Turning Points

Special Relativity

Name: __________

Class: __________

Date: __________

Time:	**218 minutes**
Marks:	**154 marks**
Comments:	

1 One of the two postulates of Einstein's theory of special relativity is that the speed of light in free space is invariant.

(a) Explain what is meant by this postulate.

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(1)

(b) State the other postulate.

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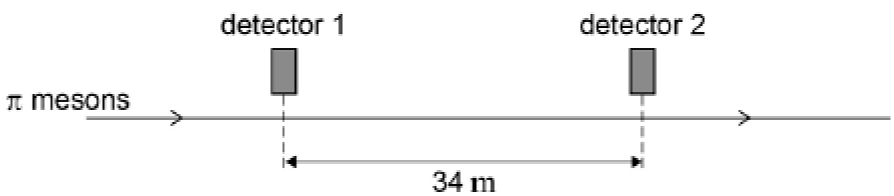
(1)

(c) Two detectors are measured to be 34 m apart by an observer in a stationary frame of reference. A beam of π mesons travel in a straight line at a speed of 0.95 c past the two detectors, as shown in the figure below.



Calculate the time taken, in the frame of reference of the observer, for a π meson to travel between the two detectors.

time = ____________________

(1)

(d) π mesons are unstable and decay with a half-life of 18 ns.

It is found in experiments that approximately 75% of the π mesons that pass the first detector decay before reaching the second detector.

Show how this provides evidence to support the theory of special relativity. In your answer compare the percentage expected by the laboratory observer with and without application of the theory of special relativity.

(5)

(Total 8 marks)

2 (a) The theory of special relativity is based on two postulates. One of these postulates is that the speed of light in free space is invariant.
State the other postulate.

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(1)

(b) An electron in the Stanford linear accelerator is accelerated to an energy of 24.0 GeV.

(i) An electron travelling with this energy has a velocity v.

Show that the value of $\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}$ is about 2.1×10^{-5}.

(3)

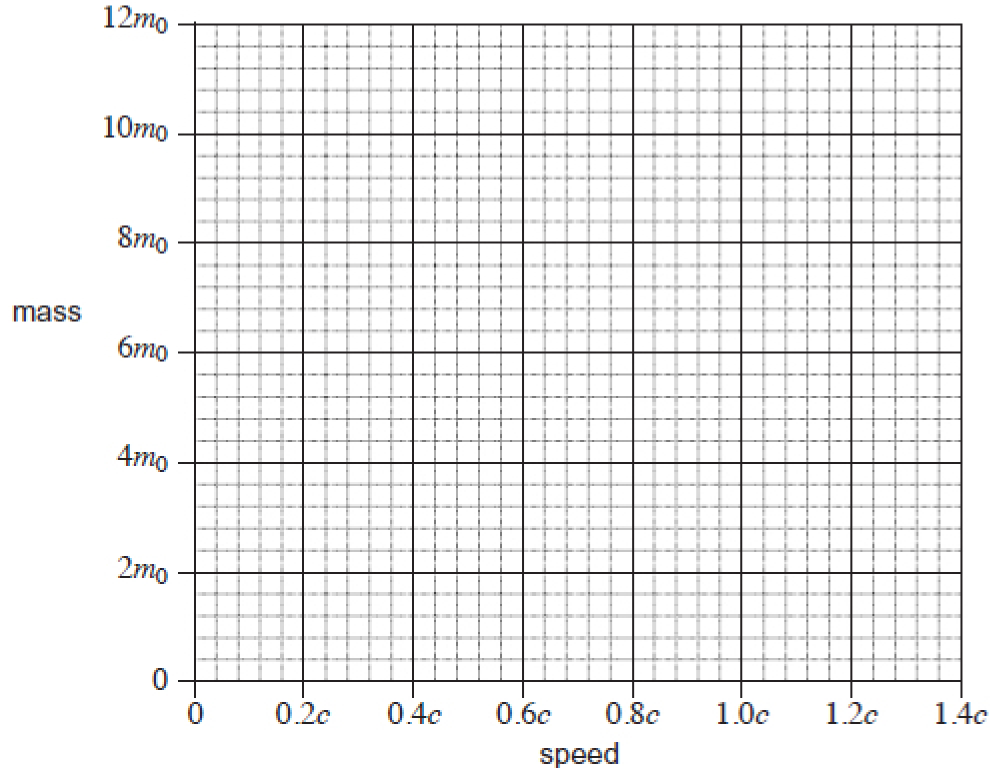
(ii) The Stanford linear accelerator has a length of 3.0 km. Assume that the electron travels for the full length of the accelerator with an energy of 24 GeV.
Calculate the length, in m, of the accelerator in the reference frame of the electron.

