=0$ and -180	**Both** $x=0$ **and** -180 $\sin x=0$ **In this part of the solution, ignore extra solutions in range.**	B1
	Note solutions are: $x=\{-180, -45, 0, 45, \}$ Ignore extra solutions outside the range		
	For all answers in degrees in (ii) M1A1A0A1ftB0 is possible		
	Allow the use of θ in place of x in (ii)		
			[5]
			Total 9

Q2.

Pearson

Question Number	Scheme		Marks
(i)	Way 1: Divides by $\cos 3\theta$ to give $\tan 3\theta = \sqrt{3}$ so $(3\theta) = 60°$	Or Way 2: Squares both sides, uses $\cos^2 3\theta + \sin^2 3\theta = 1$, obtains $\cos 3\theta = \pm\frac{1}{2}$ or $\sin 3\theta = \pm\frac{\sqrt{3}}{2}$ so $(3\theta) = 60°$	M1
	Adds 180 or 360 to previous value of angle		M1
	So $\theta = 20, 80, 140$ (all three, no extra in range)		A1 (3)
(ii)(a)	$4(1-\cos^2 x) + \cos x = 4 - k$	Applies $\sin^2 x = 1 - \cos^2 x$	M1
	Attempts to solve $4\cos^2 x - \cos x - k = 0$, to give $\cos x =$		dM1
	$\cos x = \frac{1 \pm \sqrt{1+16k}}{8}$ or $\cos x = \frac{1}{8} \pm \sqrt{\frac{1}{64} + \frac{k}{4}}$ or other correct equivalent		A1 (3)
			6

Q3.

Question Number	Scheme		Marks
	$\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45 \quad (\alpha)$		B1
	$3x - 10 = \alpha \Rightarrow x = \frac{\alpha + 10}{3}$	Uses their α to find x. Allow $x = \frac{\alpha \pm 10}{3}$ **not** $\frac{\alpha}{3} \pm 10$	M1
	$x = \frac{55}{3}$		A1
	$(3x - 10 =) 360 - \alpha$	$360 - \alpha$	M1
	$x = \frac{325}{3}$		A1
	$(3x - 10 =) 360 + \alpha$	$360 + \alpha$	M1
	$x = \frac{415}{3}$		A1

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Q4.

Question number	Scheme	Marks
(i)	$\sin(3x-15)=\frac{1}{2}$ so $3x-15=30$ (α) and $x=15$	M1 A1
	Need $3x-15=180-\alpha$ or $3x-15=540-\alpha$	M1
	Need $3x-15=180-\alpha$ and $3x-15=360+\alpha$ and $3x-15=540-\alpha$	M1
	$x=55$ or 175	A1
	$x=55, 135, 175$	A1 (6)
(ii)	At least one of $(18a-b)=0$ $(108a-b)=180$ or $(198a-b)=360$	M1
	If two of **above equations** used eliminates a or b to find one or both of these or uses period property of curve to find a	M1
	or uses other valid method to find either a or b	
	Obtains $a=2$	A1
	Obtains $b=36$	A1 (4)

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Platinum Questions

Calculators may not be used

The total mark for this section is 17

1

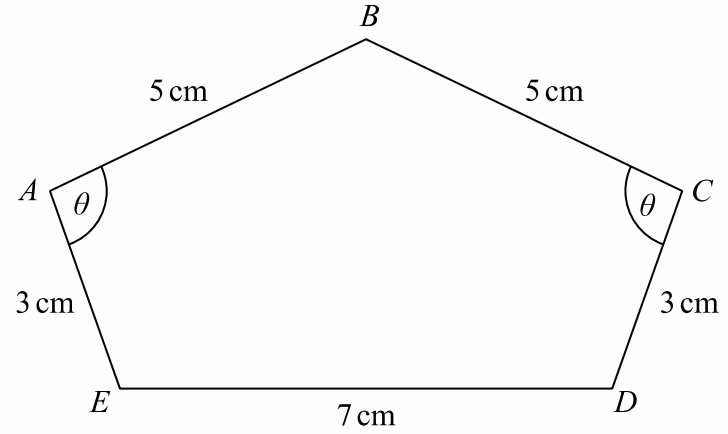


Figure 4

Figure 4 shows a shape $S(\theta)$ made up of five line segments AB, BC, CD, DE and EA.

The lengths of the sides are $AB = BC = 5$ cm, $CD = EA = 3$ cm and $DE = 7$ cm.

Angle BAE = angle $BCD = \theta$ radians.

The length of each line segment always remains the same but the value of θ can be varied so that different symmetrical shapes can be formed, with the added restriction that none of the line segments cross.

(a) Sketch $S(180^0)$, labelling the vertices clearly.

(2)

The shape $S(\phi)$ is a trapezium.

(b) Sketch $S(\phi)$ and calculate the value of ϕ.

(3)

The smallest possible value for θ is α, where $\alpha > 0$, and the largest possible value for θ is β, where $\beta > 180^0$.

(c) Show that $\alpha = \arccos\left(\frac{29}{40}\right)$. [arccos($x$) is an alternative notation for $\cos^{-1}(x)$]

(4)

(*d*) Find an expression for the value of β.

(4)

The area, in cm^2, of shape $S(\theta)$ is $R(\theta)$.

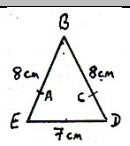
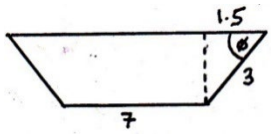
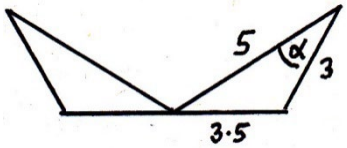
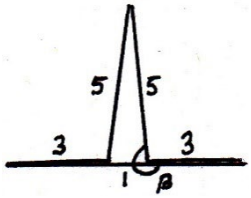
(*e*) Show that for $\alpha \leqslant \theta < 190^0$

$$R(\theta) = 15 \sin \theta + \frac{7}{4}\sqrt{87 - 120\cos\theta}$$

(4)

(Total for Question 1 is 17 marks)

Platinum Mark Scheme

Question	Scheme	Marks	Notes
7. (a)	Triangle EBD $EB = DB$ or labelling to show isos 	M1 A1 (2)	Needn't be isos D Correct labelling
(b)	Isos trapezium ($ACDE$) $\cos\phi = \frac{1.5}{3}$, so $\phi = 60^0$ 	B1 M1, A1 (3)	Sketch – with at least 1 side M1 for correct expression
(c)	$\cos\alpha = \frac{25+9-3.5^2}{2\times5\times3} = \left[\frac{29}{40}\right]$ (o.e.) So $\alpha = \arccos\left(\frac{29}{40}\right)$ **(*)** 	B1 B1 M1 A1cso (4)	Shape (o.e.) 2 or more side lengths Correct use of cos rule **In (c), (d) B1B1 can be implied by M1**
(d)	$\cos\left(\beta - 180^0\right) = \frac{0.5}{5}$ [= 0.1] $\beta = 180^0 + \arccos\left(\frac{1}{10}\right)$ or $270^0 - \frac{1}{2}\arccos\left(\frac{49}{50}\right)$ 	B1 B1 M1 A1 (4)	Shape (o.e.) 2 or more side Correct expression (can ignore $-p$)
(e)	$BE^2 = 5^2 + 3^2 - 2\times5\times3\times\cos\theta = \left[34 - 30\cos\theta\right]$ [h = height from B to ED] so $h^2 = BE^2 - 3.5^2 = \left[\frac{87-120\cos\theta}{4}\right]$ Area = $15\sin\theta + \frac{1}{2}\times7\times\sqrt{\frac{87-120\cos\theta}{4}}$, $= \underline{15\sin\theta + \frac{7}{4}\sqrt{87-120\cos\theta}}$ **(*)**	M1 M1 M1,A1 (4)	Attempt BE or BD Attempt h M1 for correct areas A1 cso

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Topic 11

Vectors

Bronze, Silver, Gold and Platinum Worksheets for AS Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

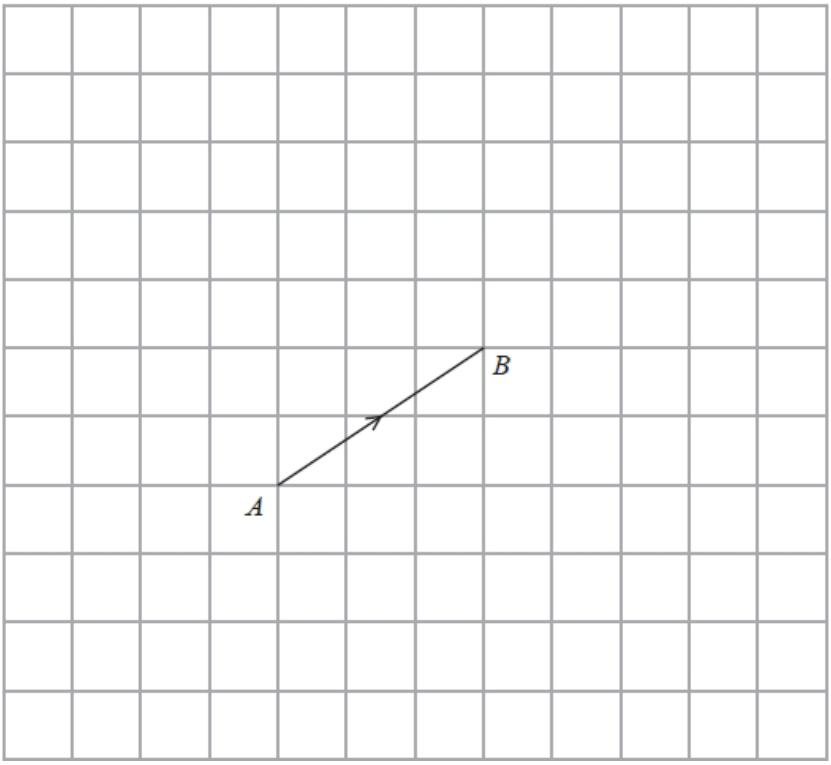
Calculators may not be used

The total mark for this section is 27

1

$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$

$\overrightarrow{AB}$ is shown on the grid



(a) On the grid, draw $\overrightarrow{BC}$.

(1)

$\overrightarrow{AD} = \overrightarrow{AB} - \overrightarrow{BC}$

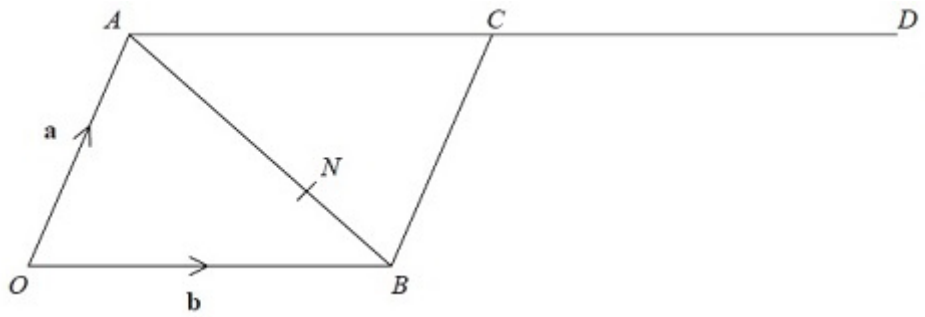
(b) On the grid, mark with a cross (X) the position of D.

Label this point D.

(2)

(Total for Question 1 is 3 marks)

Q2



$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

D is the point such that $\overrightarrow{AC} = \overrightarrow{CD}$

The point N divides AB in the ratio 2:1

(a) Write an expression for $\overrightarrow{ON}$ in terms of **a** and **b**.

(3)

(b) Prove that OND is a straight line.

(3)

(Total for Question 2 is 6 marks)

Q3

Given that the point A has position vector $4\mathbf{i} - 5\mathbf{j}$ and the point B has position vector $-5\mathbf{i} - 2\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$.

(2)

(b) Find $|\overrightarrow{AB}|$.

Give your answer as a simplified surd.

(2)

(Total for Question 2 is 4 marks)

Pearson

Q4

A particle P is moving with constant velocity $(-3\mathbf{i} + 2\mathbf{j})$ m s^{-1}. At time $t = 6$ s P is at the point with position vector $(-4\mathbf{i} - 7\mathbf{j})$ m. Find the distance of P from the origin at time $t = 2$ s.

(Total for Question 4 is 5 marks)

Q5

[In this question, the unit vectors **i** *and* **j** *are due east and due north respectively. Position vectors are relative to a fixed origin O.]*

A boat P is moving with constant velocity $(-4\mathbf{i} + 8\mathbf{j})$ km h^{-1}.

(a) Calculate the speed of P, giving your answer as a simplified surd.

(2)

When $t = 0$, the boat P has position vector $(2\mathbf{i} - 8\mathbf{j})$ km. At time t hours, the position vector of P is $\mathbf{p}$ km.

(b) Write down $\mathbf{p}$ in terms of t.

(1)

A second boat Q is also moving with constant velocity. At time t hours, the position vector of Q is $\mathbf{q}$ km, where

$$\mathbf{q} = 18\mathbf{i} + 12\mathbf{j} - t(6\mathbf{i} + 8\mathbf{j})$$

Find

(c) the value of t when P is due west of Q,

(3)

(d) the distance between P and Q when P is due west of Q.

(3)

(Total for Question 5 is 9 marks)

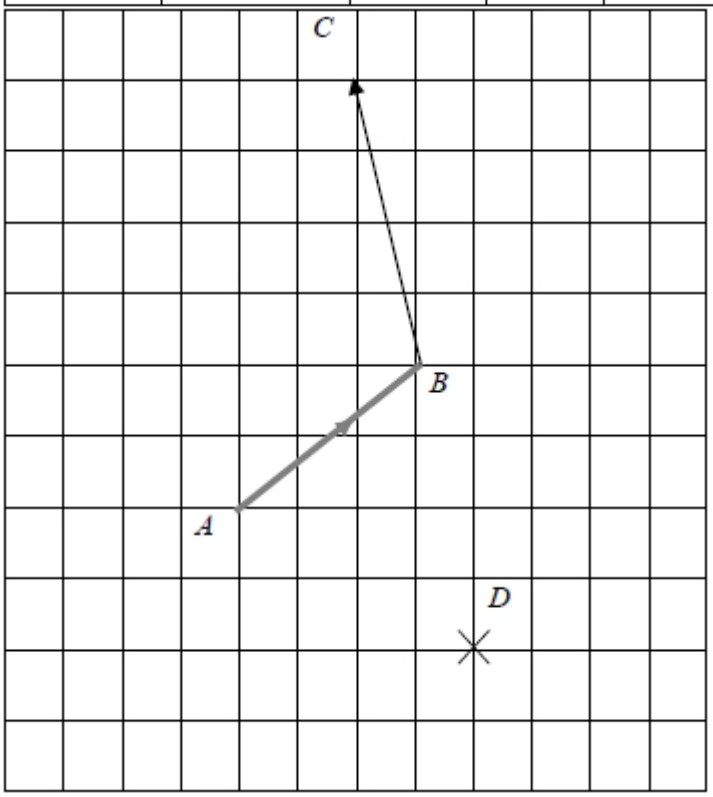
End of Questions

Pearson

Bronze Mark Scheme

Q1

Question	Working	Answer	Mark	Notes
(a)		Vector drawn	B1	for correct vector
(b)		× marked	M1	for method to find the vector $\overrightarrow{AD} = \begin{pmatrix} 3--1 \\ 2-4 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ or for $\begin{pmatrix} 1 \\ -4 \end{pmatrix}$ drawn on the grid
			A1	for ft for correct position (×) D on their diagram



Pearson

Q2

Question		Working	Answer	Mark	Notes
	(a)	$\overrightarrow{AB} = -\mathbf{a} + \mathbf{b}$ $\overrightarrow{ON} = \overrightarrow{OA} + \frac{2}{3}\overrightarrow{AB}$ $\overrightarrow{ON} = \mathbf{a} + \frac{2}{3}(-\mathbf{a} + \mathbf{b})$ $= \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$ OR $\overrightarrow{ON} = \overrightarrow{OB} + \frac{1}{3}\overrightarrow{BA}$ $\overrightarrow{ON} = \mathbf{b} + \frac{1}{3}(-\mathbf{b} + \mathbf{a})$ $= \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$	$\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$	3	M1 for correct vector equation involving $\overrightarrow{ON}$, eg. $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AN}$, may be written, partially or fully, in terms of **a** and **b**, e.g. $(\overrightarrow{ON} =)\ \mathbf{a} + \frac{2}{3}\overrightarrow{AB}$ M1 for showing answer requires $\overrightarrow{AN} = \frac{2}{3}\overrightarrow{AB}$ or $\overrightarrow{BN} = \frac{1}{3}\overrightarrow{BA}$ A1 $\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$ oe
	(b)	$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AC} + \overrightarrow{CD}$ $= \mathbf{a} + \mathbf{b} + \mathbf{b}$ $= \mathbf{a} + 2\mathbf{b}$ $\overrightarrow{OD} = 3(\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b})$ $\overrightarrow{OD} = 3\overrightarrow{ON}$	Proof	3	M1 for a correct vector statement for $\overrightarrow{OD}$ or $\overrightarrow{ND}$ in terms of **a** and **b**, e.g. $\overrightarrow{OD} = \mathbf{a} + \mathbf{b} + \mathbf{b}$ oe or $\overrightarrow{ND} = \frac{2}{3}(-\mathbf{b} + \mathbf{a}) + \mathbf{b} + \mathbf{b}$ oe A1 for correct and fully simplified vectors for $\overrightarrow{ON}$ (may be seen in (a)) **and** for $\overrightarrow{OD}$ $(= \mathbf{a} + 2\mathbf{b})$ or $\overrightarrow{ND}$ $(= \frac{2}{3}\mathbf{a} + \frac{4}{3}\mathbf{b})$ C1 (dep on A1) for statement that $\overrightarrow{OD}$ or $\overrightarrow{ND}$ is a multiple of $\overrightarrow{ON}$ (+ common point)

Pearson

Q3.

Question	Scheme	Marks	AOs
(a)	Attempts $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or similar	M1	1.1b
	$\overrightarrow{AB} = -9\mathbf{i} + 3\mathbf{j}$	A1	1.1b
		(2)	
(b)	Finds length using 'Pythagoras' $\lvert AB \rvert = \sqrt{(-9)^2 + (3)^2}$	M1	1.1b
	$\lvert AB \rvert = 3\sqrt{10}$	A1ft	1.1b
		(2)	
			(4 marks)

Notes

(a)

M1: Attempts subtraction either way around.

This may be implied by one correct component $\overrightarrow{AB} = \pm 9\mathbf{i} \pm 3\mathbf{j}$

There must be some attempt to write in vector form.

A1: cao (allow column vector notation but not the coordinate)

Correct notation should be used. Accept $-9\mathbf{i} + 3\mathbf{j}$ or $\begin{pmatrix} -9 \\ 3 \end{pmatrix}$ but not $\begin{pmatrix} -9\mathbf{i} \\ 3\mathbf{j} \end{pmatrix}$

(b)

M1: Correct use of Pythagoras theorem or modulus formula using their answer to (a)

Note that $\lvert AB \rvert = \sqrt{(9)^2 + (3)^2}$ is also correct.

Condone missing brackets in the expression $\lvert AB \rvert = \sqrt{-9^2 + (3)^2}$

Also allow a restart usually accompanied by a diagram.

A1ft: $\lvert AB \rvert = 3\sqrt{10}$ ft from their answer to (a) as long as it has both an **i** and **j** component.

It must be simplified, if appropriate. Note that $\pm 3\sqrt{10}$ would be M1 A0

Note that, in cases where there is no working, the correct answer implies M1A1 in each part of this question

Pearson

Q4.

Question Number	Scheme	Marks
	$(-4\mathbf{i}-7\mathbf{j})=\mathbf{r}+4(-3\mathbf{i}+2\mathbf{j})$ $\mathbf{r}=(8\mathbf{i}-15\mathbf{j})$ $\lvert\mathbf{r}\rvert=\sqrt{8^2+(-15)^2}=17$ m	M1 A1 A1 M1 A1 ft **[5]**

Q5.

Question Number	Scheme	Marks
(a)	$\sqrt{\left((-4)^2+8^2\right)}=\sqrt{80}\quad\left(\text{km h}^{-1}\right)$ accept exact equivalents or 8.9 or better	M1 A1 (2)
(b)	$\mathbf{p}=(2\mathbf{i}-8\mathbf{j})+t(-4\mathbf{i}+8\mathbf{j})$	B1 (1)
(c)	Equating **j** components $-8+8t=12-8t$ $t=\frac{5}{4}$ oe	M1 A1 A1 (3)
(d)	Using their t from (c) to find the **i**-cpts of **p** and **q** and subtract them $10\frac{1}{2}-(-3)=13\frac{1}{2}\quad(\text{km})$	M1 A1 ft A1 (3) **9**

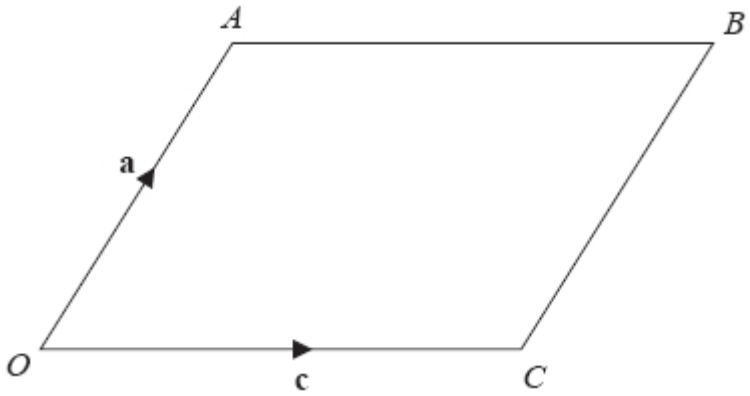
Pearson

Silver Questions

Calculators may not be used

The total mark for this section is 25

Q1



$OABC$ is a parallelogram.

