23 CAPACITORS

23.1 Capacitance

1.)

	a	b	c	d
Q (μC)	60	330	9900	6.30
V (v)	12	**2.2**	9.0	4.5
C (μF)	**5.0**	150	1100	**1.4**

Using $C = \frac{Q}{V}$

2. a) $Q = V \times C = 12.0 \times 22 \times 10^{-6} = \underline{2.64 \times 10^{-4} C}$

b) $Q = I \times t \rightarrow 2.64 \times 10^{-4} = 2.5 \times 10^{-6} \times t$ ⇒ time $= \frac{2.64 \times 10^{-4}}{2.5 \times 10^{-6}} = \underline{106 s}$

3 a) $Q = I \times t = 0.5 \times 10^{-6} \times 55 = \underline{2.75 \times 10^{-5} C}$

b) $C = Q/V = \frac{2.75 \times 10^{-5}}{5} = \underline{5.5 \times 10^{-6} F}$ or $5.5 \mu F$

4. a) $Q = I \times t = 24 \times 10^{-6} \times 38 = \underline{9.12 \times 10^{-4} C} \rightarrow$ 9.12mC, not 9.12μC (textbook)

b) $C = Q/V = \frac{9.12 \times 10^{-7}}{4.2} = \underline{2.2 \times 10^{-7} F}$ or $0.22 \mu F$

c) $Q_2 = I_2 \times t_2 = 14 \times 10^{-6} \times 50 = \underline{7.0 \times 10^{-4} C}$

d) $C = \frac{Q}{V}$ (same C, so use $\underline{2.2 \times 10^{-7}}$) $\rightarrow 2.2 \times 10^{-7} F = \frac{\cancel{7.0 \times 9.12}}{V}$ Total charge $(7.0 \times 10^{-4} + 9.1 \times 10^{-4})$

$\hookrightarrow V = \underline{7.4 V}$

23.2 Energy stored in a charged capacitor

1. a) $Q = V \times C = 3.0 \times (10 \times 10^{-6}) = \underline{3.0 \times 10^{-5} C}$

$E = \frac{1}{2} CV^2 = \frac{1}{2}(10 \times 10^{-6}) \times 3^2 = \underline{4.5 \times 10^{-5} J}$

b) $Q = V \times C = 6.0 \times (10 \times 10^{-6}) = \underline{6.0 \times 10^{-5} C}$

$E = \frac{1}{2} CV^2 = \frac{1}{2}(10 \times 10^{-6}) \times 6^2 = \underline{1.8 \times 10^{-4} J}$

2. a) $Q = V \times C = 9 \times 50{,}000 \times 10^{-6} = \underline{0.45 C}$

$E = \frac{1}{2} CV^2 = \frac{1}{2} \times (50{,}000 \times 10^{-6}) \times 9^2 = \underline{2.0 J}$

b) $P = E \div t = 2.0 / 0.2 = \underline{10 J}$

23.2

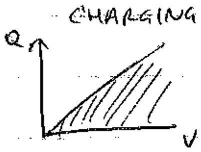
3. a) i) $Q = V \times C = 3 \times (2.2 \times 10^{-6}) = \underline{6.6 \times 10^{-6}\,C}$

$E = \frac{1}{2} C V^2 = \frac{1}{2} \times (2.2 \times 10^{-6}) \times 3^2 = \underline{9.9 \times 10^{-6}\,J}$

~~b~~ ii) $Q = V \times C = \underline{6.6 \times 10^{-6}\,C}$ (same as 3.ai)

$E = CV^2 = [a.i] \times 2 = 9.9 \times 10^{-6} \times 2 = \underline{19.8 \times 10^{-6}\,J}$

b) CHARGING CAPACITOR.



↳ current not constant.

$E = Q \times V$
= area under graph
→ $= \frac{1}{2} Q \times V$
or $\frac{1}{2} CV^2$; $\frac{1}{2}\frac{Q^2}{C}$

50% of energy supplied is wasted as thermal energy (~~elect~~ resistance) ~~in capacitor~~

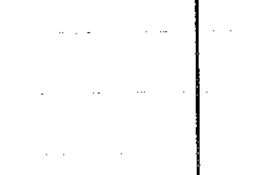
4. a) Initial charge, $Q = V \times C = 12.0 \times 4.7 \times 10^{-6}$
$= \underline{5.64 \times 10^{-5}\,C}$

$E = \frac{1}{2} CV^2 = \frac{1}{2} \times (4.7 \times 10^{-6}) \times 12^2 = \underline{3.38 \times 10^{-4}\,J}$

b) Total charge available $Q_T = \underline{5.64 \times 10^{-5}\,C}$

i) $Q_1 + Q_2 = 5.64 \times 10^{-5}\,C$

Total p.d. = 12V is same in each capacitor [parallel circuit]



$\frac{Q}{C} = V$ → for each capacitor,

$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} = 12.$

Ratios of capacitors → $C_1 : C_2$ = 2.2μF : 4.7μF (Total = 6.9μF)

∴ ratio charges → Q_1 $\left(\frac{2.2}{6.9} \times 5.64 \times 10^{-5}\right) = \underline{18\,\mu C}$ $Q \propto C$