length of accelerator = ____________________ m

(1)

(c) Draw a graph to show how the relativistic mass of an electron varies with speed as it is accelerated from rest.

Rest mass of an electron = m_0



(2)

(Total 7 marks)

3 Cosmic rays mostly consist of high-energy protons. These protons can collide with atomic nuclei in the Earth's upper atmosphere producing pions (π^-). Pions are unstable and decay into high-energy muons (μ^-).

(a) (i) Which of the following is the particle group for pions (π^-)?

Tick (✓) the correct answer.

Baryons ☐

Leptons ☐

Mesons ☐

Photons ☐

(1)

(ii) Complete the equation for the decay of a pion (π^-).

$$\pi^- \longrightarrow \mu^- + \text{..............................}$$

(1)

(b) 2.5×10^8 muons are created simultaneously above the EarthÙs surface.
These muons are unstable and have a half-life of 2.2 μs. They are created at a height of 10.7 km and travel towards the Earth's surface with a constant vertical velocity of 2.85×10^8 m s^{-1}.

(i) Show that, for the reference frame of an observer on Earth, the time taken for the muons to reach the Earth's surface is approximately 17 muon half-lives.

(2)

(ii) Estimate the number of these muons that an observer on Earth would expect to remain after 17 half-lives.

number ____________________

(2)

(iii) The number of muons that reach the Earth's surface is considerably different from the estimated number in part **(b)(ii)**.

Identify the theory that explains the difference between the estimated and observed number of muons.

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(1)

(iv) Outline why the number of muons that actually reach the Earth's surface is different from the estimated number in part **(b)(ii)**.

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(1)

(v) Calculate, for the reference frame of a muon, the time taken for the muons to travel this distance.

time ____________________ s

(3)

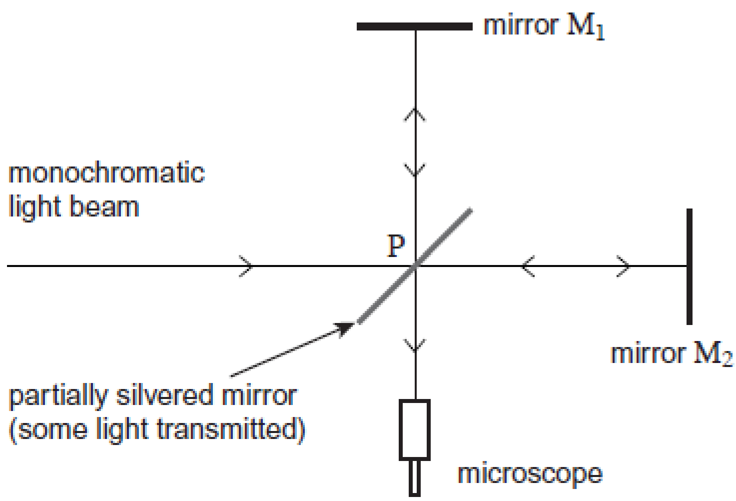
(vi) Calculate the number of muons that remain at the end of the time interval calculated in part **(b)(v)**.

number ____________________

(3)

(Total 14 marks)

4 The diagram shows the paths of light rays through a simplified version of the apparatus used by Michelson and Morley.



In the apparatus, light waves reflected by the mirrors M_1 and M_2, meet at P so that they superpose and produce interference fringes. These are observed using the microscope.

