# **Topic 12**

# **Differentiation**

# Bronze, Silver, Gold and

# Platinum Worksheets

# for AS Level Mathematics

# Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the ‘Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS’ textbook.

# Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It’s important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords “show that” or “prove”. If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

# Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

* [Bronze Questions](#BrQue)
* [Bronze Mark Scheme](#BrMS)
* [Silver Questions](#SiQue)
* [Silver Mark Scheme](#SiMS)
* [Gold Questions](#GoQu)
* [Gold Mark Scheme](#GoMS)

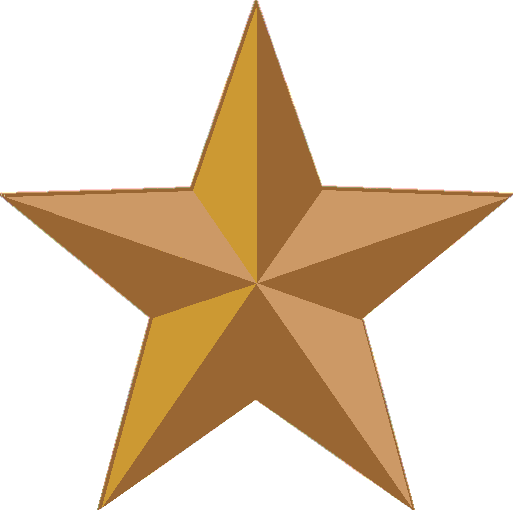
The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

* [Platinum Questions](#PlQu)
* [Platinum Mark Schemes](#PlMS)

# Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found [here](https://qualifications.pearson.com/en/qualifications/edexcel-a-levels/advanced-extension-award-mathematics-2018.html) on the Pearson Edexcel Website, or [here](https://www.mathsemporium.com/category/advanced-extension-award-mathematics/) on the Maths Emporium

**Bronze Questions **

**Calculators may not be used**

The total mark for this section is 28

**Q1**

The curve *C* has equation

*y* = 2*x*2 – 12*x* + 16

Find the gradient of the curve at the point *P* (5, 6).

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(Total for Question 1 is 4 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q2**

Given that , find 

**(Total for Question 2 is 3 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q3**

A curve has equation

*y* = 2*x*3 − 4*x* + 5

Find the equation of the tangent to the curve at the point *P*(2, 13).

Write your answer in the form *y* = *mx* + *c*, where *m* and *c* are integers to be found.

**Solutions relying on calculator technology are not acceptable.**

**(Total for Question 3 is 5 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q4**

Prove, from first principles, that the derivative of *x*3 is 3*x*2

**(Total for Question 4 is 4 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q5**



(a)  Find  giving each term in its simplest form.

**(4)**

(b)  Find 

**(2)**

**(Total for Question 5 is 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q6**

Using calculus, find the coordinates of the stationary point on the curve with equation



