**Topics 1 & 2**

**Algebra and Quadratics**

# Bronze, Silver, Gold and

# Platinum Worksheets for

# AS level Mathematics

# Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the ‘Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS’ textbook.

# Non-Calculator Questions

The new specification allows calculators to be used in all papers. We have, however, put these questions together with the intention that students can complete them without a calculator. It’s important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords “show that” or “prove”. If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

# Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

* [Bronze Questions](#BrQue)
* [Bronze Mark Scheme](#BrMS)
* [Silver Questions](#SiQue)
* [Silver Mark Scheme](#SiMS)
* [Gold Questions](#GoQu)
* [Gold Mark Scheme](#GoMS)

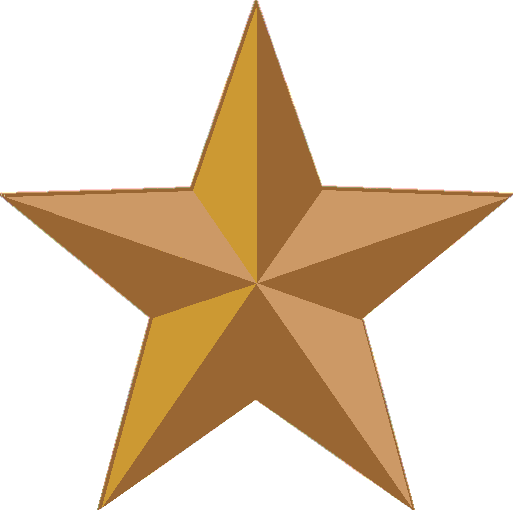
The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

* [Platinum Questions](#PlQu)
* [Platinum Mark Schemes](#PlMS)

# Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found [here](https://qualifications.pearson.com/en/qualifications/edexcel-a-levels/advanced-extension-award-mathematics-2018.html) on the Pearson Edexcel Website, or [here](https://www.mathsemporium.com/category/advanced-extension-award-mathematics/) on the Maths Emporium

**Bronze Questions **

**Calculators may not be used**

The total mark for this section is 32

**Q1**

(a)  Find the value of 

**(2)**

(b)  Simplify 

**(2)**

**(Total for Question 1 is 4 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q2**

(a) Write down the value of 

**(1)**

(b) Simplify fully 

**(3)**

**(Total for Question 2 is 4 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q3**

(a)  Simplify



giving your answer in the form  , where *a* is an integer.

**(2)**

(b)  Hence, or otherwise, simplify



giving your answer in the form , where *b* and *c* are integers and *b* ≠ 1

**(3)**

**(Total for Question 3 is 5 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q4**

Given that, find the value of *a*

**(Total for Question 4 is 3 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q5**

(a)  Evaluate , giving your answer as an integer.

**(2)**

(b)  Simplify fully 

**(2)**

**(Total for Question 5 is 4 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q6**

Express 82*x* + 3 in the form 2*y* , stating *y* in terms of *x*

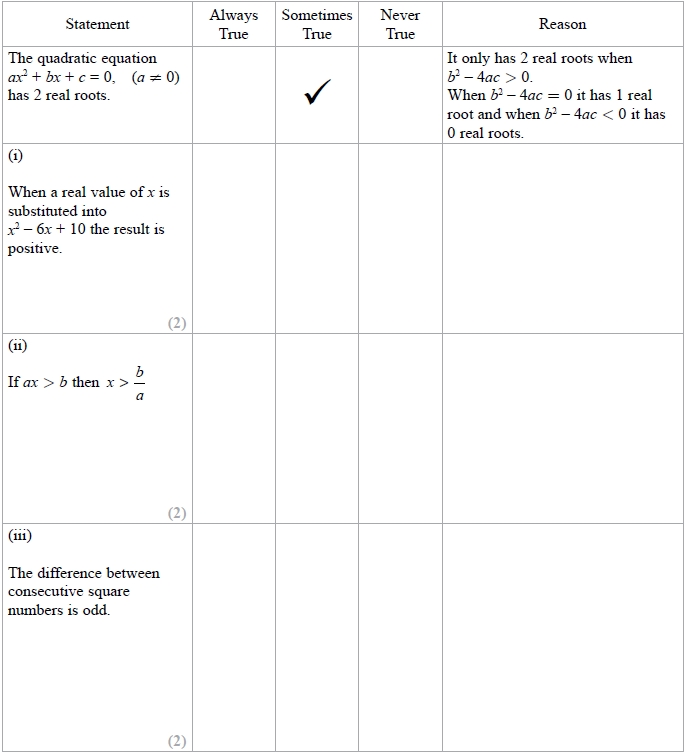
**(Total for Question 6 is 2 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q7**

Complete the table below. The first one has been done for you.

For each statement you must state if it is always true, sometimes true or never true, giving a reason in each case.



**(Total for Question 7 is 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q8**

The equation *x*2 + 3*px* + *p* = 0, where *p* is a non-zero constant, has equal roots.

Find the value of *p*.