Michelson and Morley predicted that the fringes would shift when the apparatus was rotated through 90°. They thought that this shift would enable them to measure the speed of the Earth through a substance, called the aether, that was thought to fill space.

(a) Explain why Michelson and Morley expected that the fringe positions would shift when the apparatus was rotated through 90°.

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(2)

(b) In their apparatus they made the distances PM_1 and PM_2 the same and equal to d. They used light of wavelength (λ) about 550 nm and knew that the speed of light c was 3.0×10^8 m s^{-1}. Using known astronomical data, they calculated the speed v at which they thought the Earth moved through the aether. They were then able to predict that when the apparatus was rotated through 90° the fringes should shift by a distance $0.4f$, where f was the fringe spacing.

(i) To determine v, Michelson and Morley assumed that the Sun was stationary with respect to the aether as the Earth moved through it.

Suggest, using this assumption, how the speed v of the Earth through the aether could be determined. You do not need to do the calculation.

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(1)

(ii) Michelson and Morley calculated v to be 3.0×10^4 m s^{-1}.

They worked out Δf, the magnitude of the expected shift of the fringes, using the formula $\Delta f = \frac{2v^2 d}{c^2 \lambda} f$.

Calculate the distance d they used in their experiment.

d = ____________________ m

(1)

(c) Although a shift of 0.4 f was easily detectable, no shift was observed. Explain what this null result demonstrated and its significance for Einstein in his special theory of relativity.

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(2)

(Total 6 marks)

5 A muon is an unstable particle produced by cosmic rays in the Earth's atmosphere. Muons that are produced at a height of 10.7 km above the Earth's surface, travel at a speed of 0.996 c toward Earth, where c is the speed of light. In the frame of reference of the muons, the muons have a half-life of 1.60×10^{-6} s.

(a) (i) Calculate how many muons will reach the Earth's surface for every 1000 that are produced at a height of 10.7 km.

number of muons ______________________

(3)

(ii) Which of the following statements is correct? Tick (✓) the correct answer.

	✓if correct
For an observer in a laboratory on Earth, the distance travelled by a muon that reaches the Earth is greater than the distance travelled by a muon in its frame of reference	
For an observer in a laboratory on Earth, time passes more slowly than it does for a muon in its frame of reference	
For an observer in a laboratory on Earth, the probability of a muon decaying each second is lower than it is for a muon in its frame of reference	

(1)

(b) (i) Show that the total energy of an electron that has been accelerated to a speed of $0.98c$ is about 4×10^{-13} J.

(2)

(ii) The total energy of an electron travelling at a speed of $0.97c$ is 3.37×10^{-13} J. Calculate the potential difference required to accelerate an electron from a speed of $0.97c$ to a speed of $0.98c$.

potential difference = ____________________ V

(1)

(Total 7 marks)

6 (a) One of the two postulates of Einstein's theory of special relativity is that physical laws have the same form in all inertial frames of reference. Explain in terms of velocity what is meant by an inertial frame of reference.

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(1)

(b) Light takes 4.3 years to reach the Earth from the star Alpha Centauri.

(i) A space probe is to be sent from the Earth to the star to arrive 5.0 years later, according to an observer on Earth.

Assuming the space probe's velocity is constant, calculate its speed in ms^{-1} on this journey.

speed ____________________ ms^{-1}

(1)

(ii) Calculate the time taken for this journey in years registered by a clock in the space probe.

time taken ____________________ years

(3)

(Total 5 marks)