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OC} = \mathbf{c}$

X is the midpoint of the line AC.
OCD is a straight line so that $OC : CD = k : 1$

Given that $\overrightarrow{XD} = 3\mathbf{c} - \frac{1}{2}\mathbf{a}$

find the value of k.

(Total for Question 1 is 4 marks)

Q2

Given that the point A has position vector $3\mathbf{i} - 7\mathbf{j}$ and the point B has position vector $8\mathbf{i} + 3\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$.

(2)

(b) Find $\left|\overrightarrow{AB}\right|$. Give your answer as a simplified surd.

(2)

(Total for Question 2 is 4 marks)

Q3

Three forces, $(15\mathbf{i} + \mathbf{j})$ N, $(5q\mathbf{i} - p\mathbf{j})$ N and $(-3p\mathbf{i} - q\mathbf{j})$ N, where p and q are constants, act on a particle. Given that the particle is in equilibrium, find the value of p and the value of q.

(Total for Question 3 is 6 marks)

Q4

*[In this question, the horizontal unit vectors **i** and **j** are directed due east and due north respectively.]*

The velocity, $\mathbf{v}$ m s^{-1}, of a particle P at time t seconds is given by

$$\mathbf{v} = (1 - 2t)\mathbf{i} + (3t - 3)\mathbf{j}$$

(a) Find the speed of P when $t = 0$

(3)

(b) Find the bearing on which P is moving when $t = 2$

(2)

(c) Find the value of t when P is moving

(i) parallel to $\mathbf{j}$,
(ii) parallel to $(-\mathbf{i} - 3\mathbf{j})$.

(6)

(Total for Question 4 is 11 marks)

End of Questions

Pearson

Silver Mark Scheme

Q1.

Question	Working	Answer	Mark	Notes
		$\frac{2}{5}$	P1	for first step to solve the problem e.g. $\overrightarrow{AC} = -\mathbf{a} + \mathbf{c}$ or $\overrightarrow{OX} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c}$ or demonstrates the location of D and X on the diagram
			P1	for a correct vector statement using $\overrightarrow{CD}$ eg $\overrightarrow{CD} = \overrightarrow{CX} + \overrightarrow{XD}$ or $\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$ or $\overrightarrow{OD} = \frac{7}{2}\mathbf{c}$ or $\overrightarrow{CD} = 2.5\mathbf{c}$ oe
			P1	for a correct equation or ratio using k eg equating $\overrightarrow{XD} = 3\mathbf{c} - \frac{1}{2}\mathbf{a} = \frac{1}{2}(-\mathbf{a} + \mathbf{c}) + \frac{1}{k}\mathbf{c}$ or $\frac{\overrightarrow{OD}}{\overrightarrow{OC}} = \frac{k+1}{k}$ or $k = \frac{1}{2.5}$ or using a ratio approach eg $(\overrightarrow{OC} : \overrightarrow{CD}) = k : 1 = 1 : 2.5$
			A1	cao

Q2

Question	Scheme	Marks	AOs
(a)	Attempts $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or similar	M1	1.1b
	$\overrightarrow{AB} = 5\mathbf{i} + 10\mathbf{j}$	A1	1.1b
		(2)	
(b)	Finds length using 'Pythagoras' $\lvert AB \rvert = \sqrt{(5)^2 + (10)^2}$	M1	1.1b
	$\lvert AB \rvert = 5\sqrt{5}$	A1ft	1.1b
		(2)	
			(4 marks)

Notes

(a) M1: Attempts subtraction but may omit brackets
A1: cao (allow column vector notation)

(b) M1: Correct use of Pythagoras theorem or modulus formula using their answer to (a)
A1ft: $\lvert AB \rvert = 5\sqrt{5}$ ft from their answer to (a)

Note that the correct answer implies M1A1 in each part of this question

Pearson

Q3.

Question Number	Scheme	Marks
	$(15\mathbf{i}+\mathbf{j})+(5q\mathbf{i}-p\mathbf{j})+(-3p\mathbf{i}-q\mathbf{j})=\mathbf{0}$ $3p-5q=15$ $p+q=1$ $p=2.5 \quad q=-1.5$	M1 M1 A1 M1 A1 A1 **6**
	Notes	
	First M1 for equating the sum of the three forces to zero (can be implied by subsequent working) Second M1 for equating the sum of the **i** components to zero AND the sum of the **j** components to zero oe to produce TWO equations, each one being in p and q ONLY. First A1 for TWO correct equations (in any form) **N.B.** It is possible to obtain TWO equations by using $\lambda(3p-5q-15)=\mu(p+q-1)$ with TWO different pairs of values for λ and μ, with one pair not a multiple of the other e.g $\lambda=1$, $\mu=1$ AND $\lambda=1$, $\mu=2$. Third M1(independent) for attempt (either by substitution or elimination) to produce an equation in either p ONLY or q ONLY. Second A1 for $p = 2.5$ (any equivalent form, fractions do not need to be in lowest terms) Third A1 for $q = -1.5$ (any equivalent form, fractions do not need to be in lowest terms)	

Pearson

Q4.

Question Number	Scheme	Marks
(a)	$t = 0$ gives $\mathbf{v} = \mathbf{i} - 3\mathbf{j}$	B1
	speed $= \sqrt{1^2 + (-3)^2}$	M1
	$= \sqrt{10} = 3.2$ or better	A1
		(3)
(b)	$t = 2$ gives $\mathbf{v} = (-3\mathbf{i} + 3\mathbf{j})$	M1
	Bearing is 315°	A1
		(2)
(c)(i)	$1 - 2t = 0 \Rightarrow t = 0.5$	M1 A1
(ii)	$-(3t - 3) = -3(1 - 2t)$	M1 A1
	Solving for t	**DM**1
	t = 2/3, 0.67 or better	A1
		(6)
		[11]
Notes for Question		
Q (a)	B1 for **i** – 3**j**. M1 for √ (sum of squares of cpt.s) A1 for √10, 3.2 or better	
Q (b)	M1 for clear attempt to sub $t = 2$ into given expression. A1 for 315.	
Q (c)	**(i)** First M1 for $1 - 2t = 0$. First A1 for $t = 0.5$. N.B. If they offer two solutions, by equating both the **i** and **j** components to zero, give M0. **(ii)** First M1 for $\frac{1-2t}{3t-3} = \pm(\frac{-1}{-3})$ o.e. (Must be an equation in t *only*) First A1 for a correct equation (the + sign) Second M1, dependent on first M1, for solving for t. Second A1 for 2/3, 0.67 or better.	

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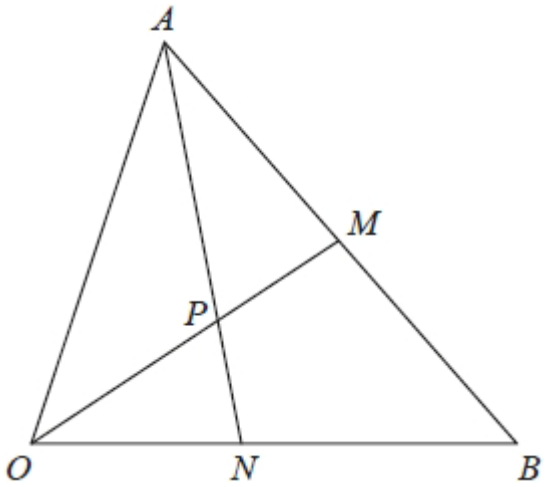
Gold Questions

Calculators may not be used

The total mark for this section is 29

Q1



OAB is a triangle.
OPM and *APN* are straight lines.
M is the midpoint of *AB*.

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

$OP : PM = 3 : 2$

Work out the ratio $ON : NB$

(Total for Question 1 is 5 marks)

Q2

[*In this question,* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship sets sail at 9 am from a port P and moves with constant velocity. The position vector of P is $(4\mathbf{i} - 8\mathbf{j})$ km. At 9.30 am the ship is at the point with position vector $(\mathbf{i} - 4\mathbf{j})$ km.

(a) Find the speed of the ship in km h^{-1}.

(4)

(b) Show that the position vector $\mathbf{r}$ km of the ship, t hours after 9 am, is given by
$\mathbf{r} = (4 - 6t)\mathbf{i} + (8t - 8)\mathbf{j}$.

(2)

At 10 am, a passenger on the ship observes that a lighthouse L is due west of the ship. At 10.30 am, the passenger observes that L is now south-west of the ship.

(c) Find the position vector of L.

(5)

(Total for Question 2 is 11 marks)

Q3

[*In this question* **I** *and* **j** *are horizontal unit vectors due east and due north respectively.*]

A hiker H is walking with constant velocity $(1.2\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}.

(a) Find the speed of H.

(2)

Figure 3

A horizontal field $OABC$ is rectangular with OA due east and OC due north, as shown in Figure 3. At twelve noon hiker H is at the point Y with position vector 100 **j** m, relative to the fixed origin O.

(b) Write down the position vector of H at time t seconds after noon.

(2)

At noon, another hiker K is at the point with position vector $(9\mathbf{i} + 46\mathbf{j})$ m. Hiker K is moving with constant velocity $(0.75\mathbf{i} + 1.8\mathbf{j})$ m s^{-1}.

(c) Show that, at time t seconds after noon,

$$\overrightarrow{HK} = [(9 - 0.45t)\,\mathbf{i} + (2.7t - 54)\,\mathbf{j}]\text{metres}.$$

(4)

Hence,

(d) show that the two hikers meet and find the position vector of the point where they meet.

(5)

(Total for Question 3 is 13 marks)

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Gold Mark Scheme

Q1

Answer	Mark	Mark scheme	Additional guidance
3 : 4	P1	starts process eg $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ oe	
	P1	for process to find $\overrightarrow{OM} = \mathbf{a} + \frac{1}{2}$ "$(\mathbf{b} - \mathbf{a})$"oe $(=\frac{1}{2}(\mathbf{a} + \mathbf{b}))$	
	P1	for process to find $\overrightarrow{AP} = -\mathbf{a} + \frac{3}{5}$ "$(\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b})$" oe **or** (indep) for $\overrightarrow{AN} = -\mathbf{a} +$ "k"$\mathbf{b}$	
	P1	process to find "k" using $\overrightarrow{AN} = -\mathbf{a} +$ "k"$\mathbf{b}$ as a multiple of $\overrightarrow{AP}$	
	A1	cao	
		ALTERNATIVE	
	P1	for producing OM to C such that AC is parallel to OB	Formal geometric reasoning relating to congruent and similar triangles is not required
	P1	for process to show that $MC = OM$, using congruent triangles ACM and BOM	
	P1	for process to find PC as a multiple of $OM/5$ $(= 7OM/5)$	
	P1	for process to find ON as a multiple of $AC(OB)$ $(= 3OB/7)$ using similar triangles ACP and NOP	
	A1	cao	

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Q2

Question Number	Scheme	Marks
(a)	$\frac{(\mathbf{i}-4\mathbf{j})-(4\mathbf{i}-8\mathbf{j})}{0.5};(\pm 6\mathbf{i}\pm 8\mathbf{j})$ $\sqrt{(\pm 6)^2+(\pm 8)^2}=10$	M1 A1 M1 A1 (4)
(b)	$\mathbf{r}=(4\mathbf{i}-8\mathbf{j})+t(-6\mathbf{i}+8\mathbf{j})$ $=(4\mathbf{i}-8\mathbf{j})-6t\mathbf{i}+8t\mathbf{j}$ $=(4-6t)\mathbf{i}+(8t-8)\mathbf{j}$ *	M1 A1 (2)
(c)	At 10 am, $\mathbf{r}=-2\mathbf{i}$ At 10.30 am, $\mathbf{r}=-5\mathbf{i}+4\mathbf{j}$ $\mathbf{l}=k\mathbf{i},\ k<-2$ $k=-5-4=-9$ $\mathbf{l}=-9\mathbf{i}$	M1 A1 A1 DM1 A1 (5) **11**

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Q3

Question Number	Scheme	Marks
(a)	$\lvert\mathbf{v}\rvert = \sqrt{1.2^2 + (-0.9)^2} = 1.5 \text{ m s}^{-1}$	M1 A1 (2)
(b)	$(\mathbf{r}_H =) 100\mathbf{j} + t(1.2\mathbf{i} - 0.9\mathbf{j})$ m	M1 A1 (2)
(c)	$(\mathbf{r}_K =) 9\mathbf{i} + 46\mathbf{j} + t(0.75\mathbf{i} + 1.8\mathbf{j})$ m	M1 A1
	$\overrightarrow{HK} = \mathbf{r}_K - \mathbf{r}_H = (9 - 0.45t)\mathbf{i} + (2.7t - 54)\mathbf{j}$ m **Printed Answer**	M1 A1 (4)
(d)	Meet when $\overrightarrow{HK} = \mathbf{0}$	
	$(9 - 0.45t) = 0$ and $(2.7t - 54) = 0$	M1 A1
	$t = 20$ from both equations	A1
	$\mathbf{r}_K = \mathbf{r}_H = (24\mathbf{i} + 82\mathbf{j})$ m	DM1 A1 cso
		(5)
		[13]

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Platinum Questions

Calculators may not be used

The total mark for this section is 16

1 Points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$, respectively, relative to an origin O, and are such that OAB is a triangle with $OA = a$ and $OB = b$.

The point C, with position vector $\mathbf{c}$, lies on the line through O that bisects the angle AOB.

(a) Prove that the vector $b\mathbf{a} - a\mathbf{b}$ is perpendicular to $\mathbf{c}$.

(4)

The point D, with position vector $\mathbf{d}$, lies on the line AB between A and B.

(b) Explain why $\mathbf{d}$ can be expressed in the form $\mathbf{d} = (1 - \lambda)\mathbf{a} + \lambda\mathbf{b}$ for some scalar λ with $0 < \lambda < 1$

(2)

(c) Given that D is also on the line OC, find an expression for λ in terms of a and b only and hence show that

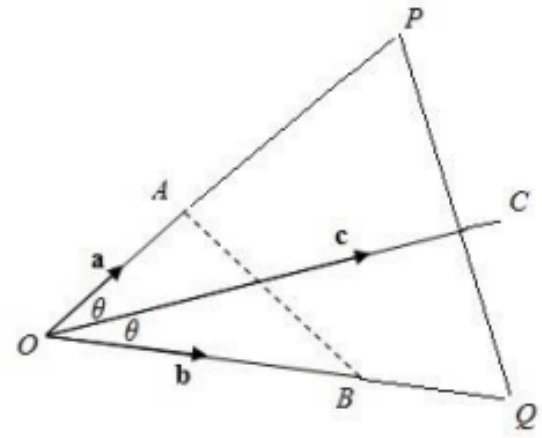
$$DA : DB = OA : OB$$

(8)

(+S2)

(Total for Question 1 is 16 marks)

Platinum Mark Scheme

Question	Scheme	Marks	Notes
5. (a)			(S+ for good diagram sketched)
	Let P and Q be points such that $\overrightarrow{OP} = b\mathbf{a}$ and $\overrightarrow{OQ} = a\mathbf{b}$.	M1	Extends OA and OB (may use unit vectors instead)
	Then $\lvert\overrightarrow{OP}\rvert = b\lvert\mathbf{a}\rvert = ba = ab = a\lvert\mathbf{b}\rvert = \lvert\overrightarrow{OQ}\rvert$ hence OPQ is isosceles.	A1	Deduce isosceles or equivalent.
	Hence the angle bisector from O is perpendicular to PQ.	M1	Use isosceles to deduce perpendicular
	But $\overrightarrow{QP} = \overrightarrow{OP} - \overrightarrow{OQ} = b\mathbf{a} - a\mathbf{b}$ and hence as C is on the angle bisector, so $b\mathbf{a} - a\mathbf{b}$ is perpendicular to $\mathbf{c}$.	A1	Draw correct conclusion.
		(4)	
(b)	$\overrightarrow{OD} = \overrightarrow{OA} + \lambda\overrightarrow{AB} \Rightarrow \mathbf{d} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$	M1	Sets up appropriate equation, either form.
	$\Rightarrow \mathbf{d} = (1-\lambda)\mathbf{a} + \lambda\mathbf{b}$ $(0 < \lambda < 1$ since D is between A and $B)$	A1 (S+)	Correctly shown (Reasoning for λ)
		(2)	
(c)	($\overrightarrow{OD} = k\mathbf{c}$ and from (a) $\mathbf{c} = K \times \frac{1}{2}\left(\overrightarrow{OP} + \overrightarrow{OQ}\right)$ hence)	M1	Makes deduction that **d** is a multiple of **p**+**q**
	$\overrightarrow{OD} = k'\left(\overrightarrow{OP} + \overrightarrow{OQ}\right)$		
	Hence $\mathbf{d} = k'\left(\overrightarrow{OP} + \overrightarrow{OQ}\right) = (1-\lambda)\mathbf{a} + \lambda\mathbf{b}$	M1	Equates their **d** to **d** from (b)
	So $k'\left(b\mathbf{a} + a\mathbf{b}\right) = (1-\lambda)\mathbf{a} + \lambda\mathbf{b}$	M1	Forms equation in **a** and **b**
	Therefore (since **a** and **b** are not parallel) $k'b = 1-\lambda$ and $k'a = \lambda$ $\Rightarrow \frac{\lambda}{a}b = 1-\lambda \Rightarrow \lambda = \frac{a}{a+b}$	M1 A1	Extracts simultaneous equations and solves for λ. (S+ for non-parallel reasoning)

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Question	Scheme	Marks	Notes
	$\overrightarrow{AD} = (1-\lambda)\mathbf{a} + \lambda\mathbf{b} - \mathbf{a} = \lambda(\mathbf{b}-\mathbf{a}) \Rightarrow AD = \lambda\lvert\mathbf{b}-\mathbf{a}\rvert$ $\overrightarrow{BD} = (1-\lambda)\mathbf{a} + \lambda\mathbf{b} - \mathbf{b} = (1-\lambda)(\mathbf{a}-\mathbf{b}) \Rightarrow BD = (1-\lambda)\lvert\mathbf{b}-\mathbf{a}\rvert$ So $\frac{AD}{BD} = \frac{\lambda}{1-\lambda}$ $= \frac{a/a+b}{b/a+b} = \frac{a}{b} = \frac{OA}{OB}$	M1 dM1 A1 **(8)**	Correct work to establish ratio (may just be quoted) Give M0 if division of vectors is used. Substitutes in for λ **Given result** established
S2	S2 mark: Award S2 for a clear and concise solution that is EITHER - fully correct with no majorly incorrect vector notation used OR - that scores 12+ and includes at least 2 S+ points but may have some poor notation and be slightly laboured Award S1 for - a clear solution that scores 10+ marks with at least one S+ point.	**(2)**	
	Notes **(a)** S+ for a clearly labelled diagram drawn showing at least **a, b** and **c** **(b)** for the explanation of why $0 < \lambda < 1$ **(c)** S+ for reason given for being able to equate coefficients, e.g. vectors cannot be parallel since *OAB* is a triangle. S+ for any innovative ways used throughout the question.		
		Total 14 + 2 marks	

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Topic 12

Differentiation

Bronze, Silver, Gold and Platinum Worksheets for AS Level Mathematics

ALWAYS LEARNING

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Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculators may not be used

The total mark for this section is 28

Q1

The curve C has equation

$$y = 2x^2 - 12x + 16$$

Find the gradient of the curve at the point P (5, 6).

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(Total for Question 1 is 4 marks)

Q2

Given that $y = x^4 + x^{\frac{1}{3}} + 3$, find $\dfrac{dy}{dx}$

(Total for Question 2 is 3 marks)

Q3

A curve has equation

$$y = 2x^3 - 4x + 5$$

Find the equation of the tangent to the curve at the point $P(2, 13)$.

Write your answer in the form $y = mx + c$, where m and c are integers to be found.

Solutions relying on calculator technology are not acceptable.

(Total for Question 3 is 5 marks)

Q4

Prove, from first principles, that the derivative of x^3 is $3x^2$

(Total for Question 4 is 4 marks)

Q5

$$y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3$$

(a) Find $\frac{dy}{dx}$ giving each term in its simplest form.

(4)

(b) Find $\frac{d^2y}{dx^2}$

(2)

(Total for Question 5 is 6 marks)

Q6

Using calculus, find the coordinates of the stationary point on the curve with equation

$$y = 2x + 3 + \frac{8}{x^2}, \quad x > 0$$

(6)

(Total for Question 6 is 6 marks)

End of Questions

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Bronze Mark Scheme

Q1.

