MASS AND ENERGY

1905: Einstein publishes 2nd paper
↳ by applying the principle of conservation of momentum:

$$m = m_0\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \quad \text{or} \quad m = \gamma m_0$$

m: relativistic mass; m_0: rest mass

↳ This equation predicts that as an object approaches the speed of light its mass increases significantly.
↳ NB: This does not mean that the amount of matter increases.
↳ Therefore, force needed to accelerate the object becomes significantly more! [$F = ma$]

↳ as $v \to c$, the relativistic mass $\to \infty$ ∴ no amount of Force will accelerate the mass further.

KEY CONSEQUENCE: No material object can ever reach the speed of light.

Experimental Evidence

- Electrons were accelerated to high speeds
- The specific charge on these electrons was measured
  ↳ specific charge: charge per unit mass [Ckg^{-1}]
- Experiments confirmed a change in mass of the electron

The famous Equation

Einstein concluded that the mass of an object would increase by tranferring Energy to the object.
↳ leading to the equation:

$$E = mc^2$$

m: relativistic mass

↳ led to equivalence of mass-energy

by subbing in the equation for m:

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{or} \quad E = \gamma m_0 c^2$$

NB: when $v = 0$:

$$E_0 = m_0 c^2$$

E_0: rest energy

MASS AND ENERGY

The famous Equation continued

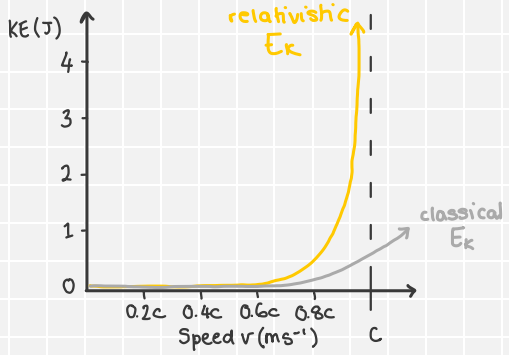
At speed v:

Total Energy = Rest Energy + Kinetic Energy

$$E = E_0 + E_K$$

or $$E_K = mc^2 - m_0c^2$$

↳ As $v \to c$, Einstein's theroy predicts that E_K increases much more rapidly than Newtons classical prediction.



Note:

→ E_K transferred to particle is:

$$W = QV$$

Q: charge on particle; V: accelerating p.d.

→ Total E can be expressed as:

$$E = QV + m_0c^2$$

Additional consequences

↳ the principle of conservation of mass-energy

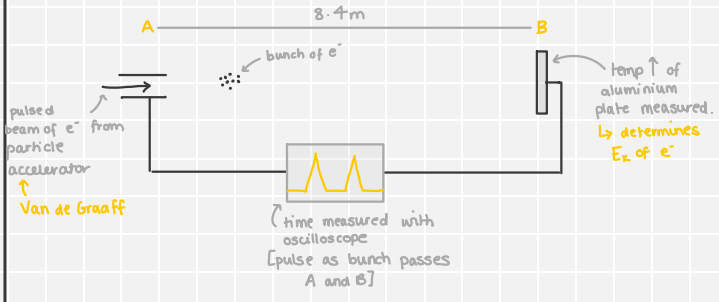
↳ mass and energy can be expressed in the same units (convert using $E=mc^2$)

↳ rest mass and rest energy of a particle are synonymous

$1kg = 8.99 \times 10^{16} J$ or $1u = 931.5 MeV$

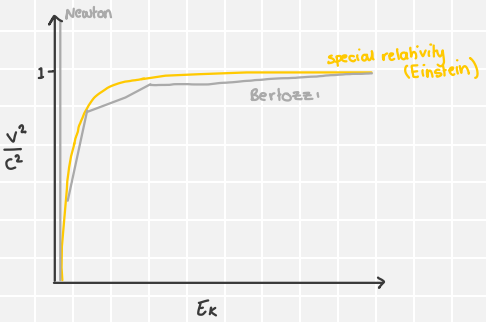
Bertozzi's ultimate speed experiment

↳ 1962 William Bertozzi used a particle accelerator to investigate the relationship between v of an electron and its E_K.



$$\text{Speed} = \frac{8.4}{\text{time between pulses}}$$

→ results were presented in graph of $\frac{v^2}{c}$ vs. E_K:



→ Bertozzi's results matched Einstein's prediction

↳ also showed that as e⁻ were accelerated through higher V, speed of e⁻ approached a limiting value of c.