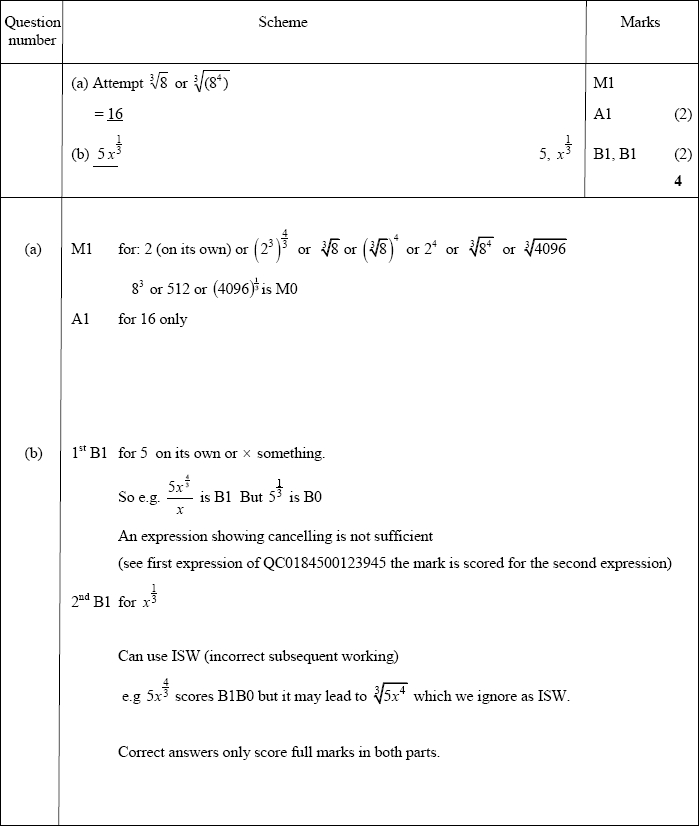
**(Total for Question 8 is 4 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

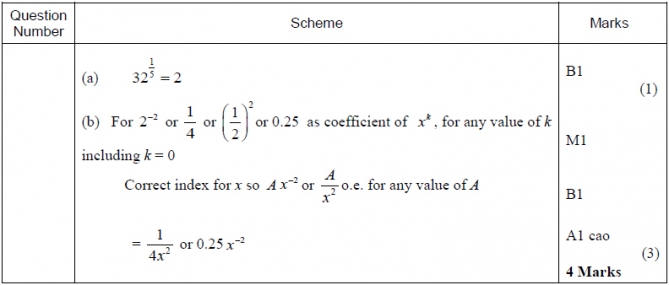
**End of Questions**

**Bronze Mark Scheme**

**Q1**



**Q2**



**Notes**

(a)   B1     Answer 2 must be in part (a) for this mark   
(b)   Look at their final answer

M1     For 2−2 or  or  or 0.25 in their answer as coefficient of *xk* for numerical value of *k*

(including *k* = 0) so final answer  is M1 B0 A0

B1     *Ax*−2 or  or equivalent e.g. *Ax* or *Ax* i.e. correct power of *x* seen in final answer

May have a bracket provided it is (*Ax*)−2 or 

A1      or  or 0.25 *x*−2 oe but must be correct power **and** coefficient combined correctly and must not be followed by a different wrong answer.

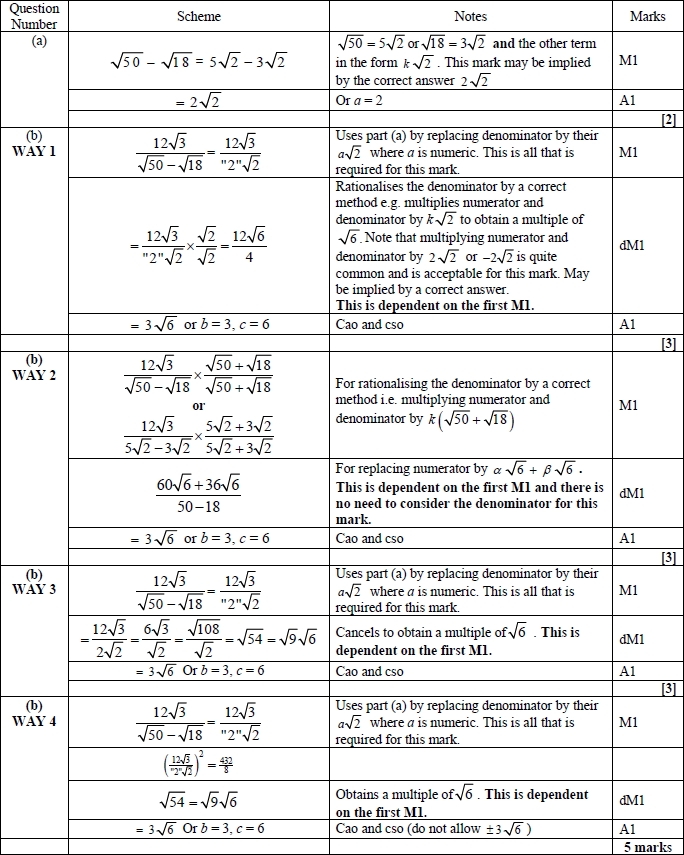
**Poor bracketing:** 2*x*−2 earns M0 B1 A0 as correct power of *x* is seen in this solution (They can recover if they follow this with  etc)

**Special case** (2*x*)−2 as a **final** answer and  can have M0 B1 A0 if the correct expanded answer is not seen

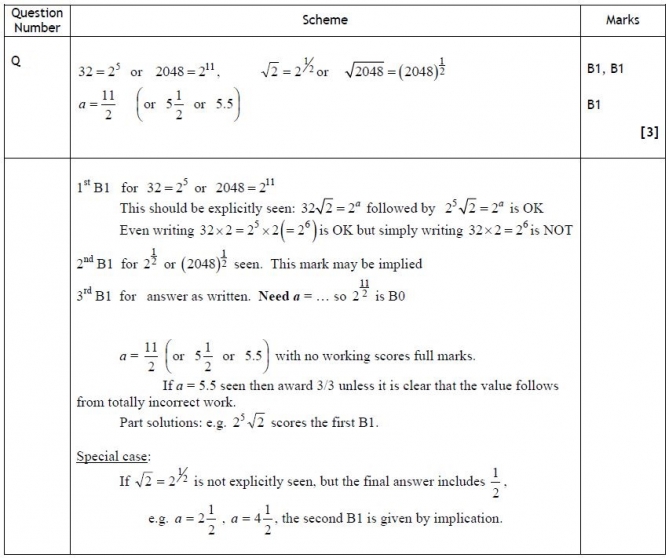
The correct answer  etc. followed by  or (2*x*)−2, treat  as final answer so M1 B1 A1 isw