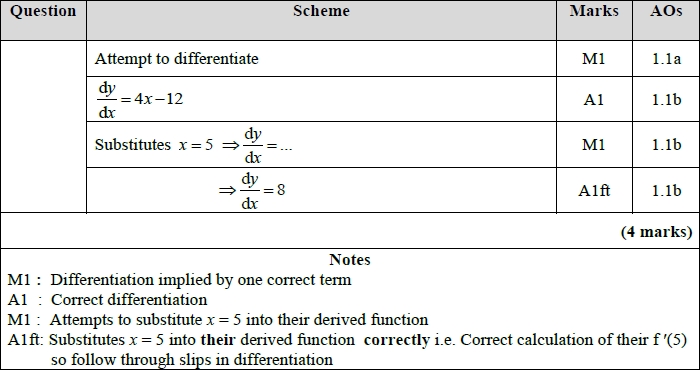
**(6)**

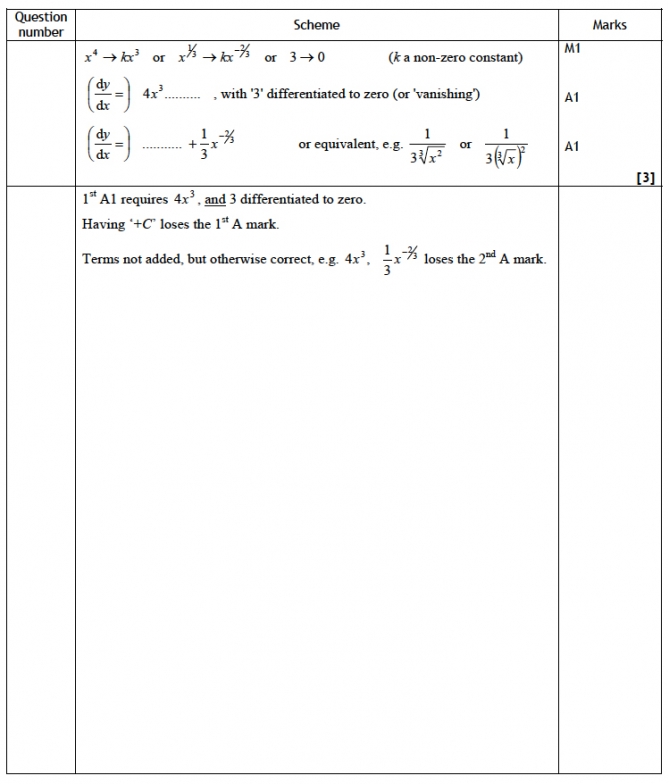
**(Total for Question 6 is 6 marks)**

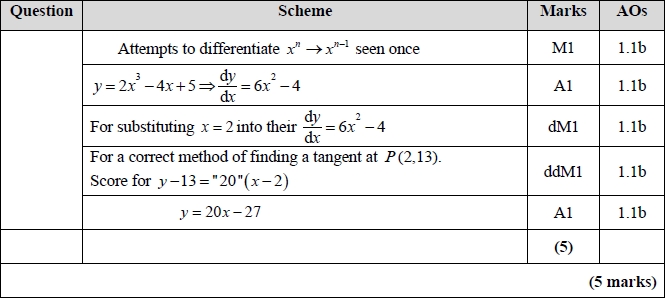
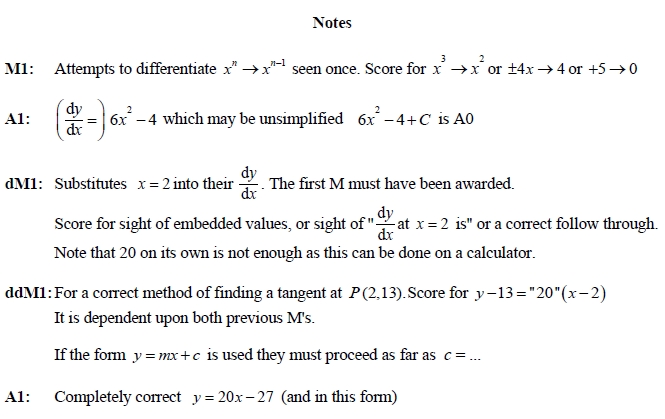
**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

