Bronze Questions

Calculators may not be used 

The total mark for this section is 32

Q1

(a) Find the value of $8^{\frac{4}{3}}$

(2)

(b) Simplify $\dfrac{15x^{\frac{4}{3}}}{3x}$

(2)

(Total for Question 1 is 4 marks)

Q2

(a) Write down the value of $32^{\frac{1}{5}}$

(1)

(b) Simplify fully $(32x^5)^{-\frac{2}{5}}$

(3)

(Total for Question 2 is 4 marks)

Q3

(a) Simplify

$$\sqrt{50} - \sqrt{18}$$

giving your answer in the form $a\sqrt{2}$, where a is an integer.

(2)

(b) Hence, or otherwise, simplify

$$\frac{12\sqrt{3}}{\sqrt{50} - \sqrt{18}}$$

giving your answer in the form $b\sqrt{c}$, where b and c are integers and $b \neq 1$

(3)

(Total for Question 3 is 5 marks)

Q4

Given that $32\sqrt{2} = 2^a$, find the value of a

(Total for Question 4 is 3 marks)

Q5

(a) Evaluate $32^{\frac{3}{5}}$, giving your answer as an integer.

(2)

(b) Simplify fully $\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}}$

(2)

(Total for Question 5 is 4 marks)

Q6

Express 8^{2x+3} in the form 2^y, stating y in terms of x

(Total for Question 6 is 2 marks)

Q7

Complete the table below. The first one has been done for you.

For each statement you must state if it is always true, sometimes true or never true, giving a reason in each case.

Statement	Always True	Sometimes True	Never True	Reason
The quadratic equation $ax^2 + bx + c = 0, \quad (a \neq 0)$ has 2 real roots.		✓		It only has 2 real roots when $b^2 - 4ac > 0$. When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots.
(i) When a real value of x is substituted into $x^2 - 6x + 10$ the result is positive. (2)				
(ii) If $ax > b$ then $x > \frac{b}{a}$ (2)				
(iii) The difference between consecutive square numbers is odd. (2)				

(Total for Question 7 is 6 marks)

Q8

The equation $x^2 + 3px + p = 0$, where p is a non-zero constant, has equal roots.

Find the value of p.

(Total for Question 8 is 4 marks)

End of Questions

Silver Questions

Calculators may not be used 

The total mark for this section is 35

Q1

(a) Find the value of $16^{\frac{1}{4}}$

(2)

(b) Simplify $x(2x^{-\frac{1}{4}})^4$

(2)

(Total for Question 1 is 4 marks)

Q2

Show that $\frac{2}{\sqrt{(12)}-\sqrt{(8)}}$ can be written in the form $\sqrt{a}-\sqrt{b}$, where a and b are integers.

(Total for Question 2 is 5 marks)

Q3

Solve

(a) $2^y = 8$

(1)

(b) $2^x \times 4^{x+1} = 8$

(4)

(Total for Question 3 is 5 marks)

Q4

Given

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}$$

express y as a function of x

(Total for Question 4 is 3 marks)

Q5

Find, using algebra, all real solutions to the equation

(i) $16a^2 = 2\sqrt{a}$

(4)

(ii) $b^4 + 7b^2 - 18 = 0$

(4)

(Total for Question 5 is 8 marks)

Q6

The equation $kx^2 + 4kx + 3 = 0$, where k is a constant, has no real roots.

Prove that

$$0 \square k < \frac{3}{4}$$

(Total for Question 6 is 4 marks)

Q7

$$f(x) = x^2 + (k+3)x + k$$

where k is a real constant.

(a) Find the discriminant of f(x) in terms of k.

(2)

(b) Show that the discriminant of f(x) can be expressed in the form $(k + a)^2 + b$, where a and b are integers to be found.

(2)

(c) Show that, for all values of k, the equation $f(x) = 0$ has real roots.

(2)

(Total for Question 7 is 6 marks)

End of Questions

Gold Questions

Calculators may not be used 

The total mark for this section is 33

Q1

Express 9^{3x+1} in the form 3^y, giving y in the form $ax + b$, where a and b are constants.

(Total for Question 1 is 2 marks)

Q2

The equation $x^2 + (k - 3)x + (3 - 2k) = 0$, where k is a constant, has two distinct real roots.

(a) Show that k satisfies

$$k^2 + 2k - 3 > 0$$

(3)

(b) Find the set of possible values of k.

(4)

(Total for Question 2 is 7 marks)

Q3

Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,

(a) show that $q^2 + 8q < 0$.

(2)

(b) Hence find the set of possible values of q.

(3)

(Total for Question 3 is 5 marks)

Q4

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(i) Solve the equation

$$x\sqrt{2} - \sqrt{18} = x$$

writing the answer as a surd in simplest form.

(3)

(ii) Solve the equation

$$4^{3x-2} = \frac{1}{2\sqrt{2}}$$

(3)

(Total for Question 4 is 6 marks)

Q5

Given that $y = 2^x$,

(a) express 4^x in terms of y.

(1)

(b) Hence, or otherwise, solve

$$8(4^x) - 9(2^x) + 1 = 0$$

(4)

(Total for Question 5 is 5 marks)

Q6

$$f(x) = x^2 - 8x + 19$$

(a) Express $f(x)$ in the form $(x + a)^2 + b$, where a and b are constants.

(2)

The curve C with equation $y = f(x)$ crosses the y-axis at the point P and has a minimum point at the point Q.

(b) Sketch the graph of C showing the coordinates of point P and the coordinates of point Q.

(3)

(c) Find the distance PQ, writing your answer as a simplified surd.

(3)

(Total for Question 6 is 8 marks)

End of Questions

Platinum Questions

Calculators may not be used 

The total mark for this section is 18

1 A student was attempting to prove that $x = \frac{1}{2}$ is the only real root of

$$x^3 + \tfrac{3}{4}x - \tfrac{1}{2} = 0.$$

The attempted solution was as follows.

$$x^3 + \tfrac{3}{4}x = \tfrac{1}{2}$$

$\therefore$ $$x(x^2 + \tfrac{3}{4}) = \tfrac{1}{2}$$

$\therefore$ $$x = \tfrac{1}{2}$$

or $$x^2 + \tfrac{3}{4} = \tfrac{1}{2}$$

i.e. $$x^2 = -\tfrac{1}{4}$$ no solution

$\therefore$ only real root is $x = \frac{1}{2}$

(a) Explain clearly the error in the above attempt.

(2)

(b) Give a correct proof that $x = \frac{1}{2}$ is the only real root of $x^3 + \frac{3}{4}x - \frac{1}{2} = 0$.

(3)

The equation

$$x^3 + \beta x - \alpha = 0 \qquad \text{(I)}$$

where α, β are real, $\alpha \neq 0$, has a real root at $x = \alpha$.

(c) Find and simplify an expression for β in terms of α and prove that α is the only real root provided $|\alpha| < 2$.

(6)

An examiner chooses a positive number α so that α is the only real root of equation (I) but the incorrect method used by the student produces 3 distinct real "roots".

(d) Find the range of possible values for α.

(7)

(Total for Question 1 is 19 marks)

End of Questions

Bronze Questions

Calculators may not be used 

The total mark for this section is 29

Q1

Find the set of values of x for which

(a) $3(x - 2) < 8 - 2x$

(2)

(b) $(2x - 7)(1 + x) < 0$

(3)

(c) both $3(x - 2) < 8 - 2x$ **and** $(2x - 7)(1 + x) < 0$

(1)

(Total for Question 1 is 6 marks)

Q2

Find the set of values of x for which

(a) $2(3x + 4) > 1 - x$

(2)

(b) $3x^2 + 8x - 3 < 0$

(4)

(Total for Question 2 is 6 marks)

Q3

Solve the simultaneous equations

$$y - 3x + 2 = 0$$
$$y^2 - x - 6x^2 = 0$$

(Total for Question 3 is 7 marks)

Q4

A rectangular room has a width of x m.

The length of the room is 4 m longer than its width.

Given that the perimeter of the room is greater than 19.2 m,

(a) show that $x > 2.8$

(3)

Given also that the area of the room is less than 21 m^2,

(b) (i) write down an inequality, in terms of x, for the area of the room.

(ii) Solve this inequality.

(4)

(c) Hence find the range of possible values for x.

(1)

(Total for Question 4 is 8 marks)

End of Questions

Silver Questions

Calculators may not be used 

The total mark for this section is 34

Q1

Find the set of values of x for which

(a) $4x - 3 > 7 - x$

(2)

(b) $2x^2 - 5x - 12 < 0$

(4)

(c) **both** $4x - 3 > 7 - x$ **and** $2x^2 - 5x - 12 < 0$

(1)

Q2

Given the simultaneous equations

$$2x + y = 1$$
$$x^2 - 4ky + 5k = 0$$

where k is a non zero constant,

(a) show that

$$x^2 + 8kx + k = 0$$

(2)

Given that $x^2 + 8kx + k = 0$ has equal roots,

(b) find the value of k.

(3)

(c) For this value of k, find the solution of the simultaneous equations.

(3)

(Total for Question 2 is 8 marks)

Q3

Solve the simultaneous equations

$$x + y = 2$$
$$4y^2 - x^2 = 11$$

(Total for Question 3 is 7 marks)

Q4

The equation

$$(k + 3)\,x^2 + 6x + k = 5, \text{ where } k \text{ is a constant,}$$

has two distinct real solutions for x.

(a) Show that k satisfies

$$k^2 - 2k - 24$$

(4)

(b) Hence find the set of possible values of k.

(3)

(Total for Question 4 is 7 marks)

Q5

(i) Show that $x^2 - 8x + 17 > 0$ for all real values of x

(3)

(ii) "If I add 3 to a number and square the sum, the result is greater than the square of the original number."

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

(Total for Question 5 is 5 marks)

End of Questions

Gold Questions

Calculators may not be used 

The total mark for this section is 27

Q1

The equation

$$x^2 + kx + 8 = k$$

has no real solutions for x.

(a) Show that k satisfies $k^2 + 4k - 32 < 0$.

(3)

(b) Hence find the set of possible values of k.

(4)

(Total for Question 1 is 7 marks)

Q2

Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,

(a) show that $q^2 + 8q < 0$.

(2)

(b) Hence find the set of possible values of q.

(3)

(Total for Question 2 is 5 marks)

Q3

The equation $20x^2 = 4kx - 13kx^2 + 2$, where k is a constant, has no real roots.

(a) Show that k satisfies the inequality

$$2k^2 + 13k + 20 < 0$$

(4)

(b) Find the set of possible values for k.

(4)

(Total for Question 3 is 8 marks)

Q4

(a) By eliminating y from the equations

$$y = x - 4$$
$$2x^2 - xy = 8,$$

show that

$$x^2 + 4x - 8 = 0$$

(2)

(b) Hence, or otherwise, solve the simultaneous equations

$$y = x - 4,$$
$$2x^2 - xy = 8,$$

giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers.

(5)

(Total for Question 4 is 7 marks)

End of Questions

Platinum Questions

Calculators may not be used

The total mark for this section is 13

1 (*a*) Find the set of values of k for which the equation

$$\frac{x^2+3x+8}{x^2+x-2}=k$$

has no real roots.

(6)

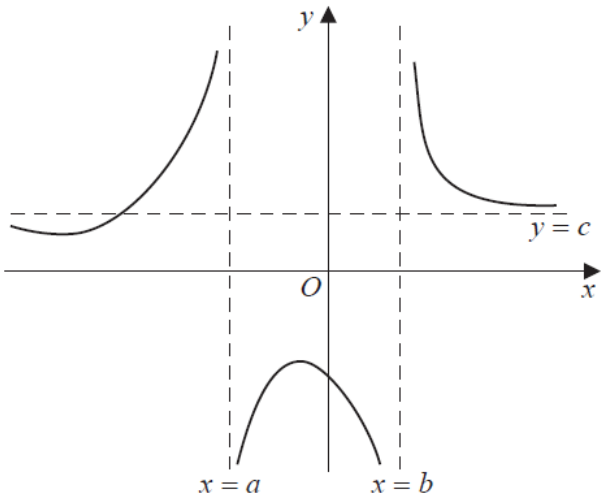


Figure 3

Figure 3 shows a sketch of the curve C_1 with equation $y = f(x)$ where $f(x) = \frac{x^2+3x+8}{x^2+x-2} = k$

The curve has asymptotes $x = a$, $x = b$ and $y = c$, where a, b and c are integers.

(*b*) Find the value of a, the value of b and the value of c.

(4)

(*c*) Find the coordinates of the points of intersection of C_1 with the line $y = 2$

(3)

(Total for Question 1 is 13 marks)

Bronze Questions

Calculators may not be used

The total mark for this section is 29

Q1

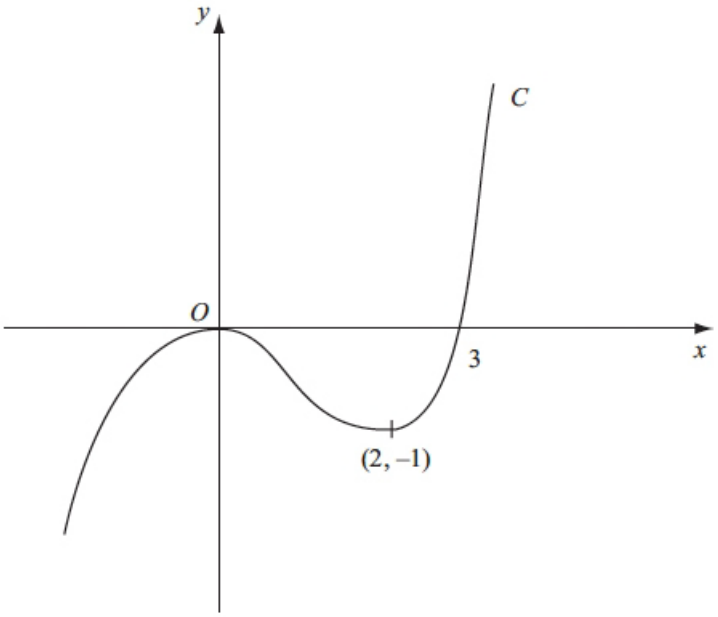


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$. There is a maximum at $(0, 0)$, a minimum at $(2, -1)$ and C passes through $(3, 0)$.

On separate diagrams sketch the curve with equation

(a) $y = f(x + 3)$,

(3)

(b) $y = f(-x)$.

(3)

On each diagram show clearly the coordinates of the maximum point, the minimum point and any points of intersection with the x-axis.

(Total for Question 1 is 6 marks)

Q2

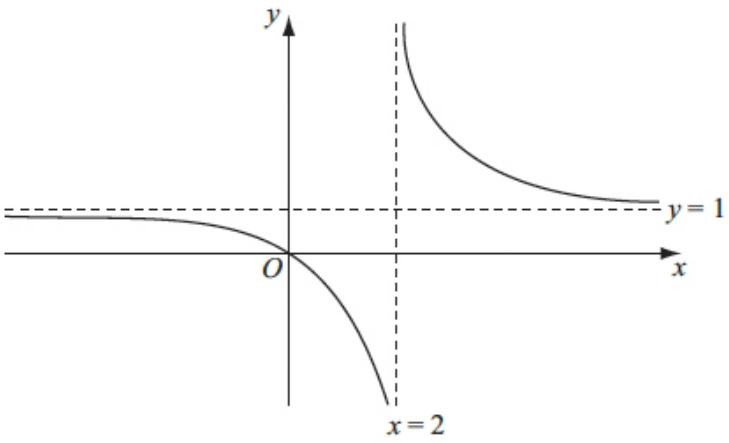


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x)\frac{x}{x-2}, \quad x \neq 2$$

The curve passes through the origin and has two asymptotes, with equations $y = 1$ and $x = 2$, as shown in Figure 1.