Question	Scheme	Marks	AOs
	Attempt to differentiate	M1	1.1a
	$\frac{dy}{dx} = 4x - 12$	A1	1.1b
	Substitutes $x = 5 \Rightarrow \frac{dy}{dx} = ...$	M1	1.1b
	$\Rightarrow \frac{dy}{dx} = 8$	A1ft	1.1b
			(4 marks)

Notes

M1 : Differentiation implied by one correct term
A1 : Correct differentiation
M1 : Attempts to substitute $x = 5$ into their derived function
A1ft: Substitutes $x = 5$ into **their** derived function **correctly** i.e. Correct calculation of their f ′(5) so follow through slips in differentiation

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Q2.

Question number	Scheme	Marks
	$x^4 \to kx^3$ or $x^{\frac{1}{3}} \to kx^{-\frac{2}{3}}$ or $3 \to 0$ (k a non-zero constant)	M1
	$\left(\frac{dy}{dx} =\right)\ 4x^3 .....$, with '3' differentiated to zero (or 'vanishing')	A1
	$\left(\frac{dy}{dx} =\right)\ ..... + \frac{1}{3}x^{-\frac{2}{3}}$ or equivalent, e.g. $\frac{1}{3\sqrt[3]{x^2}}$ or $\frac{1}{3\left(\sqrt[3]{x}\right)^2}$	A1 [3]
	1st A1 requires $4x^3$, and 3 differentiated to zero. Having '$+C$' loses the 1st A mark. Terms not added, but otherwise correct, e.g. $4x^3$, $\frac{1}{3}x^{-\frac{2}{3}}$ loses the 2nd A mark.	

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Q3.

Question	Scheme	Marks	AOs
	Attempts to differentiate $x^n \to x^{n-1}$ seen once	M1	1.1b
	$y = 2x^3 - 4x + 5 \Rightarrow \frac{dy}{dx} = 6x^2 - 4$	A1	1.1b
	For substituting $x = 2$ into their $\frac{dy}{dx} = 6x^2 - 4$	dM1	1.1b
	For a correct method of finding a tangent at $P(2,13)$. Score for $y - 13 = "20"(x - 2)$	ddM1	1.1b
	$y = 20x - 27$	A1	1.1b
		(5)	
			(5 marks)

Notes

M1: Attempts to differentiate $x^n \to x^{n-1}$ seen once. Score for $x^3 \to x^2$ or $\pm 4x \to 4$ or $+5 \to 0$

A1: $\left(\frac{dy}{dx} =\right) 6x^2 - 4$ which may be unsimplified $6x^2 - 4 + C$ is A0

dM1: Substitutes $x = 2$ into their $\frac{dy}{dx}$. The first M must have been awarded.

Score for sight of embedded values, or sight of "$\frac{dy}{dx}$ at $x = 2$ is" or a correct follow through.

Note that 20 on its own is not enough as this can be done on a calculator.

ddM1: For a correct method of finding a tangent at $P(2,13)$. Score for $y - 13 = "20"(x - 2)$

It is dependent upon both previous M's.

If the form $y = mx + c$ is used they must proceed as far as $c = ...$

A1: Completely correct $y = 20x - 27$ (and in this form)

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Q4.

Question	Scheme	Marks	AOs
	Considers $\frac{(x+h)^3-x^3}{h}$	B1	2.1
	Expands $(x+h)^3=x^3+3x^2h+3xh^2+h^3$	M1	1.1b
	so gradient (of chord) $=\frac{3x^2h+3xh^2+h^3}{h}=3x^2+3xh+h^2$	A1	1.1b
	States as $h\to 0$, $3x^2+3xh+h^2\to 3x^2$ so derivative $=3x^2$ *	A1*	2.5
			(4 marks)

B1: Gives the correct fraction for the gradient of the chord either $\frac{(x+h)^3-x^3}{h}$ or $\frac{(x+\delta x)^3-x^3}{\delta x}$

It may also be awarded for $\frac{(x+h)^3-x^3}{x+h-x}$ oe. It may be seen in an expanded form

It does not have to be linked to the gradient of the chord

M1: Attempts to expand $(x+h)^3$ or $(x+\delta x)^3$ Look for two correct terms, most likely $x^3+...+h^3$ This is independent of the B1

A1: Achieves gradient (of chord) is $3x^2+3xh+h^2$ or exact un simplified equivalent such as $3x^2+2xh+xh+h^2$. Again, there is no requirement to state that this expression is the gradient of the chord

A1*: CSO. Requires correct algebra and making a link between the gradient of the chord and the gradient of the curve. See below how the link can be made. The words "gradient of the chord" do not need to be mentioned but derivative, $f'(x)$, $\frac{dy}{dx}$, y' should be. Condone invisible brackets for the expansion of $(x+h)^3$ as long as it is only seen at the side as intermediate working.

Requires either

- $f'(x)=\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}=3x^2+3xh+h^2=3x^2$
- Gradient of chord $=3x^2+3xh+h^2$ As $h\to 0$ Gradient of chord tends to the gradient of curve so derivative is $3x^2$
- $f'(x)=\lim_{h\to 0}3x^2+3xh+h^2=3x^2$
- Gradient of **chord** $=3x^2+3xh+h^2$ when $h\to 0$ gradient of **curve** $=3x^2$
- Do not allow $h=0$ alone without limit being considered somewhere: so don't accept $h=0\Rightarrow f'(x)=3x^2+3x\times 0+0^2=3x^2$

Alternative: B1: Considers $\frac{(x+h)^3-(x-h)^3}{2h}$ M1: As above A1: $\frac{6x^2h^2+2h^3}{2h}=3x^2+h^2$

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Q5.

Question Number	Scheme	Marks
(a)	$y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3$ $\left\{\frac{dy}{dx}=\right\}\ 5(3)x^2 - 6\left(\frac{4}{3}\right)x^{\frac{1}{3}} + 2$ $= 15x^2 - 8x^{\frac{1}{3}} + 2$	M1 A1 A1 A1 [4]
(b)	$\left\{\frac{d^2y}{dx^2}=\right\}\ 30x - \frac{8}{3}x^{-\frac{2}{3}}$	M1 A1 [2] 6

	Notes
(a)	**M1:** for an attempt to differentiate $x^n \to x^{n-1}$ to one of the first three terms of $y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3$. So seeing either $5x^3 \to \pm\lambda x^2$ or $-6x^{\frac{4}{3}} \to \pm\mu x^{\frac{1}{3}}$ or $2x \to 2$ is M1. **1st A1:** for $15x^2$ only. **2nd A1:** for $-8x^{\frac{1}{3}}$ or $-8\sqrt[3]{x}$ only. **3rd A1:** for $+2$ ($+c$ included in part (a) loses this mark). **Note:** $2x^0$ is A0 unless simplified to 2.
(b)	**M1:** For differentiating $\frac{dy}{dx}$ again to give **either** • a correct follow through differentiation of their x^2 term • or for $\pm\alpha x^{\frac{1}{3}} \to \pm\beta x^{-\frac{2}{3}}$. **A1:** for any *correct* expression *on the same line* (accept un-simplified coefficients). For powers: $30x^{2-1} - \frac{8}{3}x^{\frac{1}{3}-1}$ is A0, but writing powers as one term eg: $(15\times 2x) - \frac{8}{3}x^{-\frac{4}{6}}$ is ok for A1. **Note:** Candidates leaving their answers as $\left\{\frac{dy}{dx}=\right\}15x^2 - \frac{24}{3}x^{\frac{1}{3}} + 2$ and $\left(\frac{d^2y}{dx^2}=\right)\ 30x - \frac{24}{9}x^{-\frac{2}{3}}$ are awarded M1A1A0A1 in part (a) and M1A1 in part (b). **Be careful:** $30x - \frac{8}{3}x^{-\frac{1}{3}}$ will be A0. **Note:** For an extra term appearing in part (b) on the same line, ie $30x - \frac{8}{3}x^{-\frac{2}{3}} + 2$ is M1A0 **Note:** If a candidate writes in part (a) $15x^2 - 8x^{\frac{1}{3}} + 2 + c$ and in part (b) $30x - \frac{8}{3}x^{-\frac{2}{3}} + c$ then award (a) M1A1A1A0 (b) M1A1

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Q6.

Question Number	Scheme	Marks
	$\frac{dy}{dx} = 2-16x^{-3}$	M1 A1
	$2-16x^{-3}=0$ so $x^{-3}=$ or $x^{3}=$, or $2-16x^{-3}=0$ so $x=2$	M1
	$x=2$ only (after correct derivative)	A1
	$y=2\times"2"+3+\frac{8}{"2^2"}$	M1
	$=9$	A1
		(6)
		Total 6
	Notes for Question	
	1st M1: At least one term **differentiated (not integrated)** correctly, so $2x\to 2$, or $\frac{8}{x^2}\to -16x^{-3}$, or $3\to 0$	
	A1: This answer or equivalent e.g. $2-\frac{16}{x^3}$	
	2nd M1: Sets $\frac{dy}{dx}$ to 0, and solves to give x^3 = value or x^{-3} = value	
	(or states x =2 with no working following correctly stated $2-16x^{-3}=0$)	
	A1: $x=2$ cso (if x = -2 is included this is A0 here)	
	3rd M1: Attempts to substitutes **their positive** x (found from attempt to differentiate) into $y=2x+3+\frac{8}{x^2}$, $x>0$	
	Or may be implied by $y=9$ or correct follow through from their positive x	
	A1: 9 cao (Does not need to be written as coordinates) (ignore the extra (-2,1) here)	

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Silver Questions

Calculators may not be used

The total mark for this section is 34

Q1

The curve C has equation

$$y = 2x - 8\sqrt{x} + 5 \, , \, x \geq 0$$

(a) Find $\frac{dy}{dx}$, giving each term in its simplest form.

(3)

The point P on C has x-coordinate equal to $\frac{1}{4}$

(b) Find the equation of the tangent to C at the point P, giving your answer in the form $y = ax + b$, where a and b are constants.

(4)

(Total for Question 1 is 7 marks)

Q2

The curve C has equation $y = 6 - 3x - \frac{4}{x^3}$, $x \neq 0$

(a) Use calculus to show that the curve has a turning point P when $x = \sqrt{2}$

(4)

(b) Find the x-coordinate of the other turning point Q on the curve.

(1)

(c) Find $\frac{d^2y}{dx^2}$.

(1)

(d) Hence or otherwise, state with justification, the nature of each of these turning points P and Q.

(3)

(Total for Question 2 is 9 marks)

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Q3

Prove, from first principles, that the derivative of $3x^2$ is $6x$

(Total for Question 3 is 4 marks)

Q4

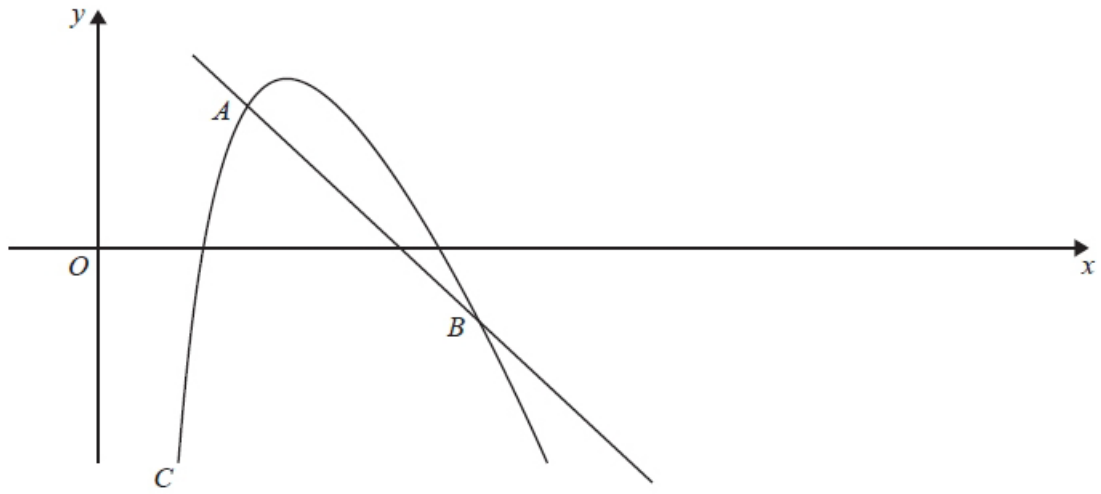

Figure 3

A sketch of part of the curve C with equation

$$y = 20 - 4x - {}^{18}\!/_{x}, \quad x > 0$$

is shown in Figure 3.

Point A lies on C and has an x coordinate equal to 2

Show that the equation of the normal to C at A is $y = -2x + 7$

(Total for Question 4 is 6 marks)

Contents

Q5

The volume V cm^3 of a box, of height x cm, is given by

$$V = 4x(5 - x)^2, \qquad 0 < x < 5$$

(a) Find $\dfrac{\mathrm{d}V}{\mathrm{d}x}$.

(4)

(b) Hence find the maximum volume of the box.

(4)

(Total for Question 5 is 8 marks)

Contents

Silver Mark Scheme

Q1.

Question Number	Scheme	Marks
	$C: y = 2x - 8\sqrt{x} + 5, \quad x \geqslant 0$	
(a)	So, $y = 2x - 8x^{\frac{1}{2}} + 5$	
	$\frac{dy}{dx} = 2 - 4x^{-\frac{1}{2}} + \{0\}$ $\quad (x > 0)$	M1 A1 A1
		[3]
(b)	(When $x = \frac{1}{4}$, $y = 2\left(\frac{1}{4}\right) - 8\sqrt{\left(\frac{1}{4}\right)} + 5$ so) $\quad y = \frac{3}{2}$	B1
	(gradient $= \frac{dy}{dx} =$) $2 - \frac{4}{\sqrt{\left(\frac{1}{4}\right)}}$ $\{= -6\}$	M1
	Either : $y - "\frac{3}{2}" = "-6"\left(x - \frac{1}{4}\right)$ **or**: $y = "-6"x + c$ and $"\frac{3}{2}" = "-6"\left(\frac{1}{4}\right) + c \Rightarrow c = "3"$	dM1
	So $\quad \underline{y = -6x + 3}$	A1
		[4]
		7 Marks

	Notes
(a)	**M1:** Evidence of differentiation, so $x^n \to x^{n-1}$ at least once so $x^1 \to 1$ or x^0 or $x^{\frac{1}{2}} \to x^{-\frac{1}{2}}$ not just $5 \to 0$
	A1: Any two of the three terms correct – do not need to see zero – the 5 disappearing is sufficient; need not be simplified.
	A1: $2 - 4x^{-\frac{1}{2}}$ Both terms correct, and simplified. Do not need to include domain $x > 0$
(b)	**B1:** Obtaining $y = 3/2$ or fractional or decimal equivalent (no working need be seen)
	M1: An attempt to substitute $x = \frac{1}{4}$ into $\frac{dy}{dx}$ to establish **gradient**. This may be implied by -6 or $m = -6$ but not $y = -6$. Can earn this M mark if they go on to use $m = \frac{1}{6}$ or use their numerical value of $\frac{dy}{dx}$
	dM1: This depends on previous method mark. Complete method for obtaining the equation of the tangent, using their **tangent** gradient and their value for y_1 (obtained from $x = ¼$, allow slip) i.e. $y - y_1 = m_T\left(x - \frac{1}{4}\right)$ with their tangent gradient and their y_1 or uses $y = mx + c$ with $\left(\frac{1}{4}, \text{their } y_1\right)$ and their tangent gradient.
	Special Cases: In (b) Finds normal could get B1 M1 M0 A0 i.e. max of 2/4

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Q2.

Question Number	Scheme		Marks
	$y = 6 - 3x - \frac{4}{x^3}$		
(a)	$\frac{dy}{dx} = -3 + \frac{12}{x^4}$ *or* $-3 + 12x^{-4}$	M1: $x^n \to x^{n-1}$ ($x^1 \to x^0$ or $x^{-3} \to x^{-4}$ or $6 \to 0$) A1: Correct derivative	M1 A1
	$\frac{dy}{dx} = 0 \Rightarrow -3 + \frac{12}{x^4} = 0 \Rightarrow x = \ldots$ **or** $\frac{dy}{dx} = -3 + \frac{12}{\sqrt{2}^4}$	$y' = 0$ and attempt to solve for x May be implied by $\frac{dy}{dx} = -3 + \frac{12}{x^4} = 0 \Rightarrow \frac{12}{x^4} = 3 \Rightarrow x = \ldots$ **or** Substitutes $x = \sqrt{2}$ into their y'	M1
	So $x^4 = 4$ and $x = \sqrt{2}$ **or** $\frac{dy}{dx} = -3 + \frac{12}{(\sqrt{2})^4}$ or $-3 + 12(\sqrt{2})^{-4} = 0$	Correct completion to answer with no errors by solving their $y' = 0$ or substituting $x = \sqrt{2}$ into their y'	A1
			(4)
(b)	$x = -\sqrt{2}$	Awrt -1.41	B1
			(1)
(c)	$\frac{d^2y}{dx^2} = \frac{-48}{x^5}$ or $-48x^{-5}$	Follow through their first derivative from part (a)	B1ft
			(1)
(d)	An appreciation that either $y'' > 0 \Rightarrow$ a minimum or $y'' < 0 \Rightarrow$ a maximum		B1
	Maximum at P as $y'' < 0$	Cso	B1
	Need a fully correct solution for this mark. y'' need not be evaluated but must be correct and there must be reference to P or to $\sqrt{2}$ and negative or < 0 and maximum. There must be no incorrect or contradictory statements (NB allow y'' = awrt-8 or -9)		
	Minimum at Q as $y'' > 0$	Cso	B1
	Need a fully correct solution for this mark. y'' need not be evaluated but must be correct **and part (b) must be correct** and there must be reference to P or to $-\sqrt{2}$ and positive or > 0 and minimum. There must be no incorrect or contradictory statements (NB allow y'' = awrt 8 or 9)		
			(3)
			[9]
	Other methods for identifying the nature of the turning points are acceptable. The first B1 is for finding values of y or dy/dx either side of $\sqrt{2}$ or their x at Q and the second and third B1's for fully correct solutions to identify the maximum/minimum.		

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Q3.

Question	Scheme	Marks	AOs
	Considers $\frac{3(x+h)^2-3x^2}{h}$	B1	2.1
	Expands $3(x+h)^2=3x^2+6xh+3h^2$	M1	1.1b
	so gradient $=\frac{6xh+3h^2}{h}=6x+3h$ or $\frac{6x\delta x+3(\delta x)^2}{\delta x}=6x+3\delta x$	A1	1.1b
	States as $h\to 0$, gradient $\to 6x$ so in the limit derivative $=6x$ *	A1*	2.5
			(4 marks)

Notes

B1: gives correct fraction as in the scheme above or $\frac{3(x+\delta x)^2-3x^2}{\delta x}$

M1: Expands the bracket as above or $3(x+\delta x)^2=3x^2+6x\delta x+3(\delta x)^2$

A1: Substitutes correctly into earlier fraction and simplifies

A1*: Completes the proof, as above (may use $\delta x \to 0$), considers the limit and states a conclusion with no errors

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Q4.

Question Number	Scheme	Marks
(a)	Substitutes $x=2$ into $y=20-4\times2-\frac{18}{2}$ and gets 3	B1
	$\frac{dy}{dx}=-4+\frac{18}{x^2}$	M1 A1
	Substitute $x=2\Rightarrow\frac{dy}{dx}=\left(\frac{1}{2}\right)$ then finds negative reciprocal (-2)	dM1
	Method 1 States or uses $y-3=-2(x-2)$ or $y=-2x+c$ with their (2, 3) to deduce that $y=-2x+7$ *	**Method 2** Or: Check that (2, 3) lies on the line $y=-2x+7$ Deduce equation of normal as it has the **same gradient** and passes through **a common point**	dM1 A1*
		(6)

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Q5.

