# **Topic 13**

# **Integration**

# Bronze, Silver, Gold and

# Platinum Worksheets

# for AS Level Mathematics

# Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the ‘Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS’ textbook.

# Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It’s important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords “show that” or “prove”. If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

# Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

* [Bronze Questions](#BrQue)
* [Bronze Mark Scheme](#BrMS)
* [Silver Questions](#SiQue)
* [Silver Mark Scheme](#SiMS)
* [Gold Questions](#GoQu)
* [Gold Mark Scheme](#GoMS)

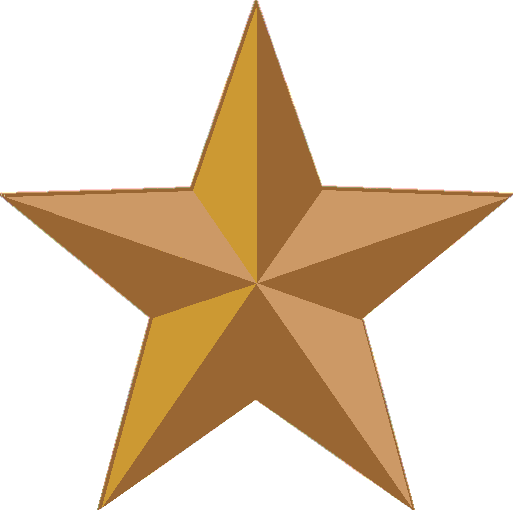
The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

* [Platinum Questions](#PlQu)
* [Platinum Mark Schemes](#PlMS)

# Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found [here](https://qualifications.pearson.com/en/qualifications/edexcel-a-levels/advanced-extension-award-mathematics-2018.html) on the Pearson Edexcel Website, or [here](https://www.mathsemporium.com/category/advanced-extension-award-mathematics/) on the Maths Emporium

**Bronze Questions **

**Calculators may not be used**

The total mark for this section is 30

**Q1**

Find **, giving each term in its simplest form.

**(Total for Question 1 is 4 marks)**

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**Q2**

Find



giving each term in its simplest form.

**(Total for Question 2 is 5 marks)**

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**Q3**

Find



giving each term in its simplest form.

**(Total for Question 3 is 4 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q4**

A curve with equation *y* = f(*x*) passes through the point (2, 10). Given that



find the value of f(1).

**(Total for Question 4 is 5 marks)**

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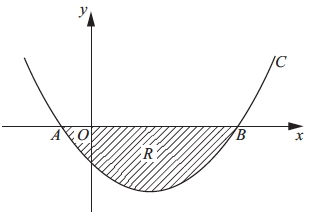
**Q5**

Use calculus to find the exact value of 

**(Total for Question 5 is 5 marks)**

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**Q6**



**Figure 1**

Figure 1 shows a sketch of part of the curve *C* with equation

*y* = (*x* + 1)(*x* − 5)

The curve crosses the *x*-axis at the points *A* and *B*.

(a) Write down the *x*-coordinates of *A* and *B*.

**(1)**

The finite region *R*, shown shaded in Figure 1, is bounded by *C* and the *x*-axis.

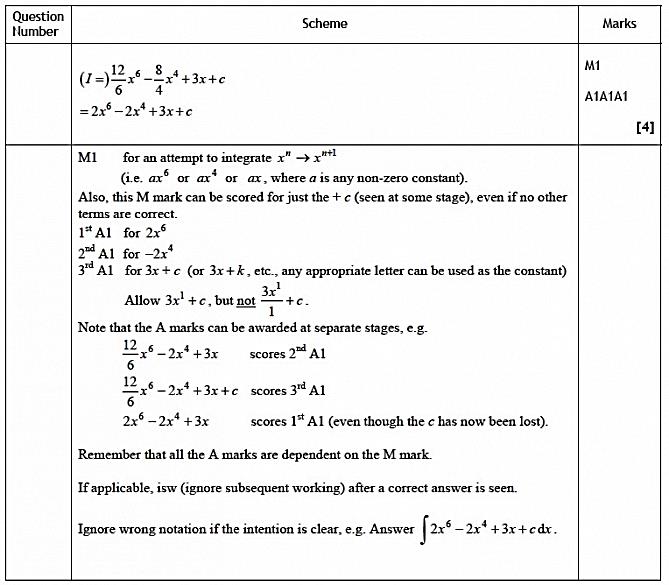
(b) Use integration to find the area of *R*.