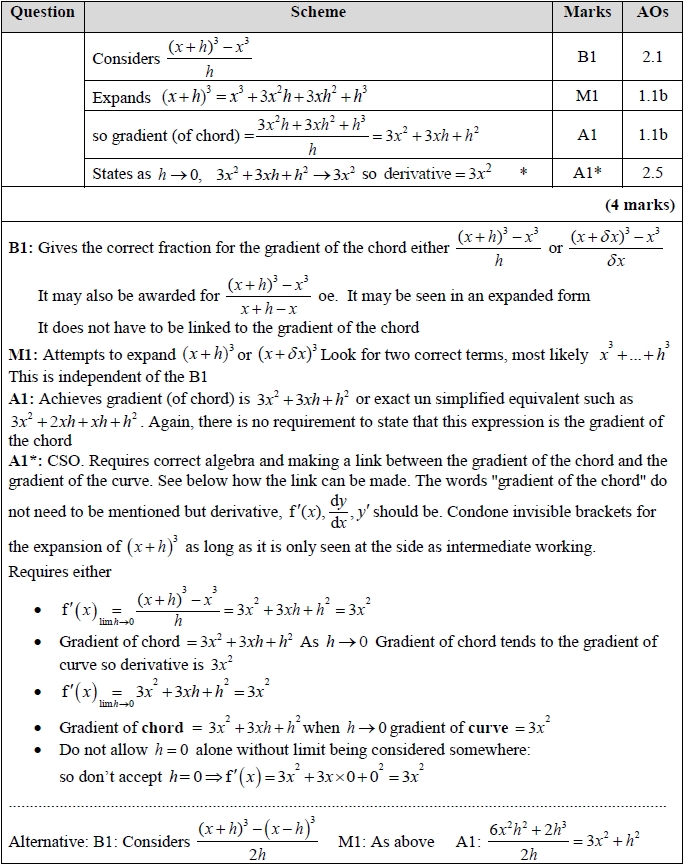
**End of Questions**

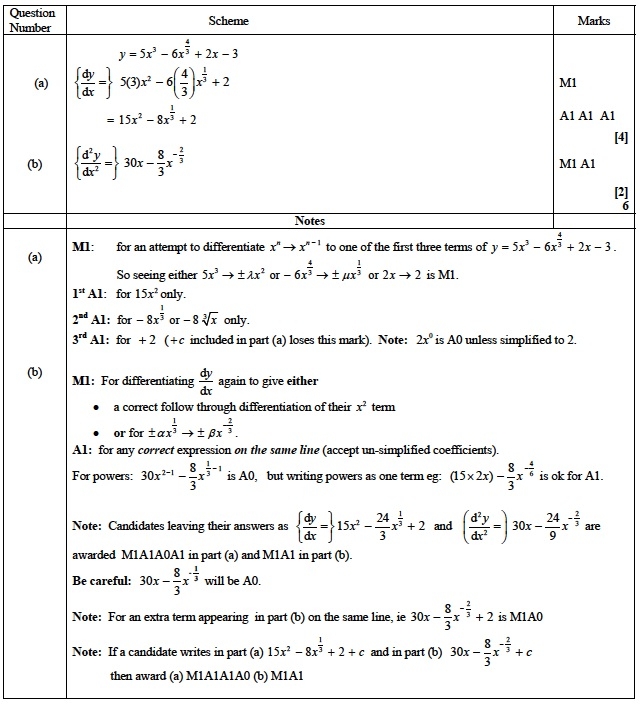
**Bronze Mark Scheme**

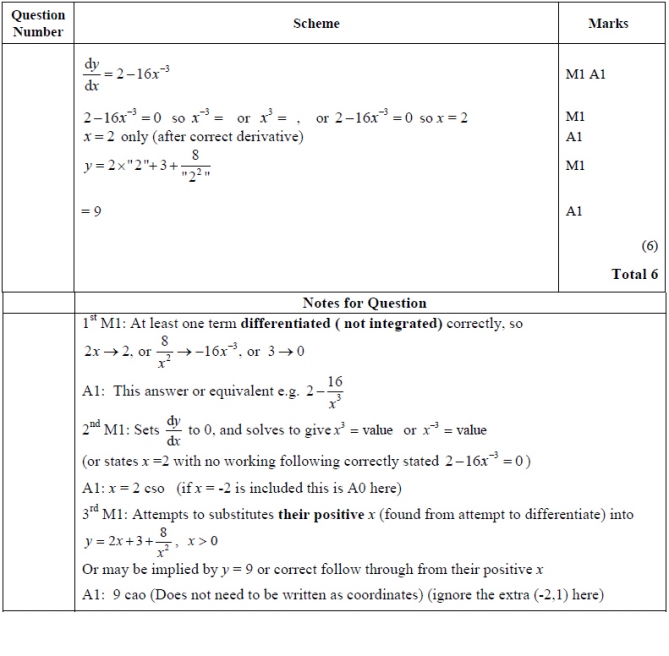
**Q1.**

**Q2.**

**Q3.**

**Q4.**

**Q5.**

**Q6.**

**Silver Questions **

**Calculators may not be used**

The total mark for this section is 34

**Q1**

The curve *C* has equation

*y* = 2*x* − 8√ *x* + 5 , *x* ≥ 0

(a)  Find , giving each term in its simplest form.

**(3)**

The point *P* on *C* has *x*-coordinate equal to 

(b)  Find the equation of the tangent to *C* at the point *P*, giving your answer in the form

*y* = *ax* + *b*, where *a* and *b* are constants.

**(4)**

**(Total for Question 1 is 7 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q2**The curve *C* has equation, *x* ≠ 0

(a)  Use calculus to show that the curve has a turning point *P* when *x* = √2

**(4)**

(b)  Find the *x*-coordinate of the other turning point *Q* on the curve.

**(1)**

(c)  Find .

**(1)**

(d)  Hence or otherwise, state with justification, the nature of each of these turning points *P* and *Q*.

**(3)**

**(Total for Question 2 is 9 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

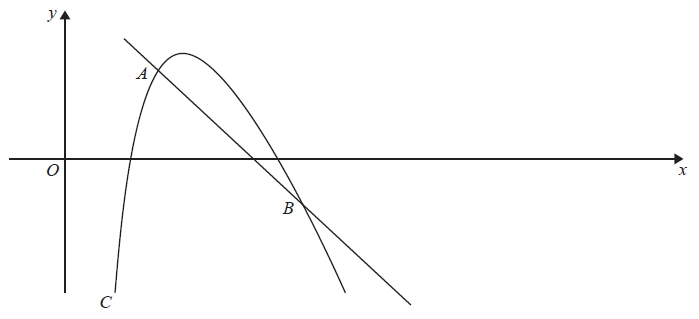
**Q3**

Prove, from first principles, that the derivative of 3*x*2 is 6*x*

**(Total for Question 3 is 4 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q4**



**Figure 3**

A sketch of part of the curve *C* with equation

*y* = 20 − 4*x* − 18⁄*x*,   *x* > 0

is shown in Figure 3.

Point *A* lies on *C* and has an *x* coordinate equal to 2

Show that the equation of the normal to *C* at *A* is *y* = −2*x* + 7

**(Total for Question 4 is 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q5**

The volume *V* cm3 of a box, of height *x* cm, is given by

*V* = 4*x*(5 − *x*)2,      0 < *x* < 5

(a) Find .