(a) Sketch the curve with equation $y = f(x - 1)$ and state the equations of the asymptotes of this curve.

(3)

(b) Find the coordinates of the points where the curve with equation $y = f(x - 1)$ crosses the coordinate axes.

(4)

(Total for Question 2 is 7 marks)

Q3

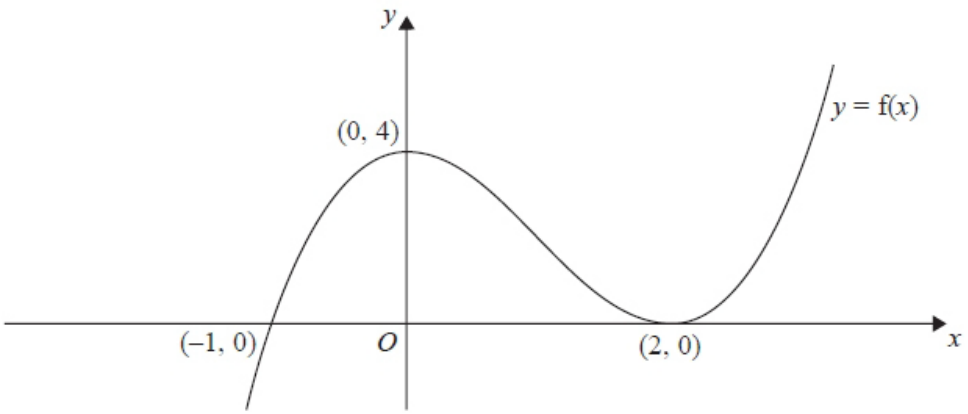


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$

The curve C passes through the point $(-1, 0)$ and touches the x-axis at the point $(2, 0)$

The curve C has a maximum at the point $(0, 4)$

(a) The equation of the curve C can be written in the form

$$y = x^3 + ax^2 + bx + c$$

where a, b and c are integers.

Calculate the values of a, b and c.

(5)

(b) Sketch the curve with equation $y = f\left(\frac{1}{2}\right)$

Show clearly the coordinates of all the points where the curve crosses or meets the coordinate axes.

(3)

(Total for Question 3 is 8 marks)

Q4

(a) Sketch the graphs of

$$y = x(x + 2)(3 - x)$$

$$y = -\frac{2}{x}$$

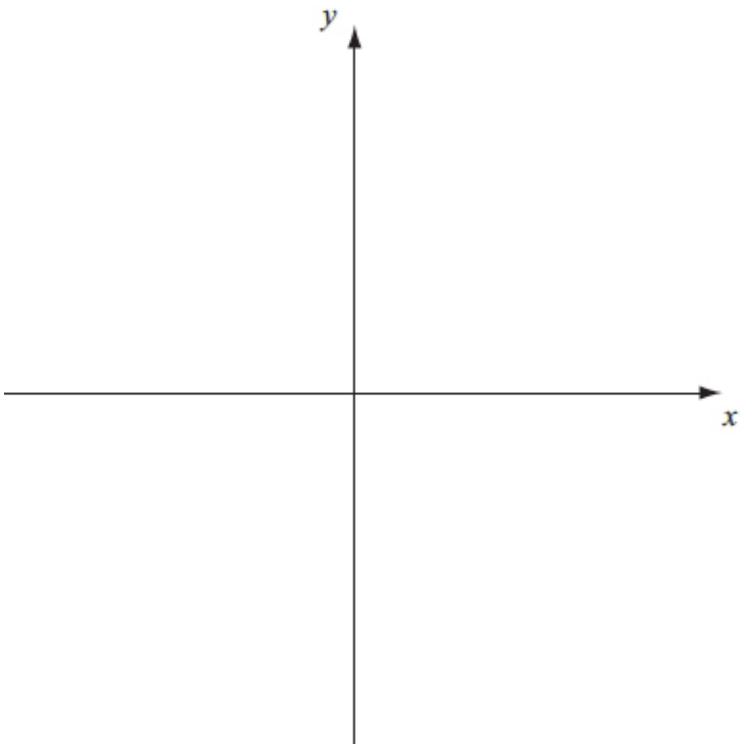
showing clearly the coordinates of all the points where the curves cross the coordinate axes.

(6)

(b) Using your sketch state, giving a reason, the number of real solutions to the equation

$$x(x+2)(3-x)+\frac{2}{x}=0$$

(2)



(Total for Question 4 is 8 marks)

End of Questions

Silver Questions

Calculators may not be used

The total mark for this section is 30

Q1

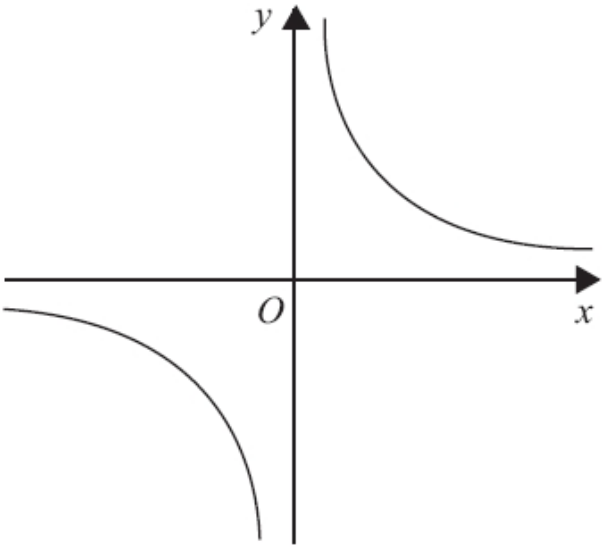


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \frac{3}{x}$, $x \neq 0$

(a) On a separate diagram, sketch the curve with equation $y = \frac{3}{x+2}$, $x \neq -2$,

showing the coordinates of any point at which the curve crosses a coordinate axis.

(3)

(b) Write down the equations of the asymptotes of the curve in part (a).

(2)

(Total for Question 1 is 5 marks)

Q2

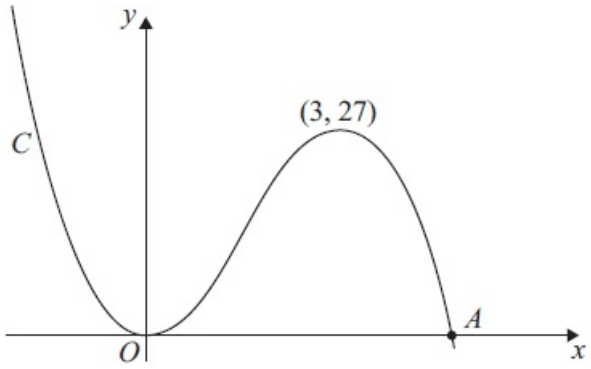


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$ where

$$f(x) = x^2(9 - 2x)$$

There is a minimum at the origin, a maximum at the point (3, 27) and C cuts the x-axis at the point A.

(a) Write down the coordinates of the point A.

(1)

(b) On separate diagrams sketch the curve with equation

(i) $y = f(x+3)$

(ii) $y = f(3x)$

On each sketch you should indicate clearly the coordinates of the maximum point and any points where the curves cross or meet the coordinate axes.

(6)

The curve with equation $y = f(x) + k$, where k is a constant, has a maximum point at (3, 10).

(c) Write down the value of k.

(1)

(Total for Question 2 is 8 marks)

Q3

(a) Factorise completely $x^3 - 6x^2 + 9x$

(3)

(b) Sketch the curve with equation

$$y = x^3 - 6x^2 + 9x$$

showing the coordinates of the points at which the curve meets the x-axis.

(4)

Using your answer to part (b), or otherwise,

(c) sketch, on a separate diagram, the curve with equation

$$y = (x - 2)^3 - 6(x - 2)^2 + 9(x - 2)$$

showing the coordinates of the points at which the curve meets the x-axis.

(2)

(Total for Question 3 is 9 marks)

Q4

The curve C has equation

$$y = \frac{k^2}{x} + 1 \qquad x \in \mathbb{R},\ x \neq 0$$

where k is a constant.

(a) Sketch C stating the equation of the horizontal asymptote.

(3)

The line l has equation $y = -2x + 5$

(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

$$2x^2 - 4x + k^2 = 0$$

(2)

(c) Hence find the exact values of k for which l is a tangent to C.

(3)

(Total for Question 4 is 8 marks)

Gold Questions

Calculators may not be used

The total mark for this section is 35

Q1

The point $P\,(1, a)$ lies on the curve with equation $y = (x + 1)^2(2 - x)$.

(a) Find the value of a.

(1)

(b) Sketch the curves with the following equations:

(i) $y = (x + 1)^2\ (2 - x)$,

(ii) $y = \dfrac{2}{x}$

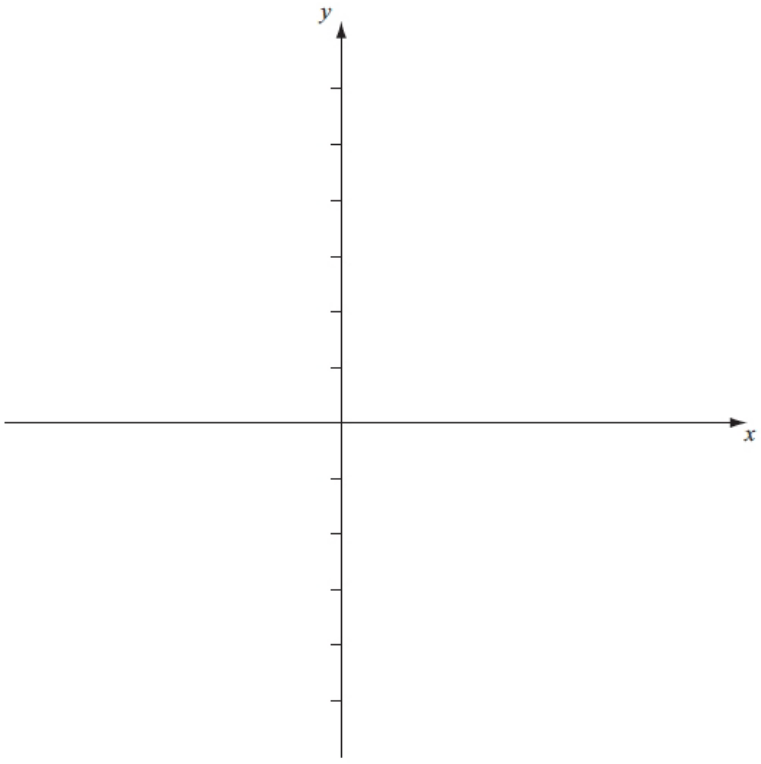
On your diagram show clearly the coordinates of any points at which the curves meet the axes.

(5)

(c) With reference to your diagram in part (b) state the number of real solutions to the equation

$$(x+1)^2\left(2-x\right)=\frac{2}{x}$$

(1)



(Total for Question 1 is 7 marks)

Q2

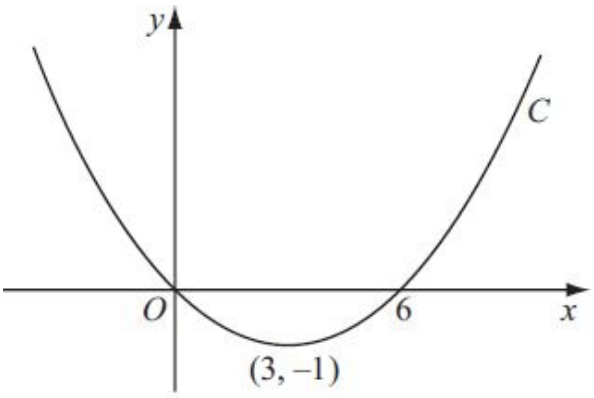


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$

The curve C passes through the origin and through $(6, 0)$

The curve C has a minimum at the point $(3, -1)$.

On separate diagrams, sketch the curve with equation

(a) $y = f(2x)$,

(3)

(b) $y = -f(x)$,

(3)

(c) $y = f(x + p)$, where p is a constant and $0 < p < 3$.

(4)

(Total for Question 1 is 10 marks)

Q3

(a) Factorise completely $x^3 + 10x^2 + 25x$

(2)

(b) Sketch the curve with equation

$$y = x^3 + 10x^3 + 25x$$

showing the coordinates of the points at which the curve cuts or touches the x-axis.

(2)

The point with coordinates $(-3, 0)$ lies on the curve with equation

$$y = (x + a)^3 + 10(x + a)^2 + 25(x + a)$$

where a is a constant.

(c) Find the two possible values of a.

(3)

(Total for Question 3 is 7 marks)

Q4

(a) On separate axes sketch the graphs of

(i) $y = -3x + c$, where c is a positive constant,

(ii) $y = \frac{1}{x} + 5$

On each sketch show the coordinates of any point at which the graph crosses the y-axis and the equation of any horizontal asymptote.

(4)

Given that $y = -3x + c$, where c is a positive constant, meets the curve $y = \frac{1}{x} + 5$ at two distinct points,

(b) show that $(5 - c)^2 > 2$

(3)

(c) Hence find the range of possible values for c

(4)

(Total for Question 4 is 11 marks)

Bronze Questions

Calculators may not be used 

The total mark for this section is 34

Q1

The line l passes through the points A (3, 1) and B (4, – 2).

Find an equation for l

(Total for Question 1 is 6 marks)

Q2

The line L_1 has equation $4x + 2y - 3 = 0$

(a) Find the gradient of L_1

(2)

The line L_2 is perpendicular to L_1 and passes through the point (2, 5)

(b) Find the equation of L_2 in the form $y = mx + c$, where m and c are constants.

(3)

(Total for Question 2 is 5 marks)

Q3

The line l_1 has equation $4y - 3x = 10$

The line l_2 passes through the points (5, –1) and (–1, 8)

Determine, giving full reasons for your answer, whether lines l_1 and l_2 are parallel, perpendicular or neither.

(Total for Question 3 is 4 marks)

Q4

The line l_1 has equation $y = -2x + 3$

The line l_2 is perpendicular to l_1 and passes through the point (5, 6).

(a) Find an equation for l_2 in the form $ax + by + c = 0$, where a , b and c are integers.

(3)

The line l_2 crosses the x-axis at the point A and the y-axis at the point B.

(b) Find the x-coordinate of A and the y-coordinate of B.

(2)

Given that O is the origin,

(c) find the area of the triangle OAB.

(2)

(Total for Question 4 is 7 marks)

Q5

The point A (–6, 4) and the point B (8, –3) lie on the line L.

(a) Find an equation for L in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

(b) Find the distance AB, giving your answer in the form $k\sqrt{5}$, where k is an integer.

(3)

(Total for Question 5 is 7 marks)

Q6

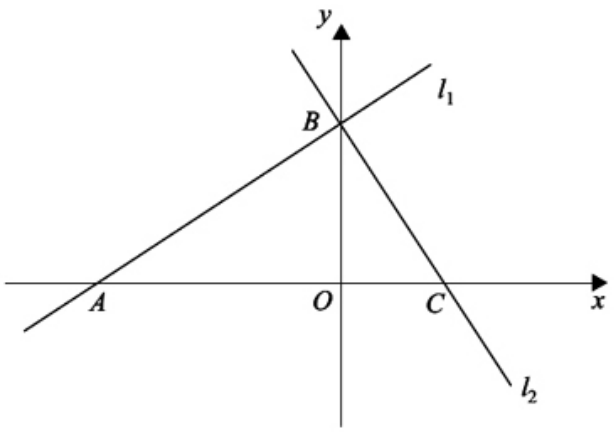


Figure 1

The line l_1 has equation $2x - 3y + 12 = 0$

(a) find the gradient of l_1.