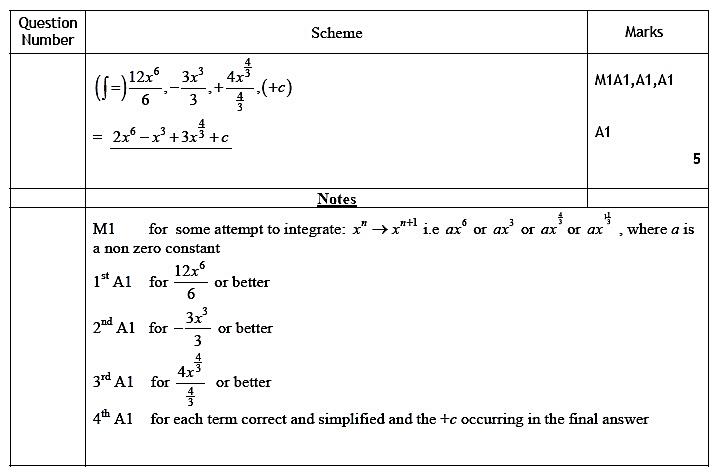
**(6)**

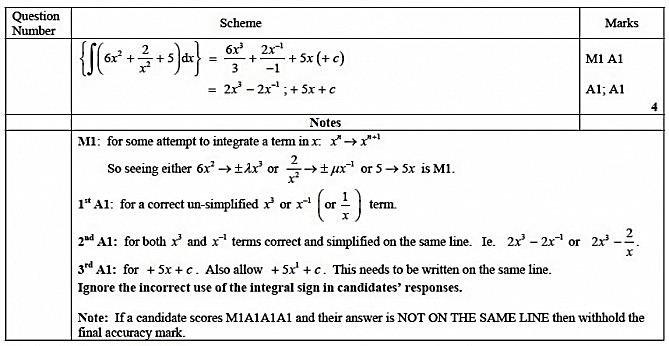
**(Total for Question 7 is 7 marks)**

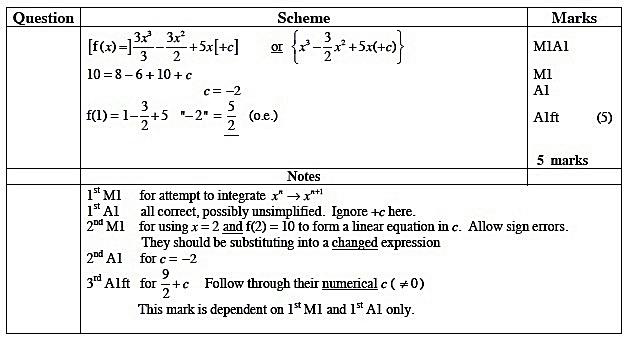
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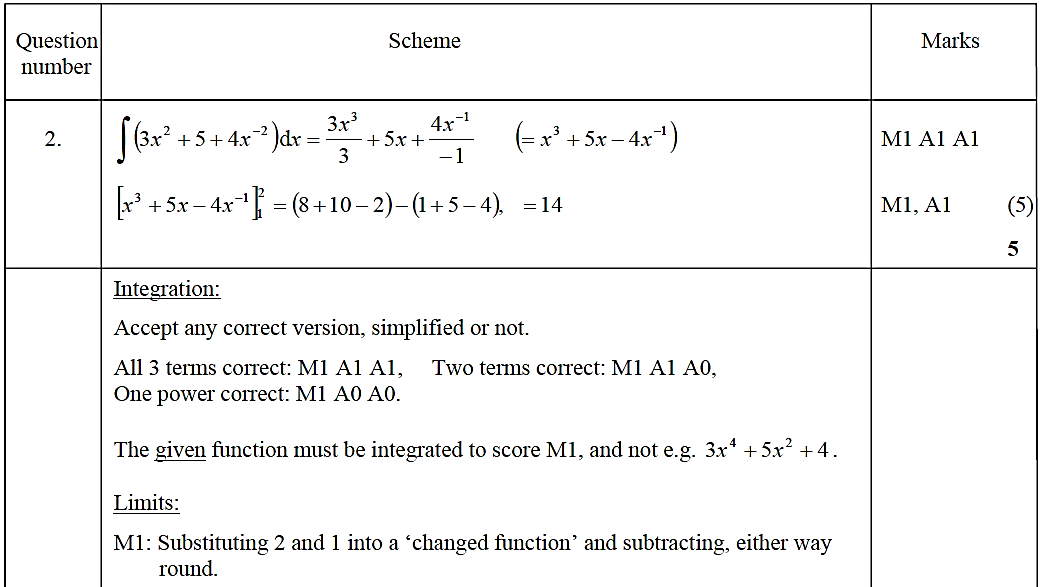
**Bronze Mark Scheme**