But the correct answer  etc clearly followed by the wrong 2*x*−2 or 4*x*−2, gets M1 B1 A0 do not ignore subsequent wrong work here

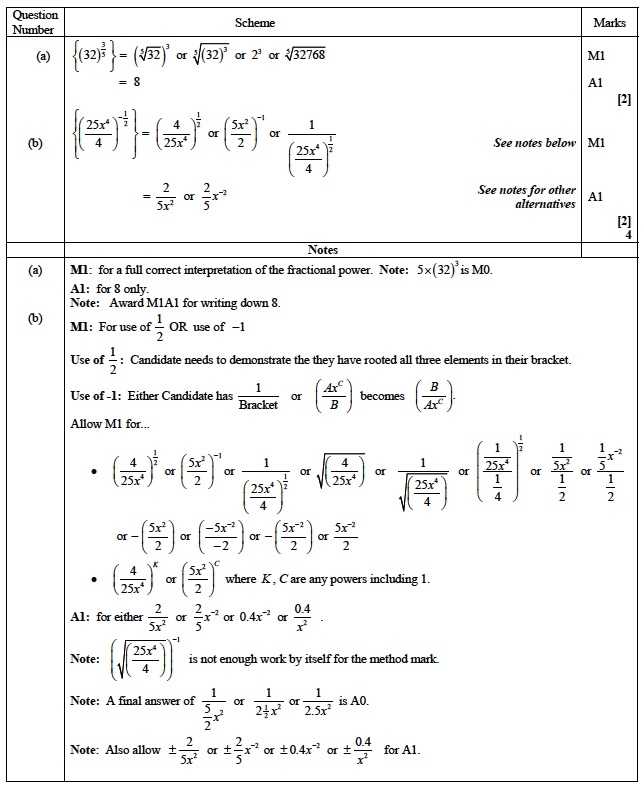
**Q3**



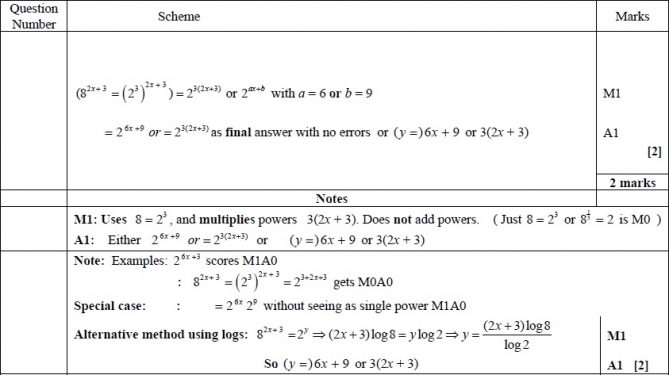
**Q4**



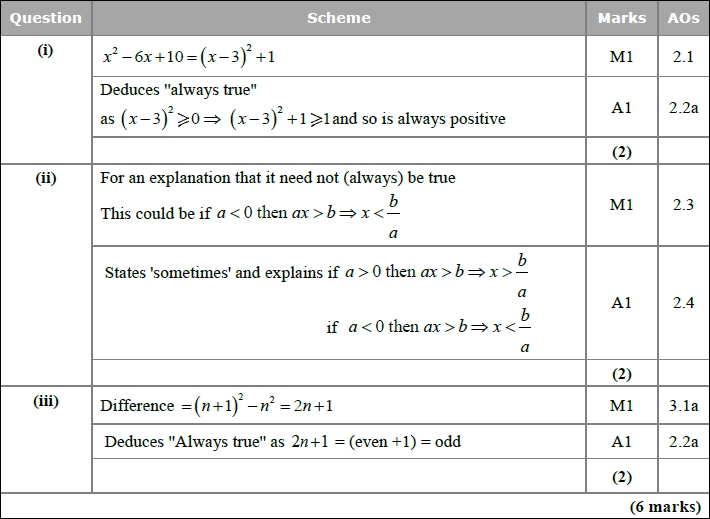
**Q5**

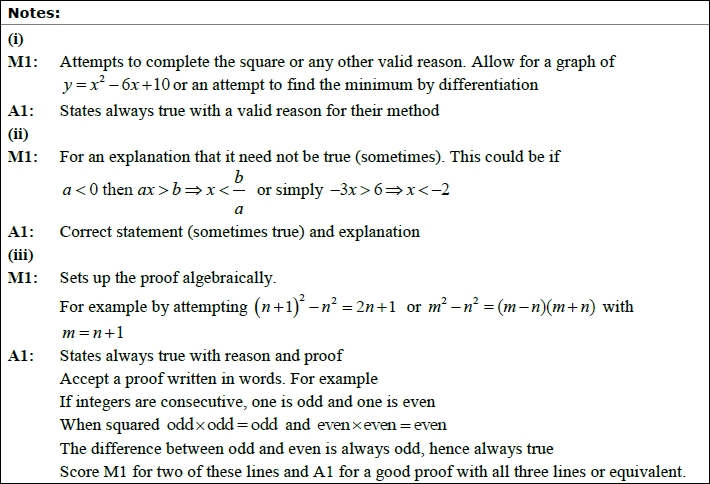


**Q6**

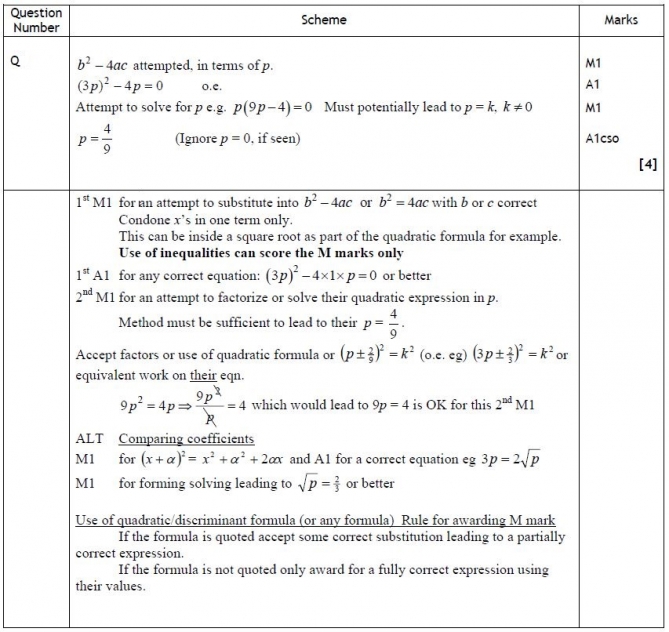


**Q7**





**Q8**



**Silver Questions **

**Calculators may not be used**

The total mark for this section is 35

**Q1**

(a) Find the value of 

**(2)**

(b) Simplify 

**(2)**

**(Total for Question 1 is 4 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q2**

Show that  can be written in the form , where *a* and *b* are integers.

**(Total for Question 2 is 5 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q3**

Solve

(a) 

**(1)**

(b) 

**(4)**

**(Total for Question 3 is 5 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q4**

Given



express *y* as a function of *x*