**(4)**

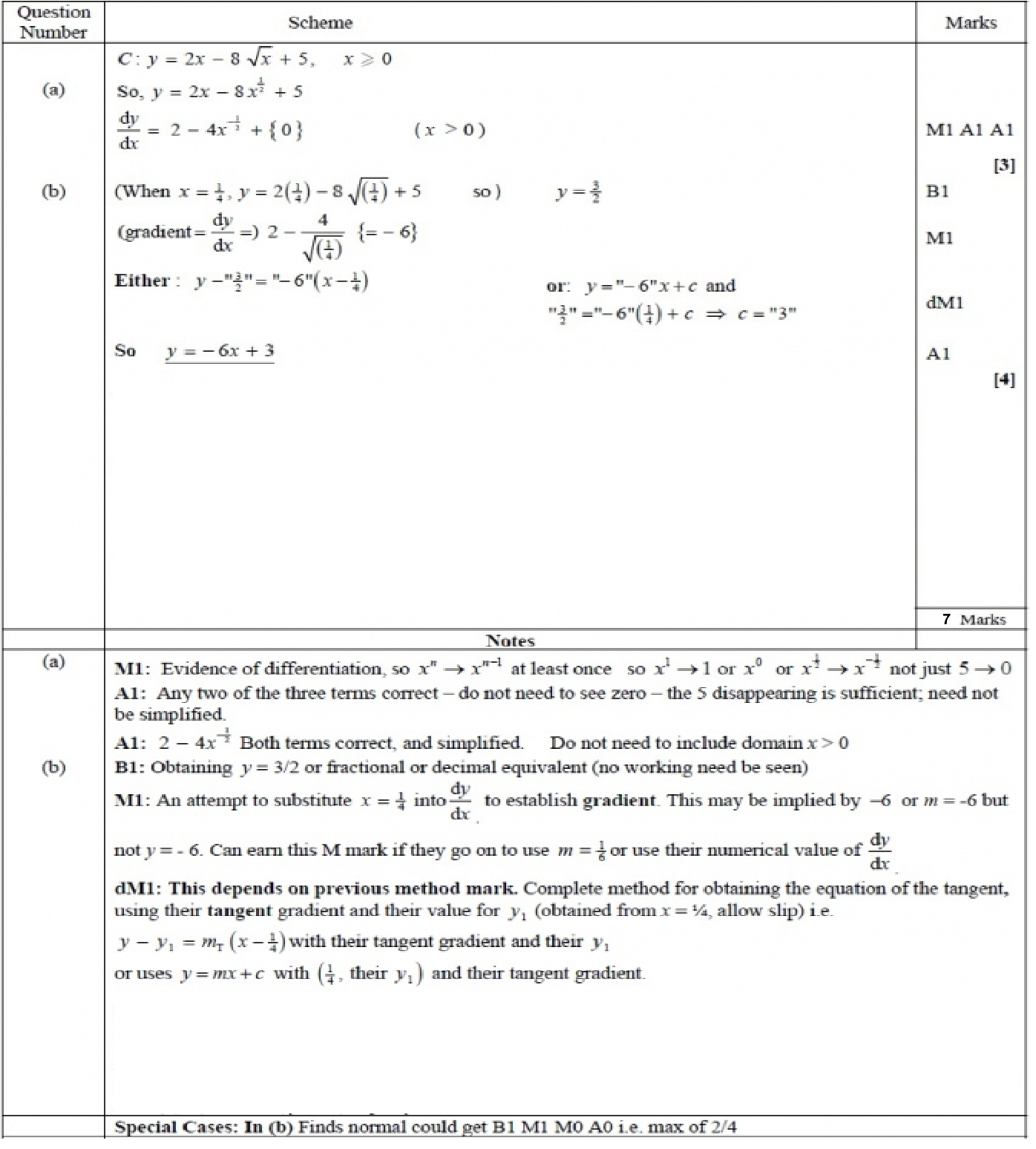
(b) Hence find the maximum volume of the box.