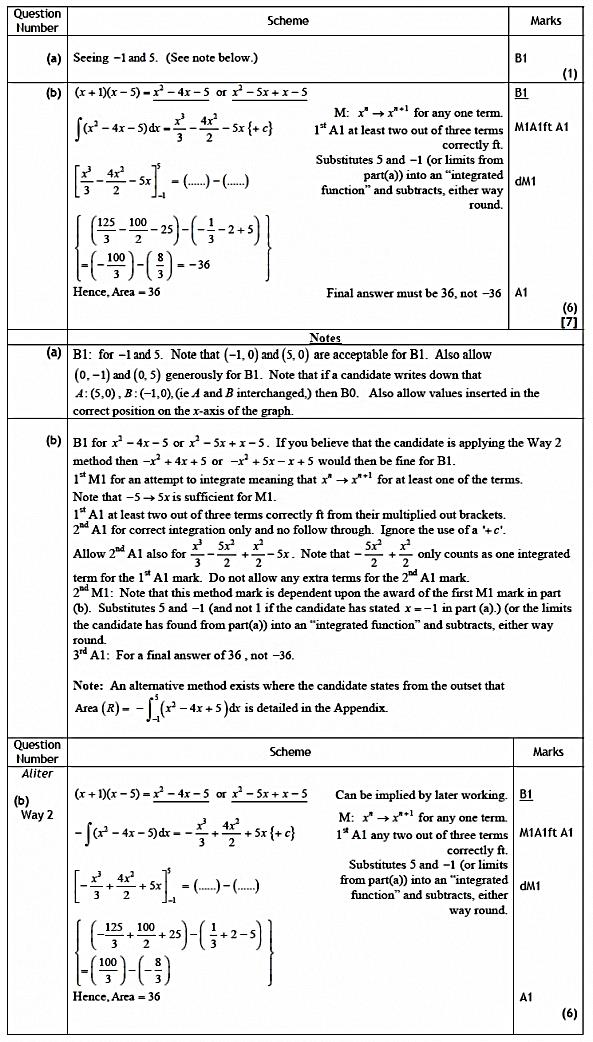
 **Q1.**

**Q2.**

**Q3.**

**Q4.**

**Q5.**



**Q6.**

**Silver Questions **

**Calculators may not be used**

The total mark for this section is 37

**Q1**

A curve has equation *y* = f(*x*) and passes through the point (4, 22).

Given that

f '(*x*) = 3*x*2 − 3*x* − 7,

use integration to find f(*x*), giving each term in its simplest form.

**(Total for Question 1 is 5 marks)**

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**Q2**

The gradient of a curve *C* is given by



(a)  Show that 

**(2)**

The point (3, 20) lies on *C*.

(b)  Find an equation for the curve *C* in the form *y* = f(*x*).

**(6)**

**(Total for Question 2 is 8 marks)**

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**Q3**

A curve has equation *y* = f(*x*). The point *P* with coordinates (9, 0) lies on the curve.

Given that



(a)  find f(*x*).

