# **Topic 10**

# **Trigonometric Identities**

# **and Equations**

# Bronze, Silver, Gold and

# Platinum Worksheets for

# AS Level Mathematics

# Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the ‘Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS’ textbook.

# Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It’s important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords “show that” or “prove”. If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

# Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

* [Bronze Questions](#BrQue)
* [Bronze Mark Scheme](#BrMS)
* [Silver Questions](#SiQue)
* [Silver Mark Scheme](#SiMS)
* [Gold Questions](#GoQu)
* [Gold Mark Scheme](#GoMS)

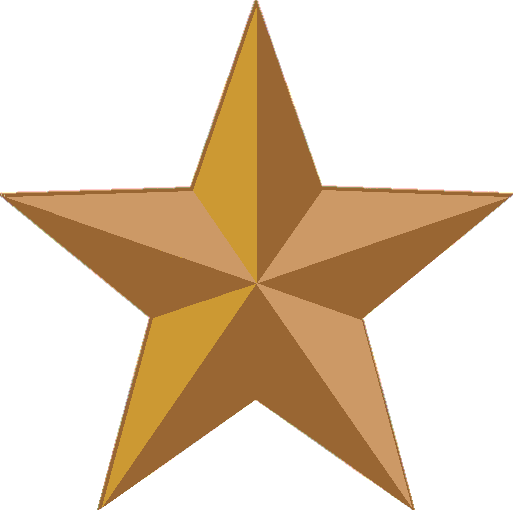
The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

* [Platinum Questions](#PlQu)
* [Platinum Mark Schemes](#PlMS)

# Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found [here](https://qualifications.pearson.com/en/qualifications/edexcel-a-levels/advanced-extension-award-mathematics-2018.html) on the Pearson Edexcel Website, or [here](https://www.mathsemporium.com/category/advanced-extension-award-mathematics/) on the Maths Emporium

**Bronze Questions **

**Calculators may not be used**

The total mark for this section is 26

**Q1**

(a) Show that the equation

5 sin *x* = 1 + 2 cos2*x*

can be written in the form

2 sin2*x* + 5 sin *x* − 3 = 0

**(2)**

(b) Solve, for 0 *x* < 360°,

2 sin2*x* + 5 sin *x* − 3 = 0

**(4)**

**(Total for Question 1 is 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q2**

Show that the equation

cos2*x* = 8sin2*x* − 6sin *x*

can be written in the form

(3sin *x* − 1)2 = 2

**(Total for Question 2 is 3 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q3**

(a)  Show that



**(4)**

(b)  Hence, or otherwise, solve, for 0 ≤ *x* ≤ 360°, the equation



**(3)**

**(Total for Question 3 is 7 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q4**

Solve, for 0 ≤ *x* < 360°,

(a)   

**(4)**

(b)  .