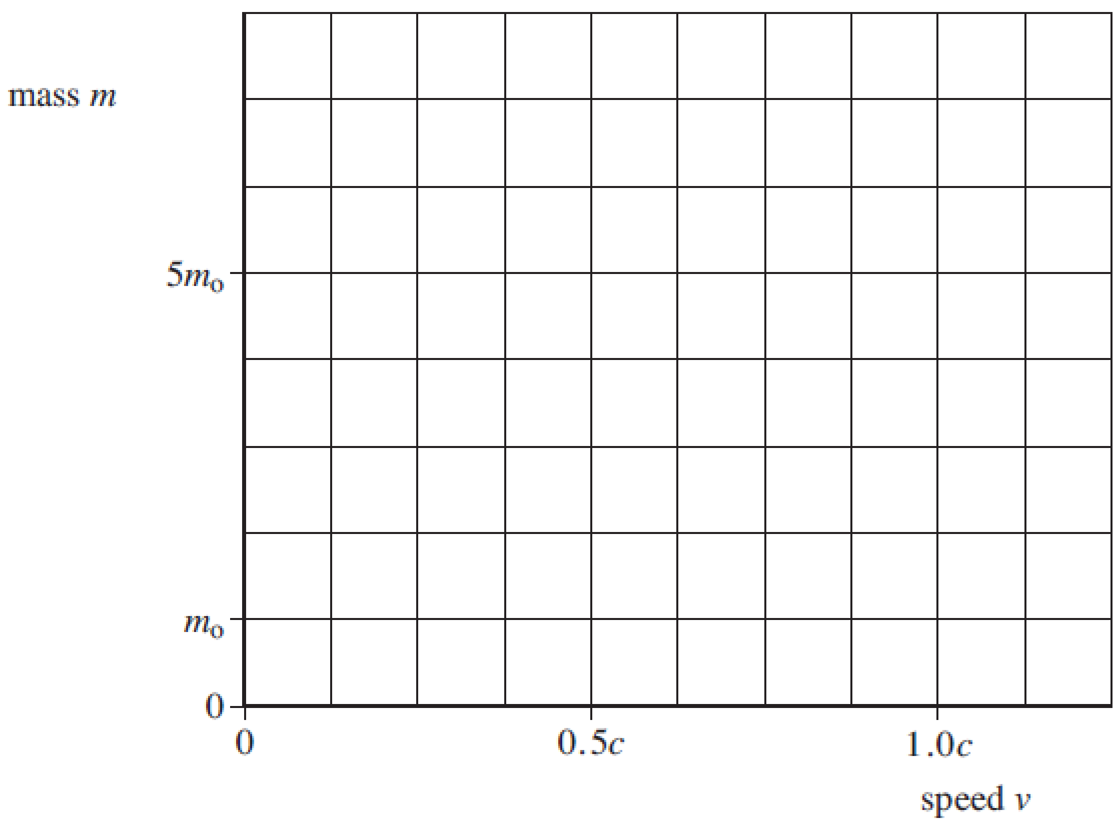
7 (a) Calculate the speed of a particle at which its mass is twice its rest mass.

speed ____________________ m s^{-1}

(2)

(b) Use the axes below to show how the mass m of a particle changes from its rest mass m_o as its speed v increases from zero.

Mark and label on the graph the point **P** where the mass of the particle is twice its rest mass.



(3)

(c) By considering the relationship between the energy of a particle and its mass, explain why the theory of special relativity does not allow a matter particle to travel as fast as light.