**(6)**

(b)  Find the *x*-coordinates of the two points on *y* = f(*x*) where the gradient of the curve is equal to 10

**(4)**

**(Total for Question 3 is 10 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q4**

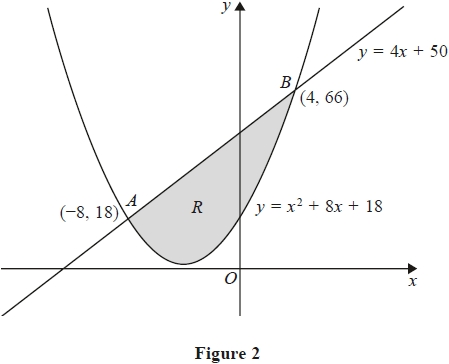


Figure 2 shows the line with equation *y* = 4*x* + 50 and the curve with equation *y* = *x*2 + 8*x* + 18. The line cuts the curve at the points *A* (−8, 18) and *B* (4, 66).

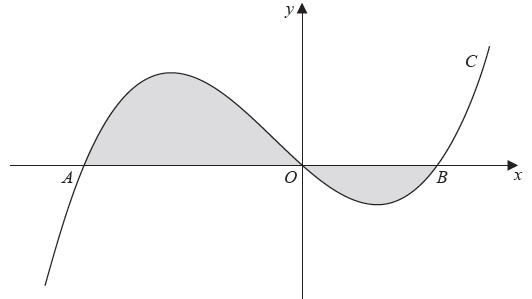
The shaded region *R* is bounded by the line and the curve, as shown in Figure 2.

Using calculus, find the area of *R*.

**(Total for Question 4 is 6 marks)**

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**Q5**



**Figure 3**

Figure 3 shows a sketch of part of the curve *C* with equation

*y* = *x*(*x* + 4)(*x* − 2)

The curve *C* crosses the *x*-axis at the origin *O* and at the points *A* and *B*.

(a)    Write down the *x*-coordinates of the points *A* and *B*.

**(1)**

The finite region, shown shaded in Figure 3, is bounded by the curve *C* and the *x*-axis.

(b)    Use integration to find the total area of the finite region shown shaded in Figure 3.