(1)

The line l_1 crosses the x-axis at the point A and the y-axis at the point B, as shown in Figure 1.

The line l_2 is perpendicular to l_1 and passes through B.

(b) Find an equation of l_2.

(3)

The line l_2 crosses the x-axis at the point C.

(c) Find the area of triangle ABC.

(4)

(Total for Question 6 is 8 marks)

End of Questions

Silver Questions

Calculators may not be used

The total mark for this section is 31

Q1

The points P and Q have coordinates $(-1, 6)$ and $(9, 0)$ respectively.

The line l is perpendicular to PQ and passes through the mid-point of PQ.

Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

(Total for Question 1 is 5 marks)

Q2

The straight line L_1 passes through the points $(-1, 3)$ and $(11, 12)$.

(a) Find an equation for L_1 in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The line L_2 has equation $3y + 4x - 30 = 0$.

(b) Find the coordinates of the point of intersection of L_1 and L_2.

(3)

(Total for Question 2 is 7 marks)

Q3

The line L_1 has equation $2y - 3x - k = 0$, where k is a constant.

Given that the point A (1, 4) lies on L_1, find

(a) the value of k,

(1)

(b) the gradient of L_1.

(2)

The line L_2 passes through A and is perpendicular to L_1

(c) Find an equation of L_2 giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The line L_2 crosses the x-axis at the point B.

(d) Find the coordinates of B.

(2)

(e) Find the exact length of AB.

(2)

(Total for Question 3 is 11 marks)

Q4

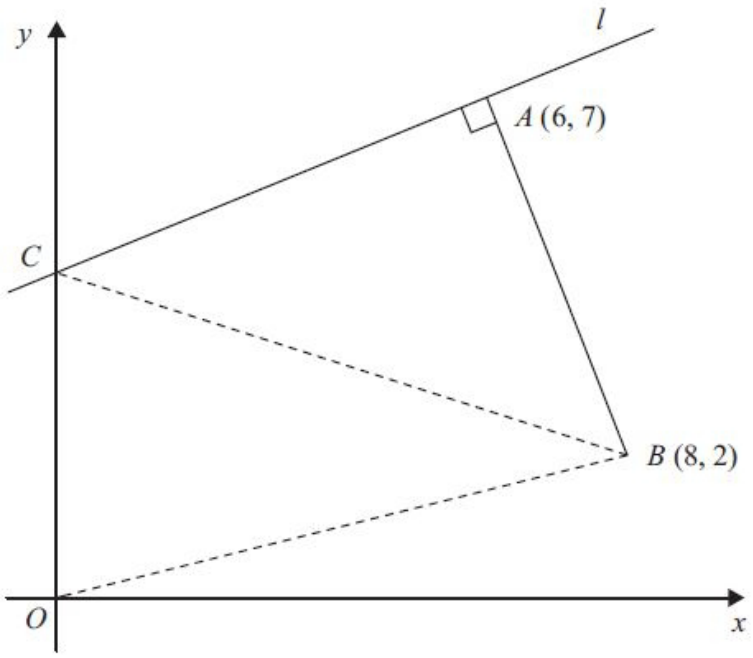


Figure 1

The points A and B have coordinates (6, 7) and (8, 2) respectively.

The line l passes through the point A and is perpendicular to the line AB, as shown in Figure 1.

(a) Find an equation for l in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Given that l intersects the y-axis at the point C, find

(b) the coordinates of C,

(2)

(c) the area of ΔOCB, where O is the origin.

(2)

(Total for Question 4 is 8 marks)

End of Questions

Gold Questions

Calculators may not be used 

The total mark for this section is 27

Q1

(a) Find an equation of the line joining $A(7, 4)$ and $B(2, 0)$, giving your answer in the form $ax+by+c = 0$, where a, b and c are integers.

(3)

(b) Find the length of AB, leaving your answer in surd form.

(2)

The point C has coordinates $(2, t)$, where $t > 0$, and $AC = AB$.

(c) Find the value of t.

(1)

(d) Find the area of triangle ABC.

(2)

(Total for Question 1 is 8 marks)

Q2

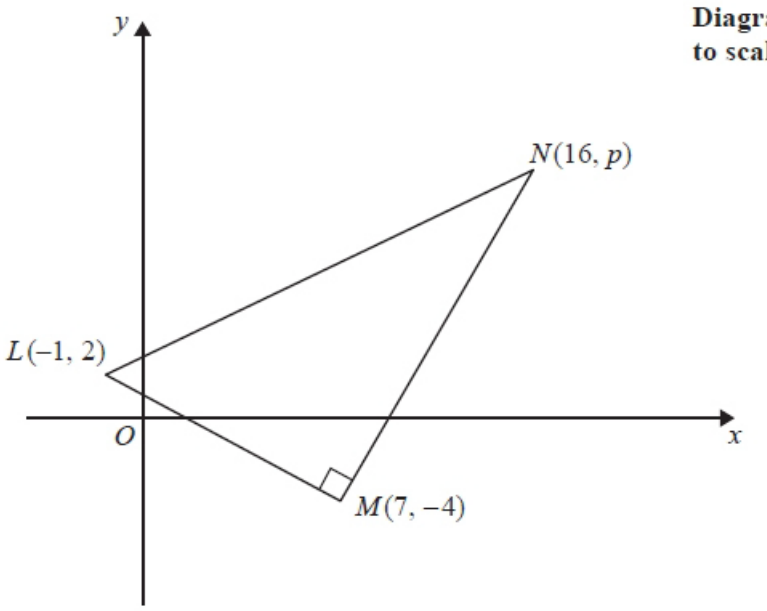


Figure 2

Figure 2 shows a right angled triangle *LMN*.

The points *L* and *M* have coordinates (−1, 2) and (7, −4) respectively.

(a) Find an equation for the straight line passing through the points *L* and *M*.

Give your answer in the form $ax + by + c = 0$, where *a*, *b* and *c* are integers.

(4)

Given that the coordinates of point *N* are (16, *p*), where *p* is a constant, and angle LMN = 90°,

(b) find the value of *p*.

(3)

Given that there is a point *K* such that the points *L, M, N*, and *K* form a rectangle,

(c) find the *y* coordinate of *K*.

(2)

(Total for Question 2 is 9 marks)

Q3

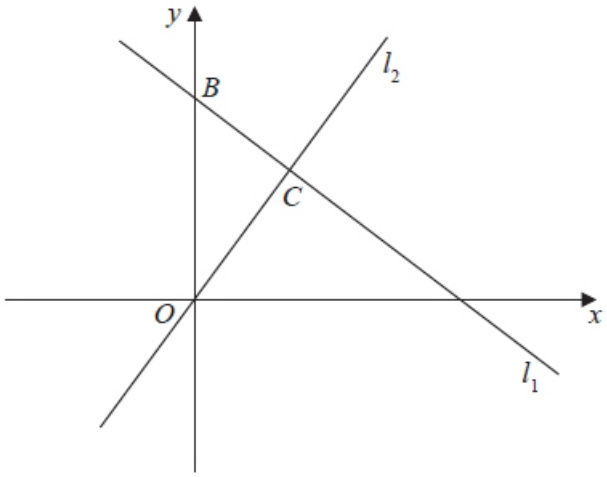


Figure 2

The line l_1, shown in Figure 2 has equation $2x + 3y = 26$

The line l_2 passes through the origin O and is perpendicular to l_1

(a) Find an equation for the line l_2

(4)

The line l_2 intersects the line l_1 at the point C

Line l_1 crosses the y-axis at the point B as shown in Figure 2.

(b) Find the area of triangle OBC

Give your answer in the form $\frac{a}{b}$, where a and b are integers to be determined.

(6)

(Total for Question 3 is 10 marks)

End of Questions

Bronze Questions

Calculators may not be used

The total mark for this section is 29

Q1

A circle C has centre $(-1, 7)$ and passes through the point $(0, 0)$. Find an equation for C

(Total for Question 1 is 4 marks)

Q2

The circle C has equation $x^2 + y^2 + 4x - 2y - 11 = 0$

Find

(a) the coordinates of the centre of C,

(2)

(b) the radius of C,

(2)

(c) the coordinates of the points where C crosses the y-axis, giving your answers as simplified surds.

(4)

(Total for Question 2 is 8 marks)

Q3

The points A and B have coordinates $(-2, 11)$ and $(8, 1)$ respectively.

Given that AB is a diameter of the circle C,

(a) show that the centre of C has coordinates $(3, 6)$,

(1)

(b) find an equation for C.

(4)

(c) Verify that the point $(10, 7)$ lies on C.

(1)

(d) Find an equation of the tangent to C at the point $(10, 7)$, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

(Total for Question 3 is 10 marks)

Q4

The circle C, with centre A, passes through the point P with coordinates $(-9, 8)$ and the point Q with coordinates $(15, -10)$.

Given that PQ is a diameter of the circle C,

(a) find the coordinates of A

(2)

(b) find an equation for C

(3)

A point R also lies on the circle C.
Given that the length of the chord PR is 20 units,

(c) find the length of the shortest distance from A to the chord PR.
Give your answer as a surd in its simplest form.

(2)

(Total for Question 4 is 7 marks)

End of Questions

Silver Questions

Calculators may not be used

The total mark for this section is 33

Q1

The circle C has equation

$$x^2 + y^2 - 20x - 24y + 195 = 0$$

The centre of C is at the point M

(a) Find

(i) the coordinates of the point M

(ii) the radius of the circle C

(5)

N is the point with coordinates (25, 32)

(b) Find the length of the line MN

(2)

The tangent to C at a point P on the circle passes through point N

(c) Find the length of the line NP

(2)

(Total for Question 1 is 9 marks)

Q2

The circle C has centre (3, 1) and passes through the point $P(8, 3)$.

(a) Find an equation for C.

(4)

(b) Find an equation for the tangent to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

(Total for Question 2 is 9 marks)

Q3

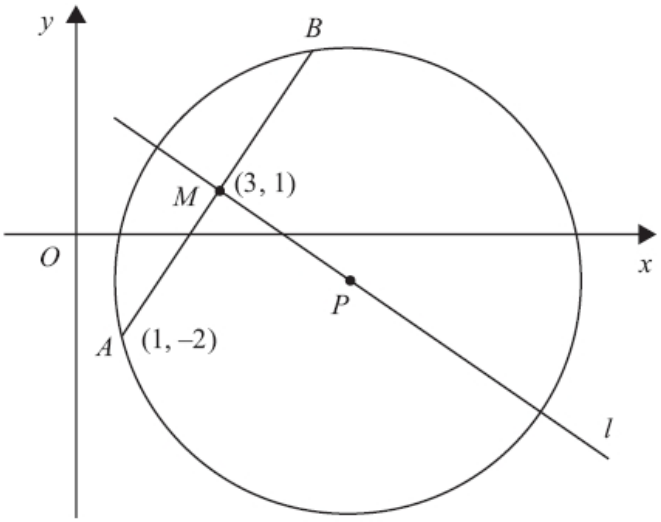


Figure 3

The points A and B lie on a circle with centre P, as shown in Figure 3.
The point A has coordinates $(1, -2)$ and the mid-point M of AB has coordinates $(3, 1)$.
The line l passes through the points M and P.

(a) Find an equation for l.

(4)

Given that the x-coordinate of P is 6,

(b) use your answer to part (a) to show that the y-coordinate of P is -1,

(1)

(c) find an equation for the circle.

(4)

(Total for Question 3 is 9 marks)

Q4

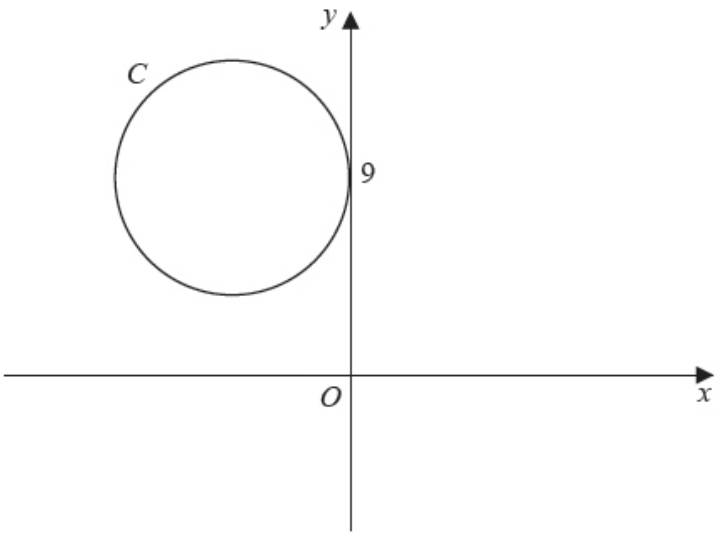


Figure 4

The circle C has radius 5 and touches the y-axis at the point (0, 9), as shown in Figure 4.

(a) Write down an equation for the circle C, that is shown in Figure 4.

(3)

A line through the point $P(8, -7)$ is a tangent to the circle C at the point T

(b) Find the length of PT

(3)

(Total for Question 4 is 6 marks)

End of Questions

Gold Questions

Calculators may not be used

The total mark for this section is 35

Q1

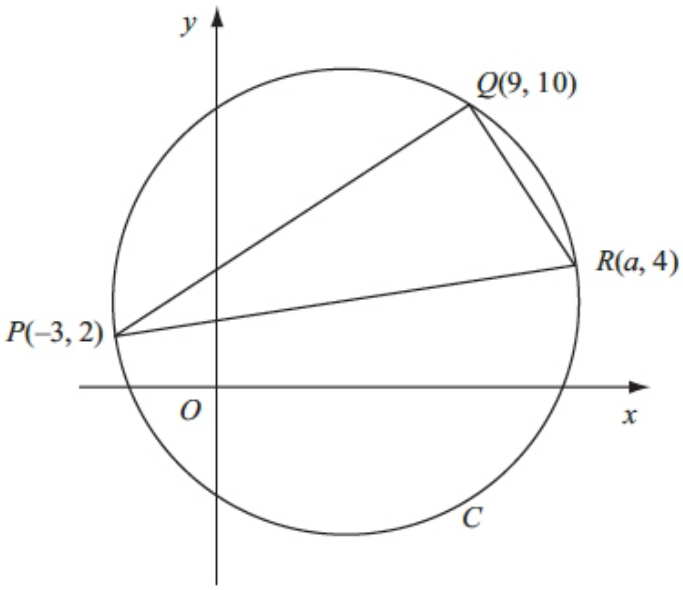


Figure 2

The points P(−3, 2), Q(9, 10) and R(a, 4) lie on the circle C, as shown in Figure 2. Given that PR is a diameter of C,

(a) show that $a = 13$

(3)

(b) find an equation for C

(5)

(2)

(Total for Question 1 is 10 marks)

Q2

The circle C has centre $A(2,1)$ and passes through the point $B(10, 7)$

(a) Find an equation for C

(4)

The line l_1 is the tangent to C at the point B

(b) Find an equation for l_1

(4)

The line l_2 is parallel to l_1 and passes through the mid-point of AB

Given that l_2 intersects C at the points P and Q,

(c) find the length of PQ, giving your answer in its simplest surd form.