Question Number	Scheme	Marks
(a)	$V = 4x(5-x)^2 = 4x(25-10x+x^2)$	
	So, $V = 100x - 40x^2 + 4x^3$ $\pm \alpha x \pm \beta x^2 \pm \gamma x^3$, where $\alpha, \beta, \gamma \neq 0$	M1
	$V = 100x - 40x^2 + 4x^3$	A1
	$\frac{dV}{dx} = 100 - 80x + 12x^2$ At least two of their expanded terms differentiated correctly.	M1
	$100 - 80x + 12x^2$	A1 cao
		(4)
(b)	$100 - 80x + 12x^2 = 0$ Sets their $\frac{dV}{dx}$ from part (a) $= 0$	M1
	$\{\Rightarrow 4(3x^2 - 20x + 25) = 0 \Rightarrow 4(3x-5)(x-5) = 0\}$	
	$\{\text{As } 0 < x < 5\}\ x = \frac{5}{3}$ $x = \frac{5}{3}$ or $x =$ awrt 1.67	A1
	$x = \frac{5}{3}$, $V = 4\left(\frac{5}{3}\right)\left(5 - \frac{5}{3}\right)^2$ Substitute candidate's value of x where $0 < x < 5$ into a formula for V.	dM1
	So, $V = \frac{2000}{27} = 74\frac{2}{27} = 74.074\ldots$ Either $\frac{2000}{27}$ or $74\frac{2}{27}$ or awrt 74.1	A1
		(4)
	Notes	
(a)	1st M1 for a three term cubic in the form $\pm \alpha x \pm \beta x^2 \pm \gamma x^3$. Note that an un-combined $\pm \alpha x \pm \lambda x^2 \pm \mu x^2 \pm \gamma x^3$, $\alpha, \lambda, \mu, \gamma \neq 0$ is fine for the 1st M1. 1st A1 for either $100x - 40x^2 + 4x^3$ or $100x - 20x^2 - 20x^2 + 4x^3$. 2nd M1 for any two of their expanded terms differentiated correctly. NB: If expanded expression is divided by a constant, then the 2nd M1 can be awarded for at least two terms are correct. Note for un-combined $\pm \lambda x^2 \pm \mu x^2$, $\pm 2\lambda x \pm 2\mu x$ counts as one term differentiated correctly. 2nd A1 for $100 - 80x + 12x^2$, cao. **Note:** See appendix for those candidates who apply the product rule of differentiation.	

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Gold Questions

Calculators may not be used

The total mark for this section is 33

Q1

The curve C has equation $y = 12\sqrt{(x)} - x^{\frac{3}{2}} - 10$, $x > 0$

(a) Use calculus to find the coordinates of the turning point on C.

(7)

(b) Find $\frac{d^2y}{dx^2}$

(2)

(c) State the nature of the turning point.

(1)

(Total for Question 1 is 10 marks)

Q2

The curve C has equation

$$y = 9 - 4x - \frac{8}{x}, \qquad x > 0$$

The point P on C has x-coordinate equal to 2.

(a) Show that the equation of the tangent to C at the point P is $y = 1 - 2x$.

(6)

(b) Find an equation of the normal to C at the point P.

(3)

(Total for Question 2 is 9 marks)

Q3

$$y = x^2 - k\sqrt{x}, \text{ where } k \text{ is a constant.}$$

(a) Find $\frac{dy}{dx}$

(2)

(b) Given that y is decreasing at $x = 4$, find the set of possible values of k.

(2)

(Total for Question 3 is 4 marks)

Q4

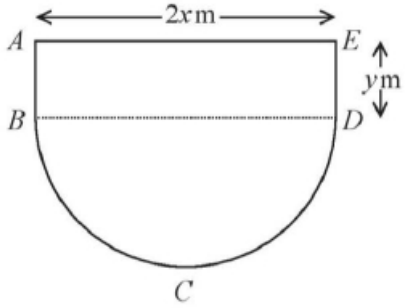

Figure 4

Figure 4 shows the plan view of the design for a swimming pool.

The shape of this pool $ABCDEA$ consists of a rectangular section $ABDE$ joined to a semicircular section BCD as shown in Figure 4.

Given that $AE = 2x$ metres, $ED = y$ metres and the area of the pool is 250 m^2,

(a) show that the perimeter, P metres, of the pool is given by

$$P = 2x + \frac{250}{x} + \frac{\pi x}{2}$$

(4)

(b) Explain why $0 < x < \sqrt{\frac{500}{\pi}}$

(2)

(c) Find the minimum perimeter of the pool, giving your answer in exact form.

(4)

(Total for Question 4 is 10 marks)

Contents

Gold Mark Scheme

Q1.

Question Number	Scheme	Marks
(a)	$\left[y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}} - 10\right]$	
	$[y' =]$ $6x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$	M1 A1
	Puts their $\frac{6}{x^{\frac{1}{2}}} - \frac{3}{2}x^{\frac{1}{2}} = 0$	M1
	So $x = \frac{12}{3} = 4$ (If $x = 0$ appears also as solution then lose A1)	M1, A1
	$x = 4, \Rightarrow y = 12 \times 2 - 4^{\frac{3}{2}} - 10$, so $y = 6$	dM1,A1 (7)
(b)	$y'' = -3x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{1}{2}}$	M1A1 (2)
(c)	[Since $x > 0$] It is a maximum	B1 (1) [10]

(a) 1st M1 for an attempt to differentiate a fractional power $x^n \rightarrow x^{n-1}$
A1 a.e.f – can be unsimplified
2nd M1 for forming a suitable equation using their $y' = 0$
3rd M1 for correct processing of fractional powers leading to $x = \ldots$ (Can be implied by $x = 4$)
A1 is for $x = 4$ only. If $x = 0$ also seen and not discarded they lose this mark only.
4th M1 for substituting their value of x back into y to find y value. Dependent on three previous M marks. Must see evidence of the substitution with attempt at fractional powers to give M1A0, but $y = 6$ can imply M1A1

(b) M1 for differentiating their y' again
A1 should be simplified

(c) B1 . Clear conclusion needed and must follow correct y'' It is dependent on previous A mark (Do not need to have found x earlier).

(Treat parts (a),(b) and (c) together for award of marks)

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Q2.

Question Number	Scheme	Marks
(a)	$\left(\frac{dy}{dx}=\right)-4+8x^{-2}$ (4 or $8x^{-2}$ for M1… sign can be wrong)	M1A1
	$x=2\Rightarrow \quad m=-4+2=-2$	M1
	$y=9-8-\frac{8}{2}=-3$ The first 4 marks could be earned in part (b)	B1
	Equation of tangent is: $y+3=-2(x-2)\rightarrow \quad y=1-2x \quad (*)$	M1 A1cso (6)
(b)	Gradient of normal $=\frac{1}{2}$	B1ft
	Equation is: $\frac{y+3}{x-2}=\frac{1}{2}$ or better equivalent, e.g. $y=\frac{1}{2}x-4$	M1A1 (3)
		[9]

(a)

1st M1 for 4 or $8x^{-2}$ (ignore the signs).

1st A1 for both terms correct (including signs).

2nd M1 for substituting $x=2$ into their $\frac{dy}{dx}$ (must be different from their y)

B1 for $y_P=-3$, but not if clearly found from the given equation of the tangent.

3rd M1 for attempt to find the equation of tangent at P, follow through their m and y_P.
Apply general principles for straight line equations (see end of scheme).
NO DIFFERENTIATION ATTEMPTED: Just assuming $m=-2$ at this stage is M0

2nd A1cso for correct work leading to printed answer (allow equivalents with $2x$, y, and 1 terms… such as $2x+y-1=0$).

(b)

B1ft for correct use of the perpendicular gradient rule. Follow through their m, but if $m\neq -2$ there must be clear evidence that the m is thought to be the gradient of the tangent.

M1 for an attempt to find normal at P using their changed gradient and their y_P.
Apply general principles for straight line equations (see end of scheme).

A1 for any correct form as specified above (correct answer only).

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Q3.

Question Number	Scheme	Marks
	(a) $\left(\frac{dy}{dx} =\right) 2x - \frac{1}{2}kx^{-\frac{1}{2}}$ (Having an extra term, e.g. $+C$, is A0)	M1 A1 (2)
	(b) Substituting $x = 4$ into their $\frac{dy}{dx}$ and 'compare with zero' (The mark is allowed for : $<, >, =, \leq, \geq$) $8 - \frac{k}{4} < 0 \quad k > 32 \quad (\text{or } 32 < k)$ Correct inequality needed	M1 A1 (2) 4
	(a) M: $x^2 \rightarrow cx$ or $k\sqrt{x} \rightarrow cx^{\frac{1}{2}}$ (c constant, $c \neq 0$) (b) Substitution of $x = 4$ into y scores M0. However, $\frac{dy}{dx}$ is sometimes called y, and in this case the M mark can be given. $\frac{dy}{dx} = 0$ may be 'implied' for M1, when, for example, a value of k or an inequality solution for k is found. Working must be seen to justify marks in (b), i.e. $k > 32$ alone is M0 A0.	

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Q4.

Question	Scheme	Marks	AOs
(a)	Sets $2xy+\frac{\pi x^2}{2}=250$	B1	2.1
	Obtain $y=\frac{250-\frac{\pi x^2}{2}}{2x}$ and substitute into P	M1	1.1b
	Use $P=2x+2y+\pi x$ with their y substituted	M1	2.1
	$P=2x+\frac{250}{x}-\frac{\pi x^2}{2x}+\pi x=2x+\frac{250}{x}+\frac{\pi x}{2}$ *	A1*	1.1b
		(4)	
(b)	$x>0$ and $y>0$ (distance) $\Rightarrow \frac{250-\frac{\pi x^2}{2}}{2x}>0$ or $250-\frac{\pi x^2}{2}>0$ o.e.	M1	2.4
	As x and y are distances they are positive so $0<x<\sqrt{\frac{500}{\pi}}$ *	A1*	3.2a
		(2)	
(c)	Differentiates P with negative index correct in $\frac{dP}{dx}$; $x^{-1}\rightarrow x^{-2}$	M1	3.4
	$\frac{dP}{dx}=2-\frac{250}{x^2}+\frac{\pi}{2}$	A1	1.1b
	Sets $\frac{dP}{dx}=0$ and proceeds to $x=$	M1	1.1b
	Substitutes their x into $P=2x+\frac{250}{x}+\frac{\pi x}{2}$ to give perimeter $=(4+\pi)\sqrt{\left(\frac{500}{4+\pi}\right)}$ m.	A1	1.1b
		(4)	
			(10 marks)

Notes

(a) B1 : Correct area equation
M1 : Rearranges **their** area equation to make y the subject of the formula and attempt to use with an expression for P
M1 : Use correct equation for perimeter with their y substituted
A1*: Completely correct solution to obtain and state printed answer

(b) M1 : States $x>0$ and $y>0$ and uses their expression from (a) to form inequality
A1*: Explains that x and y are positive because they are distances, and uses correct expression for y to give the printed answer correctly.

(c) M1: Attempt to differentiate P (deals with negative power of x correctly)
A1 : Correct differentiation
M1 : Sets derived function equal to zero and obtains $x=$
A1: The value of x may not be seen (it is $\sqrt{\left(\frac{500}{4+\pi}\right)}$).
Need to see $(4+\pi)\sqrt{\left(\frac{500}{4+\pi}\right)}$ with units included for the perimeter.

Pearson

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Platinum Questions

Calculators may not be used

The total mark for this section is 17

1

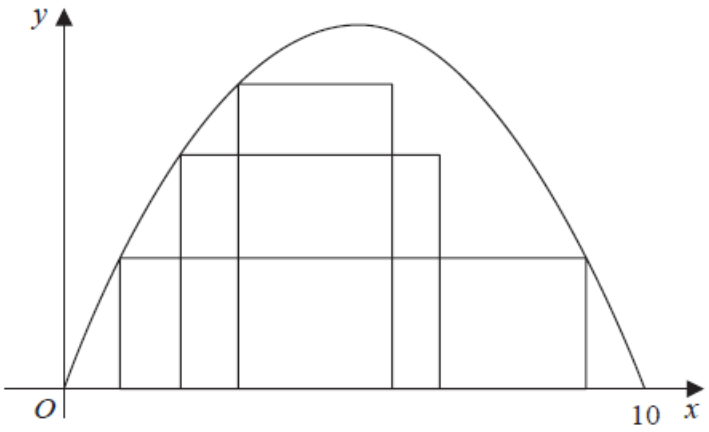

Figure 2

Figure 2 shows a sketch of the parabola with equation $y\ \ y = \frac{1}{2}x(10-x), 0 \le x \le 10$

This question concerns rectangles that lie under the parabola in the first quadrant. The bottom edge of each rectangle lies along the x-axis and the top left vertex lies on the parabola. Some examples are shown in Figure 2.

Let the x coordinate of the top left vertex be a.

(a) Explain why the width, w, of such a rectangle must satisfy $w \le 10 - 2a$

(2)

(b) Find the value of a that gives the maximum area for such a rectangle.

(5)

Given that the rectangle must be a square,

(c) find the value of a that gives the maximum area for such a square.

(3)

Given that the area of the rectangles is fixed as 36

(d) find the range of possible values for a

(6)

(+S1)

(Total for Question 7 is 17 marks)

Contents

Platinum Mark Scheme

1(a)	The rectangle must lie under the parabola, so maximum width will occur when the top right vertex also lies on the parabola. Ie recognises the symmetry and forms an equation. Allow a suitable sketch as evidence.	**M1**	1
	By symmetry about the line $x=5$, this occurs at $\left(10-a, \frac{1}{2}a(10-a)\right)$, hence width satisfies w „ $10-a-a=10-2a$ * Must be convincing reason.	**A1***	2
		(2)	
(b)	Maximum area must occur for a full width rectangle, ie when $w=10-2a$	**B1**	2
	Thus max area occurs for $A=\frac{1}{2}a(10-a)\times(10-2a)$	**M1**	3
	Attempts $\frac{\mathrm{d}A}{\mathrm{d}a}=\frac{1}{2}(10-2a)\times(10-2a)+\frac{1}{2}(10a-a^2)\times-2\left(=3a^2-30a+50\right)$ and sets $\frac{\mathrm{d}A}{\mathrm{d}a}=0$ and attempts to find a	**M1**	3
	$\Rightarrow 3(a-5)^2-3\times25+50=0\Rightarrow a=5\pm\sqrt{\frac{25}{3}}=5\pm\frac{5}{\sqrt{3}}$ Any correct method to solve the quadratic.	**M1**	3
	(But need $0<a<5$ to give a valid rectangle and as area is zero at either end of this interval so)	**(S+)**	
	max area occurs when $a=5-\frac{5\sqrt{3}}{3}$ (oe simplified)	**A1**	2
		(5)	
(c)	Max square area needs $10-2a=\frac{1}{2}a(10-a)\Rightarrow a=\ldots$ Sets up correct equation.	**M1**	3
	$20-4a=10a-a^2\Rightarrow a^2-14a+20=(a-7)^2-49+20=0$ $\Rightarrow a=7\pm\sqrt{29}$ Solves the quadratic, any valid means.	**dM1**	3
	But need $0<a<5$ (and $\sqrt{29}<7$) so $a=7-\sqrt{29}$ Selects correct root.	**(S+) A1***	3
		(3)	
(d)	If area is 36, then width is given by $w=\frac{36}{\frac{1}{2}a(10-a)}=\frac{72}{10a-a^2}$ (oe) Therefore need solutions to $\frac{72}{10a-a^2}$„ $10-2a$ OR need solutions to $\frac{1}{2}\left(a+\frac{72}{10a-a^2}\right)\left(10-\left(a+\frac{72}{10a-a^2}\right)\right)$„ $\frac{1}{2}a(10-a)$ or other valid inequality	**B1**	1

	in a set up e.g. $10a-a^2 \backslash \ldots 10b-b^2 \Rightarrow (b-a)(b+a)+10(a-b)\ldots 0$ $\Rightarrow 10-(a+b)\ldots 0$ (as $a \neq b$) followed by substitution of $b=a+\frac{72}{10a-a^2}$ This mark is for a correct reasoning of the required inequality, If no reason is given and equation is it is B0, but all other marks are possible.		
	Forms a suitable cubic using the maximum width and height (may be equation or inequation.	**M1**	3
	$\Rightarrow a^3-15a^2+50a-36\ldots 0$ — Correct cubic achieved as equation or inequation.	**A1**	3
	Identifies $(a-1)$ as factor (factor theorem) and attempts to factorise $\Rightarrow (a-1)\left(a^2-14a+36\right)\ldots 0$	**M1**	3
	$a^2-14a+36=(a-7)^2-49+36 \Rightarrow$ CVs are $a=1, 7\pm\sqrt{13}$ — Finds CVs	**M1**	3
	(positive cubic with roots $1<7-\sqrt{13}\,(<5)<7+\sqrt{13}$ (as $3<\sqrt{13}<4$)) So possible values of a are $1 \text{„} \ a \text{„} \ \backslash 7-\sqrt{13}$	**(S+)** **A1**	2
		(6)	
S1	S1 mark: Award S1 for a clear and concise solution that is - fully correct with no S- point or - that scores 13+ and includes an S+ point and no S-.	**(1)**	2
			(16 + 1 marks)

Notes:

(b) S+ for explaining clearly why the root outside $0<a<5$ is rejected.
(c) S+ for justifying the root lies in acceptable domain for a.
(c) S- for a cumbersome strategy. **S+** for justification of roots/which are in valid domain.

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Topic 13

Integration

Bronze, Silver, Gold and Platinum Worksheets for AS Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculators may not be used

The total mark for this section is 30

Q1

Find $\int (12x^5 - 8x^3 + 3)\,dx$, giving each term in its simplest form.

(Total for Question 1 is 4 marks)

Q2

Find

$$\int (12x^5 - 3x^2 + 4x^{\frac{1}{3}})\,dx$$

giving each term in its simplest form.

(Total for Question 2 is 5 marks)

Q3

Find

$$\int (6x^2 + \frac{2}{x^2} + 5)\,dx$$

giving each term in its simplest form.

(Total for Question 3 is 4 marks)

Q4

A curve with equation $y = f(x)$ passes through the point (2, 10). Given that

$$f'(x) = 3x^2 - 3x + 5$$

find the value of f(1).

(Total for Question 4 is 5 marks)

Q5

Use calculus to find the exact value of $\int_1^2 \left(3x^2 + 5 + \frac{4}{x^2}\right) dx$

(Total for Question 5 is 5 marks)

Q6

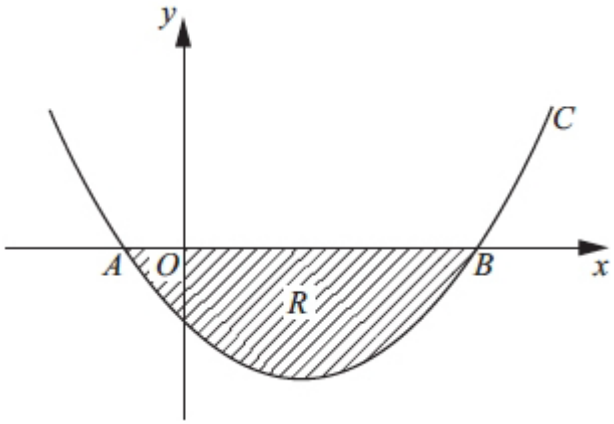

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = (x + 1)(x - 5)$$

The curve crosses the x-axis at the points A and B.

(a) Write down the x-coordinates of A and B.

(1)

The finite region R, shown shaded in Figure 1, is bounded by C and the x-axis.

(b) Use integration to find the area of R.

(6)

(Total for Question 7 is 7 marks)

Bronze Mark Scheme

Q1.

Question Number	Scheme	Marks
	$(I=)\frac{12}{6}x^6-\frac{8}{4}x^4+3x+c$ $=2x^6-2x^4+3x+c$	M1 A1A1A1 **[4]**
	M1 for an attempt to integrate $x^n \to x^{n+1}$ (i.e. ax^6 or ax^4 or ax, where a is any non-zero constant). Also, this M mark can be scored for just the $+c$ (seen at some stage), even if no other terms are correct. 1st A1 for $2x^6$ 2nd A1 for $-2x^4$ 3rd A1 for $3x+c$ (or $3x+k$, etc., any appropriate letter can be used as the constant) Allow $3x^1+c$, but not $\frac{3x^1}{1}+c$. Note that the A marks can be awarded at separate stages, e.g. $\frac{12}{6}x^6-2x^4+3x$ scores 2nd A1 $\frac{12}{6}x^6-2x^4+3x+c$ scores 3rd A1 $2x^6-2x^4+3x$ scores 1st A1 (even though the c has now been lost). Remember that all the A marks are dependent on the M mark. If applicable, isw (ignore subsequent working) after a correct answer is seen. Ignore wrong notation if the intention is clear, e.g. Answer $\int 2x^6-2x^4+3x+c\,dx$.	

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Q2.

Question Number	Scheme	Marks
	$\left(\int =\right)\frac{12x^6}{6},-\frac{3x^3}{3},+\frac{4x^{\frac{4}{3}}}{\frac{4}{3}},(+c)$ $= \underline{2x^6 - x^3 + 3x^{\frac{4}{3}} + c}$	M1A1,A1,A1 A1 5
	Notes	
	M1 for some attempt to integrate: $x^n \to x^{n+1}$ i.e ax^6 or ax^3 or $ax^{\frac{4}{3}}$ or $ax^{1\frac{1}{3}}$, where a is a non zero constant 1st A1 for $\frac{12x^6}{6}$ or better 2nd A1 for $-\frac{3x^3}{3}$ or better 3rd A1 for $\frac{4x^{\frac{4}{3}}}{\frac{4}{3}}$ or better 4th A1 for each term correct and simplified and the $+c$ occurring in the final answer	

Q3.