**(Total for Question 4 is 3 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q5**

Find, using algebra, all real solutions to the equation

(i)  16*a*2 = 

**(4)**

(ii)  *b*4 + 7*b*2 – 18 = 0

**(4)**

**(Total for Question 5 is 8 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q6**

The equation *kx*2 + 4*kx* + 3 = 0, where *k* is a constant, has no real roots.

Prove that

0 *k* < 

**(Total for Question 6 is 4 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q7**



where *k* is a real constant.

(a)  Find the discriminant of f(*x*) in terms of *k*.

**(2)**

(b)  Show that the discriminant of f(*x*) can be expressed in the form  (*k* + *a*)2 + *b*, where *a* and *b* are integers to be found.

**(2)**

(c)  Show that, for all values of *k*, the equation  f(*x*) = 0  has real roots.

**(2)**

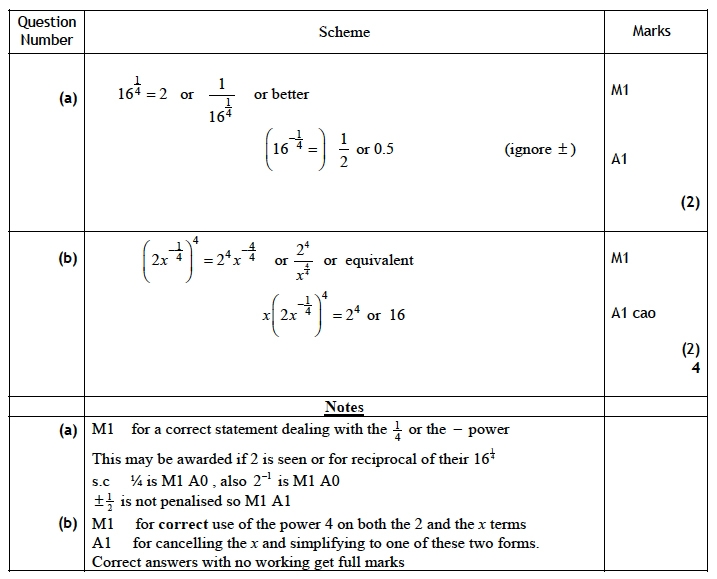
**(Total for Question 7 is 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