(3)

(Total for Question 2 is 11 marks)

Q3

The circle C has equation

$$x^2 + y^2 - 6x + 10y + 9 = 0$$

(a) Find

(i) the coordinates of the centre of C
(ii) the radius of C

(3)

The line with equation $y = kx$, where k is a constant, cuts C at two distinct points.

(b) Find the range of values for k

(6)

(Total for Question 3 is 9 marks)

Q4

A circle C with centre at (– 2, 6) passes through the point (10, 11).
The circle C also passes through the point (10, 1) and has the equation
$(x + 2)^2 + (y - 6)^2 = 13^2$

The tangent to the circle C at the point (10, 11) meets the y axis at the point P
and the tangent to the circle C at the point (10, 1) meets the y axis at the point Q.
Show that the distance PQ is 58 explaining your method clearly.

(Total for Question 4 is 7 marks)

End of Questions

Platinum Questions

Calculators may not be used 

The total mark for this section is 20

1 A point P lies on the curve with equation

$$x^2 + y^2 - 6x + 8y = 24.$$

Find the greatest and least possible values of the length OP, where O is the origin.

(6)

(Total for Question 1 is 6 marks)

---

2 The line with equation $y = mx$ is a tangent to the circle C_1 with equation

$$(x + 4)^2 + (y - 7)^2 = 13.$$

(a) Show that m satisfies the equation

$$3m^2 + 56m + 36 = 0.$$

(4)

The tangents from the origin O to C_1 touch C_1 at the points A and B.

(b) Find the coordinates of the points A and B.

(8)

Another circle C_2 has equation $x^2 + y^2 = 13$. The tangents from the point $(4, -7)$ to C_2 touch it at the points P and Q.

(c) Find the coordinates of either the point P or the point Q.

(2)

(Total for Question 2 is 20 marks)

End of Questions

Bronze Questions

Calculators may not be used

The total mark for this section is 26

Q1

$$f(x) = 2x^3 - 7x^2 - 5x + 4$$

(a) Find the remainder when f(x) is divided by ($x - 1$)

(2)

(b) Use the factor theorem to show that (x+1) is a factor of f(x)

(2)

(c) Factorise f(x) completely.

(4)

(Total for Question 1 is 9 marks)

Q2

(a) Find the remainder when

$$x^3 - 2x^2 - 4x + 8$$

is divided by

(i) $x - 3$

(ii) $x + 2$

(3)

(b) Hence, or otherwise, find all the solutions to the equation

$$x^3 - 2x^2 - 4x + 8 = 0$$

(4)

(Total for Question 2 is 7 marks)

Q3

$$f(x) = x^4 + x^3 + 2x^2 + ax + b$$

where a and b are constants.

When $f(x)$ is divided by $(x - 1)$, the remainder is 7

(a) Show that $a + b = 3$

(2)

When $f(x)$ is divided by $(x + 2)$, the remainder is -8

(b) Find the value of a and the value of b

(5)

(Total for Question 3 is 7 marks)

Q4

Given $n \in \mathbb{Z}$, prove that $n^3 + 2$ is not divisible by 8

(Total for Question 4 is 4 marks)

End of Questions

Silver Questions

Calculators may not be used

The total mark for this section is 29

Q1

$$f(x) = 4x^3 - 12x^2 + 2x - 6$$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$

(2)

(b) Hence show that 3 is the only real root of the equation $f(x) = 0$

(4)

(Total for Question 1 is 6 marks)

Q2

$$f(x) = 2x^3 - 7x^2 - 10x + 24$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$

(2)

(b) Factorise $f(x)$ completely.

(4)

(Total for Question 2 is 6 marks)

Q3

$$f(x) = (3x - 2)(x - k) - 8$$

where k is a constant.

(a) Write down the value of f(k)

(1)

When f(x) is divided by $(x - 2)$ the remainder is 4

(b) Find the value of k

(2)

(c) Factorise f(x) completely.

(3)

(Total for Question 3 is 6 marks)

Q4

$$f(x) = x^3 + ax^2 + bx + 3$$

where a and b are constants.

Given that when f (x) is divided by $(x + 2)$ the remainder is 7,

(a) show that $2a - b = 6$

(2)

Given also that when f (x) is divided by $(x - 1)$ the remainder is 4,

(b) find the value of a and the value of b.

(4)

(Total for Question 4 is 6 marks)

Q5

(a) Prove that for all positive values of a and b

$$\frac{4a}{b} + \frac{b}{a} \geqslant 4$$

(4)

(b) Prove, by counter example, that this is not true for all values of a and b

(1)

(Total for Question 5 is 5 marks)

End of Questions

Gold Questions

Calculators may not be used 

The total mark for this section is 27

Q1

$$f(x) = -6x^3 - 7x^2 + 40x + 21$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$

(2)

(b) Factorise $f(x)$ completely.

(4)

(Total for Question 1 is 6 marks)

Q2

$$f(x) = x^4 + 5x^3 + ax + b,$$

where a and b are constants.

The remainder when $f(x)$ is divided by $(x - 2)$ is equal to the remainder when $f(x)$ is divided by $(x + 1)$.

(a) Find the value of a.

(5)

Given that $(x + 3)$ is a factor of $f(x)$,

(b) find the value of b.

(3)

(Total for Question 2 is 8 marks)

Q3

$$f(x) = 2x^3 - 13x^2 + 8x + 48$$

(a) Prove that $(x - 4)$ is a factor of $f(x)$

(2)

(b) Hence, using algebra, show that the equation $f(x) = 0$ has only two distinct roots.

(4)

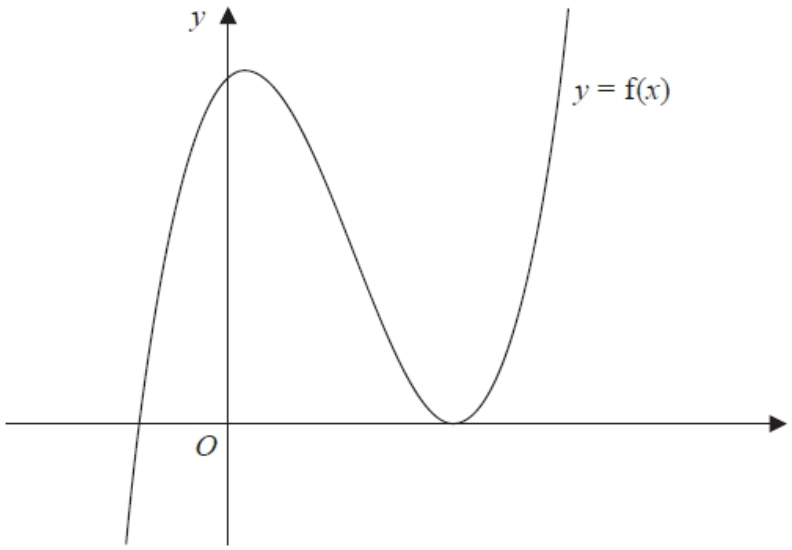


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$.

(c) Deduce, giving reasons for your answer, the number of real roots of the equation

$$2x^3 - 13x^2 + 8x + 46 = 0$$

(2)

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(d) find the two possible values of k.

(2)

(Total for Question 3 is 10 marks)

Q4

(a) Prove that for all positive values of x and y

$$\sqrt{xy} \leqslant \frac{x+y}{2}$$

(2)

(b) Prove by counter example that this is not true when x and y are both negative.

(1)

(Total for Question 4 is 3 marks)

End of Questions

Platinum Questions

Calculators may not be used 

The total mark for this section is 9

1 (*a*) Show that $(x + 1)$ is a factor of $2x^3 + 3x^2 - 1$

(1)

(*b*) Solve the equation

$$\sqrt{x^2 + 2x + 5} = x + \sqrt{2x + 3}$$

(8)

(Total for Question 1 is 9 marks)

End of Questions

Bronze Questions

Calculators may not be used 

The total mark for this section is 30

Q1

Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(2 - 3x)^5$ giving each term in its simplest form.

(Total for Question 1 is 4 marks)

Q2

(a) Find the first 4 terms, in ascending powers of x, in the binomial expansion of

$$(1 + kx)^{10}$$

where k is a positive constant. Give each term in its simplest form.

(3)

Given that, in this expansion, the coefficients of x and x^3 are equal,

(b) find the exact value of k,

(3)

(c) find the coefficient of x^2

(1)

(Total for Question 2 is 7 marks)

Q3

(a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(2 + kx)^7$$

where k is a non-zero constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^3 in this expansion is 1890

(b) find the value of k.

(3)

(Total for Question 3 is 7 marks)

Q4

Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{x}{4}\right)^{10}$$

giving each term in its simplest form.

(Total for Question 4 is 4 marks)

Q5

Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(3 - 2x)^5$, giving each term in its simplest form.

(Total for Question 5 is 4 marks)

Q6

Given that $\binom{40}{4} = \frac{40!}{4!b!}$,

(a) write down the value of b.

(1)

In the binomial expansion of $(1 + x)^{40}$, the coefficients of x^4 and x^5 are p and q respectively.

(b) Find the value of $\frac{q}{p}$

(3)

(Total for Question 6 is 4 marks)

End of Questions

Silver Questions

Calculators may not be used

The total mark for this section is 25

Q1

Find the first 3 terms, in ascending powers of x, in the binomial expansion of

$$(2 - 5x)^6$$

Give each term in its simplest form.

(Total for Question 1 is 4 marks)

Q2

(a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1 + ax)^7$, where a is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^2 in this expansion is 525,

(b) find the possible values of a.

(2)

(Total for Question 2 is 6 marks)

Q3

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 + kx)^7$$

where k is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^2 is 6 times the coefficient of x,

(b) find the value of k.

(2)

(Total for Question 3 is 6 marks)

Q4

(a) Use the binomial theorem to find all the terms of the expansion of

$$(2 + 3x)^4$$

Give each term in its simplest form.

(4)

(b) Write down the expansion of

$$(2 - 3x)^4$$

in ascending powers of x, giving each term in its simplest form.

(1)

(Total for Question 4 is 5 marks)

Q5

Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{1}{2}x\right)^8$$

giving each term in its simplest form.

(Total for Question 5 is 4 marks)

End of Questions

Gold Questions

Calculators may not be used 

The total mark for this section is 33

Q1

Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(1+\frac{3x}{2}\right)^8$$

giving each term in its simplest form.

(Total for Question 1 is 4 marks)

Q2

(a) Find the first 4 terms of the expansion of $\left(1+\frac{x}{2}\right)^{10}$ in ascending powers of x, giving each term in its simplest form.

(4)

(b) Use your expansion to estimate the value of $(1.005)^{10}$, giving your answer to 5 decimal places.

(3)

(Total for Question 2 is 7 marks)

Q3

(a) Find the first four terms, in ascending powers of x, in the binomial expansion of $(1 + kx)^6$, where k is a non-zero constant.

(3)

Given that, in this expansion, the coefficients of x and x^2 are equal, find

(b) the value of k,

(2)

(c) the coefficient of x^3 .

(1)

(Total for Question 3 is 6 marks)

Q4

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 9x)^4$$

giving each term in its simplest form.

(4)

$$f(x) = (1 + kx)(2 - 9x)^4, \text{ where } k \text{ is a constant}$$

The expansion, in ascending powers of x, of $f(x)$ up to and including the term in x^2 is

$$A - 232x + Bx^2$$

where A and B are constants.

(b) Write down the value of A.

(1)

(c) Find the value of k.

(2)

(d) Hence find the value of B.

(2)

(Total for Question 4 is 9 marks)

Q5

(a) Find the first 4 terms of the binomial expansion, in ascending powers of x, of

$$\left(1+\frac{x}{4}\right)^8$$

giving each term in its simplest form.

(4)

(b) Use your expansion to estimate the value of $(1.025)^8$, giving your answer to 4 decimal places.

(3)

(Total for Question 5 is 7 marks)

End of Questions

Platinum Questions

Calculators may not be used 

The total mark for this section is 15

1 In the binomial expansion of

$$\left(1+\frac{12n}{5}x\right)^n$$

the coefficients of x^2 and x^3 are equal and non-zero.

(*a*) Find the possible values of n.

(4)

(*b*) State, giving a reason, which value of n gives a valid expansion when $x = \frac{1}{2}$

(2)

(Total for Question 1 is 6 marks)

---

2 In the binomial expansion of

$$(1-4x)^p, \quad |x| < \tfrac{1}{4},$$

the coefficient of x^2 is equal to the coefficient of x^4 and the coefficient of x^3 is positive.

Find the value of p.

(Total for Question 2 is 9 marks)

End of Questions

Bronze Questions

Calculators may not be used

The total mark for this section is 25

Q1

Find the exact value of tan 30° × sin 60°
Give your answer in its simplest form.

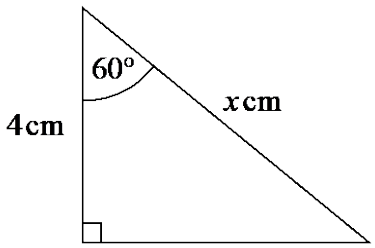
(Total for Question 1 is 2 marks)

Q2

(a) Write down the exact value of tan 45°

(1)

Here is a right-angled triangle.



cos 60° = 0.5

(b) Work out the value of x.

(2)

(Total for Question 2 is 3 marks)

Q3

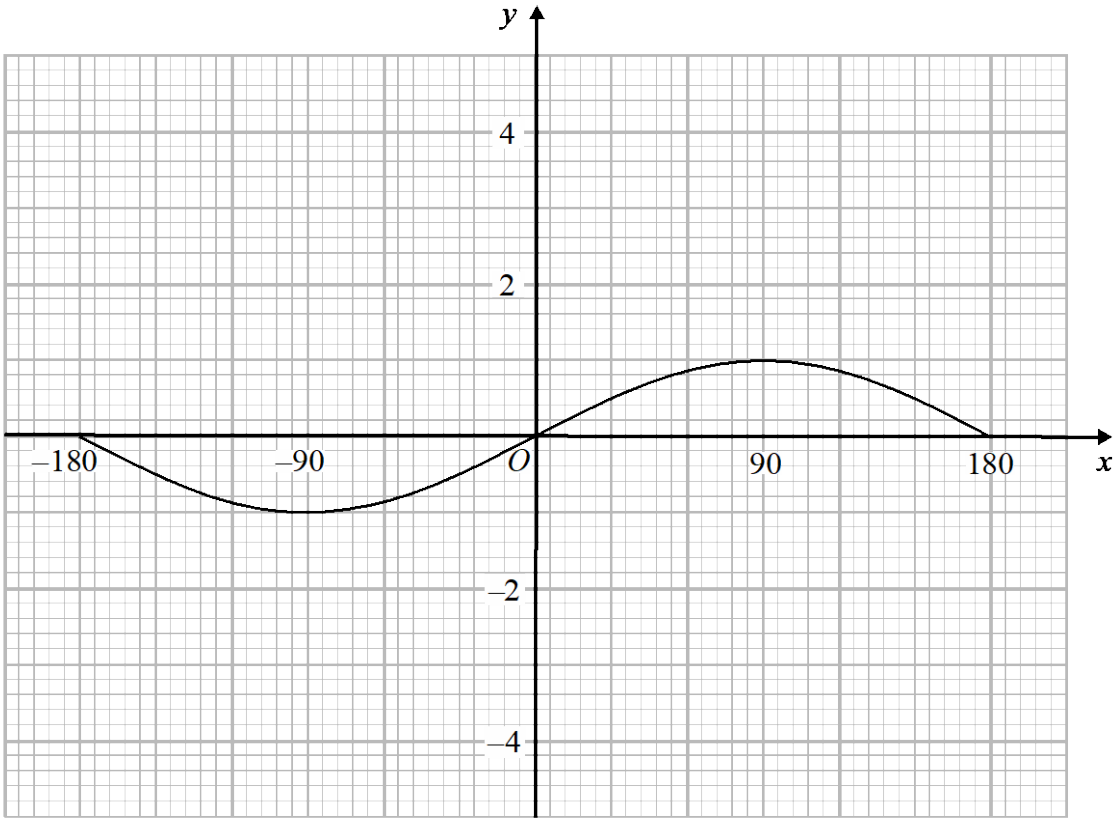
In the triangle ABC, $AB = 1\text{m}$, $AC = \sqrt{3}\text{m}$, angle $ABC = 60°$ and angle $BCA = x°$

Find the two possible values for x.

