# **Topic 4**

# **Graphs and transformations**

# Bronze, Silver, Gold

# Worksheets for

# AS Level Mathematics

# Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the ‘Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS’ textbook.

# Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It’s important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords “show that” or “prove”. If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

# Quick Links

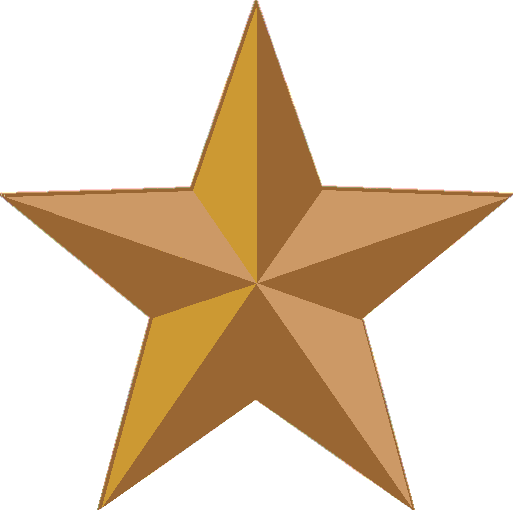
(Press Ctrl, as you click with your mouse to follow these links)

* [Bronze Questions](#BrQue)
* [Bronze Mark Scheme](#BrMS)
* [Silver Questions](#SiQue)
* [Silver Mark Scheme](#SiMS)
* [Gold Questions](#GoQu)
* [Gold Mark Scheme](#GoMS)

# Extension and Enrichment

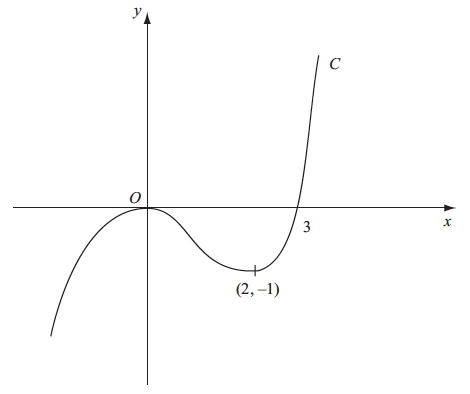
If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found [here](https://qualifications.pearson.com/en/qualifications/edexcel-a-levels/advanced-extension-award-mathematics-2018.html) on the Pearson Edexcel Website, or [here](https://www.mathsemporium.com/category/advanced-extension-award-mathematics/) on the Maths Emporium

**Bronze Questions **

**Calculators may not be used**

The total mark for this section is 29

**Q1**

**Figure 1**

Figure 1 shows a sketch of the curve *C* with equation *y* = f(*x*). There is a maximum at (0, 0), a minimum at (2, −1) and *C* passes through (3, 0).

On separate diagrams sketch the curve with equation

(a) *y* = f(*x* + 3),

**(3)**

(b) *y* = f(−*x*).

**(3)**

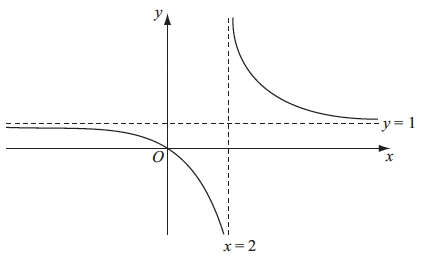
On each diagram show clearly the coordinates of the maximum point, the minimum

point and any points of intersection with the *x*-axis.

**(Total for Question 1 is 6 marks)**

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**Q2**



**Figure 1**

Figure 1 shows a sketch of the curve with equation *y* = f (*x*) where



The curve passes through the origin and has two asymptotes, with equations *y* =1

and *x* = 2 , as shown in Figure 1.

(a) Sketch the curve with equation *y* = f (*x* − 1) and state the equations of the asymptotes

of this curve.

**(3)**

(b) Find the coordinates of the points where the curve with equation *y* = f (*x* - 1) crosses

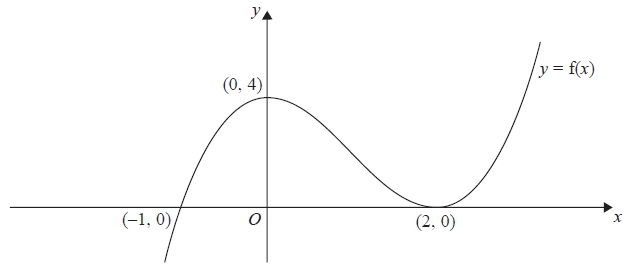
the coordinate axes.