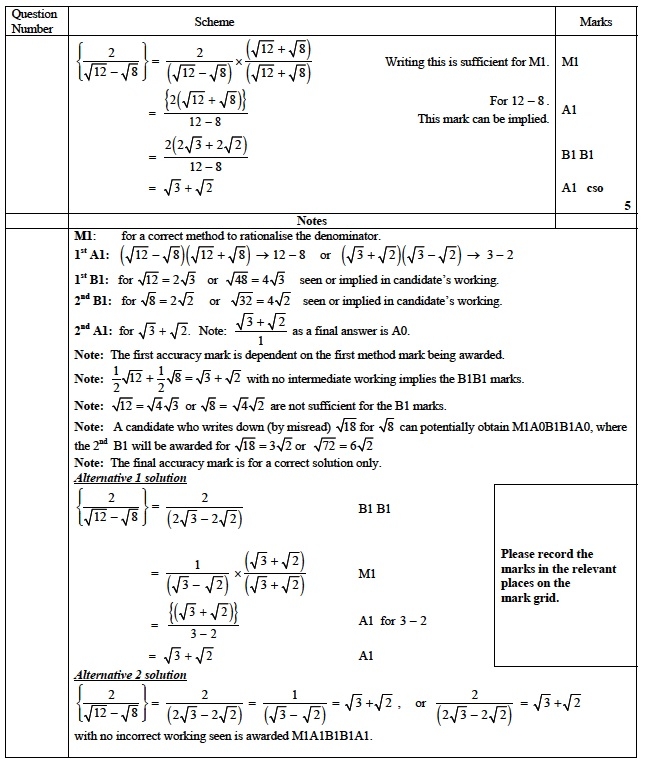
**End of Questions**

**Silver Mark Scheme**

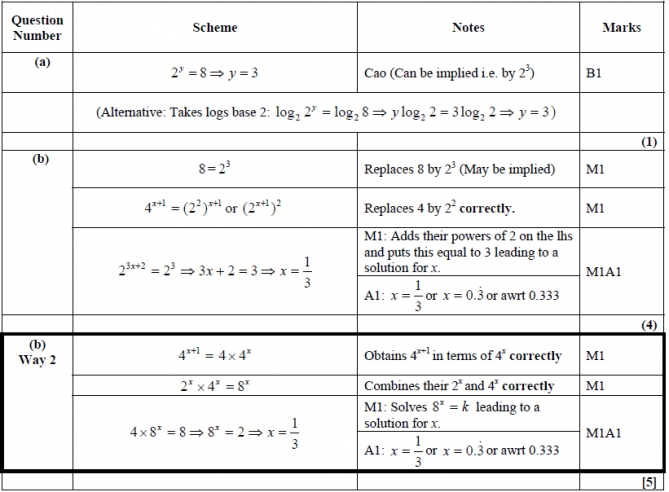
**Q1**



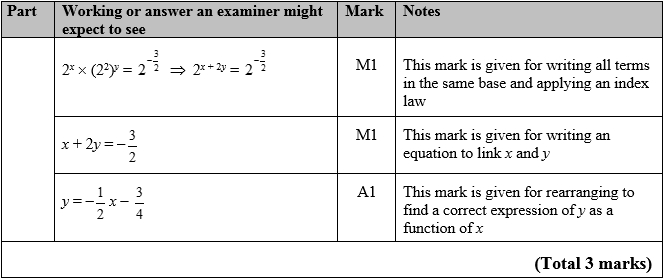
**Q2**



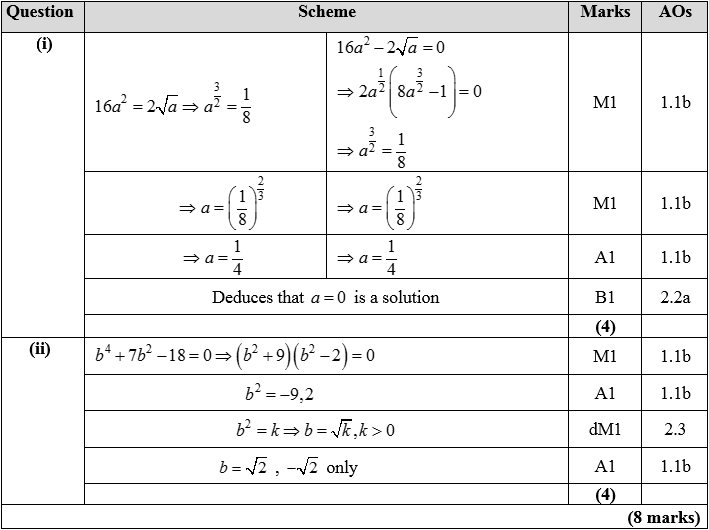
**Q3**

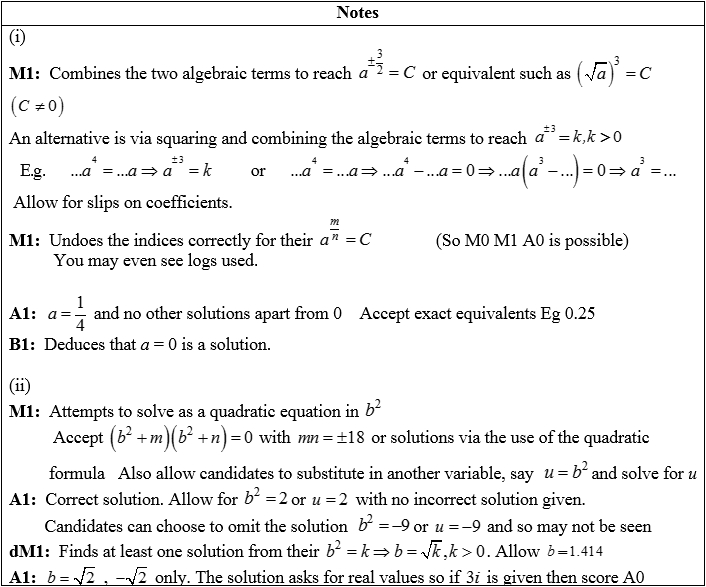


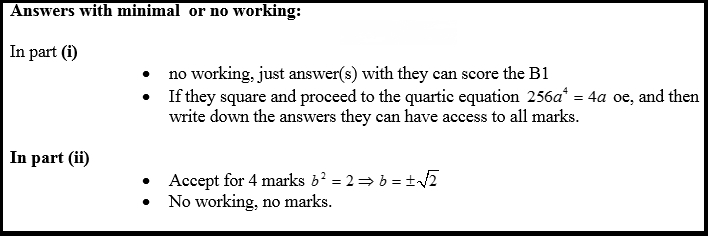
**Q4**



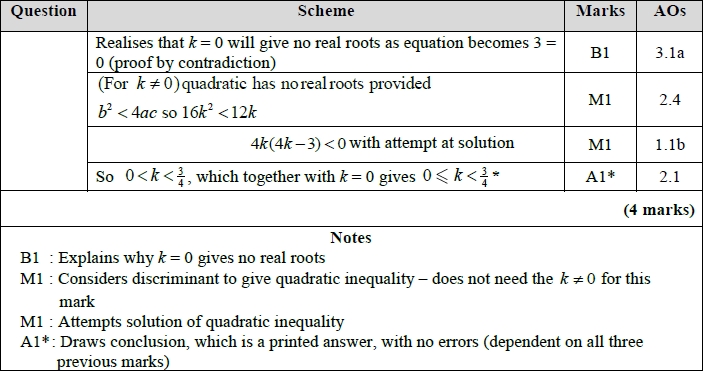
**Q5**



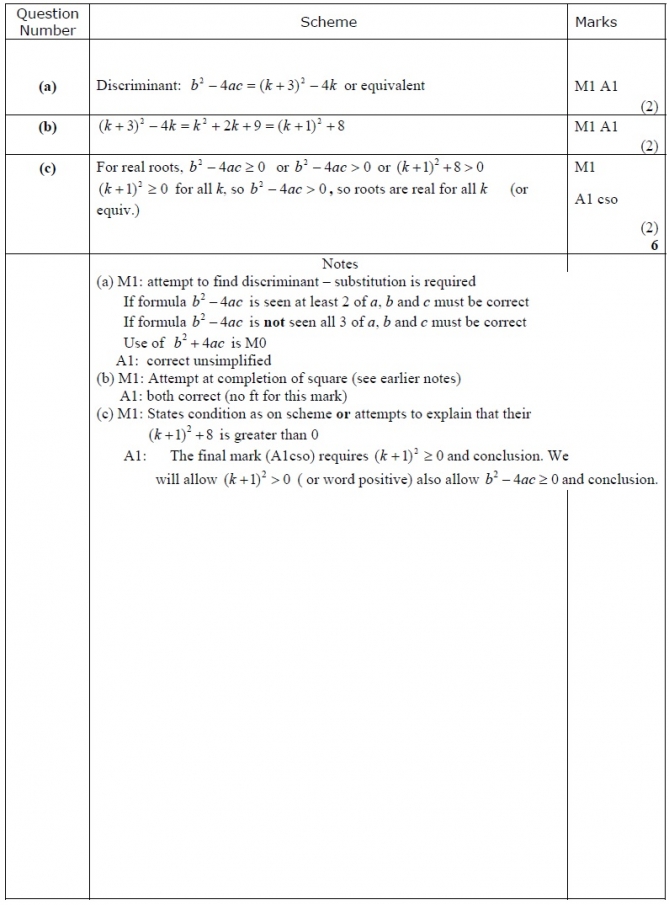




**Q6**



**Q7**



**Gold Questions **

**Calculators may not be used**

The total mark for this section is 33

**Q1**

Express 93*x* + 1 in the form 3*y*, giving *y* in the form *ax* + *b*, where *a* and *b* are constants.

**(Total for Question 1 is 2 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q2** 

The equation *x*2 + (*k* − 3)*x* + (3 − 2*k*) = 0, where *k* is a constant, has two distinct real roots.

(a) Show that *k* satisfies

*k*2 + 2*k* − 3 > 0

**(3)**

(b) Find the set of possible values of *k*.

**(4)**

**(Total for Question 2 is 7 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q3**

Given that the equation 2*qx*2 + *qx* – 1 = 0, where *q* is a constant, has no real roots,

(a)  show that *q*2 + 8*q* < 0.