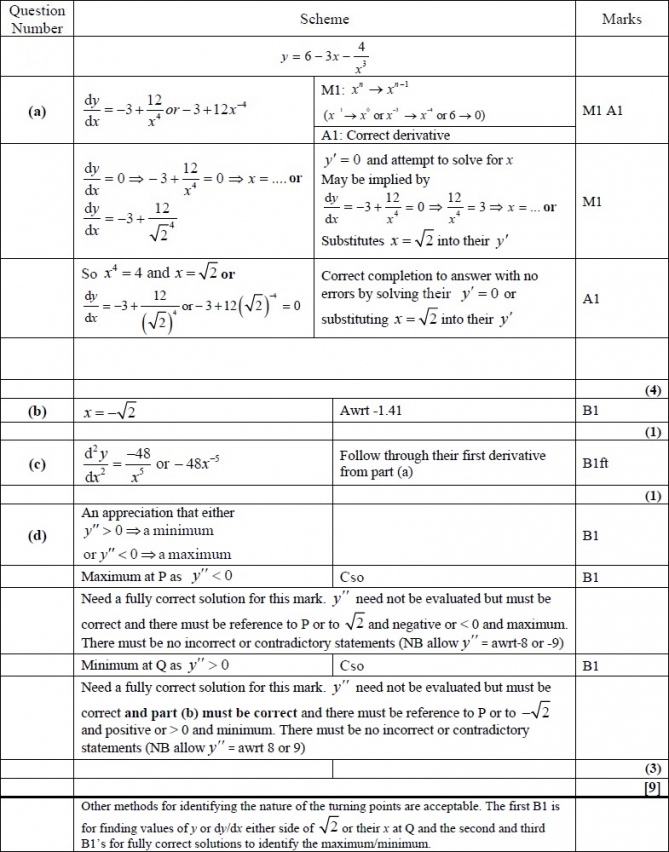
**(4)**

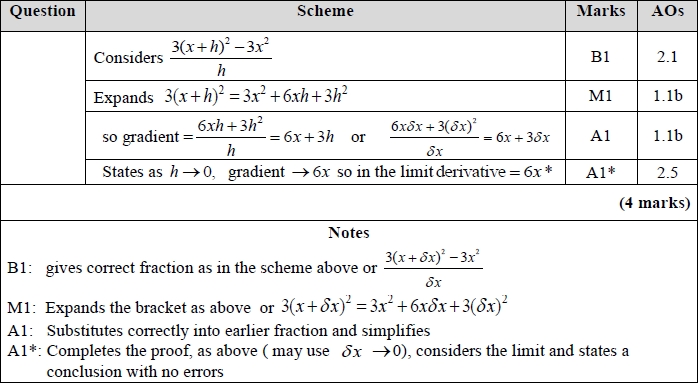
**(Total for Question 5 is 8 marks)**

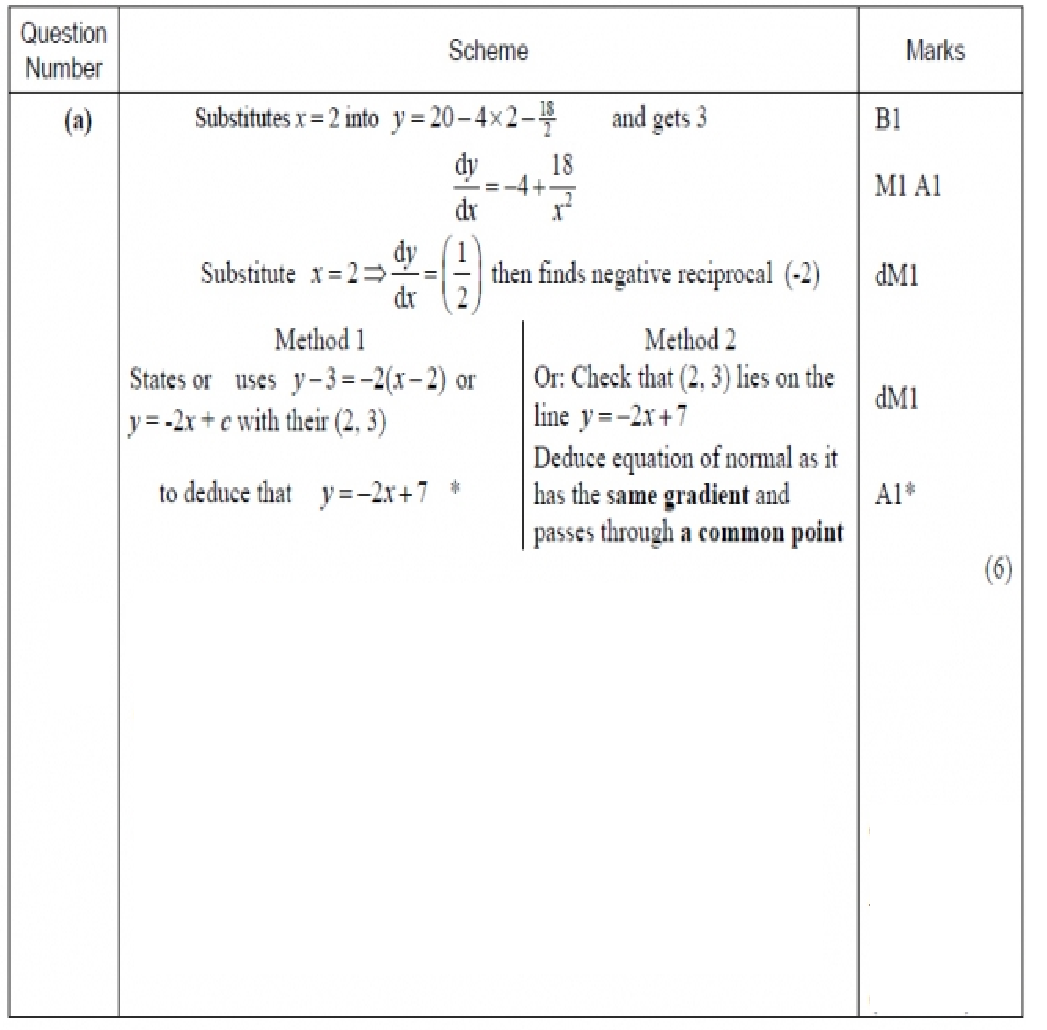
**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