**(6)**

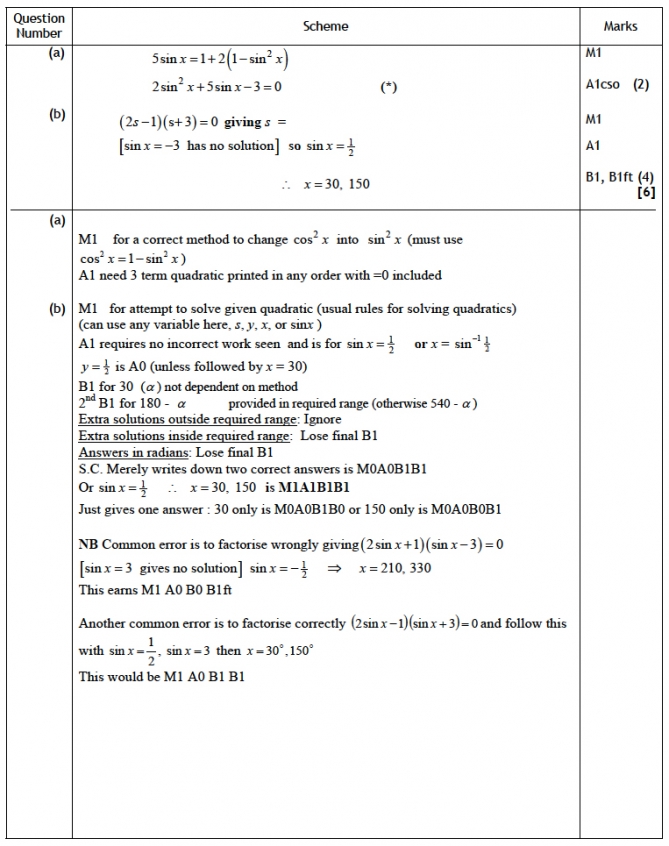
**(Total for Question 4 is 10 marks)**

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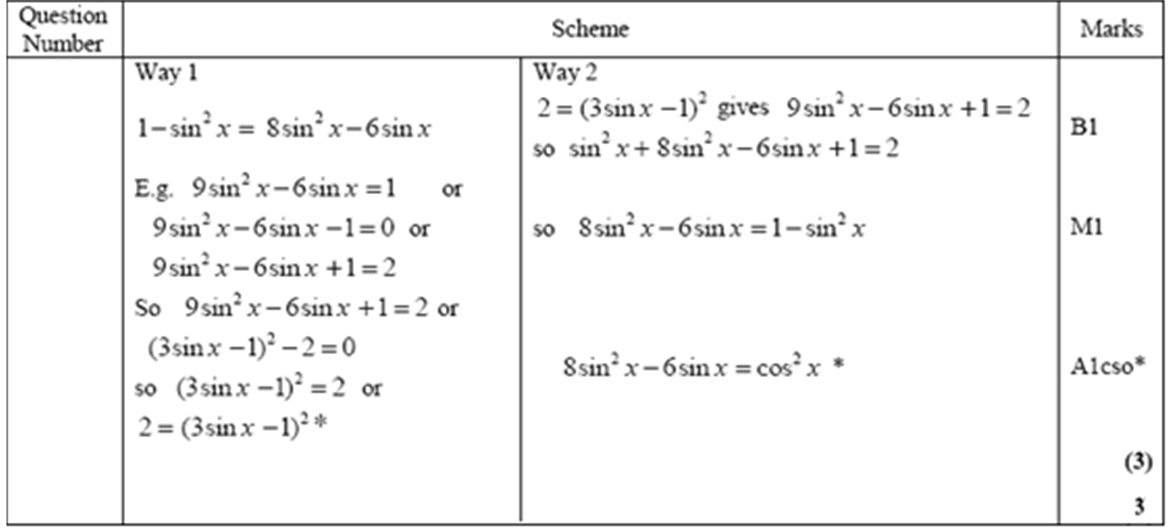
**End of Questions**