Question Number	Scheme	Marks
	$\left\{\int\left(6x^2+\frac{2}{x^2}+5\right)dx\right\} = \frac{6x^3}{3}+\frac{2x^{-1}}{-1}+5x\,(+c)$ $= 2x^3 - 2x^{-1}\,;+5x+c$	M1 A1 A1; A1 4
	Notes	
	M1: for some attempt to integrate a term in x: $x^n \to x^{n+1}$ So seeing either $6x^2 \to \pm\lambda x^3$ or $\frac{2}{x^2} \to \pm\mu x^{-1}$ or $5 \to 5x$ is M1. **1st A1**: for a correct un-simplified x^3 or x^{-1} $\left(\text{or } \frac{1}{x}\right)$ term. **2nd A1**: for both x^3 and x^{-1} terms correct and simplified on the same line. Ie. $2x^3 - 2x^{-1}$ or $2x^3 - \frac{2}{x}$. **3rd A1**: for $+5x+c$. Also allow $+5x^1+c$. This needs to be written on the same line. **Ignore the incorrect use of the integral sign in candidates' responses.** **Note**: If a candidate scores M1A1A1A1 and their answer is NOT ON THE SAME LINE then withhold the final accuracy mark.	

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Q4.

Question	Scheme	Marks
	$[f(x)=]\frac{3x^3}{3}-\frac{3x^2}{2}+5x[+c]$ or $\left\{x^3-\frac{3}{2}x^2+5x(+c)\right\}$	M1A1
	$10=8-6+10+c$	M1
	$c=-2$	A1
	$f(1)=1-\frac{3}{2}+5$ "-2" $=\underline{\frac{5}{2}}$ (o.e.)	A1ft (5)
		5 marks
	Notes	
	1st M1 for attempt to integrate $x^n \to x^{n+1}$ 1st A1 all correct, possibly unsimplified. Ignore $+c$ here. 2nd M1 for using $x=2$ and f(2) = 10 to form a linear equation in c. Allow sign errors. They should be substituting into a changed expression 2nd A1 for $c=-2$ 3rd A1ft for $\frac{9}{2}+c$ Follow through their numerical c ($\neq 0$) This mark is dependent on 1st M1 and 1st A1 only.	

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Q5.

Scheme	Marks
$\int\left(3x^2+5+4x^{-2}\right)dx=\frac{3x^3}{3}+5x+\frac{4x^{-1}}{-1}\qquad\left(=x^3+5x-4x^{-1}\right)$	M1 A1 A1
$\left[x^3+5x-4x^{-1}\right]_1^2=(8+10-2)-(1+5-4),\quad=14$	M1, A1 (5)
	5
Integration: Accept any correct version, simplified or not. All 3 terms correct: M1 A1 A1, Two terms correct: M1 A1 A0, One power correct: M1 A0 A0. The given function must be integrated to score M1, and not e.g. $3x^4+5x^2+4$. Limits: M1: Substituting 2 and 1 into a 'changed function' and subtracting, either way round.	

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Q6.

Question Number	Scheme	Marks
(a)	Seeing -1 and 5. (See note below.)	B1 (1)
(b)	$(x+1)(x-5) = \underline{x^2-4x-5}$ or $\underline{x^2-5x+x-5}$	B1
	$\int (x^2-4x-5)\,dx = \frac{x^3}{3} - \frac{4x^2}{2} - 5x \{+c\}$	M: $x^n \to x^{n+1}$ for any one term. 1st A1 at least two out of three terms correctly ft. M1A1ft A1
	$\left[\frac{x^3}{3} - \frac{4x^2}{2} - 5x\right]_{-1}^{5} = (.....) - (.....)$	Substitutes 5 and -1 (or limits from part(a)) into an "integrated function" and subtracts, either way round. dM1
	$\left\{\left(\frac{125}{3} - \frac{100}{2} - 25\right) - \left(-\frac{1}{3} - 2 + 5\right)\right\}$ $= \left(-\frac{100}{3}\right) - \left(\frac{8}{3}\right) = -36$	
	Hence, Area = 36	Final answer must be 36, not -36 A1 (6) [7]

Notes

(a) B1: for -1 and 5. Note that $(-1, 0)$ and $(5, 0)$ are acceptable for B1. Also allow $(0, -1)$ and $(0, 5)$ generously for B1. Note that if a candidate writes down that $A: (5,0)$, $B: (-1,0)$, (ie A and B interchanged,) then B0. Also allow values inserted in the correct position on the x-axis of the graph.

(b) B1 for x^2-4x-5 or $x^2-5x+x-5$. If you believe that the candidate is applying the Way 2 method then $-x^2+4x+5$ or $-x^2+5x-x+5$ would then be fine for B1.

1st M1 for an attempt to integrate meaning that $x^n \to x^{n+1}$ for at least one of the terms.

Note that $-5 \to 5x$ is sufficient for M1.

1st A1 at least two out of three terms correctly ft from their multiplied out brackets.

2nd A1 for correct integration only and no follow through. Ignore the use of a '$+c$'.

Allow 2nd A1 also for $\frac{x^3}{3} - \frac{5x^2}{2} + \frac{x^2}{2} - 5x$. Note that $-\frac{5x^2}{2} + \frac{x^2}{2}$ only counts as one integrated term for the 1st A1 mark. Do not allow any extra terms for the 2nd A1 mark.

2nd M1: Note that this method mark is dependent upon the award of the first M1 mark in part (b). Substitutes 5 and -1 (and not 1 if the candidate has stated $x = -1$ in part (a).) (or the limits the candidate has found from part(a)) into an "integrated function" and subtracts, either way round.

3rd A1: For a final answer of 36, not -36.

Note: An alternative method exists where the candidate states from the outset that Area $(R) = -\int_{-1}^{5} (x^2-4x+5)\,dx$ is detailed in the Appendix.

Question Number	Scheme	Marks
Aliter (b) Way 2	$(x+1)(x-5) = \underline{x^2-4x-5}$ or $\underline{x^2-5x+x-5}$	Can be implied by later working. B1
	$-\int (x^2-4x-5)\,dx = -\frac{x^3}{3} + \frac{4x^2}{2} + 5x \{+c\}$	M: $x^n \to x^{n+1}$ for any one term. 1st A1 any two out of three terms correctly ft. M1A1ft A1
	$\left[-\frac{x^3}{3} + \frac{4x^2}{2} + 5x\right]_{-1}^{5} = (.....) - (.....)$	Substitutes 5 and -1 (or limits from part(a)) into an "integrated function" and subtracts, either way round. dM1
	$\left\{\left(-\frac{125}{3} + \frac{100}{2} + 25\right) - \left(\frac{1}{3} + 2 - 5\right)\right\}$ $= \left(\frac{100}{3}\right) - \left(-\frac{8}{3}\right)$	
	Hence, Area = 36	A1 (6)

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Silver Questions

Calculators may not be used

The total mark for this section is 37

Q1

A curve has equation $y = f(x)$ and passes through the point (4, 22).

Given that

$$f'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7,$$

use integration to find $f(x)$, giving each term in its simplest form.

(Total for Question 1 is 5 marks)

Q2

The gradient of a curve C is given by

$$\frac{dy}{dx} = \frac{(x^2+3)^2}{x^2}, \quad x \neq 0$$

(a) Show that $\frac{dy}{dx} = x^2 + 6 + 9x^{-2}$

(2)

The point (3, 20) lies on C.

(b) Find an equation for the curve C in the form $y = f(x)$.

(6)

(Total for Question 2 is 8 marks)

Q3

A curve has equation $y = f(x)$. The point P with coordinates $(9, 0)$ lies on the curve.

Given that

$$f'(x) = \frac{x+9}{\sqrt{x}} \quad x > 0$$

(a) find $f(x)$.

(6)

(b) Find the x-coordinates of the two points on $y = f(x)$ where the gradient of the curve is equal to 10

(4)

(Total for Question 3 is 10 marks)

Q4

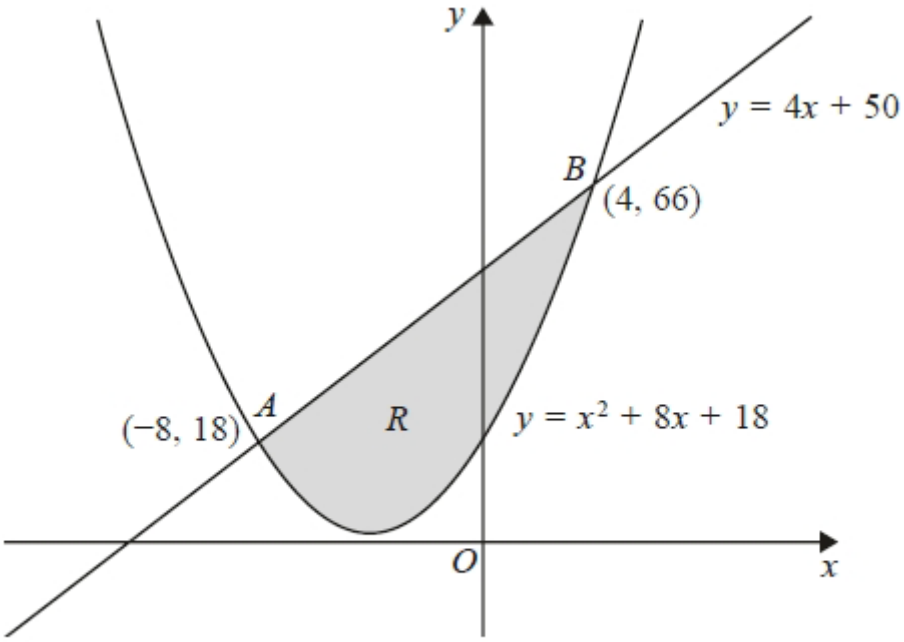


Figure 2

Figure 2 shows the line with equation $y = 4x + 50$ and the curve with equation $y = x^2 + 8x + 18$. The line cuts the curve at the points A $(-8, 18)$ and B $(4, 66)$.

The shaded region R is bounded by the line and the curve, as shown in Figure 2.

Using calculus, find the area of R.

(Total for Question 4 is 6 marks)

Pearson

Q5

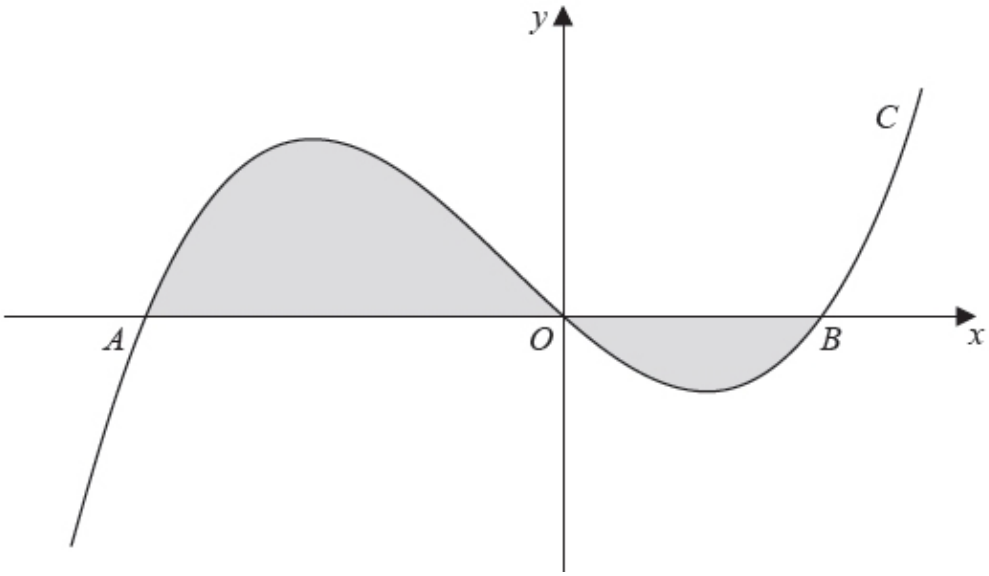


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = x(x + 4)(x - 2)$$

The curve C crosses the x-axis at the origin O and at the points A and B.

(a) Write down the x-coordinates of the points A and B.

(1)

The finite region, shown shaded in Figure 3, is bounded by the curve C and the x-axis.

(b) Use integration to find the total area of the finite region shown shaded in Figure 3.

(7)

(Total for Question 5 is 8 marks)

Pearson

Silver Mark Scheme

Q1.

Question Number	Scheme	Marks
	$(f(x)=)\frac{3x^3}{3}-\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}-7x(+c)$	M1
	$= x^3-2x^{\frac{3}{2}}-7x \quad (+c)$ $f(4)=22 \Rightarrow 22=64-16-28+c$ $c=2$	A1A1 M1 A1cso (5) [5]
	1st M1 for an attempt to integrate (x^3 or $x^{\frac{3}{2}}$ seen). The x term is insufficient for this mark and similarly the $+c$ is insufficient. 1st A1 for $\frac{3}{3}x^3$ or $-\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}$ (An unsimplified or simplified correct form) 2nd A1 for all three x terms correct and simplified… (the simplification may be seen later). The $+c$ is not required for this mark. Allow $-7x^1$, but not $-\frac{7x^1}{1}$. 2nd M1 for an attempt to use $x=4$ and $y=22$ in a changed function (even if differentiated) to form an equation in c. 3rd A1 for $c=2$ with no earlier incorrect work (a final expression for f(x) is not required).	

Q2.

Question Number	Scheme	Marks
(a)	$(x^2+3)^2=x^4+3x^2+3x^2+3^2$	M1
	$\frac{(x^2+3)^2}{x^2}=\frac{x^4+6x^2+9}{x^2}=x^2+6+9x^{-2}$ (*)	A1 cso (2)
(b)	$y=\frac{x^3}{3}+6x+\frac{9}{-1}x^{-1}(+c)$	M1 A1 A1
	$20=\frac{27}{3}+6\times3-\frac{9}{3}+c$	M1
	$c=-4$	A1
	$[y=]\frac{x^3}{3}+6x-9x^{-1}-4$	A1 ft (6)
		(8 marks)

Pearson

Q3.

Question Number	Scheme	Notes	Marks
(a)	$f'(x)=\frac{x+9}{\sqrt{x}}=\frac{x}{\sqrt{x}}+\frac{9}{\sqrt{x}}=x^{\frac{1}{2}}+9x^{-\frac{1}{2}}$	M1: Correct attempt to split into 2 separate terms or fractions. May be implied by one correct term. Divides by $x^{\frac{1}{2}}$ or multiplies by $x^{-\frac{1}{2}}$. A1: $x^{\frac{1}{2}}+9x^{-\frac{1}{2}}$ or equivalent	M1A1
	$f(x)=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+9\frac{x^{\frac{1}{2}}}{\frac{1}{2}}(+c)$	M1: Independent method mark for $x^n \rightarrow x^{n+1}$ on separate terms A1: Allow un-simplified answers. No requirement for + c here	M1A1
	$\frac{(9)^{\frac{3}{2}}}{\frac{3}{2}}+9\frac{(9)^{\frac{1}{2}}}{\frac{1}{2}}+c=0\Rightarrow c=...$	Substitutes $x=9$ and $y=0$ into their integrated expression leading to a value for c. If no c at this stage M0A0 follows unless their method implies that they are correctly finding a constant of integration.	M1
	$f(x)=\frac{2}{3}x^{\frac{3}{2}}+18x^{\frac{1}{2}}-72$	There is no requirement to simplify their f(x) so accept any correct un-simplified form.	A1
			(6)
(b)	$f'(x)=\frac{x+9}{\sqrt{x}}=10\Rightarrow x+9=10\sqrt{x}$	Sets $f'(x)=\frac{x+9}{\sqrt{x}}=10$ and multiplies by $\sqrt{x}$. The terms in x must be in the numerator. E.g. allow $\frac{x+9}{10}=\sqrt{x}$	M1
	They must be setting either the original $f'(x)=10$ or an equivalent correct expression = 10		
	$(\sqrt{x}-9)(\sqrt{x}-1)=0\Rightarrow\sqrt{x}=...$	Correct attempt to solve a relevant 3TQ in $\sqrt{x}$ leading to solution for $\sqrt{x}$. **Dependent on the previous M1.**	dM1
	$x=81, x=1$	Note that the $x=1$ solution could be just written down and is B1but must come from a correct equation.	A1, B1
			(4)
			[10]
Alternative to part (b)	$(\frac{x+9}{\sqrt{x}})^2=10^2\Rightarrow x^2+18x+81=100x$	Sets $\frac{x+9}{\sqrt{x}}=10$, squares and multiplies by x. They must be setting either the original $f'(x)=10$ or an equivalent correct expression = 10	M1
	$(x-81)(x-1)=0\Rightarrow x=..$	Correct attempt to solve a relevant 3TQ leading to solution for x. **Dependent on the previous M1.**	dM1
	$x=81, x=1$	Note that the $x=1$ solution could be just written down and is B1but must come from a correct equation.	A1, B1

Pearson

Q4.

Question Number	Scheme	Marks
	Way 1: Area of trapezium $= \frac{1}{2}(a+b)\times h = \frac{1}{2}(18+66)\times"(4--8)" =$ or may use combination of rectangle and triangle to find trapezium area or may use integration $\int_{-8}^{4}(4x+50)\mathrm{d}x = \left[2x^2+50x\right]_{-8}^{4} = (232)-(128-400) =$	M1
	504 (may be implied by correct final answer)	A1
	$\int x^2+8x+18\mathrm{d}x = \frac{1}{3}x^3+4x^2+18x$	M1 A1
	Use limits 4 **and** -8 $[(\frac{1}{3}(4)^3+4(4)^2+18\times4)-(\frac{1}{3}(-8)^3+4.(-8)^2+18\times(-8))] = A_1$ $(=216)$	
	And uses correct combination of correct areas. Area of region = Area of trapezium $-$ A_1	dM1
	$=504-\left(\frac{472}{3}--\frac{176}{3}\right) = 288$	A1 [6]
	Way 2: Alternative method using "line – curve" **Sets up** $y = 4x+50-(x^2+8x+18)$	M1 A1
	$\int -x^2-4x+32\mathrm{d}x = -\frac{x^3}{3}-2x^2+32x$	M1A1
	Use limits 4 and -8 on this *subtracted* integration	dM1
	Obtains 288	A1 [6]

Pearson

Q5.

Question Number	Scheme	Marks
(a)	Seeing -4 and 2.	B1 (1)
(b)	$x(x+4)(x-2) = \underline{x^3+2x^2-8x}$ or $\underline{x^3-2x^2+4x^2-8x}$ (without simplifying)	B1
	$\int(x^3+2x^2-8x)\,dx = \frac{x^4}{4}+\frac{2x^3}{3}-\frac{8x^2}{2}\{+c\}$ or $\frac{x^4}{4}-\frac{2x^3}{3}+\frac{4x^3}{3}-\frac{8x^2}{2}\{+c\}$	M1A1ft
	$\left[\frac{x^4}{4}+\frac{2x^3}{3}-\frac{8x^2}{2}\right]_{-4}^{0} = (0)-\left(64-\frac{128}{3}-64\right)$ or $\left[\frac{x^4}{4}+\frac{2x^3}{3}-\frac{8x^2}{2}\right]_{0}^{2} = \left(4+\frac{16}{3}-16\right)-(0)$	dM1
	One integral $=\pm 42\frac{2}{3}$ (42.6 or awrt 42.7) or other integral $=\pm 6\frac{2}{3}$ (6.6 or awrt 6.7)	A1
	Hence Area $=$ "their$42\frac{2}{3}$"$+$"their$6\frac{2}{3}$" or Area $=$ "their$42\frac{2}{3}$"$-$"their$6\frac{2}{3}$"	dM1
	$= 49\frac{1}{3}$ or 49.3 or $\frac{148}{3}$ (NOT $-\frac{148}{3}$)	A1
	(An answer of $= 49\frac{1}{3}$ may not get the final two marks – check solution carefully)	(7) [8]
	Notes for Question	
(a)	B1: Need both -4 and 2. May see $(-4,0)$ and $(2,0)$ (correct) but allow $(0,-4)$ and $(0,2)$ or $A=-4$, $B=2$ or indeed any indication of -4 and 2 – check graph also	
(b)	B1: Multiplies out cubic correctly (terms may not be collected, but if they are, mark collected terms here) M1: Tries to integrate their expansion with $x^n \to x^{n+1}$ for at least one of the terms A1ft: completely correct integral **following through** from their CUBIC expansion (if only quadratic or quartic this is A0) dM1: (dependent on previous M) substituting EITHER $-a$ and 0 and subtracting either way round OR similarly for 0 and b. **If their limits $-a$ and b are used in ONE integral, apply the Special Case below.** A1: Obtain either $\pm 42\frac{2}{3}$ (or 42.6 or awrt 42.7) *from the integral from -4 to 0* or $\pm 6\frac{2}{3}$ (6.6 or awrt 6.7) *from the integral from 0 to 2;* NO follow through on their cubic (allow decimal or improper equivalents $\frac{128}{3}$ or $\frac{20}{3}$) isw such as subtracting from rectangles. This will be penalized in the next two marks, which will be M0A0. dM1 (depends on first method mark) **Correct method to obtain shaded area** so adds two positive numbers (areas) together or uses their **positive** value minus **their negative** value, **obtained from two separate definite integrals.** A1: Allow 49.3, 49.33, 49.333 etc. Must follow correct logical work with **no errors** seen. For full marks on this question there must be two definite integrals, from -4 to 0 and from 0 to 2, though the evaluations for 0 may not be seen. **(Trapezium rule gets no marks after first two B marks)**	
(b)	**Special Case: one integral only from $-a$ to b:** B1M1A1 available as before, then $\left[\frac{x^4}{4}+\frac{2x^3}{3}-\frac{8x^2}{2}\right]_{-4}^{2} = (4+\frac{16}{3}-16)-\left(64-\frac{128}{3}-64\right) = -6\frac{2}{3}+42\frac{2}{3}=\ldots$. dM1 for correct use of their limits $-a$ and b and **subtracting** either way round. A1 for 36: NO follow through. Final M and A marks not available. Max 5/7 for part (b)	

Pearson

Gold Questions

Calculators may not be used

The total mark for this section is 35

Q1

Given that $\frac{6x+3x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $6x^p + 3x^q$,

(a) write down the value of p and the value of q

(2)

Given that $\frac{dy}{dx} = \frac{6x+3x^{\frac{5}{2}}}{\sqrt{x}}$ and that $y = 90$ when $x = 4$,

(b) find y in terms of x, simplifying the coefficient of each term.