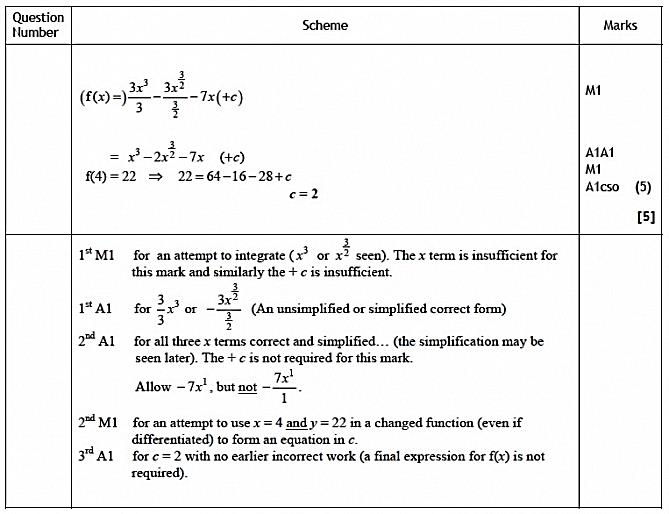
**(7)**

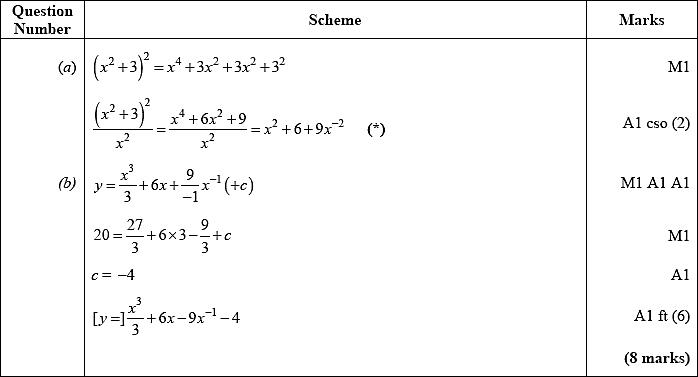
**(Total for Question 5 is 8 marks)**

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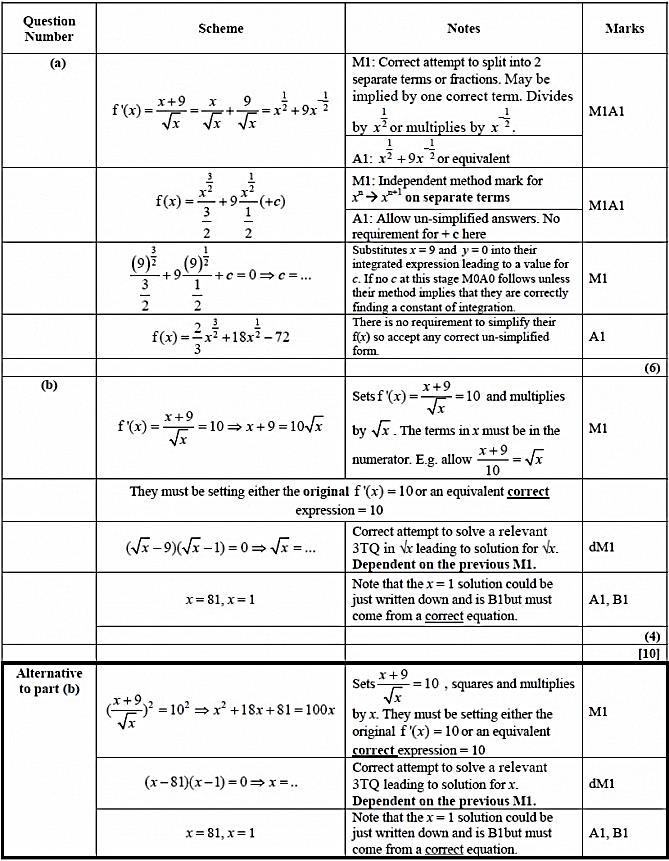
**Silver Mark Scheme**