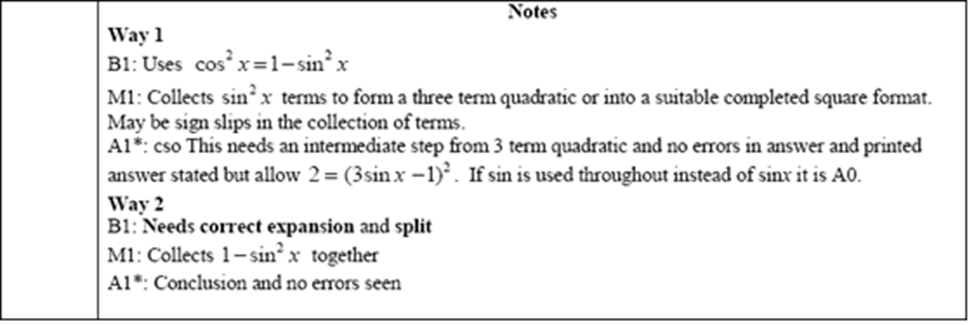
**Bronze Mark Scheme**

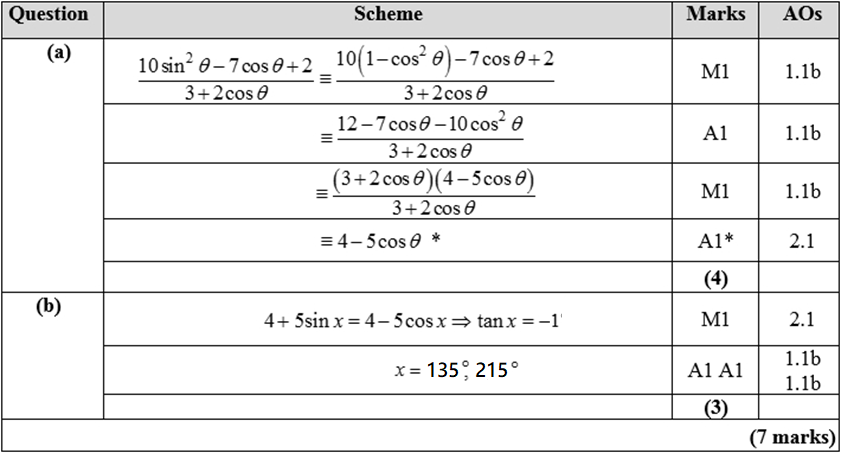
**Q1.**

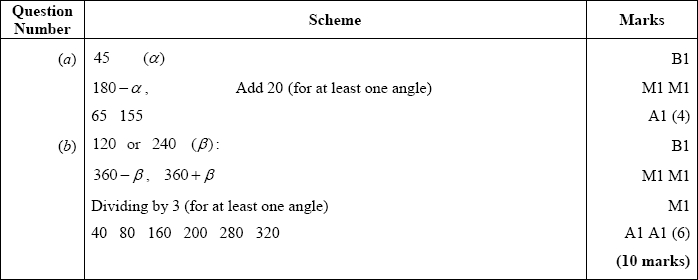


**Q2.**





**Q3.**

**Q4.**

**Silver Questions **

**Calculators may not be used**

The total mark for this section is 34

**Q1**

(i)   Solve, for −180° ≤ *θ* < 180°,



**(4)**

(ii)  Solve, for 0 ≤ *x* < 360°,

2sin *x* = tan *x*.

**(6)**

**(Total for Question 1 is 10 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q2**

(a)  Show that the equation

tan 2*x* = 2 sin 2*x*

can be written in the form

(1 − 2 cos 2*x*) sin 2*x* = 0

**(2)**

(b)  Hence solve, for 0 ≤ *x* ≤ 180°,

tan 2*x* = 2 sin 2*x*

You must show clearly how you obtained your answers.

**(5)**

**(Total for Question 2 is 7 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q3**

(a)  Show that the equation

8 sin2*θ* − 2 cos2*θ* = 3

can be written as

10 sin2*θ* = 5.

**(2)**

(b)  Hence solve, for 0° ≤ *θ* < 360°, the equation

8 sin2*θ* − 2 cos2*θ* = 3,

**(7)**

**(Total for Question 3 is 10 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q4**

 (a) Show that the equation

sin*θ* tan*θ* = cos*θ* + 1

can be written in the form

2cos2*θ* + cos*θ* − 1 = 0

**(3)**

(b) Hence solve, for 0 ≤ *θ* < 360°,

sin*θ* tan*θ* = cos*θ* + 1

showing each stage of your working.