(5)

(Total for Question 1 is 7 marks)

Q2

$$\frac{dy}{dx} = 6x^{-\frac{1}{2}} + x\sqrt{x} \text{ where } x > 0$$

Given that $y = 37$ at $x = 4$, find y in terms of x, giving each term in its simplest form.

(Total for Question 2 is 7 marks)

Q3

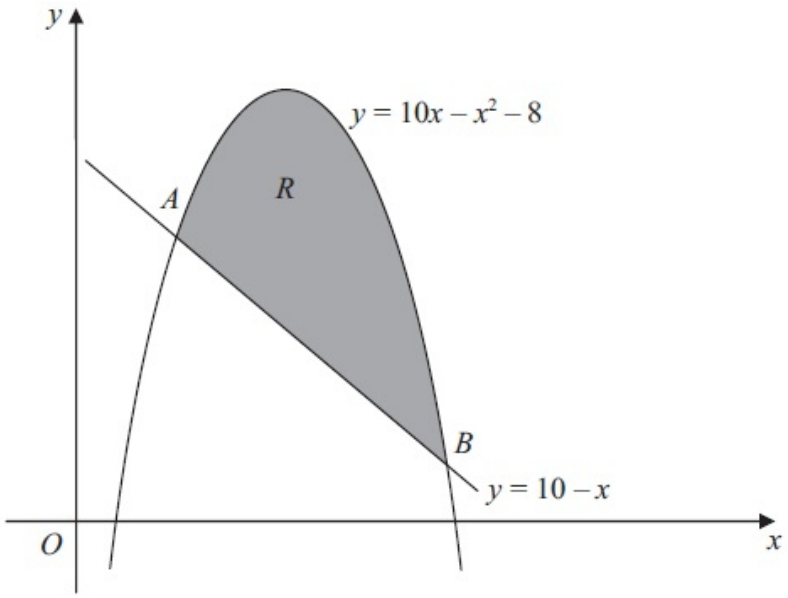


Figure 2

Figure 2 shows the line with equation $y = 10 - x$ and the curve with equation $y = 10x - x^2 - 8$
The line and the curve intersect at the points A and B, and O is the origin.

(a) Calculate the coordinates of A and the coordinates of B.

(5)

The shaded area R is bounded by the line and the curve, as shown in Figure 2.

(b) Calculate the exact area of R.

(7)

(Total for Question 3 is 12 marks)

Pearson

Q4

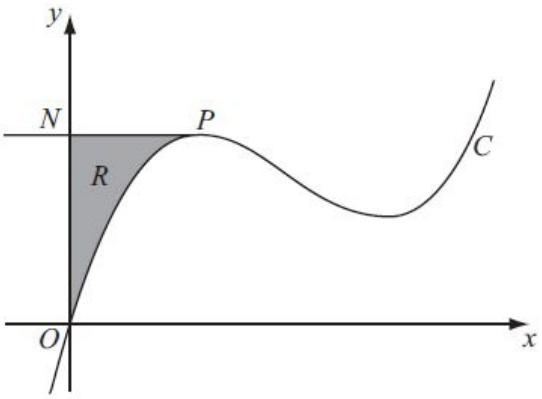


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + kx \ ,$$

where k is a constant.

The point P on C is the maximum turning point.

Given that the x-coordinate of P is 2,

(a) show that $k = 28$.

(3)

The line through P parallel to the x-axis cuts the y-axis at the point N.
The region R is bounded by C, the y-axis and PN, as shown shaded in Figure 2.

(b) Use calculus to find the exact area of R.

(6)

(Total for Question 4 is 9 marks)

Pearson

Gold Mark Scheme

Q1.

Question Number	Scheme	Marks
(a)	$p=\frac{1}{2},\ q=2$ or $6x^{\frac{1}{2}},\ 3x^2$	B1, B1 (2)
(b)	$\frac{6x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}+\frac{3x^3}{3}$ $\left(=4x^{\frac{3}{2}}+x^3\right)$ $x=4, y=90$: $32+64+C=90 \Rightarrow C=-6$ $y=4x^{\frac{3}{2}}+x^3+"their-6"$	M1 A1ft M1 A1 A1 (5)
		7
	Notes	
	(a) Accept any equivalent answers, e.g. $p = 0.5$, $q = 4/2$ (b) 1st M: Attempt to integrate $x^n \rightarrow x^{n+1}$ (for either term) 1st A: ft their p and q, but terms need not be simplified ($+C$ not required for this mark) 2nd M: Using $x = 4$ and $y = 90$ to form an equation in C. 2nd A: cao 3rd A: answer as shown with simplified correct coefficients and powers – but follow through their value for C If there is a 'restart' in part (b) it can be marked independently of part (a), but marks for part (a) cannot be scored for work seen in (b). Numerator and denominator integrated separately: First M mark **cannot** be awarded so only mark available is second M mark. So 1 out of 5 marks.	

Pearson

Q2.

Question Number	Scheme	Marks
	$\frac{dy}{dx} = 6x^{-\frac{1}{2}} + x\sqrt{x}$	
	$x\sqrt{x} = x^{\frac{3}{2}}$	B1
	$x^n \to x^{n+1}$	M1
	$y = \frac{6}{\frac{1}{2}}x^{\frac{1}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}}(+c)$	A1, A1
	Use $x=4, y=37$ to give equation in c, $37 = 12\sqrt{4} + \frac{2}{5}(\sqrt{4})^5 + c$	M1
	$\Rightarrow c = \frac{1}{5}$ or equivalent eg. 0.2	A1
	$(y) = 12x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{5}$	A1
		(7 marks)

B1 $x\sqrt{x} = x^{\frac{3}{2}}$. This may be implied by $+\frac{x^{\frac{5}{2}}}{\frac{5}{2}}$ oe in the subsequent work.

M1 $x^n \to x^{n+1}$ in at least one case so see either $x^{\frac{1}{2}}$ or $x^{\frac{5}{2}}$ or both

A1 One term integrated correctly. It does not have to be simplified Eg. $\frac{6}{\frac{1}{2}}x^{\frac{1}{2}}$ or $+\frac{x^{\frac{5}{2}}}{\frac{5}{2}}$.

No need for $+c$

A1 Other term integrated correctly. See above. No need to simplify nor for $+c$. Need to see $\frac{6}{\frac{1}{2}}x^{\frac{1}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}}$ or a simplified correct version

M1 Substitute $x=4, y=37$ to produce an equation in c.

A1 Correctly calculates $c = \frac{1}{5}$ or equivalent e.g. 0.2

A1 cso $y = 12x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{5}$. Allow $5y = 60x^{\frac{1}{2}} + 2x^{\frac{5}{2}} + 1$ and accept fully simplified equivalents.

e.g. $y = \frac{1}{5}(60x^{\frac{1}{2}} + 2x^{\frac{5}{2}} + 1)$, $y = 12\sqrt{x} + \frac{2}{5}\sqrt{x^5} + \frac{1}{5}$

Pearson

Q3.

Question number	Scheme		Marks
Method 1 (a)	Puts $10-x=10x-x^2-8$ and rearranges to give three term quadratic	Or puts $y=10(10-y)-(10-y)^2-8$ and rearranges to give three term quadratic	M1
	Solves their "$x^2-11x+18=0$" using acceptable method as in general principles to give $x=$	Solves their "$y^2-9y+8=0$" using acceptable method as in general principles to give $y=$	M1
	Obtains $x=2$, $x=9$ (may be on diagram or in part (b) in limits)	Obtains $y=8$, $y=1$ (may be on diagram)	A1
	Substitutes their x into a given equation to give $y=$ (may be on diagram)	Substitutes their y into a given equation to give $x=$ (may be on diagram or in part (b))	M1
	$y=8$, $y=1$	$x=2$, $x=9$	A1 (5)
(b)	$\int(10x-x^2-8)\,dx=\frac{10x^2}{2}-\frac{x^3}{3}-8x\,\{+c\}$		M1 A1 A1
	$\left[\frac{10x^2}{2}-\frac{x^3}{3}-8x\right]_2^9=(.....)-(.....)$		dM1
	$=90-\frac{4}{3}=88\frac{2}{3}$ or $\frac{266}{3}$		
	Area of trapezium $=\frac{1}{2}(8+1)(9-2)=31.5$		B1
	So area of R is $88\frac{2}{3}-31.5=57\frac{1}{6}$ or $\frac{343}{6}$		M1A1 cao (7)
			12 marks
Notes (a)	First **M1**: See scheme Second **M1**: See notes relating to solving quadratics Third **M1**: This may be awarded if one substitution is made Two correct Answers following tables of values, or from Graphical calculator are 5/5 **Just one pair of correct coordinates – no working or from table is M0M0A0M1A0**		
(b)	**M1**: $x^n\rightarrow x^{n+1}$ for any one term. **1st A1**: at least two out of three terms correct **2nd A1**: All three correct **dM1**: Substitutes 9 and 2 (or limits from part(a)) into an "integrated function" and subtracts, either way round (NB: If candidate **changes all signs** to get $\int(-10x+x^2+8)\,dx=-\frac{10x^2}{2}+\frac{x^3}{3}+8x\,\{+c\}$ This is M1 A1 A1 Then uses limits dM1 and trapezium is B1 Needs to *change sign of value obtained* from integration for final M1A1 so $-88\frac{2}{3}-31.5$ is M0A0) **B1**: Obtains 31.5 for area under line using any correct method (could be integration) or triangle minus triangle $\frac{1}{2}\times8\times8-\frac{1}{2}$ or rectangle plus triangle [may be implied by correct 57 1/6] **M1**: Their Area under curve – Their Area under line (if integrate both need same limits) **A1**: Accept 57.16recurring but not 57.16 **PTO for Alternative method**		

Pearson

Method 2 for (b)	Area of R $= \int_2^9 (10x - x^2 - 8) - (10 - x)\,dx$ $\int_2^9 -x^2 + 11x - 18\,dx$ $= -\frac{x^3}{3} + \frac{11x^2}{2} - 18x \{+c\}$ $\left[-\frac{x^3}{3} + \frac{11x^2}{2} - 18x\right]_2^9 = (\ldots\ldots) - (\ldots\ldots)$	3^{rd} M1 (in (b)): Uses difference between two functions in integral. M: $x^n \to x^{n+1}$ for any one term. A1 at least two out of these three simplified terms Correct integration. (Ignore + c). Substitutes 9 and 2 (or limits from part(a)) into an "integrated function" and subtracts, either way round.	M1 A1 A1 dM1
	This mark is implied by final answer which rounds to 57.2 ***See above working(allow bracketing errors) to decide to award 3^{rd} M1 mark for (b) here:*** $40.5 - (-16\frac{1}{2}) = 57\frac{1}{2}$ cao		B1 M1 A1 (7)
Special case of above method	$\int_2^9 x^2 - 11x + 18\,dx = \frac{x^3}{3} - \frac{11x^2}{2} + 18x \{+c\}$ $\left[\frac{x^3}{3} - \frac{11x^2}{2} + 18x\right]_2^9 = (\ldots\ldots) - (\ldots\ldots)$ This mark is implied by final answer which rounds to 57.2 (not -57.2) Difference of functions implied (see above expression) $40.5 - (-16\frac{1}{2}) = 57\frac{1}{2}$ cao		M1A1A1 DM1 B1 M1 A1 (7)
Special Case 2	Integrates expression in y e.g. "$y^2 - 9y + 8 = 0$": This can have first M1 in part (b) and no other marks. (It is not a method for finding this area)		
Notes	Take away trapezium again having used Method 2 loses last two marks Common Error: Integrates $-x^2 + 9x - 18$ is likely to be M1A1A0dM1B0M1A0 Integrates $2 - 11x - x^2$ is likely to e M1A0A0dM1B0M1A0 Writing $\int_2^9 (10x - x^2 - 8) - (10 - x)\,dx$ only earns final M mark		

Pearson

Q4.

Question Number	Scheme	Marks
	(a) $\frac{dy}{dx} = 3x^2 - 20x + k$ (Differentiation is required)	M1 A1
	At $x = 2$, $\frac{dy}{dx} = 0$, so $12 - 40 + k = 0$ $k = 28$ (*)	A1 cso
	N.B. The '= 0' must be seen at some stage to score the final mark.	
	Alternatively: (using $k = 28$)	
	$\frac{dy}{dx} = 3x^2 - 20x + 28$ (M1 A1)	
	'Assuming' $k = 28$ only scores the final cso mark if there is justification that $\frac{dy}{dx} = 0$ at $x = 2$ represents the maximum turning point.	(3)
	(b) $\int (x^3 - 10x^2 + 28x)\,dx = \frac{x^4}{4} - \frac{10x^3}{3} + \frac{28x^2}{2}$ Allow $\frac{kx^2}{2}$ for $\frac{28x^2}{2}$	M1 A1
	$\left[\frac{x^4}{4} - \frac{10x^3}{3} + 14x^2\right]_0^2 = \ldots$ $\left(= 4 - \frac{80}{3} + 56 = \frac{100}{3}\right)$	M1
	(With limits 0 to 2, substitute the limit 2 into a 'changed function')	
	y-coordinate of $P = 8 - 40 + 56 = 24$ Allow if seen in part (a)	B1
	(The B1 for 24 may be scored by implication from later working)	
	Area of rectangle $= 2 \times$ (their y-coordinate of P)	
	Area of R = (their 48) $-\left(\text{their } \frac{100}{3}\right)$ $= \frac{44}{3}$ $\left(14\frac{2}{3} \text{ or } 14.\dot{6}\right)$	M1 A1
	If the subtraction is the 'wrong way round', the final A mark is lost.	(6)
		9

	(a) M: $x^n \to cx^{n-1}$ (c constant, $c \neq 0$) for one term, seen in part (a). (b) 1st M: $x^n \to cx^{n+1}$ (c constant, $c \neq 0$) for one term. Integrating the gradient function loses this M mark. 2ndM: Requires use of limits 0 and 2, with 2 substituted into a 'changed function'. (It may, for example, have been differentiated). Final M: Subtract their values either way round. This mark is dependent on the use of calculus and a correct method attempt for the area of the rectangle. A1: Must be exact, not 14.67 or similar, but isw after seeing, say, $\frac{44}{3}$. Alternative: (effectively finding area of rectangle by integration) $\int \{24 - (x^3 - 10x^2 + 28x)\}dx = 24x - \left(\frac{x^4}{4} - \frac{10x^3}{3} + \frac{28x^2}{2}\right)$, etc. This can be marked equivalently, with the 1st A being for integrating the same 3 terms correctly. The 3rd M (for subtraction) will be scored at the same stage as the 2nd M. If the subtraction is the 'wrong way round', the final A mark is lost.

Pearson

Platinum Questions

Calculators may not be used

The total mark for this section is 10

1 (a) On the same diagram, sketch $y = x$ and $y = \sqrt{x}$, for $x \geq 0$, and mark clearly the coordinates of the points of intersection of the two graphs.

(2)

(b) With reference to your sketch, explain why there exists a value a of x ($a > 1$) such that

$$\int_0^a x \, \mathrm{d}x = \int_0^a \sqrt{x} \, \mathrm{d}x .$$

(2)

(c) Find the exact value of a.

(4)

(d) Hence, or otherwise, find a non-constant function f(x) and a constant b ($b \neq 0$) such that

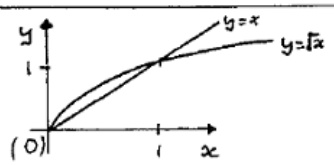
$$\int_{-b}^b \mathrm{f}(x) \, \mathrm{d}x = \int_{-b}^b \sqrt{[\mathrm{f}(x)]} \, \mathrm{d}x .$$

(2)

(Total for Question 1 is 10 marks)

Platinum Mark Scheme

(2.) (a)

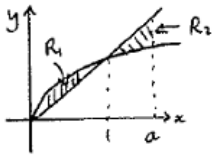


Relative shapes — B1

0 or (0,0) implied and (1,1) — B1

On axes ⇒ OK. (2)

(b) $R_1 = \frac{1}{6}$



As a increases from 1 R_2 increases

Choose a so that $R_2 = R_1$ then areas are the same.

Diagram with regions or mention of areas. — B1g

Full argument — B1h (2)

(c) $\int_0^a x\,dx = \int_0^a x^{\frac{1}{2}}dx \Rightarrow \left[\frac{x^2}{2}\right]_0^a = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^a$

Attempt both integrals – one correct — M1

A correct equation in a — A1

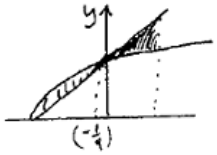
$\Rightarrow \frac{a^2}{2} = \frac{2}{3}a^{\frac{3}{2}}$

$\Rightarrow a^{\frac{3}{2}}\left(3a^{\frac{1}{2}} - 4\right) = 0 \quad \rightarrow a^{\frac{1}{2}} = \frac{4}{3}$ o.e.

Attempt to solve $\rightarrow a^{\wedge} = k$ — M1

$a = \frac{16}{9}$ — A1 (4)

(d)



Translate $\frac{1}{2}a$ ←.

$f(x) = x + \frac{8}{9}$

$x + \frac{a}{2} = f(x)$ (Any suitable f(x) & b) — B1↑

$b = \frac{8}{9}$

$\frac{a}{2} = b$ ↑ their a. — B1↑ (2)

(10)

S.C. if $b = \beta$ and $f(x) = x + \beta$ score B1 only

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Topic 14

Exponentials and Logarithms

Bronze, Silver, Gold and Platinum Worksheets for AS Level Mathematics

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Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculators may not be used

The total mark for this section is 25

Q1

Find the exact solution to the equation

$\ln x + \ln 3 = \ln 6,$

(Total for Question 1 is 2 marks)

Q2

Sketch the graph of

$$y = 3^x, \quad x \in \mathbb{R}$$

showing the coordinates of any points at which the graph crosses the axes.

(Total for Question 2 is 2 marks)

Q3

Find the value of x for which

$$\log_3(x - 2) = -1.$$

(Total for Question 2 is 2 marks)

Q4

Find the exact solutions, in their simplest form, to the equations

(a) $e^{3x-9} = 8$

(3)

(b) $\ln(2y + 5) = 2 + \ln(4 - y)$

(4)

(Total for Question 4 is 7 marks)

Q5

Given that

$$2\log_2 (x + 15) - \log_2 x = 6$$

(a) Show that

$x^2 - 34x + 225 = 0$

(5)

(b) Hence, or otherwise, solve the equation

$$2\log_2(x + 15) - \log_2 x = 6$$

(2)

(Total for Question 5 is 7 marks)

Q6

Water is being heated in an electric kettle. The temperature, θ °C, of the water t seconds after the kettle is switched on, is modelled by the equation

$$\theta = 120 - 100e^{-\lambda t}, \quad 0 \leqslant t \leqslant T$$

(a) State the value of θ when $t = 0$

(1)

Given that the temperature of the water in the kettle is 70°C when $t = 40$,

(b) find the exact value of λ, giving your answer in the form $\frac{\ln a}{b}$, where a and b are integers.

(4)

(Total for Question 6 is 5 marks)

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Bronze Mark Scheme

Q1.

Question Number	Scheme	Marks
	$\ln 3x = \ln 6$ or $\ln x = \ln\left(\frac{6}{3}\right)$ or $\ln\left(\frac{3x}{6}\right) = 0$ $x = 2$ (only this answer)	M1 A1 (cso) (2) **(2 marks)**
Notes: (a) Answer $x = 2$ with no working or no incorrect working seen: M1A1 Note: $x = 2$ from $\ln x = \frac{\ln 6}{\ln 3} = \ln 2$ M0A0 $\ln x = \ln 6 - \ln 3 \Rightarrow x = e^{(\ln 6 - \ln 3)}$ allow M1, $x = 2$ (no wrong working) A1		

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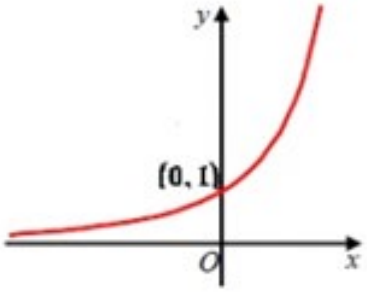
Q2.