**(4)**

**(Total for Question 2 is 7 marks)**

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**Q3**



**Figure 1**

Figure 1 shows a sketch of the curve *C* with equation *y* = f(*x*)

The curve *C* passes through the point (−1, 0) and touches the *x*-axis at the point (2, 0)

The curve *C* has a maximum at the point (0, 4)

(a)  The equation of the curve *C* can be written in the form

*y* = *x*3 + *ax*2 + *bx* + *c*

where *a*, *b* and *c* are integers.

Calculate the values of *a*, *b* and *c*.

**(5)**

(b)   Sketch the curve with equation 

Show clearly the coordinates of all the points where the curve crosses or meets the

coordinate axes.

**(3)**

**(Total for Question 3 is 8 marks)**

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**Q4**

(a) Sketch the graphs of

*y* = *x*(*x* + 2)(3 − *x*)



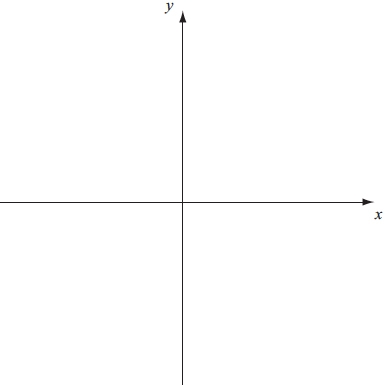
showing clearly the coordinates of all the points where the curves cross the coordinate

axes.

**(6)**

(b) Using your sketch state, giving a reason, the number of real solutions to the equation