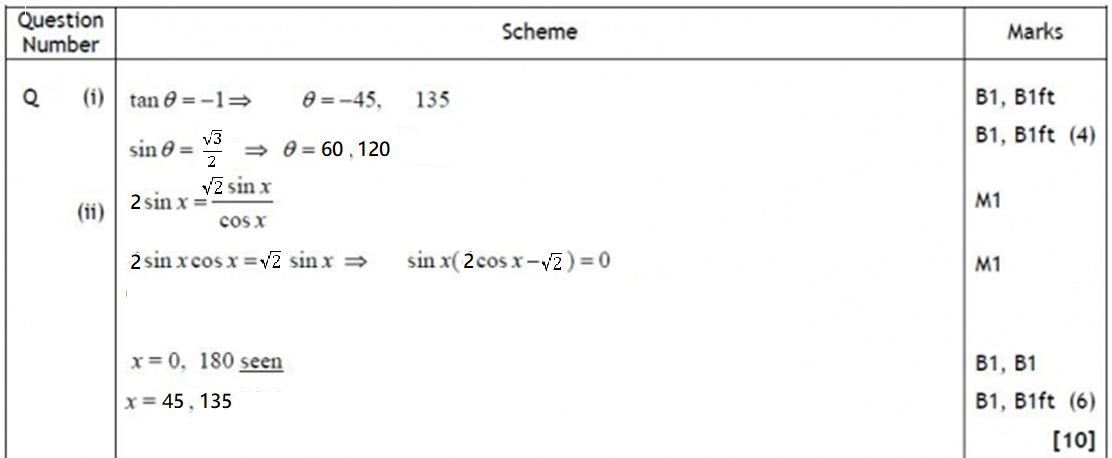
**(5)**

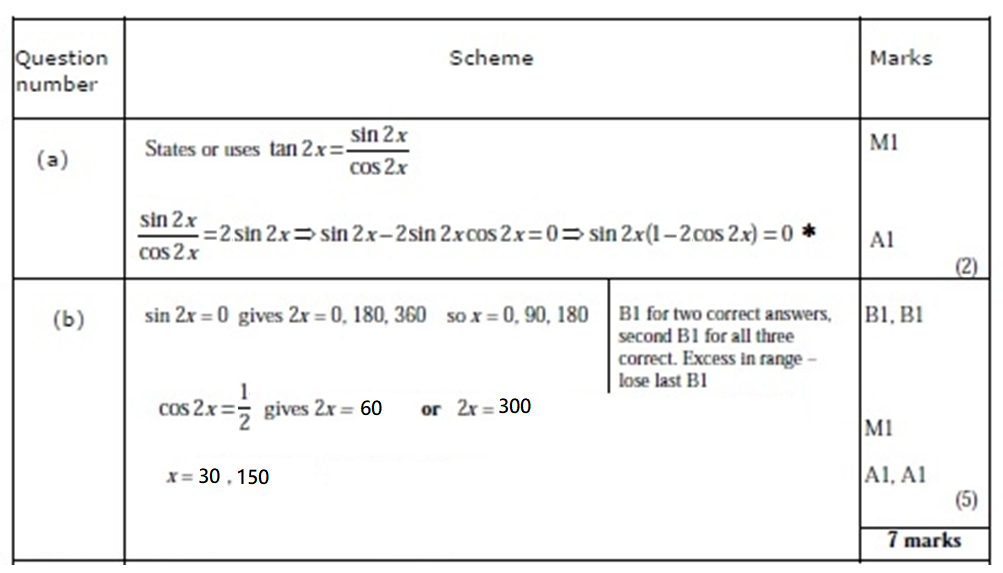
**(Total for Question 4 is 8 marks)**

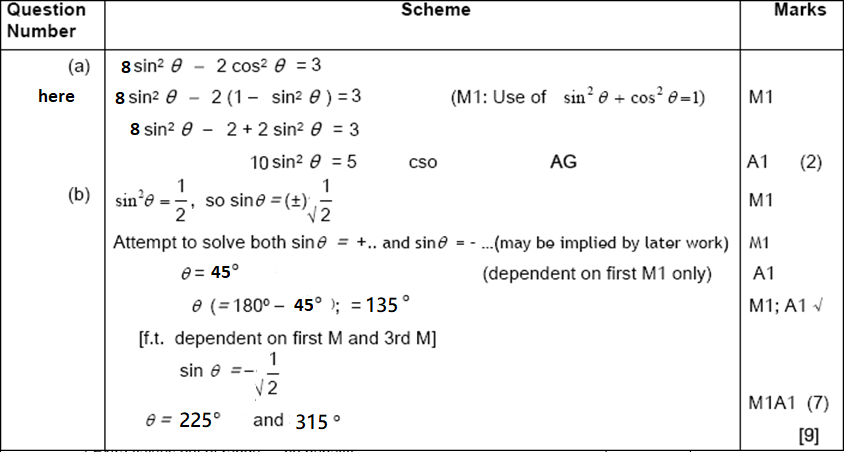
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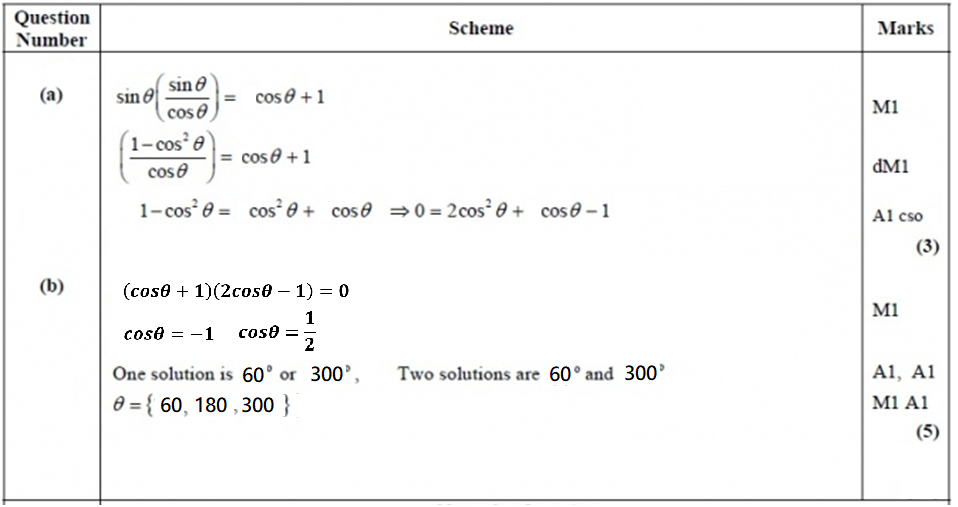
**End of Questions**

**Silver Mark Scheme**

**Q1.**

**Q2.**

**Q3.**

**Q4.**

**Gold Questions **

**Calculators may not be used**

The total mark for this section is 32

**Q1**

(i) Solve, for 0 ≤ *θ* < 360°, the equation

90sin(*θ* + 60°) = 45

You must show each step of your working.

**(4)**

(ii) Solve, for −180 ≤ *x* <180, the equation

tan *x* − sin *x* = 0

**(5)**   
  
**(Total for Question 1 is 9 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q2**

(i)   Solve, for 0 *θ* < *180*°, the equation



**(3)**

(ii)   Given that



(a)   Find cos *x* in terms of *k*.

**(3)**

**(Total for Question 2 is 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q3**

Solve, for 0 ≤ *x* < 180º,

cos(3*x* − 10º ) = 

You should show each step in your working.

**(7)**

**(Total for Question 3 is 7 marks)**

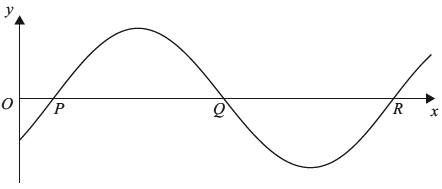
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**Q4**

(i) Find the solutions of the equation sin(3*x* − 15°) =  , for which 0 *x* 180°

**(6)**

(ii)



**Figure 4**

Figure 4 shows part of the curve with equation

*y* = sin(*ax* − *b*), where *a* > 0,   0 < *b* < 180

The curve cuts the *x*-axis at the points *P*, *Q* and *R* as shown.