Question Number	Scheme		Marks
	Graph of $y = 3^x$ and solving $3^{2x} - 9(3^x) + 18 = 0$		
		At least two of the three criteria correct. **(See notes below.)**	B1
		All three criteria correct. **(See notes below.)**	B1
		Criteria number 1: Correct shape of curve for $x \geq 0$ and at least touches the positive y-axis. **Criteria number 2**: Correct shape of curve for $x < 0$. Must not touch the x-axis or have any turning points. **Criteria number 3**: $(0, 1)$ stated or in a table or 1 marked on the y-axis. Allow $(1, 0)$ rather than $(0, 1)$ if marked in the "correct" place on the y-axis.	
			[2]
			Total 2

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Q3.

Question Number	Scheme	Marks
	$(x-2)=3^{-1}$ $\quad$ $(x-2)=3^{-1}$ or $\frac{1}{3}$	M1 oe
	$x\left\{=\frac{1}{3}+2\right\}=2\frac{1}{3}$ $\quad$ $2\frac{1}{3}$ or $\frac{7}{3}$ or $2.\dot{3}$ or awrt 2.33	A1
		[2]
	M1: Is for correctly eliminating log out of the equation. **Eg 1:** $\log_3(x-2)=\log_3(\frac{1}{3}) \Rightarrow x-2=\frac{1}{3}$ only gets M1 when the logs are correctly removed. **Eg 2:** $\log_3(x-2)=-\log_3(3) \Rightarrow \log_3(x-2)+\log_3(3)=0 \Rightarrow \log_3(3(x-2))=0$ $\Rightarrow 3(x-2)=3^0$ only gets M1 when the logs are correctly removed, but $3(x-2)=0$ would score M0. **Note:** $\log_3(x-2)=-1 \Rightarrow \log_3\left(\frac{x}{2}\right)=-1$ $\quad$ $x=3^{-1}$ would score M0 for incorrect use of logs. **Alternative: changing base** $\frac{\log_{10}(x-2)}{\log_{10}3}=-1 \Rightarrow \log_{10}(x-2)=-\log$ $\quad$ $\log_{10}(x-2)+\log_{10}3=0$ $\Rightarrow \log_{10}3(x-2)=0 \Rightarrow 3(x-2)=10^0$. point M1 is scored. A correct answer in (b) without any workir es M1A1.	

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Q4.

Question Number	Scheme	Marks
(a)	$e^{3x-9}=8 \Rightarrow 3x-9=\ln 8$ $\Rightarrow x=\dfrac{\ln 8+9}{3}, =\ln 2+3$	M1 A1, A1 (3)
(b)	$\ln(2y+5)=2+\ln(4-y)$ $\ln\left(\dfrac{2y+5}{4-y}\right)=2$ $\left(\dfrac{2y+5}{4-y}\right)=e^2$ $2y+5=e^2(4-y) \Rightarrow 2y+e^2y=4e^2-5 \Rightarrow y=\dfrac{4e^2-5}{2+e^2}$	 M1 M1 dM1, A1 (4) **7 marks**

(a)

M1 Takes ln's of both sides and uses the power law. You may even accept candidates taking logs of both sides

A1 A correct unsimplified answer $\dfrac{\ln 8+9}{3}$ or equivalent such as $\dfrac{\ln 8e^9}{3}$, $3+\ln\left(\sqrt[3]{8}\right)$, $\dfrac{\log 8}{3\log e}+3$ or even 3.69

A1 cso $\ln 2+3$. Accept $\ln 2e^3$

.....

Alt I (a)

$e^{3x-9}=8 \Rightarrow \dfrac{e^{3x}}{e^9}=8 \Rightarrow e^{3x}=8e^9 \Rightarrow 3x=\ln\left(8e^9\right)$ for M1 (Condone slips on index work and lack of bracket)

Alt II (a)

$e^{x-3}=\sqrt[3]{8} \Rightarrow x-3=\ln\left(\sqrt[3]{8}\right)$ for M1 (Condone slips on the 9. Eg $e^{x-9}=2 \Rightarrow x-9=\ln 2$)

.....

(b)

M1 Uses a correct method to combine two terms to create a single ln term.

Eg. Score for $2+\ln(4-y)=\ln\left(e^2(4-y)\right)$ or $\ln(2y+5)-\ln(4-y)=\ln\left(\dfrac{2y+5}{4-y}\right)$

Condone slips on the signs and coefficients of the terms, but not on the e^2

M1 Scored for an attempt to undo the ln's to get an equation in y This must be awarded after an attempt to combine the ln terms. Award for $\ln(g(y))=2 \Rightarrow g(y)=e^2$ and can be scored eg where $g(y)=2y+5-(4-y)$

It cannot be awarded for just $2y+5=e^2+4-y$ where the candidate attempts to undo term by term

dM1 Dependent upon **both** previous M's. It is for making y the subject. Expect to see both terms in y collected and factorised (may be implied) before reaching $y=$. Condone slips, for eg, on signs. $y=2.615$ scores this.

A1 $y=\dfrac{4e^2-5}{2+e^2}$ or equivalent such as $y=4-\dfrac{13}{2+e^2}$ ISW after you see the correct answer.

.....

Special Case: $\ln(2y+5)-\ln(4-y)=2 \Rightarrow \dfrac{\ln(2y+5)}{\ln(4-y)}=2 \Rightarrow \dfrac{2y+5}{4-y}=e^2 \Rightarrow$ Correct answer score M0 M1 M1 A0

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Q5.

Question Number	Scheme		Marks
(a)	$2\log(x+15)=\log(x+15)^2$		B1
	$\log(x+15)^2-\log x=\log\dfrac{(x+15)^2}{x}$	Correct use of $\log a-\log b=\log\dfrac{a}{b}$	M1
	$2^6=64$ or $\log_2 64=6$	64 used in the correct context	B1
	$\log_2\dfrac{(x+15)^2}{x}=6\Rightarrow\dfrac{(x+15)^2}{x}=64$	Removes logs correctly	M1
	$\Rightarrow x^2+30x+225=64x$ *or* $x+30+225x^{-1}=64$	Must see expansion of $(x+15)^2$ to score the final mark.	
	$\therefore x^2-34x+225=0$ *		A1
			(5)
(b)	$(x-25)(x-9)=0\Rightarrow x=25$ or $x=9$	M1: Correct attempt to solve the **given** quadratic as far as $x=$... A1: Both 25 and 9	M1 A1
			(2)
			[7]

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Q6.

Question Number	Scheme	Marks
(a)	$(\theta =)20$	B1 **(1)**
(b)	Sub $t = 40, \theta = 70 \Rightarrow 70 = 120 - 100e^{-40\lambda}$	
	$\Rightarrow e^{-40\lambda} = 0.5$	M1A1
	$\Rightarrow \lambda = \frac{\ln 2}{40}$	M1A1
		(4)
Alt (b)	Sub $t = 40, \theta = 70 \Rightarrow 100e^{-40\lambda} = 50$	
	$\Rightarrow \ln 100 - 40\lambda = \ln 50$	M1A1
	$\Rightarrow \lambda = \frac{\ln 100 - \ln 50}{40} = \frac{\ln 2}{40}$	M1A1
		(4)

(a)

B1 Sight of $(\theta =)20$

(b)

M1 Sub $t = 40, \theta = 70 \Rightarrow 70 = 120 - 100e^{-40\lambda}$ and proceed to $e^{\pm 40\lambda} = A$ where A is a constant. Allow sign slips and copying errors.

A1 $e^{-40\lambda} = 0.5$ or $e^{40\lambda} = 2$ or exact equivalent

M1 For undoing the e's by taking ln's and proceeding to $\lambda = ..$

May be implied by the correct decimal answer awrt 0.017 or $\lambda = \frac{\ln 0.5}{-40}$

A1 cso $\lambda = \frac{\ln 2}{40}$

Accept equivalents in the form $\frac{\ln a}{b}$, $a, b \in \mathbb{Z}$ such as $\lambda = \frac{\ln 4}{80}$

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Silver Questions

Calculators may not be used

The total mark for this section is 28

Q1

Find the exact solutions, in their simplest form, to the equations

(a) $2 \ln (2x + 1) - 10 = 0$

(2)

(b) $3^x e^{4x} = e^7$

(4)

(Total for Question 1 is 6 marks)

Q2

A student was asked to give the exact solution to the equation

$$2^{2x+4} - 9(2^x) = 0$$

The student's attempt is shown below:

$2^{2x+4} - 9(2^x) = 0$

$2^{2x} + 2^4 - 9(2^x) = 0$

Let $2^x = y$

$y^2 - 9y + 8 = 0$

$(y - 8)(y - 1) = 0$

$y = 8$ or $y = 1$

So $x = 3$ or $x = 0$

(a) Identify the two errors made by the student.

(2)

(b) Find the exact solution to the equation.

(2)

(Total for Question 2 is 4 marks)

Q3

(a) Given that

$$2\log_3(x-5)-\log_3(2x-13)=1$$

show that $x^2 - 16x + 64 = 0$.

(5)

(b) Hence, or otherwise, solve $2\log_3(x-5)-\log_3(2x-13)=1$

(2)

(Total for Question 1 is 7 marks)

Q4

Given that a and b are positive constants, solve the simultaneous equations

$$a = 3b,$$

$$\log_3 a + \log_3 b = 2.$$

Give your answers as exact numbers.

(Total for Question 2 is 6 marks)

Q5

The mass, m grams, of a leaf t days after it has been picked from a tree is given by

$$m = pe^{-kt}$$

where k and p are positive constants.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

(a) Write down the value of p.

(1)

(b) Show that $k = \frac{1}{4}\ln 3$

(4)

(Total for Question 5 is 5 marks)

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Silver Mark Scheme

Q1.

Question Number	Scheme	Marks
(a)	$2\ln(2x+1)-10=0 \Rightarrow \ln(2x+1)=5 \Rightarrow 2x+1=e^5 \Rightarrow x=..$ $\Rightarrow x=\dfrac{e^5-1}{2}$	M1 A1 (2)
(b)	$3^x e^{4x}=e^7 \Rightarrow \ln(3^x e^{4x})=\ln e^7$ $\ln 3^x+\ln e^{4x}=\ln e^7 \Rightarrow x\ln 3+4x\ln e=7\ln e$ $x(\ln 3+4)=7 \Rightarrow x=...$ $x=\dfrac{7}{(\ln 3+4)}$ oe	M1,M1 dM1 A1 (4) **6 marks**
Alt 1 (b)	$3^x e^{4x}=e^7 \Rightarrow 3^x=\dfrac{e^7}{e^{4x}}$ $3^x=e^{7-4x} \Rightarrow x\ln 3=(7-4x)\ln e$ $x(\ln 3+4)=7 \Rightarrow x=...$ $x=\dfrac{7}{(\ln 3+4)}$	M1,M1 dM1 A1 (4)
Alt 2 (b) Using logs	$3^x e^{4x}=e^7 \Rightarrow \log(3^x e^{4x})=\log e^7$ $\log 3^x+\log e^{4x}=\log e^7 \Rightarrow x\log 3+4x\log e=7\log e$ $x(\log 3+4\log e)=7\log e \Rightarrow x=...$ $x=\dfrac{7\log e}{(\log 3+4\log e)}$	M1, M1 dM1 A1 (4)
Alt 3 (b) Using $\log_3$	$3^x e^{4x}=e^7 \Rightarrow 3^x=\dfrac{e^7}{e^{4x}}$ $3^x=e^{7-4x} \Rightarrow x=(7-4x)\log_3 e$ $x(1+4\log_3 e)=7\log_3 e \Rightarrow x=...$ $x=\dfrac{7\log_3 e}{(1+4\log_3 e)}$	M1,M1 dM1 A1 (4)
Alt 4 (b) Using $3^x=e^{x\ln 3}$	$3^x e^{4x}=e^7 \Rightarrow e^{x\ln 3}e^{4x}=e^7$ $\Rightarrow e^{x\ln 3+4x}=e^7, \Rightarrow x\ln 3+4x=7$ $x(\ln 3+4)=7 \Rightarrow x=...$ $x=\dfrac{7}{(\ln 3+4)}$	M1,M1 dM1 A1 (4)

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(a)

M1 Proceeds from $2\ln(2x+1)-10=0$ to $\ln(2x+1)=5$ before taking exp's to achieve x in terms of e^5

Accept for M1 $2\ln(2x+1)-10=0 \Rightarrow \ln(2x+1)=5 \Rightarrow x=f(e^5)$

Alternatively they could use the power law before taking exp's to achieve x in terms of $\sqrt{e^{10}}$

$2\ln(2x+1)=10 \Rightarrow \ln(2x+1)^2=10 \Rightarrow (2x+1)^2=e^{10} \Rightarrow x=g\left(\sqrt{e^{10}}\right)$

A1 cso. Accept $x=\dfrac{e^5-1}{2}$ or other exact simplified alternatives such as $x=\dfrac{e^5}{2}-\dfrac{1}{2}$. Remember to isw.

The decimal answer of 73.7 will score M1A0 unless the exact answer has also been given.

The answer $\dfrac{\sqrt{e^{10}}-1}{2}$ does not score this mark unless simplified. $x=\dfrac{\pm e^5-1}{2}$ is M1A0

(b)

M1 Takes ln's or logs of both sides and applies the addition law.

$\ln(3^x e^{4x})=\ln 3^x+\ln e^{4x}$ or $\ln(3^x e^{4x})=\ln 3^x+4x$ is evidence for the addition law

If the e^{4x} was 'moved' over to the right hand side score for either e^{7-4x} or the subtraction law.

$\ln\dfrac{e^7}{e^{4x}}=\ln e^7-\ln e^{4x}$ or $3^x e^{4x}=e^7 \Rightarrow 3^x=\dfrac{e^7}{e^{4x}} \Rightarrow 3^x=e^{7-4x}$ is evidence of the subtraction law

M1 Uses the power law of logs (seen at least once in a term with x as the index Eg $3^x, e^{4x}$ or e^{7-4x}).

$\ln 3^x+\ln e^{4x}=\ln e^7 \Rightarrow x\ln 3+4x\ln e=7\ln e$ is an example after the addition law

$3^x=e^{7-4x} \Rightarrow x\log 3=(7-4x)\log e$ is an example after the subtraction law.

It is possible to score M0M1 by applying the power law after an incorrect addition/subtraction law

For example $3^x e^{4x}=e^7 \Rightarrow \ln(3^x)\times\ln(e^{4x})=\ln e^7 \Rightarrow x\ln 3\times 4x\ln e=7\ln e$

dM1 This is dependent upon **both** previous M's. Collects/factorises out term in x and proceeds to $x=$.

Condone sign slips for this mark. An unsimplified answer can score this mark.

A1 If the candidate has taken ln's then they must use $\ln e=1$ and achieve $x=\dfrac{7}{(\ln 3+4)}$ or equivalent.

If the candidate has taken log's they must be writing log as oppose to ln and achieve

$x=\dfrac{7\log e}{(\log 3+4\log e)}$ or other exact equivalents such as $x=\dfrac{7\log e}{\log 3e^4}$.

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Q2.

Question	Scheme		Marks	AOs
(a)	$2^{2x}+2^4$ is wrong in line 2 - it should be $2^{2x}\times 2^4$		B1	2.3
	In line 4, 2^4 has been replaced by 8 instead of by 16		B1	2.3
			(2)	
(b)	Way 1 $2^{2x+4}-9(2^x)=0$ $2^{2x}\times 2^4-9(2^x)=0$ Let $2^x=y$ $16y^2-9y=0$	Way 2 $(2x+4)\log 2-\log 9-x\log 2=0$	M1	2.1
	$y=\frac{9}{16}$ or $y=0$ So $x=\log_2\left(\frac{9}{16}\right)$ or $\frac{\log\left(\frac{9}{16}\right)}{\log 2}$ o.e. with no second answer.	$x=\frac{\log 9}{\log 2}-4$ o.e.	A1	1.1b
			(2)	
				(4 marks)

Notes

(a) B1: Lists error in line 2 (as above)
B1 : Lists error in line 4 (as above)
(b) M1: Correct work with powers reaching this equation
A1 : Correct answer here – there are many exact equivalents

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Q3.

Question Number	Scheme	Marks
	(a) $2\log_3(x-5) = \log_3(x-5)^2$	B1
	$\log_3(x-5)^2 - \log_3(2x-13) = \log_3\dfrac{(x-5)^2}{2x-13}$	M1
	$\log_3 3 = 1$ seen or used correctly	B1
	$\log_3\left(\dfrac{P}{Q}\right)=1 \Rightarrow P=3Q \quad \left\{\dfrac{(x-5)^2}{2x-13}=3 \Rightarrow (x-5)^2=3(2x-13)\right\}$	M1
	$x^2-16x+64=0$ (*)	A1 cso
		(5)
	(b) $(x-8)(x-8)=0 \Rightarrow x=8$ Must be seen in part (b).	M1 A1
	Or: Substitute $x=8$ into original equation and verify.	(2)
	Having additional solution(s) such as $x=-8$ loses the A mark.	
	$x=8$ with no working scores both marks.	7

(a) Marks may be awarded if equivalent work is seen in part (b).

1st M: $\log_3(x-5)^2 - \log_3(2x-13) = \dfrac{\log_3(x-5)^2}{\log_3(2x-13)}$ is M0

$2\log_3(x-5) - \log_3(2x-13) = 2\log\dfrac{x-5}{2x-13}$ is M0

2nd M: After the first mistake above, this mark is available only if there is 'recovery' to the required $\log_3\left(\dfrac{P}{Q}\right)=1 \Rightarrow P=3Q$. Even then the final mark (cso) is lost.

'Cancelling logs', e.g. $\dfrac{\log_3(x-5)^2}{\log_3(2x-13)} = \dfrac{(x-5)^2}{2x-13}$ will also lose the 2nd M.

A typical wrong solution:

$\log_3\dfrac{(x-5)^2}{2x-13}=1 \Rightarrow \log_3\dfrac{(x-5)^2}{2x-13}=3 \Rightarrow \dfrac{(x-5)^2}{2x-13}=3 \Rightarrow (x-5)^2=3(2x-13)$

(Wrong step here)

This, with no evidence elsewhere of $\log_3 3=1$, scores B1 M1 B0 M0 A0

However, $\log_3\dfrac{(x-5)^2}{2x-13}=1 \Rightarrow \dfrac{(x-5)^2}{2x-13}=3$ is correct and could lead to full marks.

(Here $\log_3 3=1$ is implied).

No log methods shown:

It is not acceptable to jump immediately to $\dfrac{(x-5)^2}{2x-13}=3$. The only mark this scores is the 1st B1 (by generous implication).

(b) M1: Attempt to solve the given quadratic equation (usual rules), so the factors $(x-8)(x-8)$ with no solution is M0.

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Q4.

Question Number	Scheme	Marks
(a)	Way 1: $\log_3(9x) = \log_3 9 + \log_3 x$ $= 2 + a$ or Way 2: $\log_3(9x) = \log_3 3^{a+2}$ $= 2 + a$	M1 A1 (2)
(b)	Way 1: $\log_3\left(\frac{x^5}{81}\right) = \log_3 x^5 - \log_3 81$ $\log x^5 = 5\log x$ or log81=4log3 or log81=4 or Way 2 $= \log_3 \frac{3^{5a}}{3^4}$ $= \log_3 3^{5a-4}$ $= 5a - 4$	M1 M1 A1 cso (3)
(c)	$\log_3(9x) + \log_3\left(\frac{x^5}{81}\right) = 3$ Method 1 $\Rightarrow 2 + a + 5a - 4 = 3$ $\Rightarrow a = \frac{5}{6}$ $\Rightarrow x = 3^{\frac{5}{6}}$ or $\log_{10} x = a\log_{10} 3$ so $x =$ $x = 2.498$ **or awrt** Method 2 $\log_3\left(9x.\frac{x^5}{81}\right) = (3 \text{ or } \log 27)$ $\log_3\left(\frac{x^6}{9}\right) = 3 \text{ or } \log 27$ $\Rightarrow \frac{x^6}{9} = 3^3 \Rightarrow x^6 = 3^5 \Rightarrow x =$ $x = 2.498$ **or awrt** If x = -2.498 appears as well or instead this is A0	M1 A1 M1 A1 (4) **Total 9**

Notes for Question

(a)
Way 1: M1: Use of log(ab) = log(a) + log(b) A1: must be $a + 2$ or $2 + a$
Way 2: Uses $x = 3^a$ to give $\log_3(9x) = \log_3 3^{a+2}$, A1 for $a + 2$ or $2 + a$

(b)
Way 1: M1: Use of log(a/b) = log(a) – log(b)
M1: Use of nlog(a) = log(a^n)
Way 2: M1 Use of correct powers of 3 in numerator and denominator
M1: Subtracts powers
A1: No errors seen

(c)
Method 1: M1: Uses (a) and (b) results to form an equation in a (may not be linear)
A1: a = awrt 0.833
M1: Finds x by use of 3 to a power, or change of base performed correctly
A1: $x = 2.498$ (accept answer which round to this value from 2.498049533…)
Method 2: M1: Use of log(ab) = log(a) + log(b) in an equation (RHS may be wrong)
A1: Equation correct and simplified
M1: Tries to undo log by 3 to power correctly, and uses root to obtain x
A1: $x = 2.498$ (accept answer which round to this value from 2.498049533…) Lose this mark if negative answer is given as well as or instead of positive answer.