**Q1.**

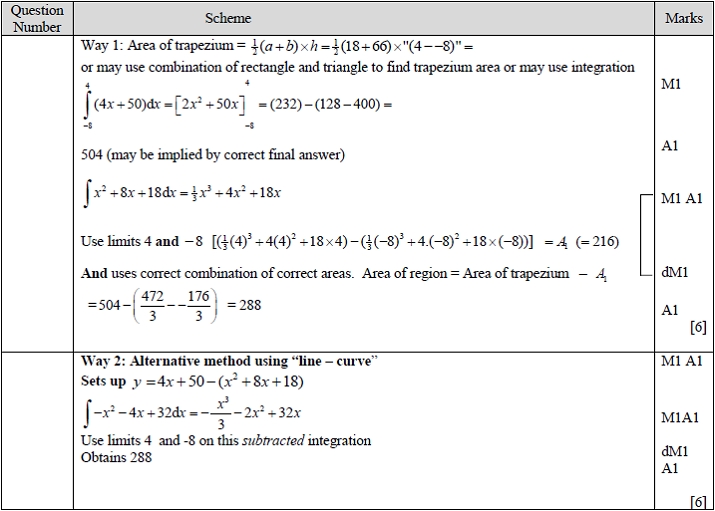
**Q2.**



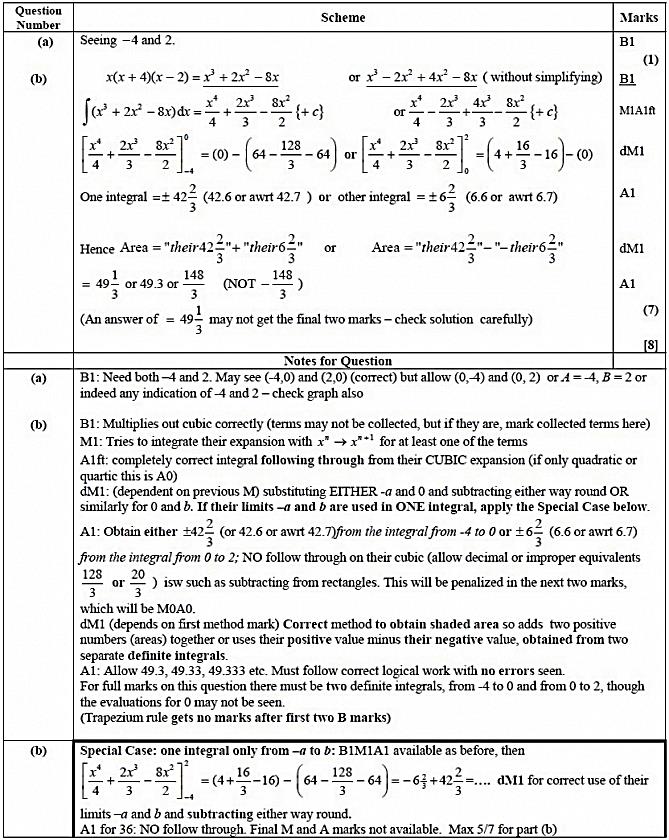
**Q3.**



**Q4.**



**Q5.**



**Gold Questions **

**Calculators may not be used**

The total mark for this section is 35

**Q1**

Given that  ** can be written in the form 6*xp* + 3*xq*,

(a)  write down the value of *p* and the value of *q*

**(2)**

Given that **  and that *y* = 90 when *x* = 4,

(b)  find *y* in terms of *x*, simplifying the coefficient of each term.

**(5)**

**(Total for Question 1 is 7 marks)**

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**Q2**

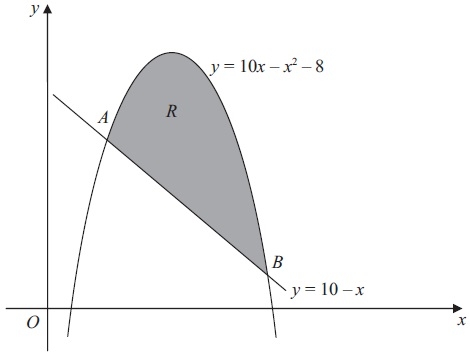
**where *x* > 0

Given that *y* = 37 at *x* = 4, find *y* in terms of *x*, giving each term in its simplest form.

**(Total for Question 2 is 7 marks)**

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**Q3**



**Figure 2**

Figure 2 shows the line with equation *y* = 10 − *x* and the curve with equation *y* = 10*x* − *x*2 − 8

The line and the curve intersect at the points *A* and *B*, and *O* is the origin.

(a)  Calculate the coordinates of *A* and the coordinates of *B*.

**(5)**

The shaded area *R* is bounded by the line and the curve, as shown in Figure 2.

(b)  Calculate the exact area of *R*.

**(7)**

**(Total for Question 3 is 12 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q4**

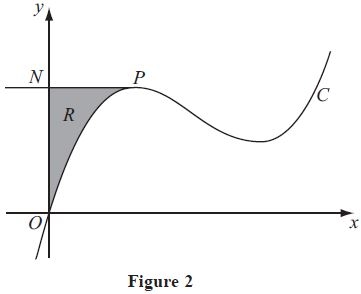


Figure 2 shows a sketch of part of the curve C with equation



where *k* is a constant.

The point *P* on *C* is the maximum turning point.

Given that the *x*-coordinate of *P* is 2,

(a)  show that *k* = 28.

**(3)**

The line through P parallel to the *x*-axis cuts the *y*-axis at the point *N*.   
The region *R* is bounded by *C*, the *y*-axis and *PN*, as shown shaded in Figure 2.

(b)  Use calculus to find the exact area of *R*.