Given that the coordinates of *P*, *Q* and *R* are (11, 0), (108, 0) and 198, 0) respectively, find the values of *a* and *b*.

**(4)**

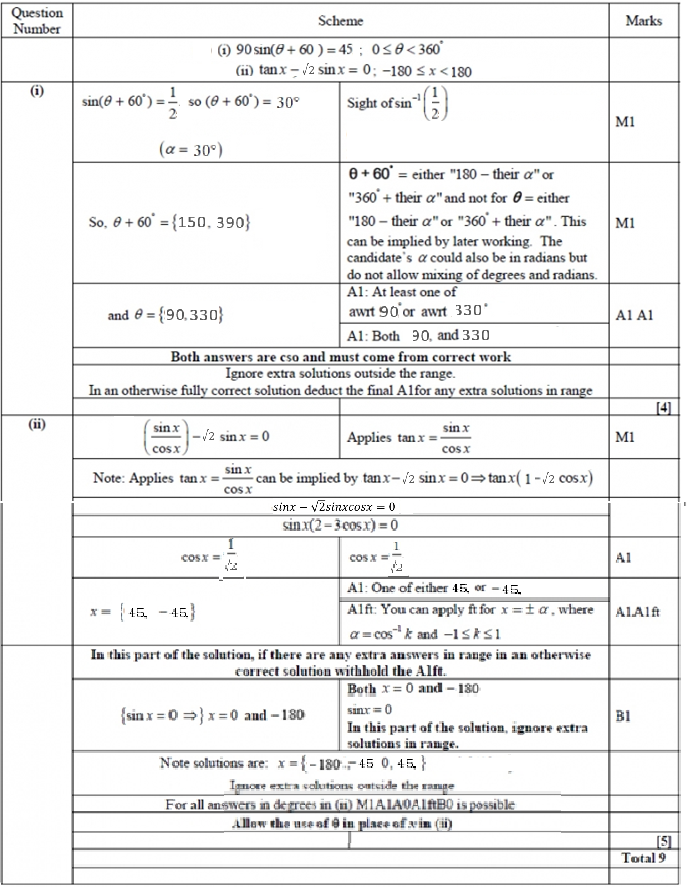
**(Total for Question 4 is 10 marks)**

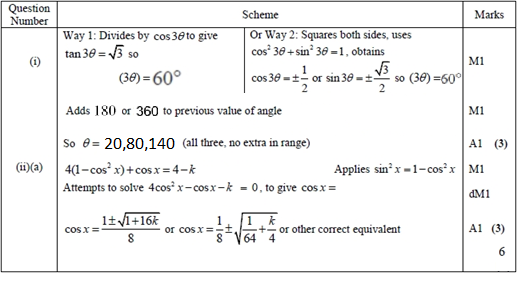
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**End of Questions**