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Question Number	Scheme	Marks
	Method 1 (Substituting a = 3b into second equation at some stage)	
	Using a law of logs correctly (anywhere) e.g. $\log_3 ab = 2$	M1
	Substitution of $3b$ for a (or a/3 for b) e.g. $\log_3 3b^2 = 2$	M1
	Using base correctly on **correctly derived** $\log_3 p = q$ e.g. $3b^2 = 3^2$	M1
	First correct value $b = \sqrt{3}$ (allow $3^{½}$)	A1
	Correct method to find other value (dep. on at least first M mark)	M1
	Second answer $a = 3b = 3\sqrt{3}$ or $\sqrt{27}$	A1
	Method 2 (Working with two equations in $\log_3 a$ and $\log_3 b$)	
	" Taking logs" of first equation and " separating" $\log_3 a = \log_3 3 + \log_3 b$ $(= 1 + \log_3 b)$	M1
	Solving simultaneous equations to find $\log_3 a$ or $\log_3 b$ [$\log_3 a = 1½$, $\log_3 b = ½$]	M1
	Using base correctly to find a or b	M1
	Correct value for a or b $a = 3\sqrt{3}$ or $b = \sqrt{3}$	A1
	Correct method for second answer, dep. on first M; correct second answer [Ignore negative values]	M1;A1 **[6]**

Pearson

Notes:	Answers must be exact; decimal answers lose both A marks There are several variations on **Method 1**, depending on the stage at which $a = 3b$ is used, but they should all mark as in scheme. In this method, the first three method marks on Epen are for (i) First M1: correct use of log law, (ii) Second M1: substitution of $a = 3b$, (iii) Third M1: requires using base correctly on correctly derived $\log_3 p = q$ *Three examples of applying first 4 marks in Method 1:* (i) $\log_3 3b + \log_3 b = 2$ gains second M1 $\log_3 3 + \log_3 b + \log_3 b = 2$ gains first M1 $(2\log_3 b = 1,\ \log_3 b = ½)$ no mark yet $b = 3^{½}$ gains third M1, and if correct A1 (ii) $\log_3(ab) = 2$ gains first M1 $ab = 3^2$ gains third M1 $3b^2 = 3^2$ gains second M1 (iii) $\log_3 3b^2 = 2$ has gained first 2 M marks $\Rightarrow 2\log_3 3b = 2$ or similar type of error $\Rightarrow \log_3 3b = 1 \Rightarrow 3b = 3$ does not gain third M1, as $\log_3 3b = 1$ not derived correctly	

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Q5

Question Number	Scheme	Marks
(a)	$p=7.5$	B1 (1)
(b)	$2.5 = 7.5e^{-4k}$ $e^{-4k} = \frac{1}{3}$ $-4k = \ln(\frac{1}{3})$ $-4k = -\ln(3)$ $k = \frac{1}{4}\ln(3)$ See notes for additional correct solutions and the last A1	M1 M1 dM1 A1* (4) 5 Marks

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Gold Questions

Calculators may not be used

The total mark for this section is 30

Q1

(a) Find the positive value of x such that

$$\log_x 64 = 2$$

(2)

(b) Solve for x

$$\log_2 (11 - 6x) = 2 \log_2 (x - 1) + 3$$

(6)

(Total for Question 1 is 8 marks)

Q2

Find algebraically the exact solutions to the equation $2^x e^{3x+1} = 10$

Give your answer to (b) in the form $\frac{a + \ln b}{c + \ln d}$ where a, b, c and d are integers.

(Total for Question 2 is 5 marks)

Q3

(i)

$$2\log(x + a) = \log(16a^6), \text{ where } a \text{ is a positive constant}$$

Find x in terms of a, giving your answer in its simplest form.

(3)

(ii)

$$\log_3(9y + b) - \log_3(2y - b) = 2, \text{ where } b \text{ is a positive constant}$$

Find y in terms of b, giving your answer in its simplest form.

(4)

(Total for Question 3 is 7 marks)

Q4

(a) Find the value of y such that

$$\log_2 y = -3$$

(2)

(b) Find the values of x such that

$$\frac{\log_2 32 + \log_2 16}{\log_2 x} = \log_2 x$$

(5)

(Total for Question 4 is 7 marks)

Q5

Rabbits were introduced onto an island. The number of rabbits, P, t years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t}, \quad t \in \mathbb{R}, t \geqslant 0$$

(a) Write down the number of rabbits that were introduced to the island.

(1)

(b) Find the number of years it would take for the number of rabbits to first exceed 1000.

(2)

(Total for Question 5 is 3 marks)

Gold Mark Scheme

Q1.

Question Number	Scheme	Marks
(a)	$\log_x 64 = 2 \Rightarrow 64 = x^2$	M1
	So $x = 8$	A1 (2)
(b)	$\log_2(11-6x) = \log_2(x-1)^2 + 3$	M1
	$\log_2\left[\frac{11-6x}{(x-1)^2}\right] = 3$	M1
	$\frac{11-6x}{(x-1)^2} = 2^3$	M1
	$\{11-6x = 8(x^2-2x+1)\}$ and so $0 = 8x^2 - 10x - 3$	A1
	$0 = (4x+1)(2x-3) \Rightarrow x = \ldots$	dM1
	$x = \frac{3}{2}, \left[-\frac{1}{4}\right]$	A1 (6)
		[8]

(a)	M1 for getting out of logs A1 Do not need to see $x = -8$ appear and get rejected. Ignore $x = -8$ as extra solution. $x = 8$ with no working is M1 A1
(b)	1st M1 for using the $n\log x$ rule 2nd M1 for using the $\log x - \log y$ rule or the $\log x + \log y$ rule as appropriate 3rd M1 for using 2 to the power– need to see 2^3 or 8 (May see $3 = \log_2 8$ used) **If all three M marks have been earned and logs are still present in equation do not give final M1.** So solution stopping at $\log_2\left[\frac{11-6x}{(x-1)^2}\right] = \log_2 8$ would earn M1M1M0 1st A1 for a correct 3TQ 4th dependent M1 for attempt to solve or factorize their 3TQ to obtain $x = \ldots$ (mark depends on three previous M marks) 2nd A1 for 1.5 (ignore -0.25) s.c 1.5 only – no working – is 0 marks
(a)	Alternatives Change base : (i) $\frac{\log_2 64}{\log_2 x} = 2$, so $\log_2 x = 3$ and $x = 2^3$, is M1 or (ii) $\frac{\log_{10} 64}{\log_{10} x} = 2$, $\log x = \frac{1}{2}\log 64$ so $x = 64^{\frac{1}{2}}$ is M1 then $x = 8$ is A1 **BUT** $\log x = 0.903$ so $x = 8$ is M1A0 (loses accuracy mark) (iii) $\log_{64} x = \frac{1}{2}$ so $x = 64^{\frac{1}{2}}$ is M1 then $x = 8$ is A1

Pearson

Q2.

Question Number	Scheme	Marks
	Take $\log_e$'s to give $\ln 2^x + \ln e^{3x+1} = \ln 10$	M1
	$x \ln 2 + (3x+1)\ln e = \ln 10$	M1
	$x(\ln 2 + 3\ln e) = \ln 10 - \ln e \Rightarrow x = ..$	dM1
	and uses $\ln e = 1$	M1
	$x = \dfrac{-1+\ln 10}{3+\ln 2}$	A1
		(5)
	Note that the 4th M mark may occur on line 2	
		(5 marks)

Notes for Question Continued

(b)

M1 Takes logs of both sides **and** splits LHS using addition law. If one of the terms is taken to the other side it can be awarded for taking logs of both sides **and** using the subtraction law.

M1 Taking both powers down using power rule. It is not wholly dependent upon the first M1 but logs of both sides must have been taken. Below is an example of M0M1

$$\ln 2^x \times \ln e^{3x+1} = \ln 10 \Rightarrow x \ln 2 \times (3x+1)\ln e = \ln 10$$

dM1 This is dependent upon both previous two M's being scored. It can be awarded for a full method to solve their linear equation in x. The terms in x must be collected on one side of the equation and factorised. You may condone slips in signs for this mark but the process must be correct and leading to $x = \ldots$

M1 Uses $\ln e = 1$. This could appear in line 2, but it must be part of their equation and not just a statement.

Another example where it could be awarded is $e^{3x+1} = \dfrac{10}{2^x} \Rightarrow 3x+1 = \ldots$

A1 Obtains answer $x = \dfrac{-1+\ln 10}{3+\ln 2} = \left(\dfrac{\ln 10 - 1}{3+\ln 2}\right) = \left(\dfrac{\log_e 10 - 1}{3+\log_e 2}\right)$ *oe*. **DO NOT ISW HERE**

Note 1: If the candidate takes $\log_{10}$'s of both sides can score M1M1dM1M0A0 for 3 out of 5.

$$\text{Answer} = x = \frac{-\log e + \log 10}{3\log e + \log 2} = \left(\frac{-\log e + 1}{3\log e + \log 2}\right)$$

Note 2: If the candidate writes $x = \dfrac{-1+\log 10}{3+\log 2}$ without reference to natural logs then award M4 but with hold the last A1 mark, scoring 4 out of 5.

Pearson

Question Number	Scheme	Marks
Alt 1 to (b)	Writes lhs in e's $2^x e^{3x+1} = 10 \Rightarrow e^{x\ln 2} e^{3x+1} = 10$	1st M1
	$\Rightarrow e^{x\ln 2+3x+1} = 10, \quad x\ln 2+3x+1 = \ln 10$	2nd M1, 4th M1
	$x(\ln 2+3) = \ln 10 - 1 \Rightarrow x = ..$	dM1
	$x = \dfrac{-1+\ln 10}{3+\ln 2}$	A1 **(5)**

Notes for Question Alt 1

M1 Writes the lhs of the expression in e's. Seeing $2^x = e^{x\ln 2}$ in their equation is sufficient

M1 Uses the addition law on the lhs to produce a single exponential

dM1 Takes ln's of both sides to produce and attempt to solve a linear equation in x
You may condone slips in signs for this mark but the process must be correct leading to $x = ..$

M1 Uses ln e = 1. This could appear in line 2

Pearson

Q3.

Question Number	Scheme	Marks
(i)	Use of power rule so $\log(x+a)^2 = \log 16a^6$ or $2\log(x+a) = 2\log 4a^3$ or $\log(x+a) = \log(16a^6)^{\frac{1}{2}}$	M1
	Removes logs and square roots, **or** halves then removes logs to give $(x+a) = 4a^3$ Or $x^2 + 2ax + a^2 - 16a^6 = 0$ followed by factorisation or formula to give $x = \sqrt{16a^6} - a$	M1
	$(x=)\ 4a^3 - a$ (depends on previous M's and must be this expression or equivalent)	A1cao **(3)**
(ii) Way 1	$\log_3 \frac{(9y+b)}{(2y-b)} = 2$ Applies quotient law of logarithms	M1
	$\frac{(9y+b)}{(2y-b)} = 3^2$ Uses $\log_3 3^2 = 2$	M1
	$(9y+b) = 9(2y-b) \Rightarrow y =$ Multiplies across and makes y the subject	M1
	$y = \frac{10}{9}b$	A1cso **(4)**
Way 2	Or : $\log_3(9y+b) = \log_3 9 + \log_3(2y-b)$ 2nd M mark	M1
	$\log_3(9y+b) = \log_3 9(2y-b)$ 1st M mark	M1
	$(9y+b) = 9(2y-b) \Rightarrow y = \frac{10}{9}b$ Multiplies across and makes y the subject	M1 A1cso **(4)**
		[7]

Notes

(i)

1st M1: Applies power law of logarithms correctly to one side of the equation

M1: Correct log work in correct order. If they square and obtain a quadratic the algebra should be correct. The marks is for $x+a = \sqrt{16a^6}$ isw so allow $x+a = \pm 4a^3$ for Method mark. Also allow $x+a = 4a^4$ or $x+a = \pm 4a^{5.5}$ or even $x+a = 16a^3$ as there is evidence of attempted square root. May see the correct $x+a = 10^{(\log 4 + 3\log a)}$ *so* $x = -a + 10^{(\log 4 + 3\log a)}$ which gains M1A0 unless followed by the answer in the scheme.

A1: Do not allow $x = \pm 4a^3 - a$ for accuracy mark. You may see the factorised $a(2a + 1)(2a - 1)$ o.e.

(ii)

M1: Applying the subtraction or addition law of logarithms correctly to make **two** log **terms into one** log term in y

M1: Uses $\log_3 3^2 = 2$

3rd M1: Obtains **correct** linear equation in y usually the one in the scheme and attempts $y =$

A1cso: $y = \frac{10}{9}b$ or correct equivalent after **completely correct** work.

Special case:

$\frac{\log_3(9y+b)}{\log_3(2y-b)} = 2$ is M0 unless clearly crossed out and replaced by the correct $\log_3 \frac{(9y+b)}{(2y-b)} = 2$

Candidates may then write $\frac{(9y+b)}{(2y-b)} = 3^2$ and proceed to the *correct* answer – allow M0M1M1A0 as the answer requires a completely correct solution.

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Q4.

Question Number	Scheme	Marks
Q (a)	$\log_2 y = -3 \Rightarrow y = 2^{-3}$	M1
	$y = \frac{1}{8}$ or 0.125	A1 (2)
(b)	$32 = 2^5$ or $16 = 2^4$ or $512 = 2^9$	M1
	[or $\log_2 32 = 5\log_2 2$ or $\log_2 16 = 4\log_2 2$ or $\log_2 512 = 9\log_2 2$]	
	[or $\log_2 32 = \frac{\log_{10} 32}{\log_{10} 2}$ or $\log_2 16 = \frac{\log_{10} 16}{\log_{10} 2}$ or $\log_2 512 = \frac{\log_{10} 512}{\log_{10} 2}$]	
	$\log_2 32 + \log_2 16 = 9$	A1
	$(\log x)^2 = \ldots$ or $(\log x)(\log x) = \ldots$ (May not be seen explicitly, so M1 may be implied by later work, and the base may be 10 rather than 2)	M1
	$\log_2 x = 3 \Rightarrow x = 2^3 = 8$	A1
	$\log_2 x = -3 \Rightarrow x = 2^{-3} = \frac{1}{8}$	A1ft (5) [7]

(a) M1 for <u>getting out of logs</u> correctly.
If done by change of base, $\log_{10} y = -0.903\ldots$ is insufficient for the M1, but $y = 10^{-0.903}$ scores M1.
A1 for the <u>exact</u> answer, e.g. $\log_{10} y = -0.903 \Rightarrow y = 0.12502..$ scores M1 (implied) A0.
<u>Correct answer</u> with no working scores both marks.
<u>Allow</u> both marks for implicit statements such as $\log_2 0.125 = -3$.

(b) 1st M1 for expressing 32 or 16 or 512 as a power of 2, or for a change of base enabling evaluation of $\log_2 32$, $\log_2 16$ or $\log_2 512$ by calculator. (Can be implied by 5, 4 or 9 respectively).
1st A1 for 9 (exact).
2nd M1 for getting $(\log_2 x)^2$ = constant. The constant can be a log or a sum of logs.
If written as $\log_2 x^2$ instead of $(\log_2 x)^2$, allow the M mark <u>only</u> if subsequent work implies correct interpretation.
2nd A1 for 8 (exact). Change of base methods leading to a non-exact answer score A0.
3rd A1ft for an answer of $\frac{1}{\text{their } 8}$. An ft answer may be non-exact.

<u>Possible mistakes</u>:
$\log_2(2^9) = \log_2(x^2) \Rightarrow x^2 = 2^9 \Rightarrow x = \ldots$ scores M1A1(implied by 9)M0A0A0
$\log_2 512 = \log_2 x \times \log_2 x \Rightarrow x^2 = 512 \Rightarrow x = \ldots$ scores M0A0(9 never seen)M1A0A0
$\log_2 48 = (\log_2 x)^2 \Rightarrow (\log_2 x)^2 = 5.585 \Rightarrow x = 5.145,\ x = 0.194$ scores M0A0M1A0A1ft

<u>No working</u> (or 'trial and improvement'):
$x = 8$ scores M0 A0 M1 A1 A0

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Q5.

Question Number	Scheme	Marks
Q	$P = 80e^{-\frac{t}{5}}$	
(a)	$t = 0 \Rightarrow P = 80e^{-\frac{0}{5}} = 80(1) = \underline{80}$ $\underline{80}$	B1 (1)
(b)	$P = 1000 \Rightarrow 1000 = 80e^{-\frac{t}{5}} \Rightarrow \frac{1000}{80} = e^{-\frac{t}{5}}$ Substitutes $P = 1000$ and rearranges equation to make $e^{-\frac{t}{5}}$ the subject.	M1
	$\therefore t = 5\ln\left(\frac{1000}{80}\right)$ awrt $5\ln\left(\frac{1000}{80}\right)$ years	A1 (2)
		[3]

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Platinum Questions

Calculators may not be used

The total mark for this section is 25

1 (a) Solve the equation

$$\sqrt{(3x + 16)} = 3 + \sqrt{(x + 1)}$$

(5)

(b) Solve the equation

$$\log_3 (x - 7) - \tfrac{1}{2} \log_3 x = 1 - \log_3 2$$

(7)

(Total for Question 1 is 12 marks)

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2 (a) Given that $x > 0$, $y > 0$, $x \neq 1$ and $n > 0$, show that

$$\log_x y = \log_{x^n} y^n$$

(2)

(b) Solve the following, leaving your answers in the form 2^p, where p is a rational number.

(i) $\log_2 u + \log_4 u^2 + \log_8 u^3 + \log_{16} u^4 = 5$

(ii) $\log_2 v + \log_4 v + \log_8 v + \log_{16} v = 5$

(iii) $\log_4 w^2 + \dfrac{3 \log_8 64}{\log_2 w} = 5$

(9)

(Total for Question 2 is 11 marks)

Platinum Mark Scheme

Q.	Scheme	Marks	Notes
1(a)	$3x+16=9+x+1+6\sqrt{x+1}$	M1	Initial squaring - both sides
	$3+x=3\sqrt{x+1}$ (o.e.)	A1	
	$9+6x+x^2=9(x+1)$ or $y=\sqrt{x+1}\rightarrow$ 3TQ in y	M1 A1	Correct collecting of terms
	$x^2-3x=0$ or $(y-2)(y-1)=0$	B1 (5)	2nd squaring o.e.
(b)	**$x=0$ or 3**	B1	Both values (S+ for checking values)
	$\frac{1}{2}\log_3 x=\log_3\sqrt{x}$	M1	
	$\log_3(x-7)-\log_3\sqrt{x}=\log_3\frac{x-7}{\sqrt{x}}$	M1A1	For use of $n\log x$ rule
	So $2x-14=3\sqrt{x}$ (o.e. all x terms on same line)	M1	For reducing xs to a single log
	$2\left(\sqrt{x}\right)^2-3\sqrt{x}-14=0$		M1 for getting out of logs
	$\left(2\sqrt{x}-7\right)\left(\sqrt{x}+2\right)=0$	A1	A1 for correct equation
	$\sqrt{x}=\frac{7}{2}$ or -2	A1 (7)	
	$x=\frac{49}{4}$	**[12]**	Attempt to solve suitable 3TQ in x or $\sqrt{x}$
			Either solution for $\sqrt{x}$ or x. Must be rational a/b
			49/4 oe only (S+ for clear reason for rejecting $x=4$)

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Qu	Scheme	Mark
2(a)	$\log_x y = k \Rightarrow x^k = y \Rightarrow y^n = \ldots$ or $\log_x y^n = nk \Rightarrow y^n = \ldots$ or base change	M1
	$y^n = \left(x^k\right)^n = x^{nk} = \left(x^n\right)^k$ therefore $\log_{x^n} y^n = k = \log_x y$ **(*)**	A1cso
		(2)
(b)(i)	LHS $= 4\log_2 u$	M1
	$\therefore \log_2 u = \frac{5}{4}$ so $\underline{u = 2^{\frac{5}{4}}}$	A1
		(2)
(ii)	$\log_{16} v^4 + \log_{16} v^2 + \log_{16} v^{\frac{4}{3}} + \log_{16} v$ or $\log_2 v + \log_2 v^{\frac{1}{2}} + \log_2 v^{\frac{1}{3}} + \log_2 v^{\frac{1}{4}}$	M1
	$= \log_{16} v^{\frac{25}{3}}$ $=$ $\log_2 v^{\frac{25}{12}}$	M1
	so $v = 2^{\frac{60}{25}} = \underline{2^{\frac{12}{5}}}$	A1
		(3)
(iii)	LHS $= \log_2 w + \dfrac{3\times 2}{\log_2 w}$	M1
	Sub $t = \log_2 w$ gives $t^2 - 5t + 6 = 0$ or $(t-3)(t-2) = 0$	M1
	$\log_2 w = 2 \Rightarrow w = \underline{2^2}$ and $\log_2 w = 3 \Rightarrow w = \underline{2^3}$ (accept 4 and 8)	A1,A1
		(4)
		(11)

Pearson