**(2)**

(b)  Hence find the set of possible values of *q*.

**(3)**

**(Total for Question 3 is 5 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q4**

**In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

(i)  Solve the equation



writing the answer as a surd in simplest form.

**(3)**

(ii)  Solve the equation



**(3)**

**(Total for Question 4 is 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q5**

Given that *y* = 2*x*,

(a)   express 4*x* in terms of *y*.

**(1)**

(b)   Hence, or otherwise, solve

8(4*x*) – 9(2*x*) + 1 = 0

**(4)**

**(Total for Question 5 is 5 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q6**

f(*x*) = *x*2 − 8*x* + 19

(a)   Express f(*x*) in the form (*x* + *a*)2 + *b*, where *a* and *b* are constants.

**(2)**

The curve *C* with equation *y* = f(*x*) crosses the *y*-axis at the point *P* and has a minimum point at the point *Q*.

(b)   Sketch the graph of *C* showing the coordinates of point *P* and the coordinates of point *Q*.

**(3)**

(c)   Find the distance *PQ*, writing your answer as a simplified surd.

**(3)**

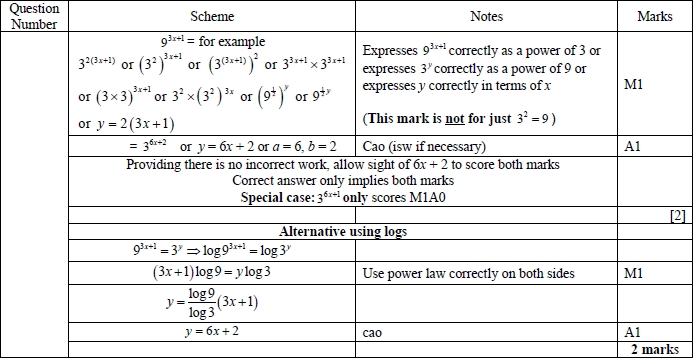
**(Total for Question 6 is 8 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