(Total for Question 3 is 4 marks)

Q4

Here is the graph of $y = \sin x°$ for $-180 \leqslant x \leqslant 180$



On the grid, sketch the graph of $y = \sin x° - 2$ for $-180 \leqslant x \leqslant 180$

(Total for Question 4 is 2 marks)

Q5

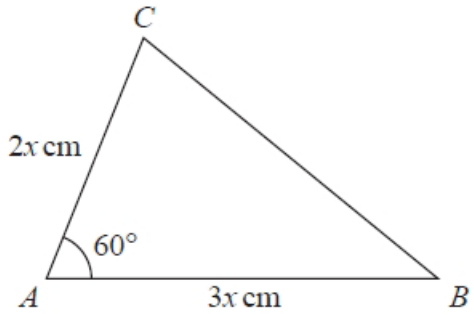


Figure 1

Figure 1 shows a sketch of a triangle ABC with $AB = 3x$ cm, $AC = 2x$ cm and angle $CAB = 60°$

Given that the area of triangle ABC is $18\sqrt{3}$ cm^2

(a) Show that $x = 2\sqrt{3}$

(3)

(b) Hence find the exact length of BC, giving your answer as a simplified surd.

(3)

(Total for Question 5 is 6 marks)

Q6

In triangle RPQ,

$RP = \sqrt{3}$ cm
$PQ = 1$ cm
Angle $PRQ = 30°$

(a) Assuming that angle PQR is an acute angle,
calculate the area of triangle RPQ.
Give your answer in exact form.

(4)

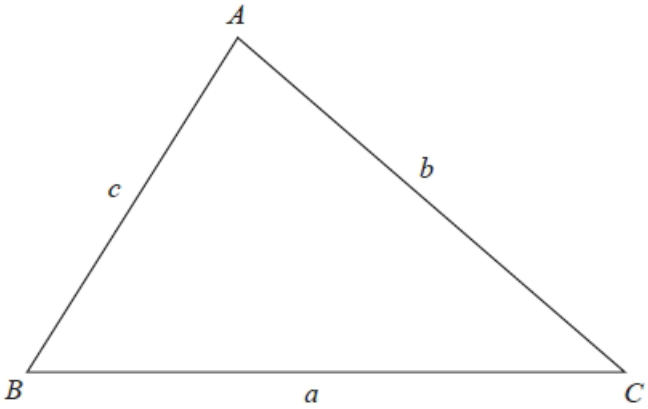
(b) If you did not know that angle PQR is an acute angle, what effect would this have on your calculation of the area of triangle RPQ?

(1)

(Total for Question 6 is 5 marks)

Q7

The diagram shows an acute-angled triangle ABC.



Prove that area of triangle $ABC = \frac{1}{2}ab\sin C$

(Total for Question 7 is 3 marks)

Silver Questions

Calculators may not be used

The total mark for this section is 29

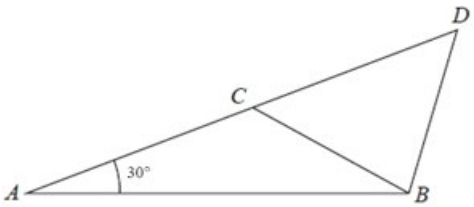
Q1

In the triangle ABC, $AB = 5\sqrt{6}$ cm, $AC = 4$ cm, angle $ABC = 45°$ and angle $BCA = x°$

Find the two possible values for x, giving your answers in exact form.

(Total for Question 1 is 4 marks)

Q2



Not to scale

Figure 1

Figure 1 shows the design for a structure used to support a roof.

The structure consists of four steel beams, AB, BD, BC and AD.

Given $AB = \sqrt{2}$m, $BC = BD = 1$m and angle $BAC = 30°$

Find, the size of angle ACB.

(Total for Question 2 is 3 marks)

Q3

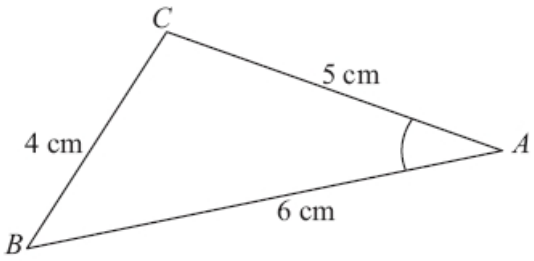


Figure 1

Figure 1 shows the triangle ABC, with $AB = 6$ cm, $BC = 4$ cm and $CA = 5$ cm.

(a) Show that $\cos A = \frac{3}{4}$.

(3)

(b) Hence, or otherwise, find the exact value of $\sin A$.

(2)

(Total for Question 3 is 5 marks)

Q4

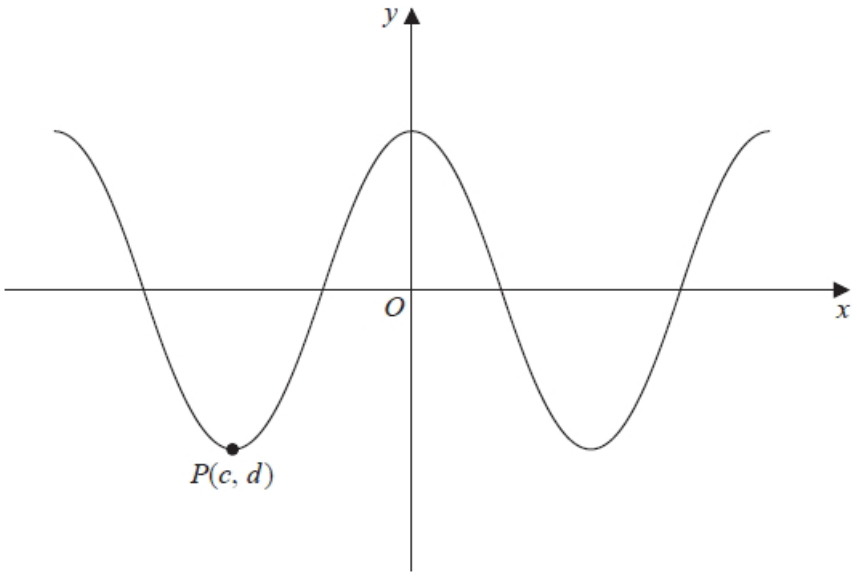


Figure 3

Figure 3 shows part of the curve with equation $y = 3 \cos x°$.

The point $P(c, d)$ is a minimum point on the curve with c being the smallest negative value of x at which
a minimum occurs.

(a) State the value of c and the value of d.

(1)

(b) State the coordinates of the point to which P is mapped by the transformation which transforms the curve with equation $y = 3 \cos x°$ to the curve with equation

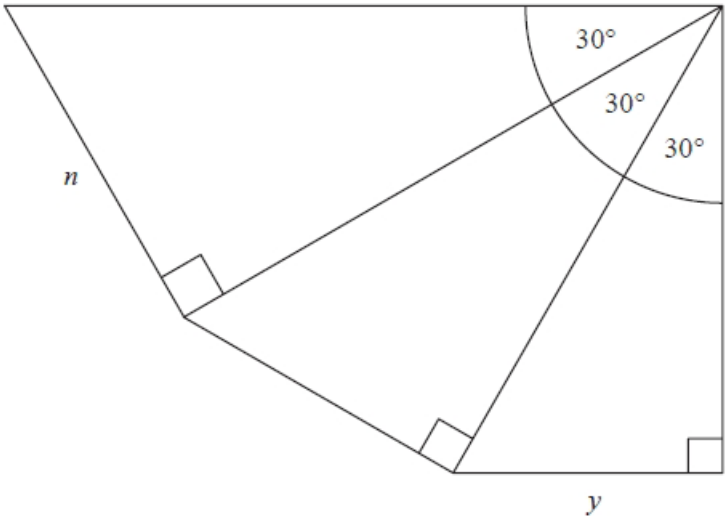
(i) $y = 3\cos\left(\frac{x°}{4}\right)$

(ii) $y = 3 \cos (x - 36)°$

(2)

(Total for Question 4 is 3 marks)

Q5



The diagram shows three right-angled triangles.

Prove that $y = \frac{3}{4}n$

(Total for Question 5 is 4 marks)

Q6

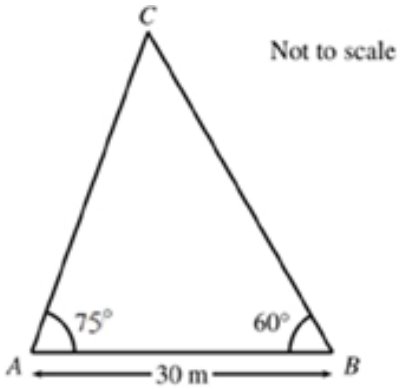


Figure 1

A triangular lawn is modelled by the triangle ABC, shown in Figure 1. The length AB is to be 30 m long.

Given that angle $BAC = 75°$ and angle $ABC = 60°$,

(a) Calculate the length AC **(2)**

Given that $BC=15+15\sqrt{3}$

(b) Calculate the area of the lawn in exact form.

(2)

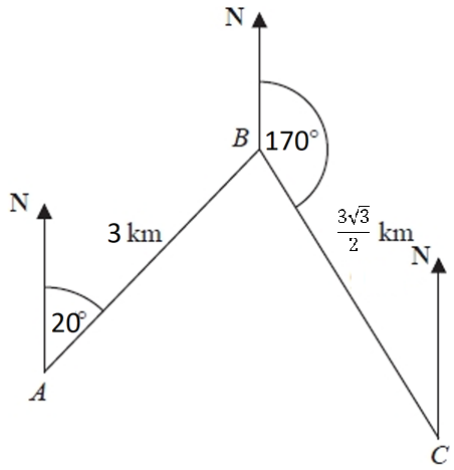
(c) Why is your answer unlikely to be accurate to the nearest square metre?

(1)

(Total for Question 6 is 5 marks)

Q7

The diagram shows the positions of three towns, Acton (A), Barston (B) and Chorlton (C).



Barston is 3 km from Acton on a bearing of 020°

Chorlton is $\frac{3\sqrt{3}}{2}$ km from Barston on a bearing of 170°

Find the bearing of Chorlton from Acton.
You must show all your working.

(Total for Question 7 is 5 marks)

Gold Questions

Calculators may not be used

The total mark for this section is 29

Q1

Figure 1

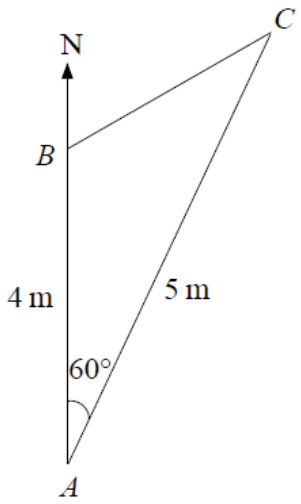


Figure 1 shows 3 yachts A, B and C which are assumed to be in the same horizontal plane. Yacht B is 4 m due north of yacht A and yacht C is 5 m from A. The bearing of C from A is 060°.

Calculate the distance between yacht B and yacht C, in exact form.

(Total for Question 1 is 3 marks)

Q2

In a triangle ABC, side AB has length 10 cm, side AC has length 5 cm, and angle $BAC = \theta$ where θ is measured in degrees. The area of triangle ABC is 15cm^2

(a) Find the two possible values of $\cos\theta$

(4)

Given that BC is the longest side of the triangle,

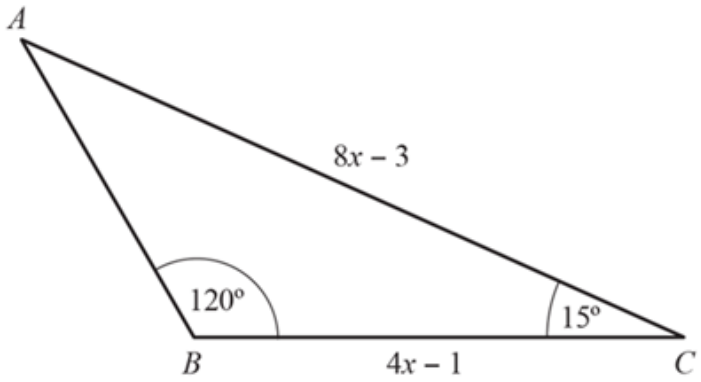
(b) find the exact length of BC.

(2)

(Total for Question 2 is 6 marks)

Q3

The diagram shows $\triangle ABC$ with $AC = 8x - 3$, $BC = 4x - 1$, $\angle ABC = 120°$ and $\angle ACB = 15°$.



(*a*) Show that the exact value of x is $\frac{9+\sqrt{6}}{20}$.

(7)

(*b*) Find the area of $\triangle ABC$, giving your answer in exact form

(3)

(Total for Question 3 is 10 marks)

Q4

A buoy is a device which floats on the surface of the sea and moves up and down as waves pass.

For a certain buoy, its height, above its position in still water, y in metres, is modelled by a sine function of the form $y = \frac{1}{2} \sin 180t°$, where t is the time in seconds.

(*a*) Sketch a graph showing the height of the buoy above its still water level for $0 \leqslant t \leqslant 10$ showing the coordinates of points of intersection with the t-axis.

(3)

(*b*) Write down the number of times the buoy is 0.4 m above its still water position during the first 10 seconds.

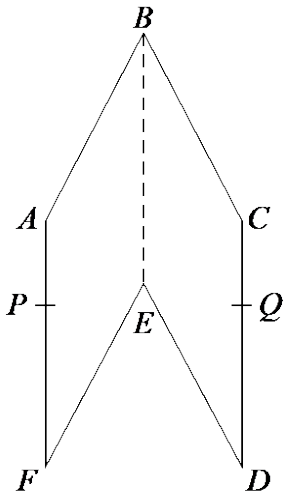
(1)

(*c*) Give one reason why this model might not be realistic.

(1)

(Total for Question 4 is 5 marks)

Q5 The diagram shows a hexagon $ABCDEF$.



$ABEF$ and $CBED$ are congruent parallelograms where $AB = BC = x$ cm.
P is the point on AF and Q is the point on CD such that $BP = BQ = 10$ cm.

