# **Topic 7**

# **Algebraic Methods**

# Bronze, Silver, Gold and

# Platinum Worksheets for

# AS Level Mathematics

# Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the ‘Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS’ textbook.

# Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It’s important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords “show that” or “prove”. If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

# Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

* [Bronze Questions](#BrQue)
* [Bronze Mark Scheme](#BrMS)
* [Silver Questions](#SiQue)
* [Silver Mark Scheme](#SiMS)
* [Gold Questions](#GoQu)
* [Gold Mark Scheme](#GoMS)

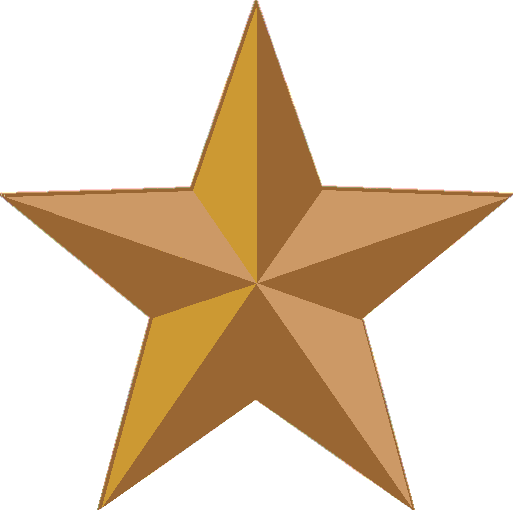
The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

* [Platinum Questions](#PlQu)
* [Platinum Mark Schemes](#PlMS)

# Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found [here](https://qualifications.pearson.com/en/qualifications/edexcel-a-levels/advanced-extension-award-mathematics-2018.html) on the Pearson Edexcel Website, or [here](https://www.mathsemporium.com/category/advanced-extension-award-mathematics/) on the Maths Emporium

**Bronze Questions **

**Calculators may not be used**

The total mark for this section is 26

**Q1**

****

(a) Find the remainder when f(*x*) is divided by (*x* −1)

**(2)**

(b)  Use the factor theorem to show that (*x*+1) is a factor of f(*x*)

**(2)**

(c)  Factorise f(*x*) completely.

**(4)**

**(Total for Question 1 is 9 marks)**

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**Q2**

(a)  Find the remainder when

*x*3 − 2*x*2 − 4*x* + 8

is divided by

(i)  *x* − 3

(ii)  *x* + 2

**(3)**

(b)  Hence, or otherwise, find all the solutions to the equation

*x*3 − 2*x*2 − 4*x* + 8 = 0

**(4)**

**(Total for Question 2 is 7 marks)**

**Q3**

f(*x*) = *x*4 + *x*3 + 2*x*2 + *ax* + *b*

where *a* and *b* are constants.

When f(*x*) is divided by (*x* − 1), the remainder is 7

(a) Show that *a* + *b* = 3

**(2)**

When f(*x*) is divided by (*x* + 2), the remainder is −8

(b) Find the value of *a* and the value of *b*

**(5)**

**(Total for Question 3 is 7 marks)**

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**Q4**

Given , prove that *n*3 + 2 is not divisible by 8

**(Total for Question 4 is 4 marks)**

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**End of Questions**

**Bronze Mark Scheme**

**Q1**

Table

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**Q2**

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**Q3**

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**Q4**

Table

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**Silver Questions **

**Calculators may not be used**

The total mark for this section is 29

**Q1**

f(*x*) = 4*x*3 – 12*x*2 + 2*x* – 6

(a)   Use the factor theorem to show that (*x* – 3) is a factor of f(*x*)

**(2)**

(b)   Hence show that 3 is the only real root of the equation f(*x*) = 0

**(4)**

**(Total for Question 1 is 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q2**

f(*x*) = 2*x*3 – 7*x*2 – 10*x* + 24

(a)  Use the factor theorem to show that (*x* + 2) is a factor of f(*x*)

**(2)**

(b)  Factorise f(*x*) completely.

**(4)**

**(Total for Question 2 is 6 marks)**

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**Q3**



where *k* is a constant.

(a) Write down the value of f(*k*)

**(1)**

When f(*x*) is divided by (*x* − 2) the remainder is 4

(b) Find the value of *k*

**(2)**

(c) Factorise f(*x*) completely.

**(3)**

**(Total for Question 3 is 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q4**

f (*x*) = *x*3 + *ax*2 + *bx* + 3

where *a* and *b* are constants.

Given that when f (*x*) is divided by (*x* + 2) the remainder is 7,

(a) show that 2*a* − *b* = 6

**(2)**

Given also that when f (*x*) is divided by (*x* − 1) the remainder is 4,

(b) find the value of *a* and the value of *b*.

**(4)**

**(Total for Question 4 is 6 marks)**

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**Q5**

 (a)  Prove that for all positive values of *a* and *b*

 4

**(4)**

(b)  Prove, by counter example, that this is not true for all values of *a* and *b*

**(1)**

**(Total for Question 5 is 5 marks)**

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**End of Questions**

**Silver Mark Scheme**

Q1

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**Q2**

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**Q3**

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**Q4**

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**Q5**

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**Gold Questions **

**Calculators may not be used**

The total mark for this section is 27

**Q1**

f(*x*) = −6*x*3 − 7*x*2 + 40*x* + 21

(a) Use the factor theorem to show that (*x* + 3) is a factor of f(*x*)

**(2)**

(b)   Factorise f(*x*) completely.

**(4)**

**(Total for Question 1 is 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q2**

f (*x*) = *x*4 + 5*x*3 + *ax* + *b*,

where *a* and *b* are constants.

The remainder when f(*x*) is divided by (*x* − 2) is equal to the remainder when f(*x*) is divided by (*x* + 1).

(a) Find the value of *a*.

**(5)**

Given that (*x* + 3) is a factor of f(*x*),

(b) find the value of *b*.

**(3)**

**(Total for Question 2 is 8 marks)**

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**Q3**

f(*x*) = 2*x*3 – 13*x*2 + 8*x* + 48

(a) Prove that (*x* – 4) is a factor of f(*x*)

**(2)**

(b)  Hence, using algebra, show that the equation f(*x*) = 0 has only two distinct roots.

**(4)**

Diagram

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Figure 2 shows a sketch of part of the curve with equation *y* = f(*x*).

(c)  Deduce, giving reasons for your answer, the number of real roots of the equation

2*x*3 – 13*x*2 + 8*x* + 46 = 0

**(2)**

Given that *k* is a constant and the curve with equation *y* = f(*x* + *k*) passes through the origin,

(d) find the two possible values of *k*.

**(2)**

**(Total for Question 3 is 10 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q4**

(a)   Prove that for all positive values of *x* and *y*

**(2)**

(b)   Prove by counter example that this is not true when *x* and *y* are both negative.

**(1)**

**(Total for Question 4 is 3 marks)**

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**End of Questions**

**Gold Mark Scheme**

**Q1**

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**Q2**

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**Q3**

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**Q4**

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**Platinum Questions **

**Calculators may not be used**

The total mark for this section is 9

**1** (*a*) Show that (*x* + 1) is a factor of 2*x*3 + 3*x*2 – 1

**(1)**

(*b*) Solve the equation



**(8)**

**(Total for Question 1 is 9 marks)**

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**End of Questions**

**Platinum Mark Scheme**

**1**