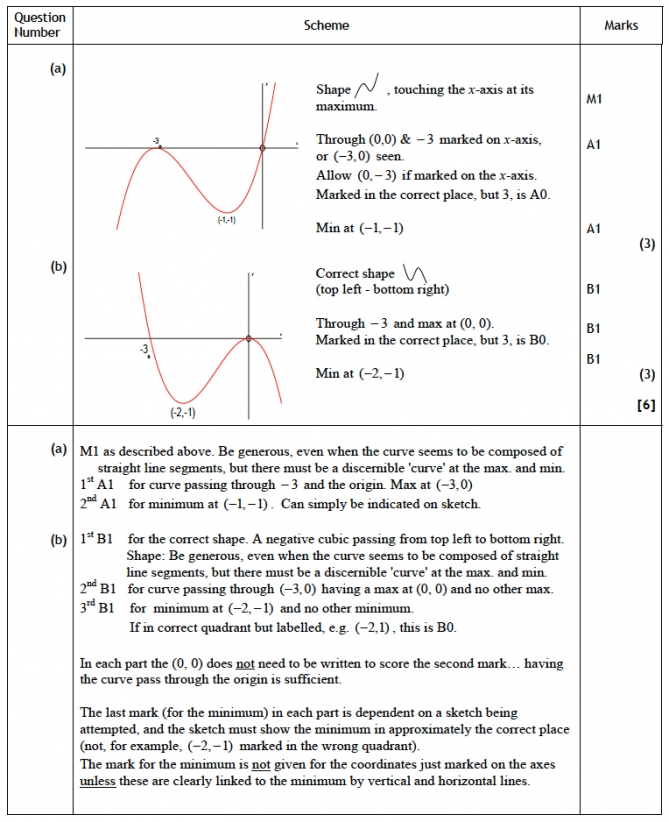
**(2)**

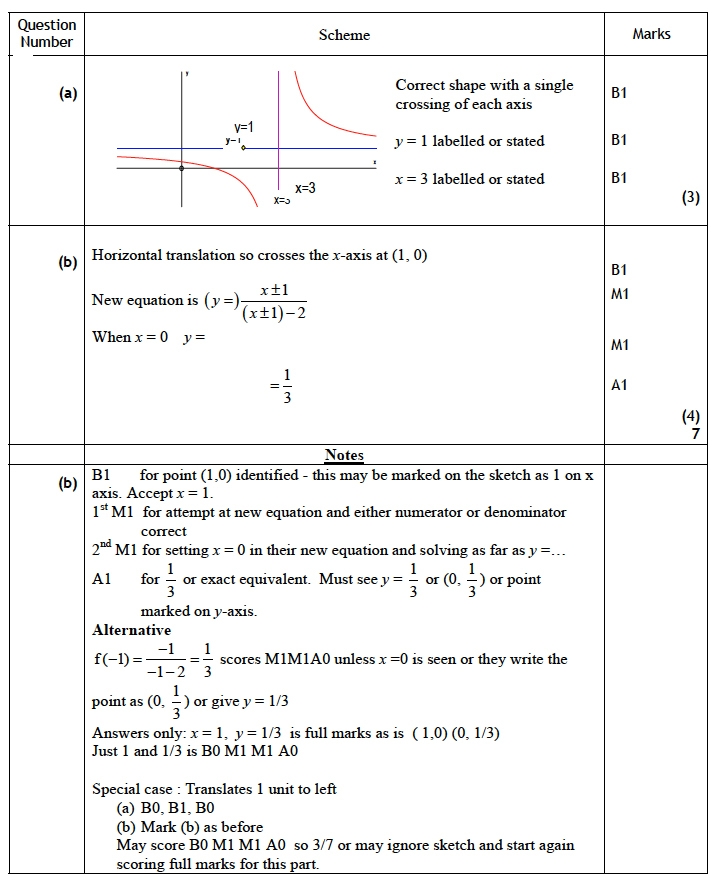
**(Total for Question 4 is 8 marks)**

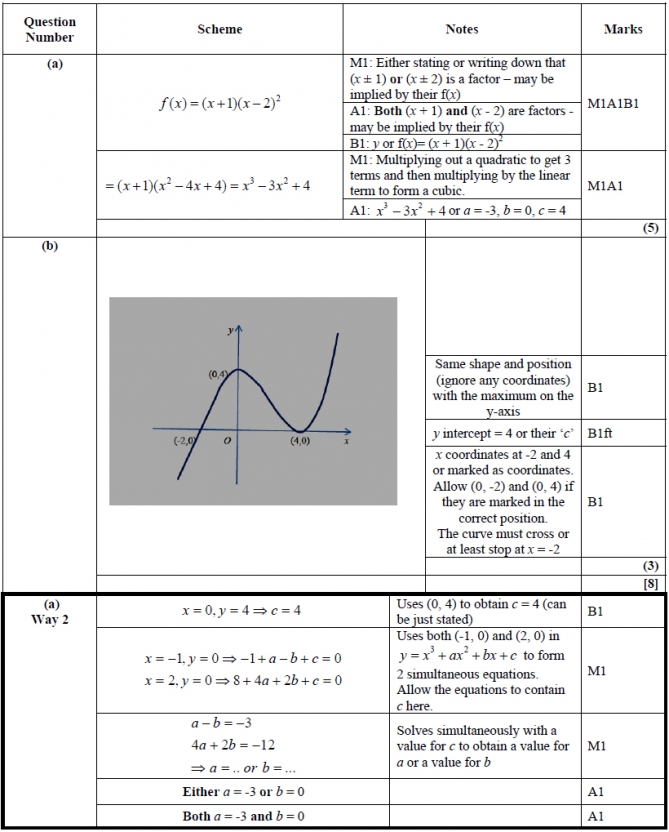
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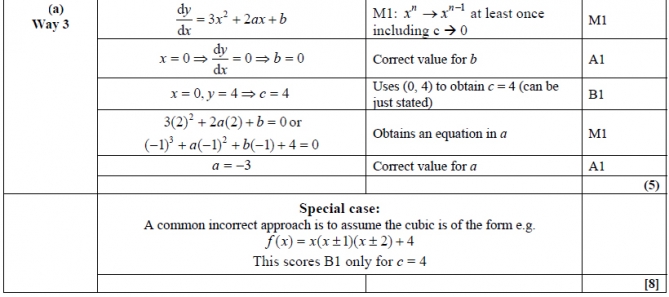
**End of Questions**