Given that angle $ABC = 30°$,

prove that $\cos PBQ = 1 - \dfrac{(2-\sqrt{3})}{200}x^2$

(Total for Question 5 is 5 marks)

Bronze Questions

Calculators may not be used 

The total mark for this section is 26

Q1

(a) Show that the equation

$$5 \sin x = 1 + 2 \cos^2 x$$

can be written in the form

$$2 \sin^2 x + 5 \sin x - 3 = 0$$

(2)

(b) Solve, for $0 \leqslant x < 360°$,

$$2 \sin^2 x + 5 \sin x - 3 = 0$$

(4)

(Total for Question 1 is 6 marks)

Q2

Show that the equation

$$\cos^2 x = 8\sin^2 x - 6\sin x$$

can be written in the form

$$(3\sin x - 1)^2 = 2$$

(Total for Question 2 is 3 marks)

Q3

(a) Show that

$$\frac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta} = 4 - 5\cos\theta$$

(4)

(b) Hence, or otherwise, solve, for $0 \leq x \leq 360°$, the equation

$$\frac{10\sin^2 x - 7\cos x + 2}{3 + 2\cos x} = 4 + 5\sin x$$

(3)

(Total for Question 3 is 7 marks)

Q4

Solve, for $0 \leq x < 360°$,

(a) $\sin(x - 20^\circ) = \frac{1}{\sqrt{2}}$,

(4)

(b) $\cos 3x = -\frac{1}{2}$.

(6)

(Total for Question 4 is 10 marks)

End of Questions

Silver Questions

Calculators may not be used 

The total mark for this section is 34

Q1

(i) Solve, for $-180° \leq \theta < 180°$,

$$(1+\tan\theta)(2\sin\theta-\sqrt{3})=0$$

(4)

(ii) Solve, for $0 \leq x < 360°$,

$$2\sin x = \sqrt{2}\tan x.$$

(6)

(Total for Question 1 is 10 marks)

Q2

(a) Show that the equation

$$\tan 2x = 2\sin 2x$$

can be written in the form

$$(1-2\cos 2x)\sin 2x = 0$$

(2)

(b) Hence solve, for $0 \leq x \leq 180°$,

$$\tan 2x = 2\sin 2x$$

You must show clearly how you obtained your answers.

(5)

(Total for Question 2 is 7 marks)

Q3

(a) Show that the equation

$$8\sin^2\theta - 2\cos^2\theta = 3$$

can be written as

$$10\sin^2\theta = 5.$$

(2)

(b) Hence solve, for $0° \leq \theta < 360°$, the equation

$$8\sin^2\theta - 2\cos^2\theta = 3,$$

(7)

(Total for Question 3 is 10 marks)

Q4

(a) Show that the equation

$$\sin\theta\tan\theta = \cos\theta + 1$$

can be written in the form

$$2\cos^2\theta + \cos\theta - 1 = 0$$

(3)

(b) Hence solve, for $0 \leq \theta < 360°$,

$$\sin\theta\tan\theta = \cos\theta + 1$$

showing each stage of your working.

(5)

(Total for Question 4 is 8 marks)

End of Questions

Gold Questions

Calculators may not be used 

The total mark for this section is 32

Q1

(i) Solve, for $0 \leq \theta < 360°$, the equation

$$90\sin(\theta + 60°) = 45$$

You must show each step of your working.

(4)

(ii) Solve, for $-180 \leq x < 180$, the equation

$$\tan x - \sqrt{2}\sin x = 0$$

(5)

(Total for Question 1 is 9 marks)

Q2

(i) Solve, for $0 \leqslant \theta < 180°$, the equation

$$\sin 3\theta - \sqrt{3}\cos 3\theta = 0$$

(3)

(ii) Given that

$$4\sin^2 x + \cos x = 4 - k, \quad 0 \text{ „ } k \text{ „ } 3\,3\,3\,;$$

3

(a) Find cos x in terms of k.

(3)

(Total for Question 2 is 6 marks)

Q3

Solve, for $0 \leq x < 180^\circ$,

$$\cos(3x - 10^\circ) = \frac{1}{\sqrt{2}}$$

You should show each step in your working.

(7)

(Total for Question 3 is 7 marks)

Q4

(i) Find the solutions of the equation $\sin(3x - 15°) = \frac{1}{2}$, for which $0 \leqslant x \leqslant 180°$

(6)

(ii)

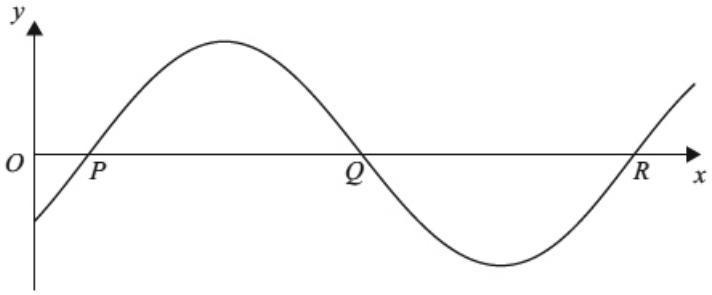


Figure 4

Figure 4 shows part of the curve with equation

$y = \sin(ax - b)$, where $a > 0$, $0 < b < 180$

The curve cuts the x-axis at the points P, Q and R as shown.

Given that the coordinates of P, Q and R are (11, 0), (108, 0) and 198, 0) respectively, find the values of a and b.

(4)

(Total for Question 4 is 10 marks)

End of Questions

Gold Mark Scheme

Platinum Questions

Calculators may not be used

The total mark for this section is 17

1

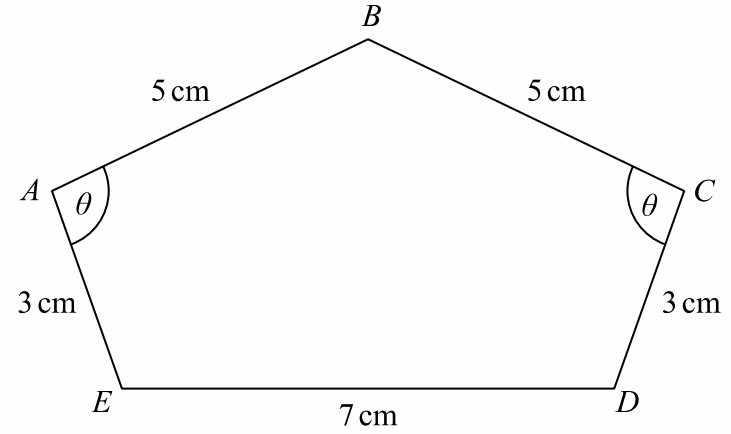


Figure 4

Figure 4 shows a shape $S(\theta)$ made up of five line segments AB, BC, CD, DE and EA.

The lengths of the sides are $AB = BC = 5$ cm, $CD = EA = 3$ cm and $DE = 7$ cm.

Angle BAE = angle $BCD = \theta$ radians.

The length of each line segment always remains the same but the value of θ can be varied so that different symmetrical shapes can be formed, with the added restriction that none of the line segments cross.

(a) Sketch $S(180^0)$, labelling the vertices clearly.

(2)

The shape $S(\phi)$ is a trapezium.

(b) Sketch $S(\phi)$ and calculate the value of ϕ.

(3)

The smallest possible value for θ is α, where $\alpha > 0$, and the largest possible value for θ is β, where $\beta > 180^0$.

(c) Show that $\alpha = \arccos\left(\frac{29}{40}\right)$. [arccos($x$) is an alternative notation for $\cos^{-1}(x)$]

(4)

(*d*) Find an expression for the value of β.

(4)

The area, in cm^2, of shape $S(\theta)$ is $R(\theta)$.

(*e*) Show that for $\alpha \leqslant \theta < 190^0$

$$R(\theta) = 15 \sin \theta + \frac{7}{4}\sqrt{87 - 120\cos\theta}$$

(4)

(Total for Question 1 is 17 marks)

Bronze Questions

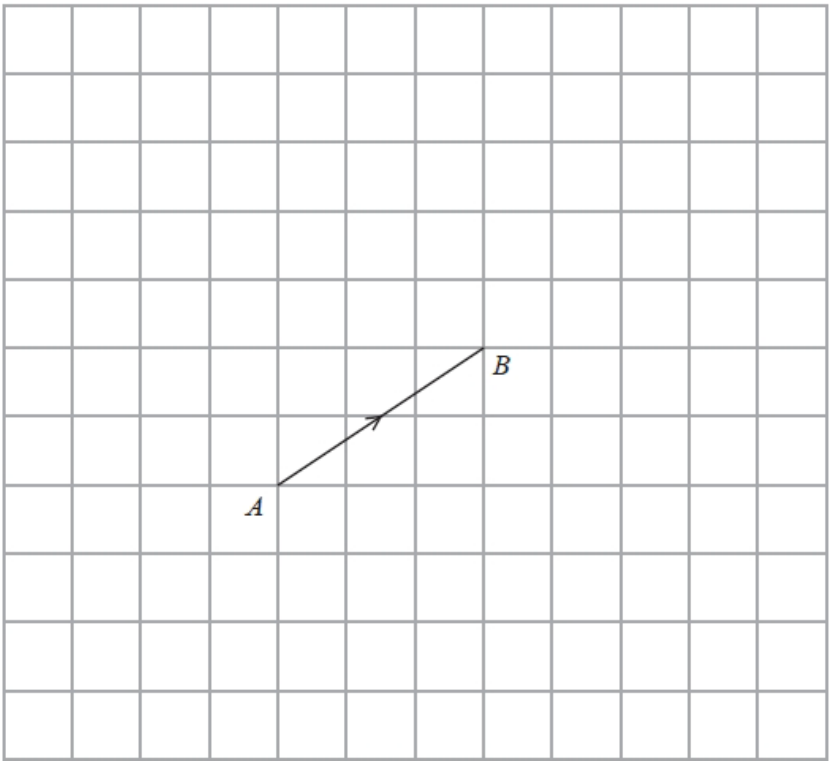
Calculators may not be used

The total mark for this section is 27

1

$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$

$\overrightarrow{AB}$ is shown on the grid



(a) On the grid, draw $\overrightarrow{BC}$.

(1)

$\overrightarrow{AD} = \overrightarrow{AB} - \overrightarrow{BC}$

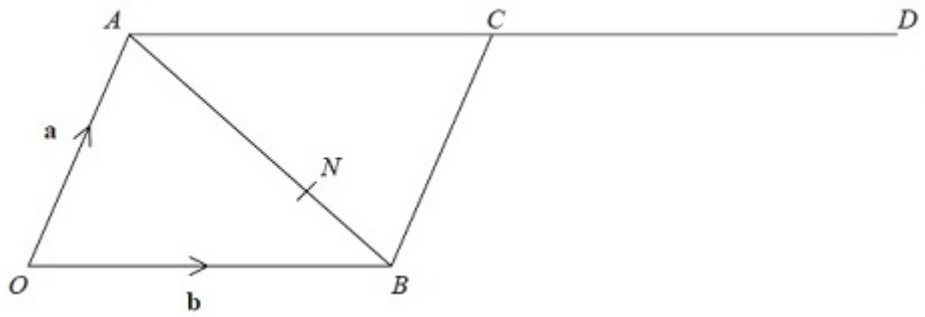
(b) On the grid, mark with a cross (X) the position of D.

Label this point D.

(2)

(Total for Question 1 is 3 marks)

Q2



$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

D is the point such that $\overrightarrow{AC} = \overrightarrow{CD}$

The point N divides AB in the ratio 2:1

(a) Write an expression for $\overrightarrow{ON}$ in terms of **a** and **b**.

(3)

(b) Prove that OND is a straight line.

(3)

(Total for Question 2 is 6 marks)

Q3

Given that the point A has position vector $4\mathbf{i} - 5\mathbf{j}$ and the point B has position vector $-5\mathbf{i} - 2\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$.

(2)

(b) Find $|\overrightarrow{AB}|$.

Give your answer as a simplified surd.

(2)

(Total for Question 2 is 4 marks)

Q4

A particle P is moving with constant velocity $(-3\mathbf{i} + 2\mathbf{j})$ m s^{-1}. At time $t = 6$ s P is at the point with position vector $(-4\mathbf{i} - 7\mathbf{j})$ m. Find the distance of P from the origin at time $t = 2$ s.

(Total for Question 4 is 5 marks)

Q5

[In this question, the unit vectors **i** *and* **j** *are due east and due north respectively. Position vectors are relative to a fixed origin O.]*

A boat P is moving with constant velocity $(-4\mathbf{i} + 8\mathbf{j})$ km h^{-1}.

(a) Calculate the speed of P, giving your answer as a simplified surd.

(2)

When $t = 0$, the boat P has position vector $(2\mathbf{i} - 8\mathbf{j})$ km. At time t hours, the position vector of P is $\mathbf{p}$ km.

(b) Write down $\mathbf{p}$ in terms of t.

(1)

A second boat Q is also moving with constant velocity. At time t hours, the position vector of Q is $\mathbf{q}$ km, where

$$\mathbf{q} = 18\mathbf{i} + 12\mathbf{j} - t(6\mathbf{i} + 8\mathbf{j})$$

Find

(c) the value of t when P is due west of Q,

(3)

(d) the distance between P and Q when P is due west of Q.

(3)

(Total for Question 5 is 9 marks)

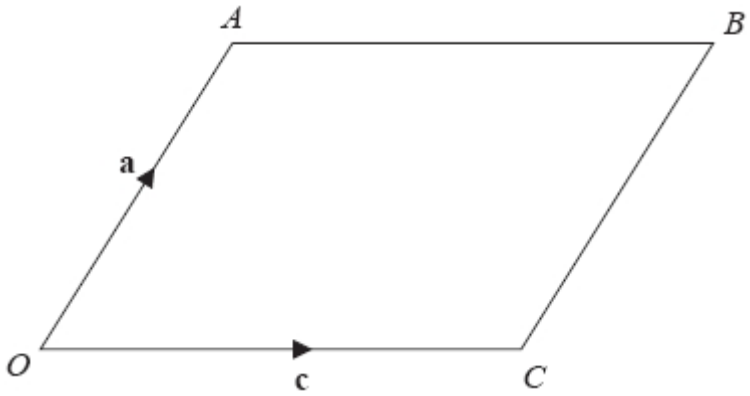
End of Questions

Silver Questions

Calculators may not be used

The total mark for this section is 25

Q1



$OABC$ is a parallelogram.

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OC} = \mathbf{c}$

X is the midpoint of the line AC.
OCD is a straight line so that $OC : CD = k : 1$

Given that $\overrightarrow{XD} = 3\mathbf{c} - \frac{1}{2}\mathbf{a}$

find the value of k.

(Total for Question 1 is 4 marks)

Q2

Given that the point A has position vector $3\mathbf{i} - 7\mathbf{j}$ and the point B has position vector $8\mathbf{i} + 3\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$.

(2)

(b) Find $\left|\overrightarrow{AB}\right|$. Give your answer as a simplified surd.

(2)

(Total for Question 2 is 4 marks)

Q3

Three forces, $(15\mathbf{i} + \mathbf{j})$ N, $(5q\mathbf{i} - p\mathbf{j})$ N and $(-3p\mathbf{i} - q\mathbf{j})$ N, where p and q are constants, act on a particle. Given that the particle is in equilibrium, find the value of p and the value of q.

(Total for Question 3 is 6 marks)

Q4

*[In this question, the horizontal unit vectors **i** and **j** are directed due east and due north respectively.]*

The velocity, $\mathbf{v}$ m s^{-1}, of a particle P at time t seconds is given by

$$\mathbf{v} = (1 - 2t)\mathbf{i} + (3t - 3)\mathbf{j}$$

(a) Find the speed of P when $t = 0$

(3)

(b) Find the bearing on which P is moving when $t = 2$

(2)

(c) Find the value of t when P is moving

(i) parallel to $\mathbf{j}$,
(ii) parallel to $(-\mathbf{i} - 3\mathbf{j})$.