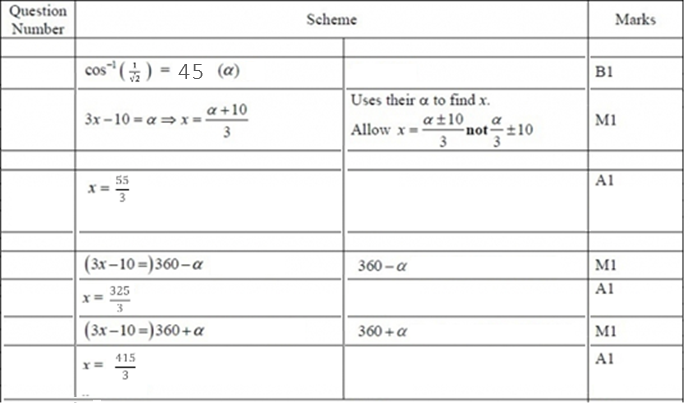
**Gold Mark Scheme**

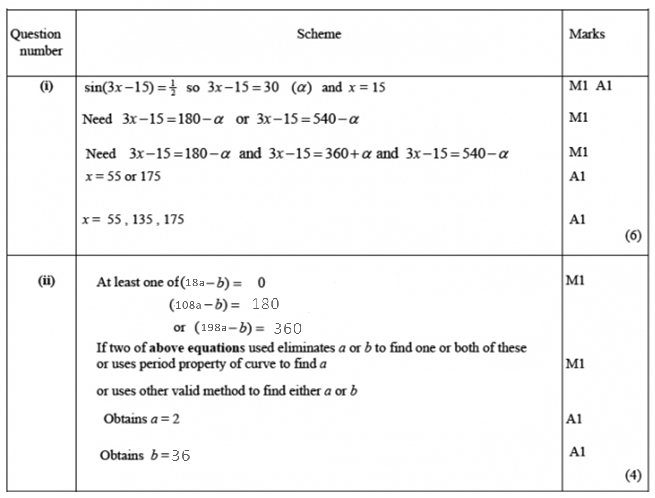
**Q1.**

 **Q2.**



**Q3.**



**Q4.**

**Platinum Questions **

**Calculators may not be used**

The total mark for this section is 17

**1**



Figure 4 shows a shape *S*(*θ*) made up of five line segments *AB*, *BC*, *CD*, *DE* and *EA*.

The lengths of the sides are *AB* = *BC* = 5 cm, *CD* = *EA* = 3 cm and *DE* = 7 cm.

Angle *BAE* = angle *BCD* = *θ* radians.

The length of each line segment always remains the same but the value of *θ* can be varied so

that different symmetrical shapes can be formed, with the added restriction that none of the line segments cross.

(*a*)Sketch *S*(*1800*), labelling the vertices clearly.

**(2)**

The shape *S*(*ϕ*) is a trapezium.

(*b*)Sketch *S*(*ϕ*) and calculate the value of *ϕ*.

**(3)**

The smallest possible value for *θ* is *α*, where *α* > 0, and the largest possible value for *θ* is *β*,

where *β* > *1800*.

(*c*)Show that *α* = arccos . [arccos(*x*) is an alternative notation for cos–1(*x*)]

**(4)**

(*d*)Find an expression for the value of *β*.

**(4)**

The area, in cm2, of shape *S*(*θ*) is *R*(*θ*).

(*e*)Show that for *α* ⩽ *θ* < *1900*

*R*(*θ*) = 15 sin *θ* + 

**(4)**

**(Total for Question 1 is 17 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Platinum Mark Scheme**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Question** | **Scheme** | | | | **Marks** | **Notes** |
| **7. (a)** | Triangle *EBD*  *EB* = *DB* or labelling to show isos |  | | | M1  A1 | Needn’t be isos D  Correct labelling |
|  |  | | | | (2) |  |
| **(b)** | Isos trapezium (*ACDE*)  , so | | |  | B1  M1,  A1 | Sketch – with at least 1 side |
| M1 for correct expression |
|  |  | | | | (3) |  |
| **(c)** | (o.e.) | |  | | B1  B1  M1 | Shape (o.e.)  2 or more side lengths |
|  | Correct use of cos rule |
|  | So  **(\*)** | | A1cso | **In (c), (d) B1B1 can be implied by M1** |
|  |  | | | | (4) |
| **(d)** | [ = 0.1] |  | | | B1  B1  M1 | Shape (o.e.)  2 or more side lengths |
| Correct expression (can ignore – *p*) |
|  | or | A1 |  |
|  |  | | | | (4) |  |
| **(e)** |  | | | | M1 | Attempt *BE* or *BD* |
|  | [*h* = height from *B* to *ED*] so | | | | M1 | Attempt *h* |
|  | Area =  **(\*)** | | | | M1,A1 | M1 for correct areas  A1 cso |
|  |  | | | | (4) |  |
|  |  | | | |  |  |