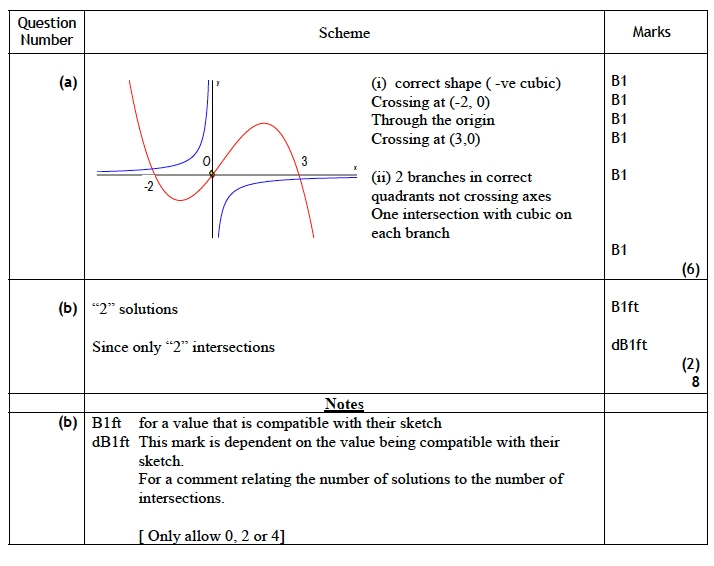
**Bronze Mark Scheme**

**Q1**  


**Q2**

**Q3**



**Q4**

**Silver Questions **

**Calculators may not be used**

The total mark for this section is 30

**Q1**

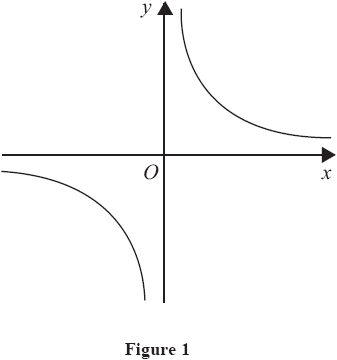


Figure 1 shows a sketch of the curve with equation , *x* ≠ 0

(a) On a separate diagram, sketch the curve with equation  ,

showing the coordinates of any point at which the curve crosses a coordinate axis.

**(3)**

(b)  Write down the equations of the asymptotes of the curve in part (a).

**(2)**

**(Total for Question 1 is 5 marks)**

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**Q2**

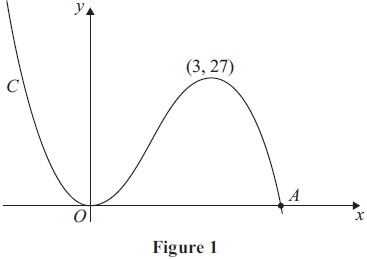


Figure 1 shows a sketch of the curve *C* with equation *y* = f(*x*) where

f(*x*) = *x*2(9 − 2*x*)

There is a minimum at the origin, a maximum at the point (3, 27) and *C* cuts the *x*-axis at

the point *A*.

(a)  Write down the coordinates of the point *A*.

**(1)**

(b)  On separate diagrams sketch the curve with equation

(i) *y* = f(*x*+3)

(ii) *y* = f(3*x*)

On each sketch you should indicate clearly the coordinates of the maximum point and

any points where the curves cross or meet the coordinate axes.

**(6)**

The curve with equation *y* = f(*x*) + *k*, where *k* is a constant, has a maximum point at (3, 10).

(c)  Write down the value of *k*.

**(1)**

**(Total for Question 2 is 8 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q3**

(a)   Factorise completely *x*3 − 6*x*2 + 9*x*

**(3)**

(b)   Sketch the curve with equation

*y* = *x*3 − 6*x*2 + 9*x*

        showing the coordinates of the points at which the curve meets the *x*-axis.

**(4)**

Using your answer to part (b), or otherwise,

(c)   sketch, on a separate diagram, the curve with equation

*y* = (*x* − 2)3 − 6(*x* − 2)2 + 9(*x* − 2)

        showing the coordinates of the points at which the curve meets the *x*-axis.

**(2)**

**(Total for Question 3 is 9 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q4**

The curve *C* has equation



where *k* is a constant.

(a)  Sketch *C* stating the equation of the horizontal asymptote.

**(3)**

The line *l* has equation *y* = –2*x* + 5

(b)  Show that the *x* coordinate of any point of intersection of *l* with *C* is given by a solution of the equation

2*x*2 – 4*x* + *k*2 = 0

**(2)**

(c)  Hence find the exact values of *k* for which *l* is a tangent to *C*.