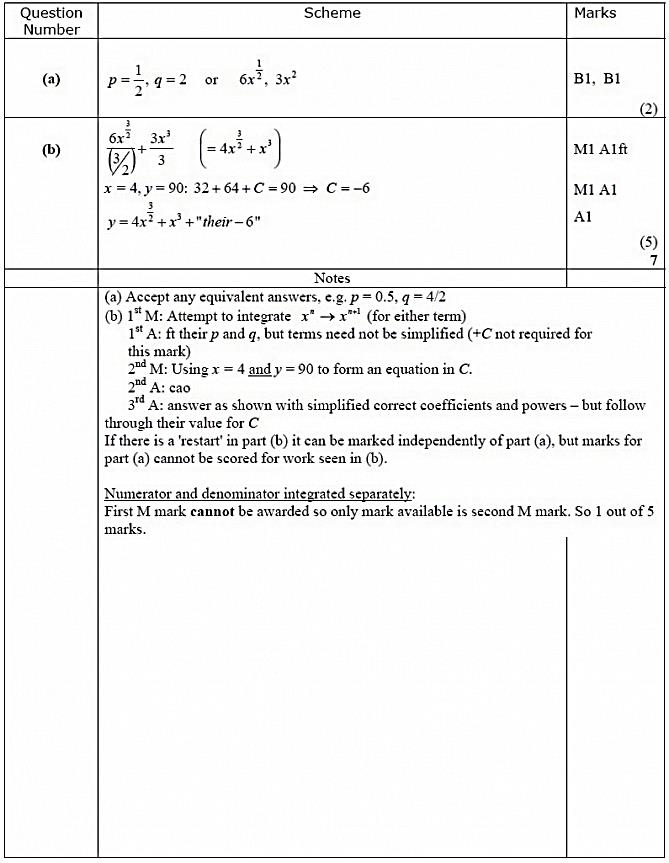
**(6)**

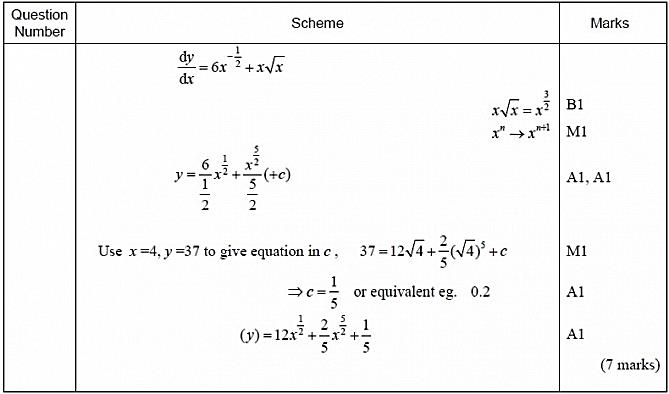
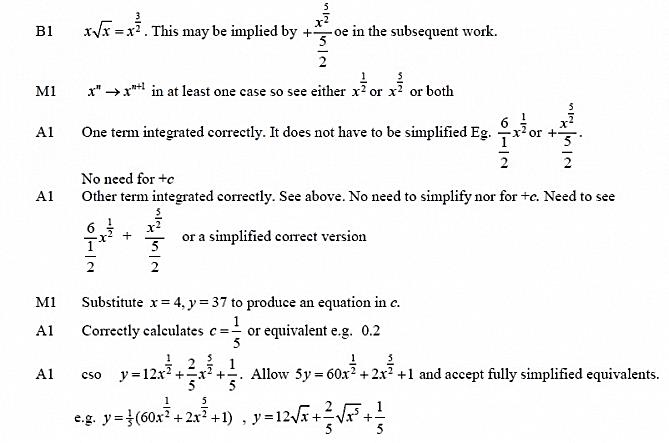
**(Total for Question 4 is 9 marks)**

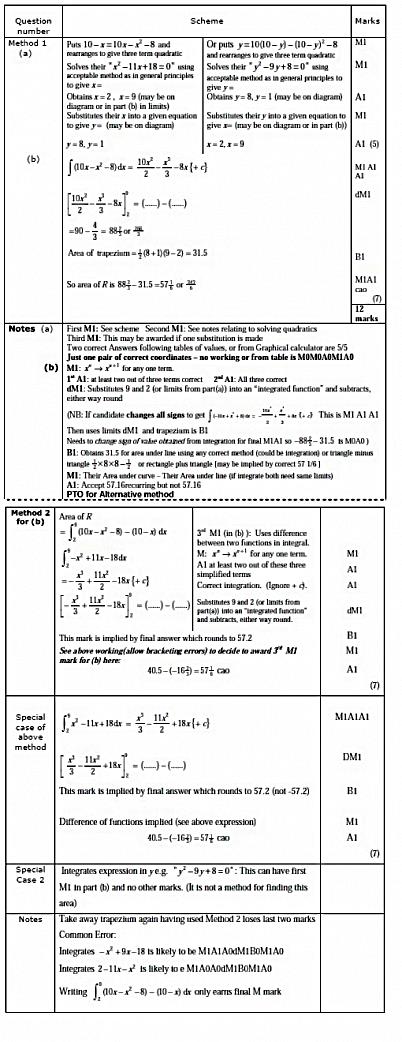
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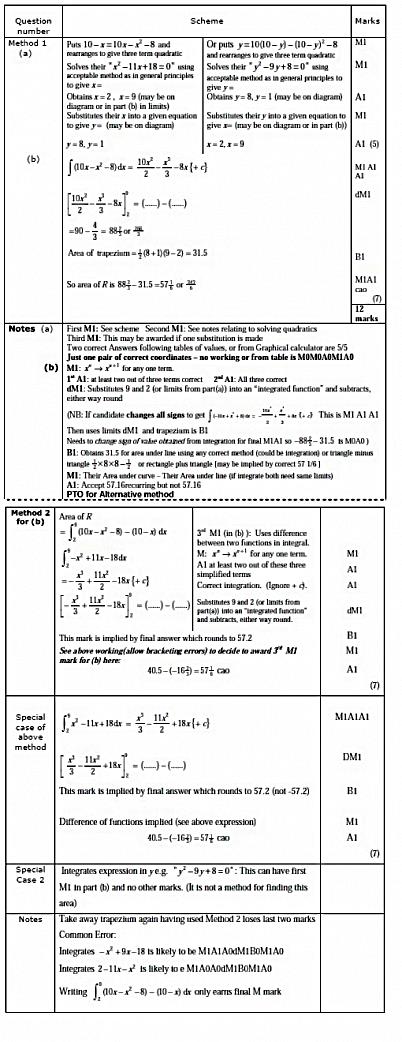
**Gold Mark Scheme**