(6)

(Total for Question 4 is 11 marks)

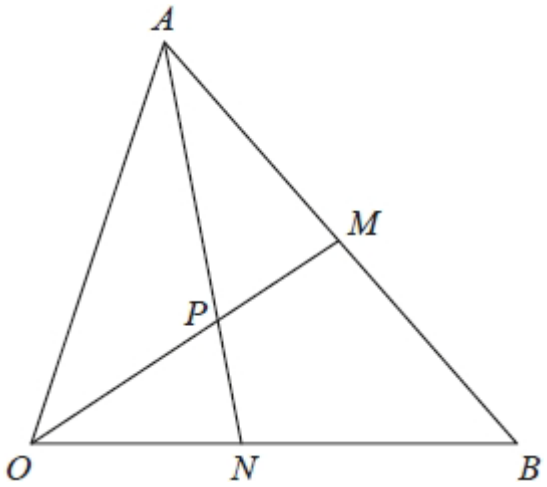
End of Questions

Gold Questions

Calculators may not be used

The total mark for this section is 29

Q1



OAB is a triangle.
OPM and *APN* are straight lines.
M is the midpoint of *AB*.

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

$OP : PM = 3 : 2$

Work out the ratio $ON : NB$

(Total for Question 1 is 5 marks)

Q2

[*In this question,* **i** *and* **j** *are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.*]

A ship sets sail at 9 am from a port P and moves with constant velocity. The position vector of P is $(4\mathbf{i} - 8\mathbf{j})$ km. At 9.30 am the ship is at the point with position vector $(\mathbf{i} - 4\mathbf{j})$ km.

(a) Find the speed of the ship in km h^{-1}.

(4)

(b) Show that the position vector $\mathbf{r}$ km of the ship, t hours after 9 am, is given by
$\mathbf{r} = (4 - 6t)\mathbf{i} + (8t - 8)\mathbf{j}$.

(2)

At 10 am, a passenger on the ship observes that a lighthouse L is due west of the ship. At 10.30 am, the passenger observes that L is now south-west of the ship.

(c) Find the position vector of L.

(5)

(Total for Question 2 is 11 marks)

Q3

[*In this question* **I** *and* **j** *are horizontal unit vectors due east and due north respectively.*]

A hiker H is walking with constant velocity $(1.2\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}.

(a) Find the speed of H.

(2)

Figure 3

A horizontal field $OABC$ is rectangular with OA due east and OC due north, as shown in Figure 3. At twelve noon hiker H is at the point Y with position vector 100 **j** m, relative to the fixed origin O.

(b) Write down the position vector of H at time t seconds after noon.

(2)

At noon, another hiker K is at the point with position vector $(9\mathbf{i} + 46\mathbf{j})$ m. Hiker K is moving with constant velocity $(0.75\mathbf{i} + 1.8\mathbf{j})$ m s^{-1}.

(c) Show that, at time t seconds after noon,

$$\overrightarrow{HK} = [(9 - 0.45t)\,\mathbf{i} + (2.7t - 54)\,\mathbf{j}]\text{metres}.$$

(4)

Hence,

(d) show that the two hikers meet and find the position vector of the point where they meet.

(5)

(Total for Question 3 is 13 marks)

Platinum Questions

Calculators may not be used 

The total mark for this section is 16

1 Points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$, respectively, relative to an origin O, and are such that OAB is a triangle with $OA = a$ and $OB = b$.

The point C, with position vector $\mathbf{c}$, lies on the line through O that bisects the angle AOB.

(a) Prove that the vector $b\mathbf{a} - a\mathbf{b}$ is perpendicular to $\mathbf{c}$.

(4)

The point D, with position vector $\mathbf{d}$, lies on the line AB between A and B.

(b) Explain why $\mathbf{d}$ can be expressed in the form $\mathbf{d} = (1 - \lambda)\mathbf{a} + \lambda\mathbf{b}$ for some scalar λ with $0 < \lambda < 1$

(2)

(c) Given that D is also on the line OC, find an expression for λ in terms of a and b only and hence show that

$$DA : DB = OA : OB$$

(8)

(+S2)

(Total for Question 1 is 16 marks)

Bronze Questions

Calculators may not be used

The total mark for this section is 28

Q1

The curve C has equation

$$y = 2x^2 - 12x + 16$$

Find the gradient of the curve at the point P (5, 6).

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(Total for Question 1 is 4 marks)

Q2

Given that $y = x^4 + x^{\frac{1}{3}} + 3$, find $\frac{dy}{dx}$

(Total for Question 2 is 3 marks)

Q3

A curve has equation

$$y = 2x^3 - 4x + 5$$

Find the equation of the tangent to the curve at the point $P(2, 13)$.

Write your answer in the form $y = mx + c$, where m and c are integers to be found.

Solutions relying on calculator technology are not acceptable.

(Total for Question 3 is 5 marks)

Q4

Prove, from first principles, that the derivative of x^3 is $3x^2$

(Total for Question 4 is 4 marks)

Q5

$$y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3$$

(a) Find $\frac{dy}{dx}$ giving each term in its simplest form.

(4)

(b) Find $\frac{d^2y}{dx^2}$

(2)

(Total for Question 5 is 6 marks)

Q6

Using calculus, find the coordinates of the stationary point on the curve with equation

$$y = 2x + 3 + \frac{8}{x^2}, \quad x > 0$$

(6)

(Total for Question 6 is 6 marks)

End of Questions

Silver Questions

Calculators may not be used

The total mark for this section is 34

Q1

The curve C has equation

$$y = 2x - 8\sqrt{x} + 5 \, , \, x \geq 0$$

(a) Find $\frac{dy}{dx}$, giving each term in its simplest form.

(3)

The point P on C has x-coordinate equal to $\frac{1}{4}$

(b) Find the equation of the tangent to C at the point P, giving your answer in the form $y = ax + b$, where a and b are constants.

(4)

(Total for Question 1 is 7 marks)

Q2

The curve C has equation $y = 6 - 3x - \frac{4}{x^3}$, $x \neq 0$

(a) Use calculus to show that the curve has a turning point P when $x = \sqrt{2}$

(4)

(b) Find the x-coordinate of the other turning point Q on the curve.

(1)

(c) Find $\frac{d^2y}{dx^2}$.

(1)

(d) Hence or otherwise, state with justification, the nature of each of these turning points P and Q.

(3)

(Total for Question 2 is 9 marks)

Contents

Q3

Prove, from first principles, that the derivative of $3x^2$ is $6x$

(Total for Question 3 is 4 marks)

Q4

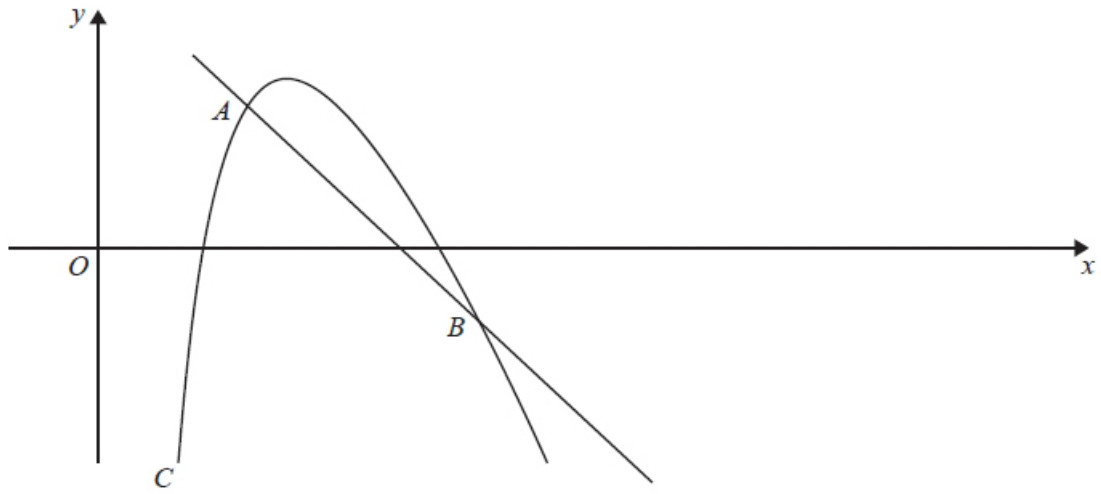


Figure 3

A sketch of part of the curve C with equation

$$y = 20 - 4x - {}^{18}\!/_{x}, \quad x > 0$$

is shown in Figure 3.

Point A lies on C and has an x coordinate equal to 2

Show that the equation of the normal to C at A is $y = -2x + 7$

(Total for Question 4 is 6 marks)

Contents

Q5

The volume V cm^3 of a box, of height x cm, is given by

$$V = 4x(5 - x)^2, \qquad 0 < x < 5$$

(a) Find $\dfrac{\mathrm{d}V}{\mathrm{d}x}$.

(4)

(b) Hence find the maximum volume of the box.

(4)

(Total for Question 5 is 8 marks)

Contents

Gold Questions

Calculators may not be used 

The total mark for this section is 33

Q1

The curve C has equation $y = 12\sqrt{(x)} - x^{\frac{3}{2}} - 10$, $x > 0$

(a) Use calculus to find the coordinates of the turning point on C.

(7)

(b) Find $\frac{d^2y}{dx^2}$

(2)

(c) State the nature of the turning point.

(1)

(Total for Question 1 is 10 marks)

Q2

The curve C has equation

$$y = 9 - 4x - \frac{8}{x}, \qquad x > 0$$

The point P on C has x-coordinate equal to 2.

(a) Show that the equation of the tangent to C at the point P is $y = 1 - 2x$.

(6)

(b) Find an equation of the normal to C at the point P.

(3)

(Total for Question 2 is 9 marks)

Q3

$$y = x^2 - k\sqrt{x}, \text{ where } k \text{ is a constant.}$$

(a) Find $\frac{dy}{dx}$

(2)

(b) Given that y is decreasing at $x = 4$, find the set of possible values of k.

(2)

(Total for Question 3 is 4 marks)

Q4

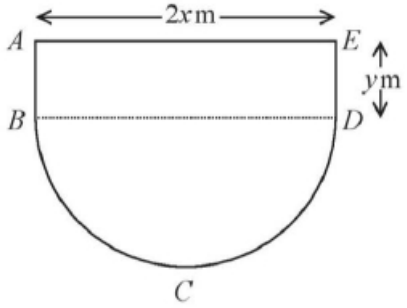


Figure 4

Figure 4 shows the plan view of the design for a swimming pool.

The shape of this pool $ABCDEA$ consists of a rectangular section $ABDE$ joined to a semicircular section BCD as shown in Figure 4.

Given that $AE = 2x$ metres, $ED = y$ metres and the area of the pool is 250 m^2,

(a) show that the perimeter, P metres, of the pool is given by

$$P = 2x + \frac{250}{x} + \frac{\pi x}{2}$$

(4)

(b) Explain why $0 < x < \sqrt{\frac{500}{\pi}}$

(2)

(c) Find the minimum perimeter of the pool, giving your answer in exact form.

(4)

(Total for Question 4 is 10 marks)

Contents

Platinum Questions

Calculators may not be used

The total mark for this section is 17

1

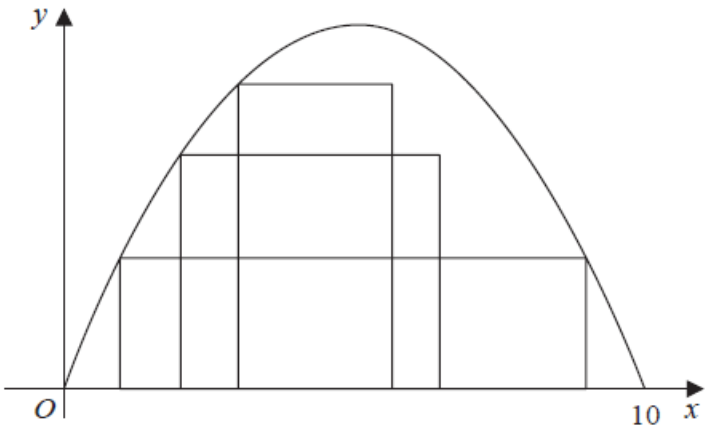


Figure 2

Figure 2 shows a sketch of the parabola with equation $y\ \ y = \frac{1}{2}x(10-x), 0 \le x \le 10$

This question concerns rectangles that lie under the parabola in the first quadrant. The bottom edge of each rectangle lies along the x-axis and the top left vertex lies on the parabola. Some examples are shown in Figure 2.

Let the x coordinate of the top left vertex be a.

(a) Explain why the width, w, of such a rectangle must satisfy $w \le 10 - 2a$

(2)

(b) Find the value of a that gives the maximum area for such a rectangle.

(5)

Given that the rectangle must be a square,

(c) find the value of a that gives the maximum area for such a square.

(3)

Given that the area of the rectangles is fixed as 36

(d) find the range of possible values for a

(6)

(+S1)

(Total for Question 7 is 17 marks)

Bronze Questions

Calculators may not be used

The total mark for this section is 30

Q1

Find $\int (12x^5 - 8x^3 + 3)\,dx$, giving each term in its simplest form.

(Total for Question 1 is 4 marks)

Q2

Find

$$\int (12x^5 - 3x^2 + 4x^{\frac{1}{3}})\,dx$$

giving each term in its simplest form.

(Total for Question 2 is 5 marks)

Q3

Find

$$\int (6x^2 + \frac{2}{x^2} + 5)\,dx$$

giving each term in its simplest form.

(Total for Question 3 is 4 marks)

Q4

A curve with equation $y = f(x)$ passes through the point (2, 10). Given that

$$f'(x) = 3x^2 - 3x + 5$$

find the value of f(1).

(Total for Question 4 is 5 marks)

Q5

Use calculus to find the exact value of $\int_1^2 \left(3x^2 + 5 + \frac{4}{x^2}\right) dx$

(Total for Question 5 is 5 marks)

Q6

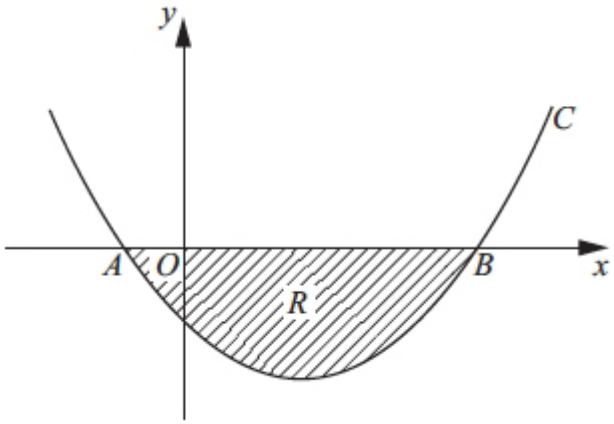


Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = (x + 1)(x - 5)$$

The curve crosses the x-axis at the points A and B.

(a) Write down the x-coordinates of A and B.

(1)

The finite region R, shown shaded in Figure 1, is bounded by C and the x-axis.

(b) Use integration to find the area of R.

(6)

(Total for Question 7 is 7 marks)

Silver Questions

Calculators may not be used

The total mark for this section is 37

Q1

A curve has equation $y = f(x)$ and passes through the point (4, 22).

Given that

$$f'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7,$$

use integration to find $f(x)$, giving each term in its simplest form.

(Total for Question 1 is 5 marks)

Q2

The gradient of a curve C is given by

$$\frac{dy}{dx} = \frac{(x^2+3)^2}{x^2}, \quad x \neq 0$$

(a) Show that $\frac{dy}{dx} = x^2 + 6 + 9x^{-2}$

(2)

The point (3, 20) lies on C.

(b) Find an equation for the curve C in the form $y = f(x)$.