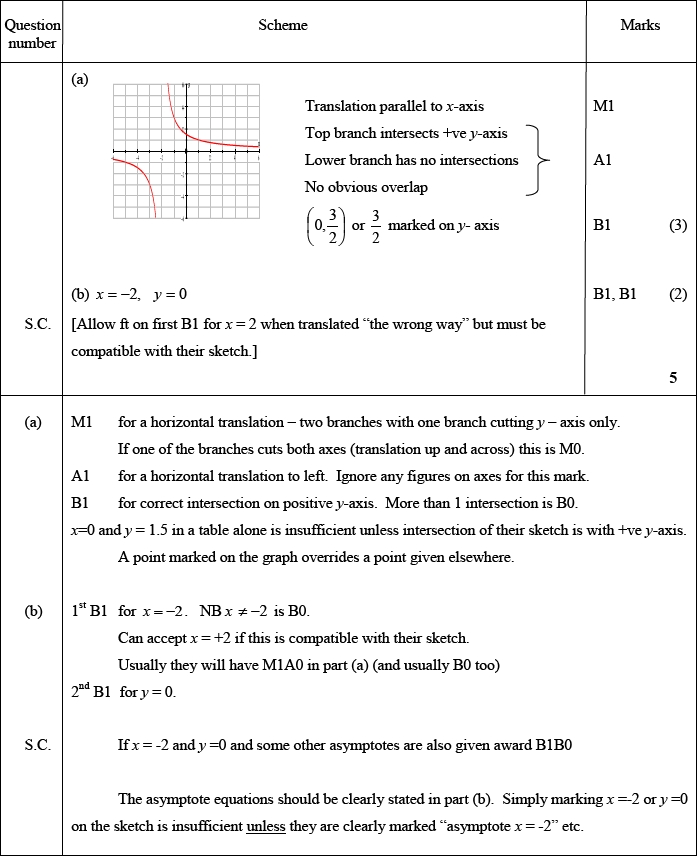
**(3)**

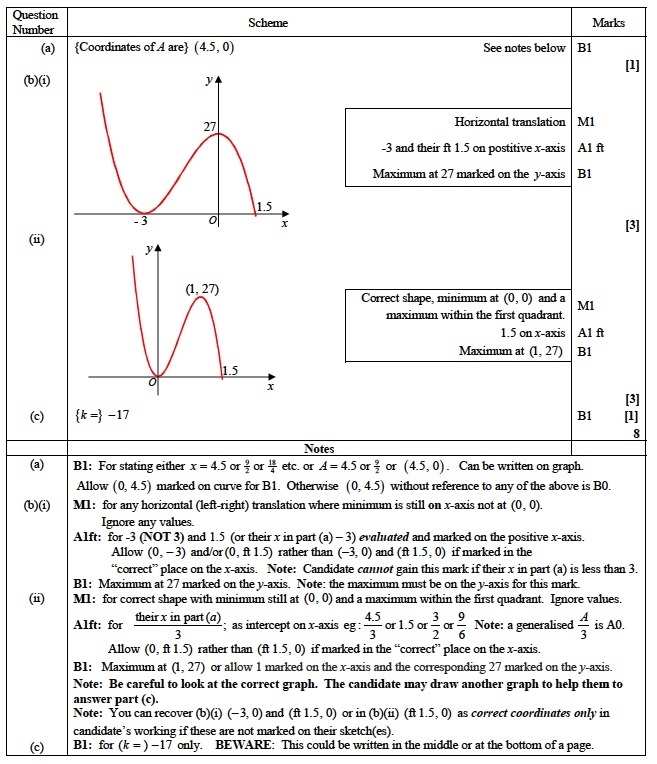
**(Total for Question 4 is 8 marks)**

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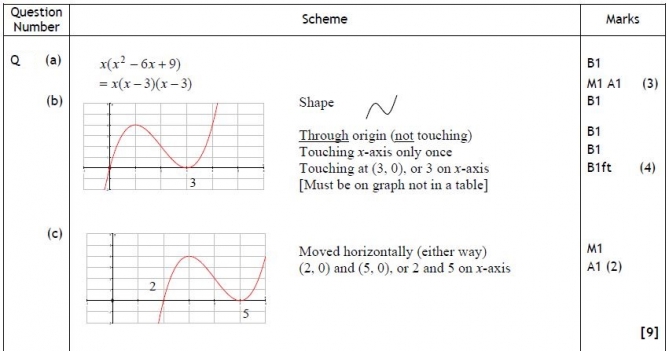
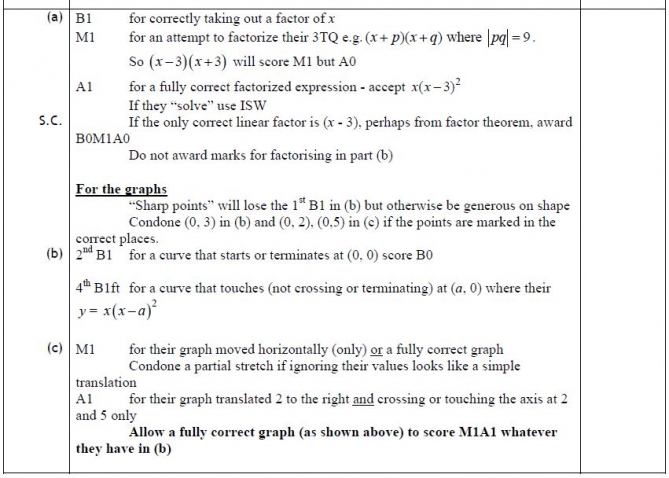
**Silver Mark Scheme**