→ Q_2 $\frac{4.7}{6.9} \times 5.64 \times 10^{-5} = \underline{36\,\mu C}$

ii) Final p.d, $V = \frac{Q_{Total}}{C_{Total}} = \frac{5.64 \times 10^{-5}}{(2.2 + 4.7)} = \underline{8.2\,V}$

c) $E_1 = \frac{1}{2} C_1 V^2 = \frac{1}{2} \times (2.2 \times 10^{-6}) \times 8.2^2 = \underline{7.3 \times 10^{-5}\,J}$

$E_2 = \frac{1}{2} C_2 V^2 = \frac{1}{2} \times (4.7 \times 10^{-6}) \times 8.2^2 = \underline{1.57 \times 10^{-4}\,J}$

23.3 Charging and discharging a capacitor through a fixed resistor

1\. a) i) $Q = V \times C = 6 \times (50 \times 10^{-6}) = \underline{3.0 \times 10^{-4} C}$

ii) Time constant $= R \times C = (100 \times 10^{3}) \times (50 \times 10^{-6}) = \underline{5s}$

b) i) $\frac{2v}{6v} = 33\%$ of it's original value.

time constant $= 5s$ to decrease to e^{-1} or 37%
$\hookrightarrow$ so it will be $\sim \underline{5s}$ (slightly longer)

ii) $\frac{V}{V_0} = e^{-t/RC}$ → time constant

$0.01 = e^{-5/RC}$ $\rightarrow \ln 0.01 = \frac{5}{R \times (50 \times 10^{-6})}$

↳ 99% discharged, 1% left

$R = \underline{22{,}000 \Omega}$

2\. a) i) $Q = V \times C = 9.0 \times (68 \times 10^{-6}) = \cancel{6.1} \ \underline{6.12 \times 10^{-4} C}$

ii) $V = I \times R \rightarrow 9.0 = I \times 20 \times 10^{3} \rightarrow I = \underline{4.5 \times 10^{-4} A}$

b) $V = V_0 e^{-t/R} = 9 \times e^{-5/(20{,}000 \times 68 \times 10^{-6})}$
$= \underline{0.228V}$

$I = V/R = \frac{0.228}{20{,}000} = \underline{1.1 \times 10^{-5} A}$

3\. a) $Q = V \times C = 6 \times (2.2 \times 10^{-6}) = \underline{1.3 \times 10^{-5} C}$
$E = 1/2\, CV^2 = 1/2 \times (2.2 \times 10^{-6}) \times 6^2 = \underline{4.0 \times 10^{-5} J}$

b) $V = V_0 e^{-t/RC}$ $\qquad R \times C = 100 \times 10^{3} \times 2.2 \times 10^{-6}$
$= 6 \times e^{-0.5/(0.22)}$ $\qquad = 0.22$ seconds
$= \underline{0.62V}$

c) $E = 1/2\, CV^2 = 1/2\, (2.2 \times 10^{-6}) \times 0.62^2$
$= \underline{4.23 \times 10^{-7} J}$

4. a i) $I = V/R = 12/200 = \underline{0.06A}$

ii) $E = \frac{1}{2}CV^2 = \frac{1}{2} \times (4.7\times10^{-6}) \times 12^2 = \underline{0.0034J}$

b) $\frac{V}{V_0} = e^{-t/R}$ $\qquad R\times C = 220\times10^3 \times 4.7\times10^{-6} = 1.034$ seconds

$\frac{3}{12} = e^{\frac{-t}{1.034}}$

$\ln\left(\frac{3}{12}\right) = \frac{t}{1.034} \rightarrow t = \ln 0.25 \times 1.034$
$= \underline{1.43s}$

c) $E = E_{\cancel{final}\,initial} - E_{final} = \overset{(4\cdot a\, ii)}{0.0034} - \frac{1}{2}CV^2$
$= 0.0034 - \frac{1}{2}\cdot(4.7\times10^{-6})\times 3^2$
$= \underline{0.00319J}$

23.4 Dielectrics

1. a) $C = \frac{A\varepsilon_0\varepsilon_r}{d}$ i) Dielectric $\rightarrow \varepsilon_r$ increases, so C increases
ii) $\hat{Q} = V\hat{C}$, so as C increases, Q increases.

b) $\hat{E} = \frac{1}{2}\hat{Q}V$, so if Q increases, E increases.

2. Circuit broken, so no current and Q stays the same
$\hat{E} = \frac{1}{2}\,Q^2/C$, as dielectric is removed C decreases (see above)
So E must increase.
↳ work done to overcome electrostatic attraction between charges on dielectric surface and plates.

3. a) $C = \frac{A\varepsilon_0\varepsilon_r}{d}$ → all constant apart from ε_r now 7.0
C with $\varepsilon_r = 1$ is 1.4pf, so $\times 7 = \underline{9.8pF}$

b) $E = \frac{1}{2}CV^2 = \frac{1}{2}\times(9.8\times10^{-12})\times 15^2 = \underline{1.103\times10^{-9}F}$ or 1.1nf

4. b) $\cancel{C = \frac{A\varepsilon_0\varepsilon_r}{d}}$

a) $E = \frac{V}{d}$ $\quad d = \frac{V}{E} = \frac{100}{700\times10^6} = \underline{1.43\times10^{-7}m}$

b) $E = \frac{1}{2}CV^2 = 0.5\times(100\times10^{-3})\times100^2 = 500J$
volume $= 1.6\times1.43\times10^{-7}$
$= 2.28\times10^{-7}$ $\qquad E$ per $V = \frac{500}{2.28\times10^{-7}} = \underline{2.18\,MJ/m^3}$