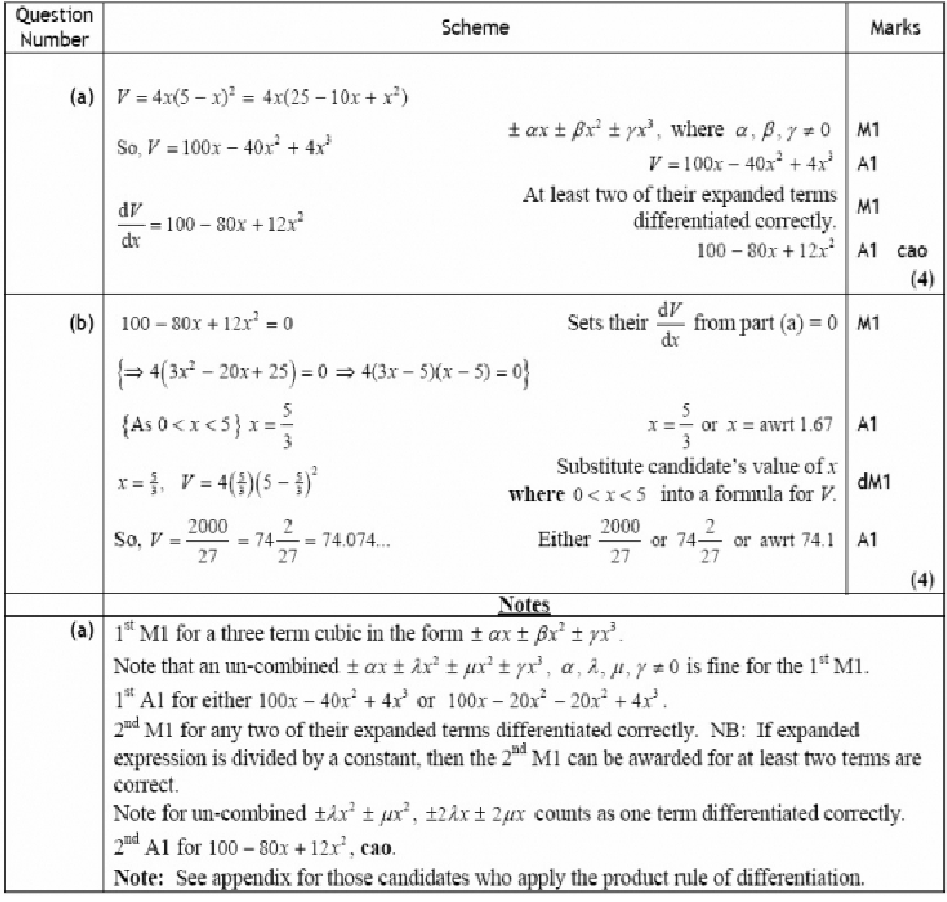
**Silver Mark Scheme**

**Q1.**

 **Q2.**

**Q3.**

**Q4.**

**Q5.**

**Gold Questions **

**Calculators may not be used**

The total mark for this section is 33

**Q1**The curve *C* has equation,      *x* > 0

(a) Use calculus to find the coordinates of the turning point on *C*.

**(7)**

(b) Find 

**(2)**

(c) State the nature of the turning point.

**(1)**

**(Total for Question 1 is 10 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q2**

The curve *C* has equation

,            *x* > 0

The point *P* on *C* has *x*-coordinate equal to 2.

(a) Show that the equation of the tangent to *C* at the point *P* is *y* = 1 − 2*x*.

**(6)**

(b) Find an equation of the normal to *C* at the point *P*.

**(3)**

**(Total for Question 2 is 9 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q3**

,  where *k* is a constant.

(a)   Find 

**(2)**

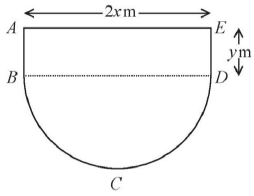
(b)   Given that *y* is decreasing at *x* = 4, find the set of possible values of *k*.

**(2)**

**(Total for Question 3 is 4 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Q4**



**Figure 4**

Figure 4 shows the plan view of the design for a swimming pool.

The shape of this pool *ABCDEA* consists of a rectangular section *ABDE* joined to a semicircular section *BCD* as shown in Figure 4.

Given that *AE* = 2*x* metres, *ED* = *y* metres and the area of the pool is 250 m2,

(a)   show that the perimeter, *P* metres, of the pool is given by



**(4)**

(b)   Explain why 0 < *x* < 

**(2)**

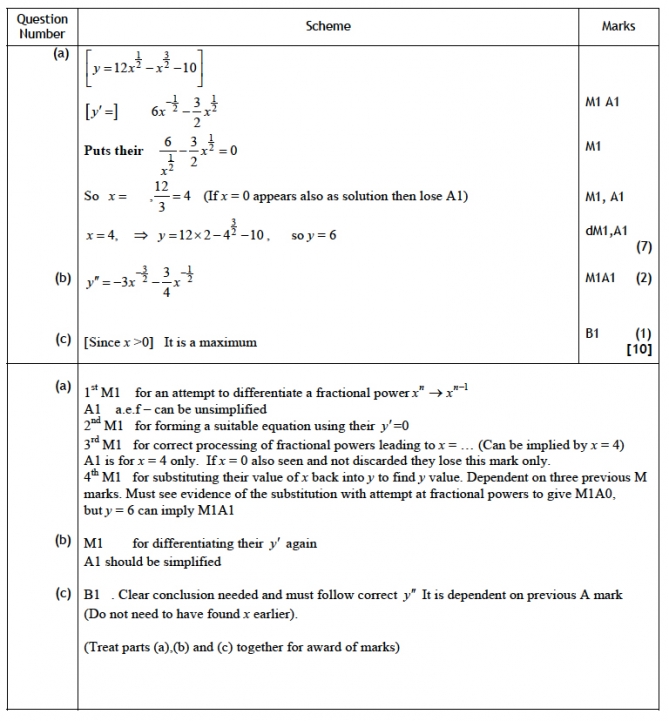
(c)   Find the minimum perimeter of the pool, giving your answer in exact form.