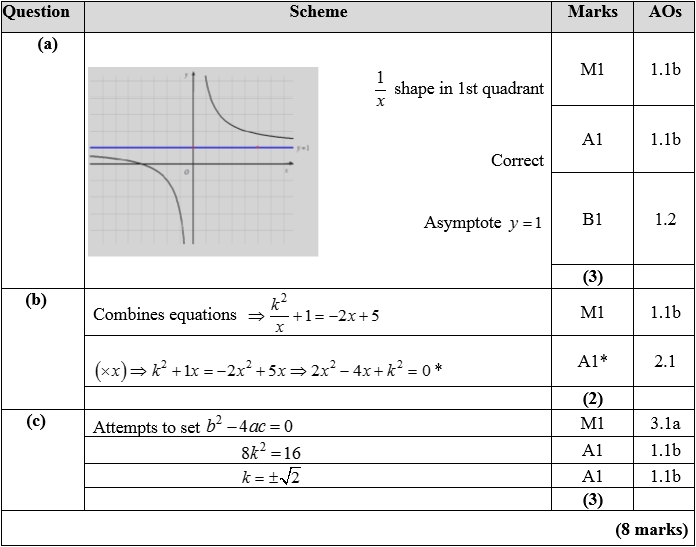
**Q1**

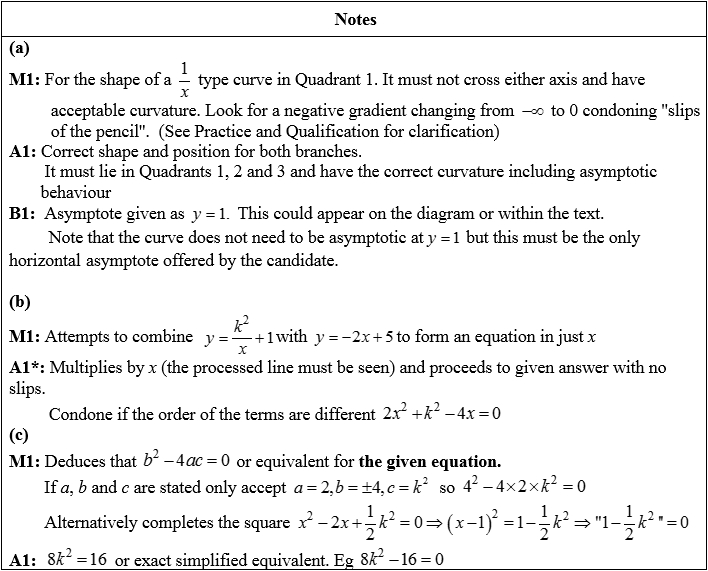


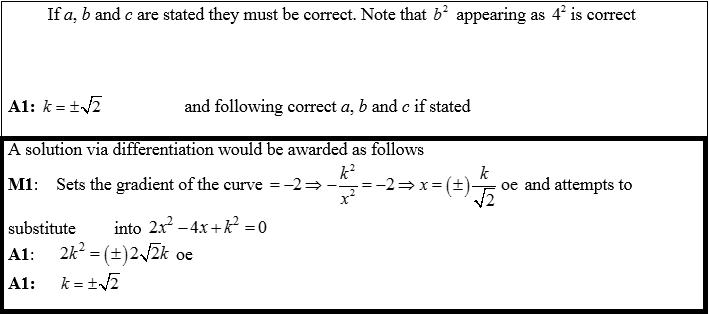
**Q2**

**Q3**

**Q4**





**Gold Questions **

**Calculators may not be used**

The total mark for this section is 35

**Q1**

The point *P* (1, *a*) lies on the curve with equation *y* = (*x* + 1)2(2 − *x*).

(a) Find the value of *a*.