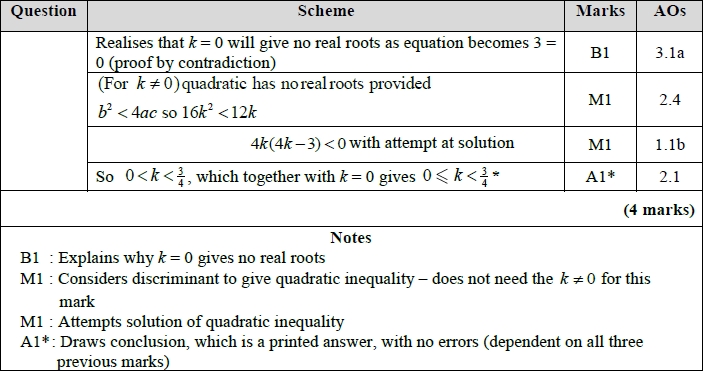
**End of Questions**

**Gold Mark Scheme**

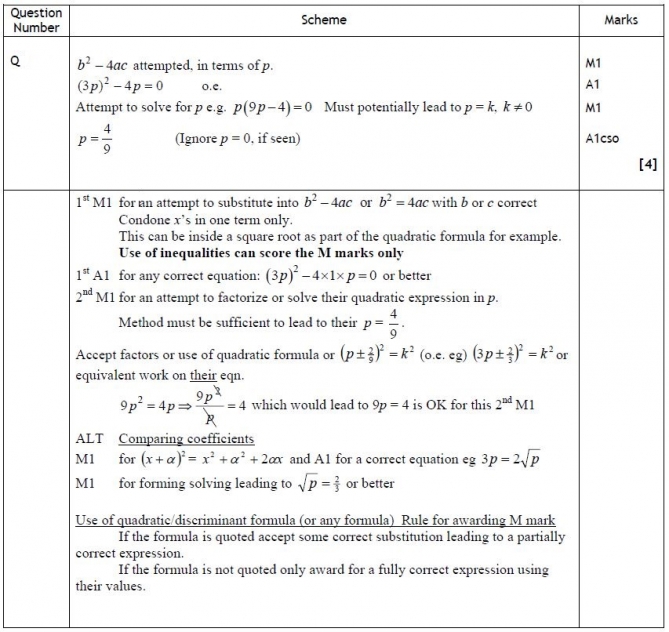
Q1



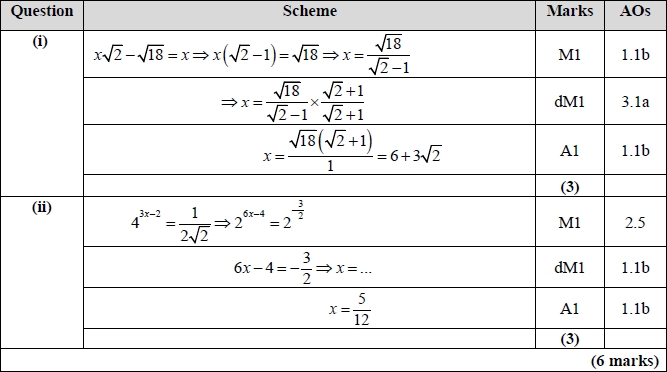
**Q2**

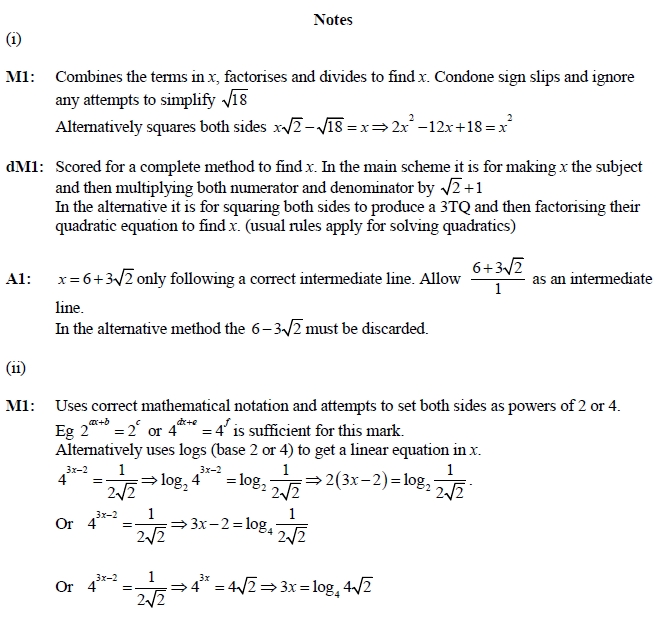


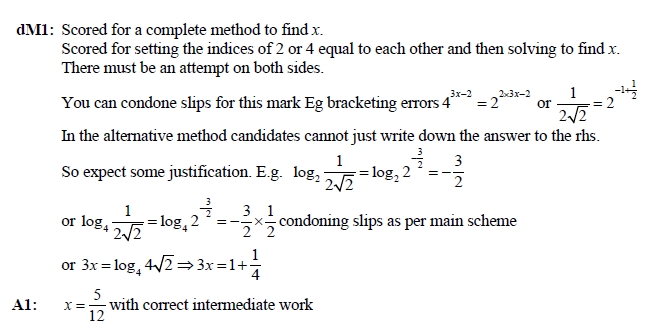
**Q3**



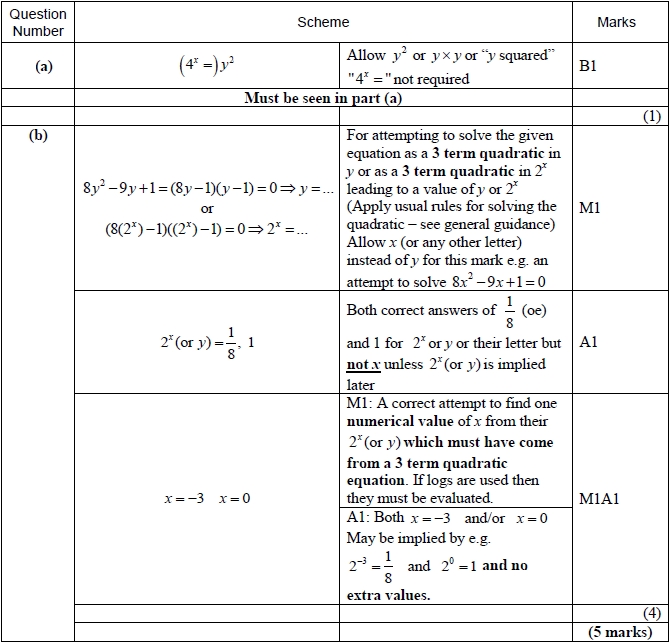
**Q4**



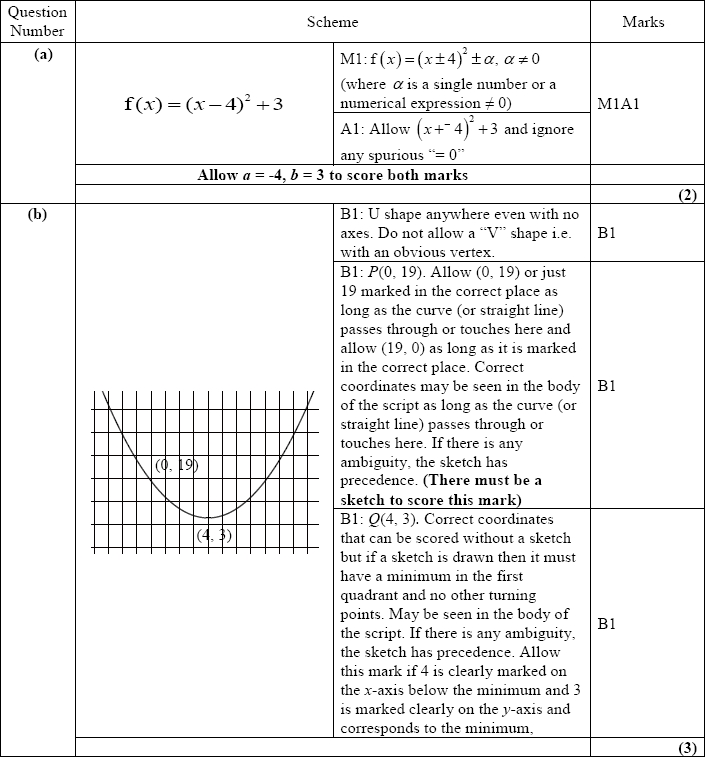


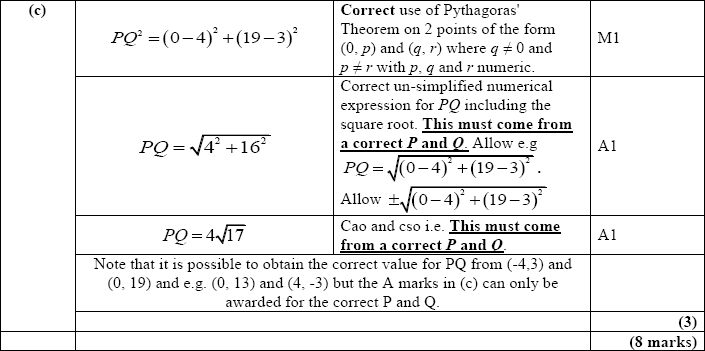


**Q5**



**Q6**





**Platinum Questions **

**Calculators may not be used**

The total mark for this section is 18

**1** A student was attempting to prove that *x* **= ** is the only real root of

*x*3 + *x* − **** = 0.

The attempted solution was as follows.

*x*3 + *x* = ****

**∴** *x*(*x*2 + ) = ****

**∴** *x* = 

or *x*2 +  = ****

i.e. *x*2 = −**** no solution

**∴** only real root is *x* = ****

(a) Explain clearly the error in the above attempt.

**(2)**

(b) Give a correct proof that *x* = **** is the only real root of*x*3 + *x* − **** = 0.

**(3)**

The equation

*x*3 + *βx* − *α* = 0 (I)

where *α*, *β* are real, *α* ≠ 0, has a real root at *x* = *α*.

(c) Find and simplify an expression for *β* in terms of *α* and prove that *α* is the only real root provided |*α*| < 2.

**(6)**

An examiner chooses a positive number *α* so that *α* is the only real root of equation (I) but the incorrect method used by the student produces 3 distinct real “roots”.

(d) Find the range of possible values for *α*.

**(7)**

**(Total for Question 1 is 19 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**End of Questions**

**Platinum Mark Scheme**

|  |  |  |