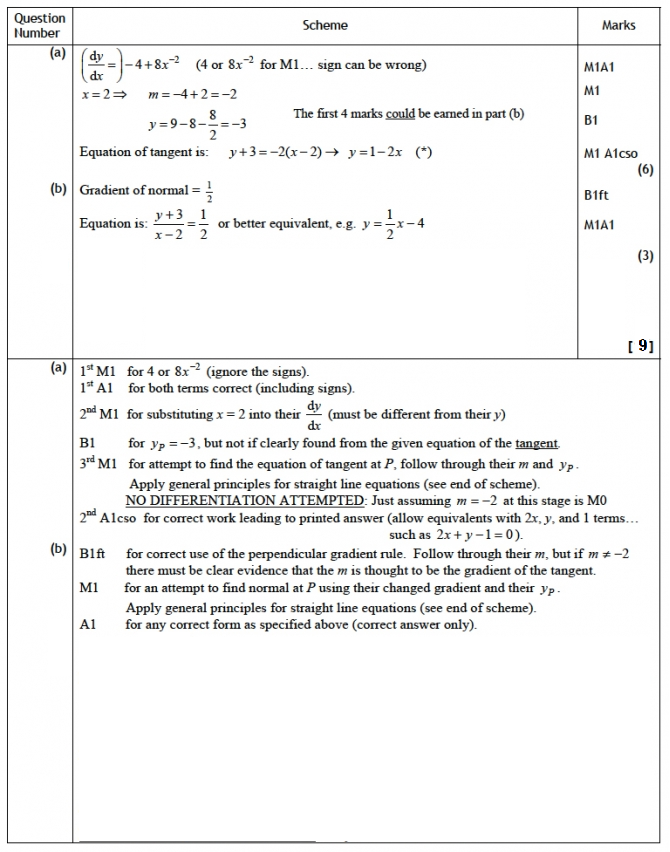
**(4)**

**(Total for Question 4 is 10 marks)**

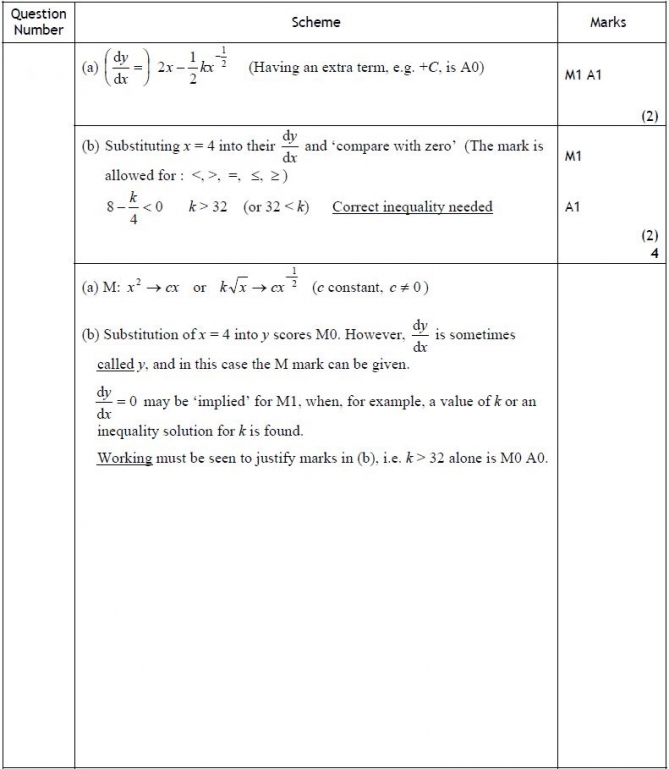
**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Gold Mark Scheme**