**(1)**

(b) Sketch the curves with the following equations:

     (i) *y* = (*x* + 1)2 (2 − *x*),

     (ii) 

     On your diagram show clearly the coordinates of any points at which the curves meet

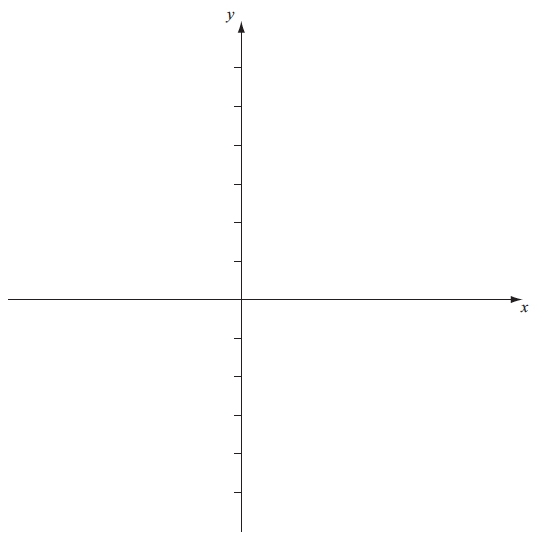
the axes.

**(5)**

(c) With reference to your diagram in part (b) state the number of real solutions to the

equation



**(1)**

**(Total for Question 1 is 7 marks)**

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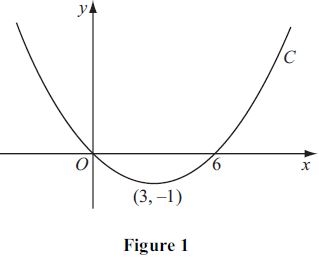
**Q2**

Figure 1 shows a sketch of the curve *C* with equation y = f (*x*)

The curve *C* passes through the origin and through (6, 0)

The curve *C* has a minimum at the point (3, −1).

On separate diagrams, sketch the curve with equation

(a)  *y* = f (2*x*),

**(3)**

(b)  *y* = −f (*x*),

**(3)**

(c)  *y* = f (*x* + *p*), where *p* is a constant and 0 < *p* < 3.

**(4)**

**(Total for Question 1 is 10 marks)**

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**Q3**

(a)   Factorise completely *x*3 + 10*x*2 + 25*x*

**(2)**

(b)   Sketch the curve with equation

*y* = *x*3 + 10*x*3 + 25*x*

showing the coordinates of the points at which the curve cuts or touches the *x*-axis.

**(2)**

The point with coordinates ( –3, 0) lies on the curve with equation

*y* = (*x* + *a*)3 + 10(*x* + *a*)2 + 25(*x* + *a*)

where *a* is a constant.

(c)   Find the two possible values of *a*.

**(3)**

**(Total for Question 3 is 7 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q4**

(a)  On separate axes sketch the graphs of

(i)   *y* = −3*x* + *c*, where *c* is a positive constant,

(ii) 

On each sketch show the coordinates of any point at which the graph crosses the *y*-axis and the equation of any horizontal asymptote.

**(4)**

Given that *y* = −3*x* + *c*, where *c* is a positive constant, meets the curve at two distinct points,