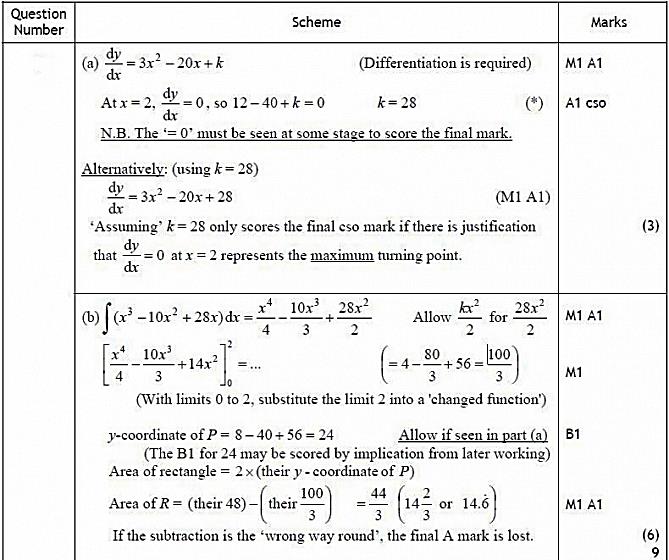
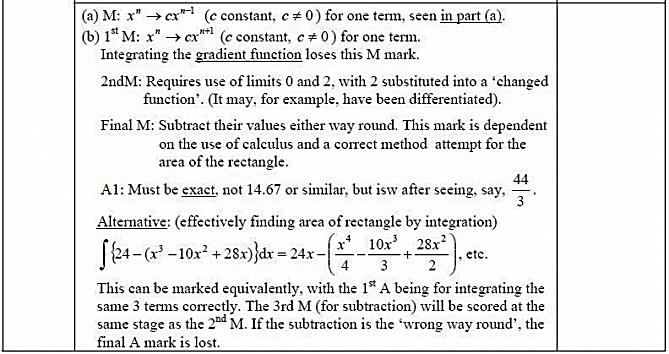
**Q1.**



**Q2.**

**Q3.**

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**Q4.**

**Platinum Questions **

**Calculators may not be used**

The total mark for this section is 10

**1** (a) On the same diagram, sketch *y* = *x* and *y* = √*x*, for *x* ≥ 0, and mark clearly the coordinates of the points of intersection of the two graphs.

**(2)**

(b) With reference to your sketch, explain why there exists a value *a* of *x* (*a* > 1) such that

 = .

**(2)**

(c) Find the exact value of *a*.

**(4)**

(d) Hence, or otherwise, find a non-constant function f(*x*) and a constant *b* (*b* ≠ 0) such that

 = .

**(2)**

**(Total for Question 1 is 10 marks)**

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**Platinum Mark Scheme**