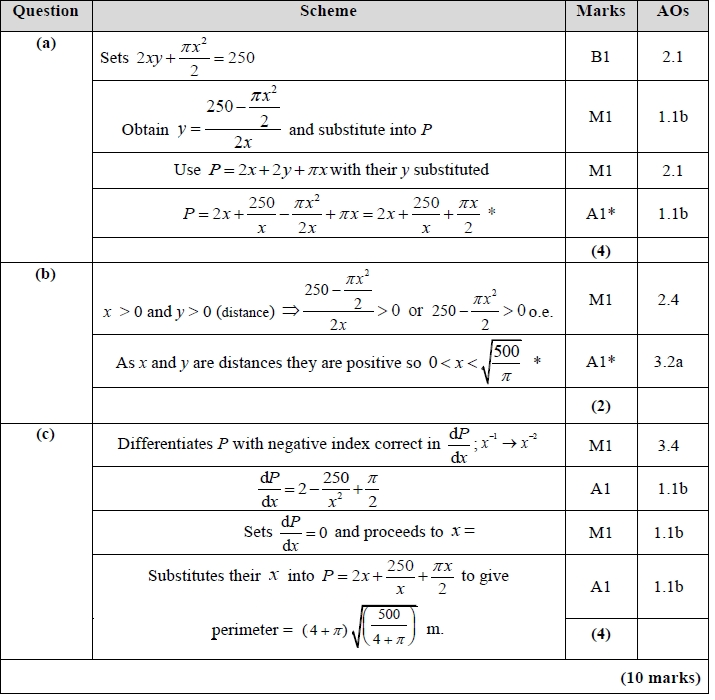
**Q1.**

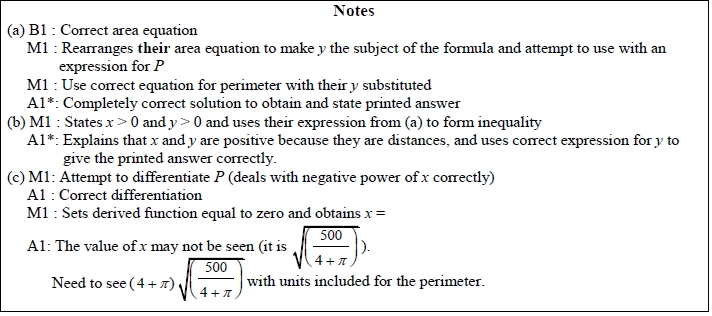
**Q2.**

**Q3.**



**Q4.**





**Platinum Questions **

**Calculators may not be used**

The total mark for this section is 17

**1**



Figure 2 shows a sketch of the parabola with equation *y *

This question concerns rectangles that lie under the parabola in the first quadrant. The

bottom edge of each rectangle lies along the *x*-axis and the top left vertex lies on the

parabola. Some examples are shown in Figure 2.

Let the *x* coordinate of the top left vertex be *a*.

(*a*)Explain why the width, *w*, of such a rectangle must satisfy *w* ≤ 10 − 2*a*

**(2)**

(*b*)Find the value of *a* that gives the maximum area for such a rectangle.

**(5)**

Given that the rectangle must be a square,

(*c*)find the value of *a* that gives the maximum area for such a square.

**(3)**

Given that the area of the rectangles is fixed as 36

(*d*)find the range of possible values for *a*

**(6)**

**(+S1)**

**(Total for Question 7 is 17 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Platinum Mark Scheme**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **1(a)** | The rectangle must lie under the parabola, so maximum width will occur when the top right vertex also lies on the parabola. Ie recognises the symmetry and forms an equation. Allow a suitable sketch as evidence. | | | | | | | **M1** | 1 |
| By symmetry about the line , this occurs at , hence width satisfies \* Must be convincing reason. | | | | | | | **A1\*** | 2 |
|  | | | | | | | **(2)** |  |
| **(b)** | Maximum area must occur for a full width rectangle, ie when | | | | | | | **B1** | 2 |
| Thus max area occurs for | | | | | | | **M1** | 3 |
| Attempts  and sets  and attempts to find *a* | | | | | | | **M1** | 3 |
|  | | | Any correct method to solve the quadratic. | | | | **M1** | 3 |
| (But need  to give a valid rectangle and as area is zero at either end of this interval so) | | | | | | | **(S+)** |  |
| max area occurs when  (oe simplified) | | | | | | | **A1** | 2 |
|  | | | | | | | **(5)** |  |
| **(c)** | Max square area needs | | | | Sets up correct equation. | | | **M1** | 3 |
|  | | | | | Solves the quadratic, any valid means. | | **dM1** | 3 |
| But need  (and ) so | | Selects correct root. | | | | | **(S+)A1\*** | 3 |
|  | | | | | | | **(3)** |  |
| **(d)** | If area is 36, then width is given by  (oe) Therefore need solutions to  OR need solutions to  or other valid inequality in *a* set up e.g.  (as ) followed by substitution of  This mark is for a correct reasoning of the required inequality, If no reason is given and equation is it is B0, but all other marks are possible. | | | | | | | **B1** | 1 |
| Forms a suitable cubic using the maximum width and height (may be equation or inequation. | | | | | | | **M1** | 3 |
|  | Correct cubic achieved as equation or inequation. | | | | | | **A1** | 3 |
| Identifies  as factor (factor theorem) and attempts to factorise | | | | | | | **M1** | 3 |
|  | | | | | | Finds CVs | **M1** | 3 |
| (positive cubic with roots  (as ))  So possible values of *a* are | | | | | | | **(S+)**  **A1** | 2 |
|  | | | | | | | **(6)** |  |
| **S1** | S1 mark: Award S1 for a clear and concise solution that is   * fully correct with no S- point or * that scores 13+ and includes an S+ point and no S-. | | | | | | | **(1)** | 2 |
| **(16 + 1 marks)** | | | | | | | | | |
| **Notes:** | | | | | | | | | |
| **(b) S+** for explaining clearly why the root outside  is rejected.  **(c) S+** for justifying the root lies in acceptable domain for *a*.  **(c) S-** for a cumbersome strategy. **S+** for justification of roots/which are in valid domain. | | | | | | | | | |