(b)   show that (5 − *c*)2 > 2

**(3)**

(c)   Hence find the range of possible values for *c*

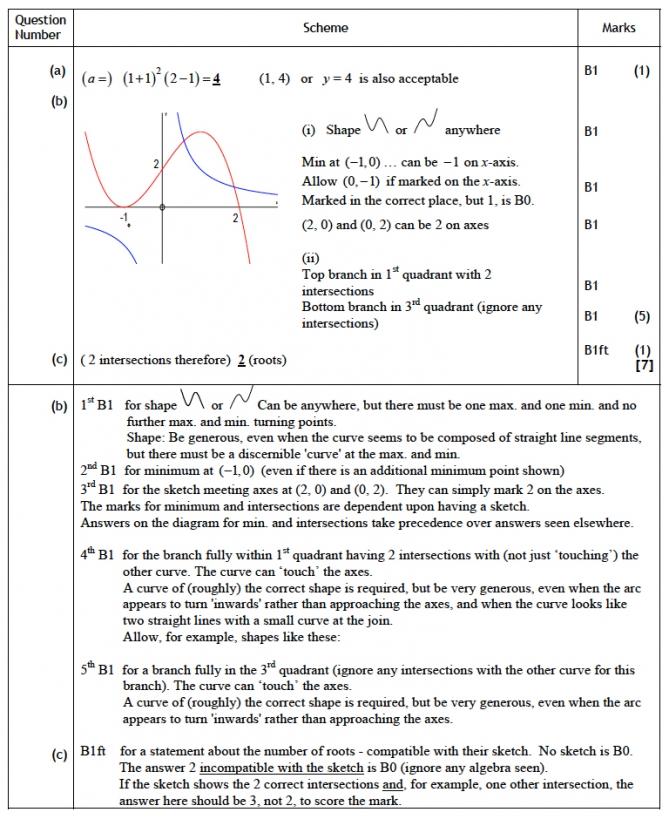
**(4)**

**(Total for Question 4 is 11 marks)**

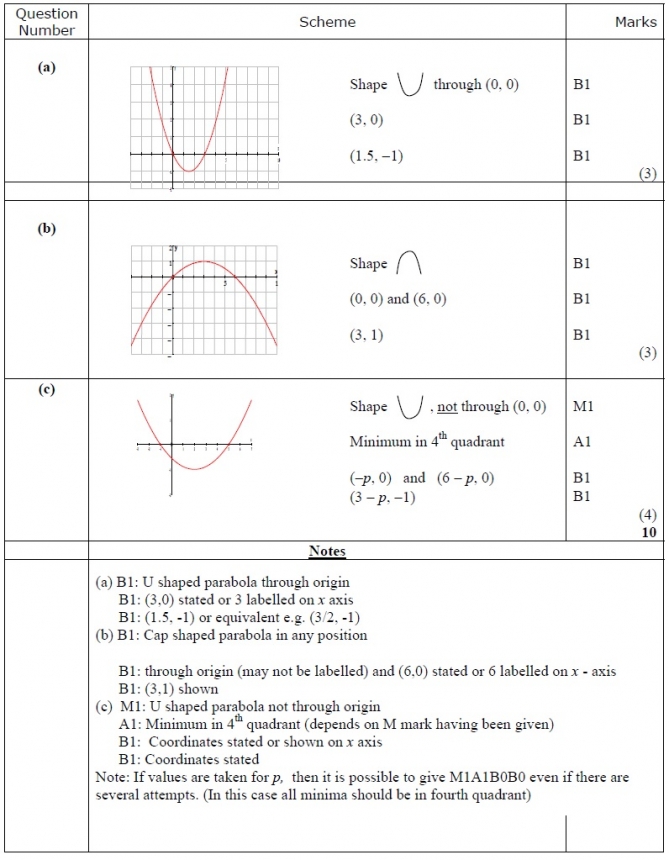
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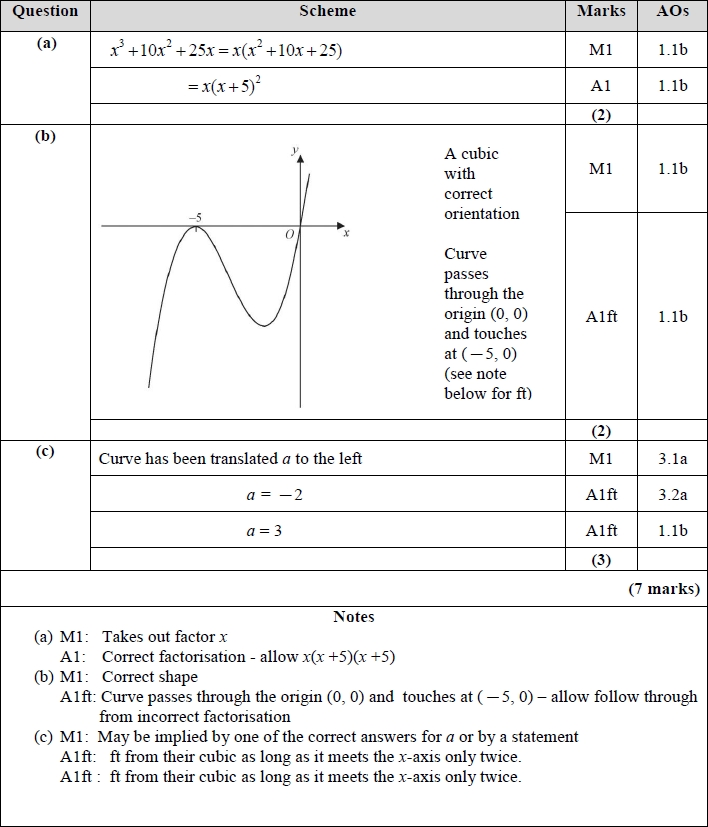
**Gold Mark Scheme**

**Q1**



**Q2**



**Q3**

**Q4**