(6)

(Total for Question 2 is 8 marks)

Q3

A curve has equation $y = f(x)$. The point P with coordinates $(9, 0)$ lies on the curve.

Given that

$$f'(x) = \frac{x+9}{\sqrt{x}} \quad x > 0$$

(a) find $f(x)$.

(6)

(b) Find the x-coordinates of the two points on $y = f(x)$ where the gradient of the curve is equal to 10

(4)

(Total for Question 3 is 10 marks)

Q4

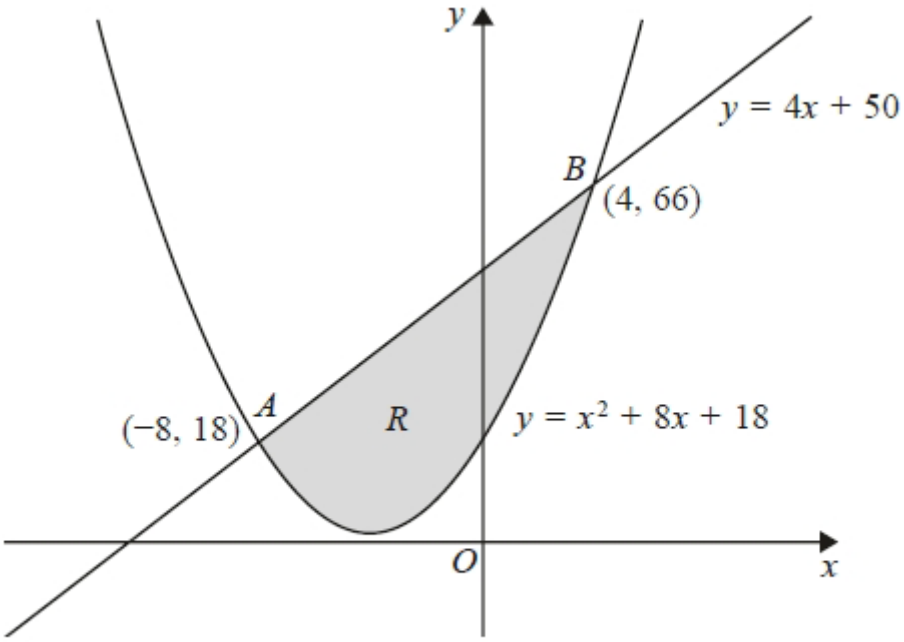


Figure 2

Figure 2 shows the line with equation $y = 4x + 50$ and the curve with equation $y = x^2 + 8x + 18$. The line cuts the curve at the points A $(-8, 18)$ and B $(4, 66)$.

The shaded region R is bounded by the line and the curve, as shown in Figure 2.

Using calculus, find the area of R.

(Total for Question 4 is 6 marks)

Q5

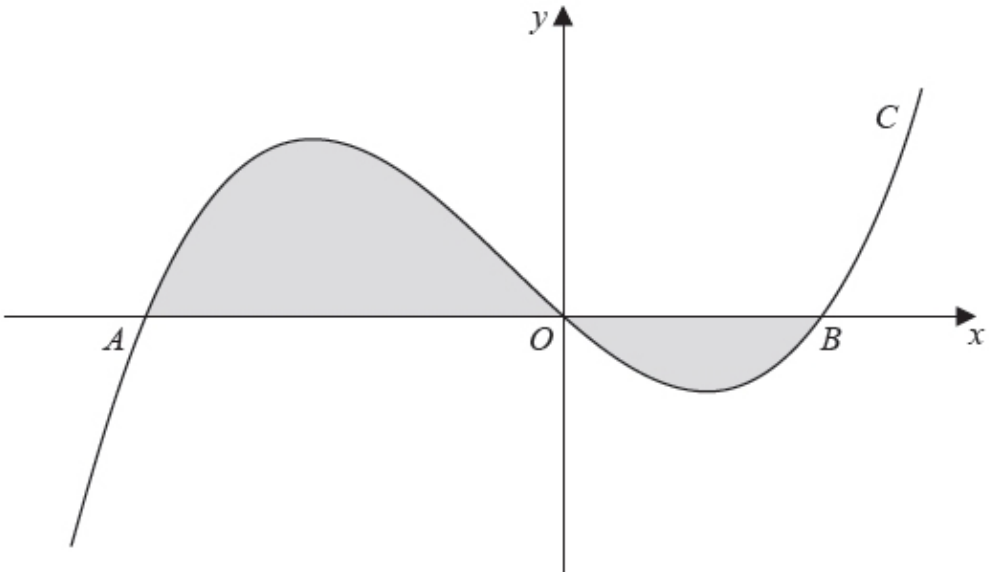


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = x(x + 4)(x - 2)$$

The curve C crosses the x-axis at the origin O and at the points A and B.

(a) Write down the x-coordinates of the points A and B.

(1)

The finite region, shown shaded in Figure 3, is bounded by the curve C and the x-axis.

(b) Use integration to find the total area of the finite region shown shaded in Figure 3.

(7)

(Total for Question 5 is 8 marks)

Gold Questions

Calculators may not be used 

The total mark for this section is 35

Q1

Given that $\dfrac{6x+3x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $6x^p + 3x^q$,

(a) write down the value of p and the value of q

(2)

Given that $\dfrac{dy}{dx} = \dfrac{6x+3x^{\frac{5}{2}}}{\sqrt{x}}$ and that $y = 90$ when $x = 4$,

(b) find y in terms of x, simplifying the coefficient of each term.

(5)

(Total for Question 1 is 7 marks)

Q2

$$\frac{dy}{dx} = 6x^{-\frac{1}{2}} + x\sqrt{x} \text{ where } x > 0$$

Given that $y = 37$ at $x = 4$, find y in terms of x, giving each term in its simplest form.

(Total for Question 2 is 7 marks)

Q3

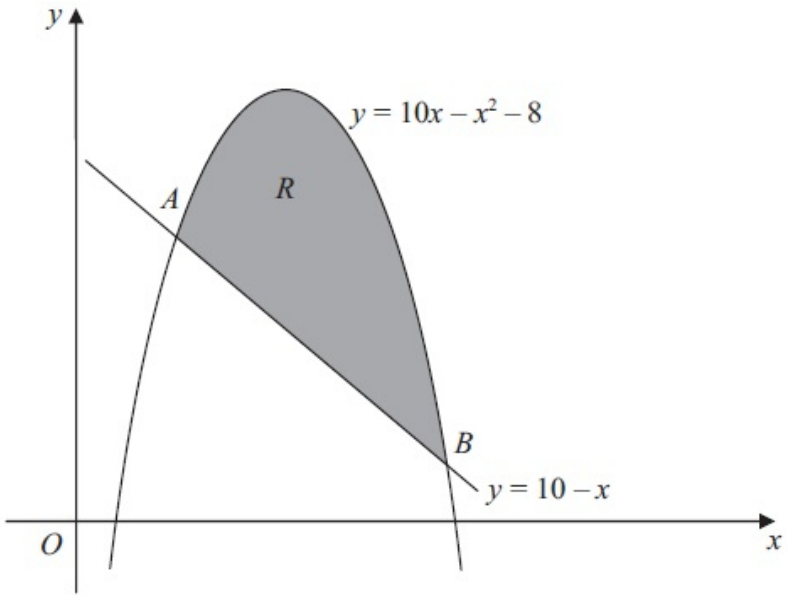


Figure 2

Figure 2 shows the line with equation $y = 10 - x$ and the curve with equation $y = 10x - x^2 - 8$
The line and the curve intersect at the points A and B, and O is the origin.

(a) Calculate the coordinates of A and the coordinates of B.

(5)

The shaded area R is bounded by the line and the curve, as shown in Figure 2.

(b) Calculate the exact area of R.

(7)

(Total for Question 3 is 12 marks)

Q4

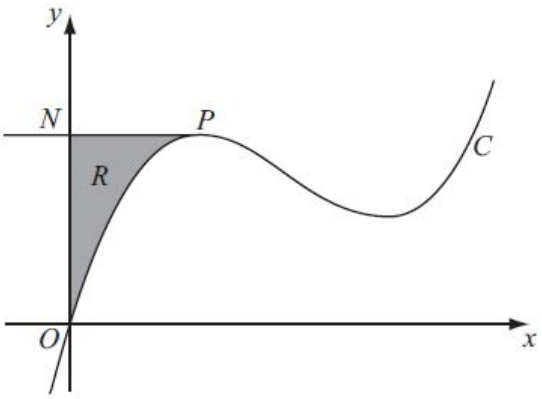


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + kx \ ,$$

where k is a constant.

The point P on C is the maximum turning point.

Given that the x-coordinate of P is 2,

(a) show that $k = 28$.

(3)

The line through P parallel to the x-axis cuts the y-axis at the point N.
The region R is bounded by C, the y-axis and PN, as shown shaded in Figure 2.

(b) Use calculus to find the exact area of R.

(6)

(Total for Question 4 is 9 marks)

Platinum Questions

Calculators may not be used 

The total mark for this section is 10

1 (a) On the same diagram, sketch $y = x$ and $y = \sqrt{x}$, for $x \geq 0$, and mark clearly the coordinates of the points of intersection of the two graphs.

(2)

(b) With reference to your sketch, explain why there exists a value a of x ($a > 1$) such that

$$\int_0^a x \, \mathrm{d}x = \int_0^a \sqrt{x} \, \mathrm{d}x .$$

(2)

(c) Find the exact value of a.

(4)

(d) Hence, or otherwise, find a non-constant function f(x) and a constant b ($b \neq 0$) such that

$$\int_{-b}^b \mathrm{f}(x) \, \mathrm{d}x = \int_{-b}^b \sqrt{[\mathrm{f}(x)]} \, \mathrm{d}x .$$

(2)

(Total for Question 1 is 10 marks)

Bronze Questions

Calculators may not be used

The total mark for this section is 25

Q1

Find the exact solution to the equation

$\ln x + \ln 3 = \ln 6,$

(Total for Question 1 is 2 marks)

Q2

Sketch the graph of

$$y = 3^x, \quad x \in \mathbb{R}$$

showing the coordinates of any points at which the graph crosses the axes.

(Total for Question 2 is 2 marks)

Q3

Find the value of x for which

$$\log_3(x - 2) = -1.$$

(Total for Question 2 is 2 marks)

Q4

Find the exact solutions, in their simplest form, to the equations

(a) $e^{3x-9} = 8$

(3)

(b) $\ln(2y + 5) = 2 + \ln(4 - y)$

(4)

(Total for Question 4 is 7 marks)

Q5

Given that

$$2\log_2 (x + 15) - \log_2 x = 6$$

(a) Show that

$x^2 - 34x + 225 = 0$

(5)

(b) Hence, or otherwise, solve the equation

$$2\log_2(x + 15) - \log_2 x = 6$$

(2)

(Total for Question 5 is 7 marks)

Q6

Water is being heated in an electric kettle. The temperature, θ °C, of the water t seconds after the kettle is switched on, is modelled by the equation

$$\theta = 120 - 100e^{-\lambda t}, \quad 0 \leqslant t \leqslant T$$

(a) State the value of θ when $t = 0$

(1)

Given that the temperature of the water in the kettle is 70°C when $t = 40$,

(b) find the exact value of λ, giving your answer in the form $\frac{\ln a}{b}$, where a and b are integers.

(4)

(Total for Question 6 is 5 marks)

Silver Questions

Calculators may not be used

The total mark for this section is 28

Q1

Find the exact solutions, in their simplest form, to the equations

(a) $2 \ln (2x + 1) - 10 = 0$

(2)

(b) $3^x e^{4x} = e^7$

(4)

(Total for Question 1 is 6 marks)

Q2

A student was asked to give the exact solution to the equation

$$2^{2x+4} - 9(2^x) = 0$$

The student's attempt is shown below:

$2^{2x+4} - 9(2^x) = 0$

$2^{2x} + 2^4 - 9(2^x) = 0$

Let $2^x = y$

$y^2 - 9y + 8 = 0$

$(y - 8)(y - 1) = 0$

$y = 8$ or $y = 1$

So $x = 3$ or $x = 0$

(a) Identify the two errors made by the student.

(2)

(b) Find the exact solution to the equation.

(2)

(Total for Question 2 is 4 marks)

Q3

(a) Given that

$$2\log_3(x-5)-\log_3(2x-13)=1$$

show that $x^2 - 16x + 64 = 0$.

(5)

(b) Hence, or otherwise, solve $2\log_3(x-5)-\log_3(2x-13)=1$

(2)

(Total for Question 1 is 7 marks)

Q4

Given that a and b are positive constants, solve the simultaneous equations

$$a = 3b,$$

$$\log_3 a + \log_3 b = 2.$$

Give your answers as exact numbers.

(Total for Question 2 is 6 marks)

Q5

The mass, m grams, of a leaf t days after it has been picked from a tree is given by

$$m = pe^{-kt}$$

where k and p are positive constants.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

(a) Write down the value of p.

(1)

(b) Show that $k = \frac{1}{4}\ln 3$

(4)

(Total for Question 5 is 5 marks)

Gold Questions

Calculators may not be used 

The total mark for this section is 30

Q1

(a) Find the positive value of x such that

$$\log_x 64 = 2$$

(2)

(b) Solve for x

$$\log_2 (11 - 6x) = 2 \log_2 (x - 1) + 3$$

(6)

(Total for Question 1 is 8 marks)

Q2

Find algebraically the exact solutions to the equation $2^x e^{3x+1} = 10$

Give your answer to (b) in the form $\frac{a + \ln b}{c + \ln d}$ where a, b, c and d are integers.

(Total for Question 2 is 5 marks)

Q3

(i)

$$2\log(x + a) = \log(16a^6), \text{ where } a \text{ is a positive constant}$$

Find x in terms of a, giving your answer in its simplest form.

(3)

(ii)

$$\log_3(9y + b) - \log_3(2y - b) = 2, \text{ where } b \text{ is a positive constant}$$

Find y in terms of b, giving your answer in its simplest form.

(4)

(Total for Question 3 is 7 marks)

Q4

(a) Find the value of y such that

$$\log_2 y = -3$$

(2)

(b) Find the values of x such that

$$\frac{\log_2 32 + \log_2 16}{\log_2 x} = \log_2 x$$

(5)

(Total for Question 4 is 7 marks)

Q5

Rabbits were introduced onto an island. The number of rabbits, P, t years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t}, \quad t \in \mathbb{R}, t \geqslant 0$$

(a) Write down the number of rabbits that were introduced to the island.

(1)

(b) Find the number of years it would take for the number of rabbits to first exceed 1000.

(2)

(Total for Question 5 is 3 marks)

Platinum Questions

Calculators may not be used 

The total mark for this section is 25

1 (a) Solve the equation

$$\sqrt{(3x + 16)} = 3 + \sqrt{(x + 1)}$$

(5)

(b) Solve the equation

$$\log_3 (x - 7) - \tfrac{1}{2} \log_3 x = 1 - \log_3 2$$

(7)

(Total for Question 1 is 12 marks)

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2 (a) Given that $x > 0$, $y > 0$, $x \neq 1$ and $n > 0$, show that

$$\log_x y = \log_{x^n} y^n$$

(2)

(b) Solve the following, leaving your answers in the form 2^p, where p is a rational number.

(i) $\log_2 u + \log_4 u^2 + \log_8 u^3 + \log_{16} u^4 = 5$

(ii) $\log_2 v + \log_4 v + \log_8 v + \log_{16} v = 5$

(iii) $\log_4 w^2 + \dfrac{3 \log_8 64}{\log_2 w} = 5$

(9)

(Total for Question 2 is 11 marks)