| --- | --- | --- |
| **Question Number** | **Scheme** | **Marks** |
|  |  |  |
| **7.** (*a*) | *pq* =  ⇒ *p* =  or *q* =  (line 3) identify; explain | B1; B1 (2) |
| (*b*) | *x*3 + *x* −  = 0 ⇒ (*x* − )(*x*2 + *x* + 1) = 0 attempt to divide | M1 |
|  | correct quadratic | A1 |
|  | i.e. *x* =  or *x*2 + *x* + 1 = 0, discriminant = ()2 – 4 | M1 |
|  | < 0 ∴ no real roots (so only root is *x* = ) | A1 cso (4) |
| (*c*) | *x* = *α* is a root ⇒  *α* 3 *+ βα* − *α* = 0, i.e.  *β* = 1 – *α* 2 (*α* ≠ 0) | M1, A1 |
|  | *x* 3 + *βx* − *α*  ≡ (*x* – *α*)[*x*2 + *α x* + 1] | M1 [A1] |
|  | Discriminant of *x*2 + *α x* + 1 is *α* 2 – 4 | M1 |
|  | ∴ *x* = *α* is the only real root if *α* 2 – 4 < 0, i.e. |*α*| < 2 (🞿) | A1 cso (6) |
| (*d*) | Student’s method: *x*(*x*2 + *β*) = *α* |  |
|  | ⇒ *x* = *α* or *x*2 + *β*  = *α* | M1 |
|  | require *α* − *β*  > 0 |  |
|  | *α* 2 + *α* − 1 > 0 |  |
|  | cvs *α* =  attempt cvs | M1 |
|  | 2 correct cvs | A1 |
|  | ∴  < α < 2 or −2 < *α* < − | A1, A1 (7) |
|  |  | **(19 marks)** |

|  |  |  |
| --- | --- | --- |
|  | STYLE INSIGHT & REASONING |  |
| (*a*) | S marks |  |
|  | For a novel or neat solution to any of questions 3—7. Apply once per question in up to 3 questions |  |
|  | **S2** if solution is fully correct in principle and accuracy | S6 (S2 × 3) |
|  | **S1** if principle is sound but includes a minor algebraic or numerical slip |  |
|  | T mark |  |
|  | **For a good and largely accurate attempt at the whole paper** | **T1** |
|  |  | **(7 marks)** |