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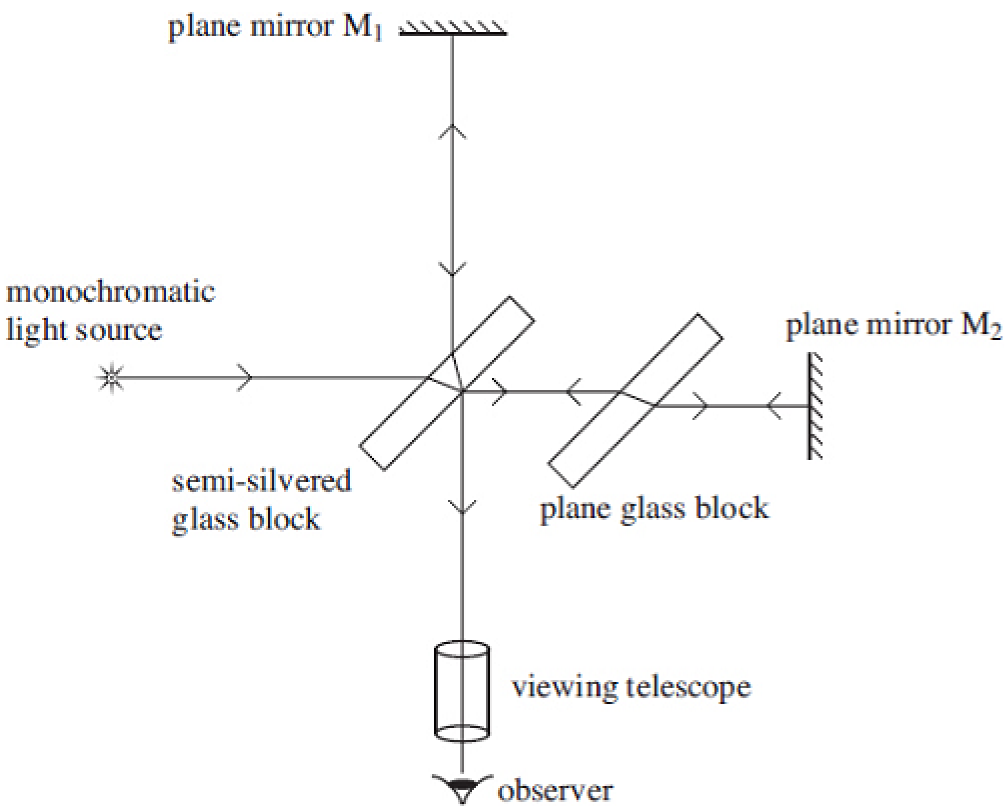
(2)
(Total 7 marks)

8 The figure below represents the Michelson-Morley interferometer. Interference fringes are seen by an observer looking through the viewing telescope.



(a) Explain why the interference fringes shift their position if the distance from either of the two mirrors to the semi-silvered block is changed.

(2)

(b) Michelson and Morley predicted that the interference fringes would shift when the apparatus was rotated through 90°. When they tested their prediction, no such fringe shift was observed.

(i) Why was it predicted that a shift of the fringes would be observed?

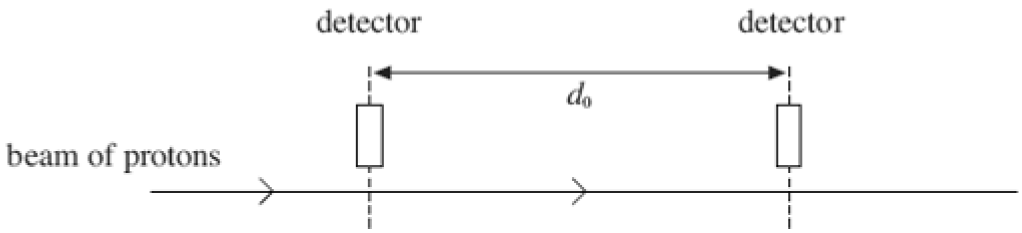
(3)

(ii) What conclusion was drawn from the observation that the fringes did not shift?

(1)

(Total 6 marks)

9 In an experiment, a beam of protons moving along a straight line at a constant speed of $1.8 \times 10^8 \text{ms}^{-1}$ took 95 ns to travel between two detectors at a fixed distance d_0 apart, as shown in the figure below.



(a) (i) Calculate the distance d_0 between the two detectors in the frame of reference of the detectors.

answer = ______________________ m

(1)

(ii) Calculate the distance between the two detectors in the frame of reference of the protons.

answer = ______________________ m

(2)

(b) A proton is moving at a speed of $1.8 \times 10^{8} \text{ms}^{-1}$

Calculate the ratio $\dfrac{\text{kinetic energy of the proton}}{\text{rest energy of the proton}}$

answer = ______________________

(5)

(Total 8 marks)

10 (a) Calculate the speed at which a matter particle has a mass equal to 10 times its rest mass.

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(3)

(b) Explain why a matter particle can not travel as fast as a photon in free space even though its kinetic energy can be increased without limit.

(3)

(Total 6 marks)

11 (a) In a science fiction film, a space rocket travels away from the Earth at a speed of 0.994 c, where c is the speed of light in free space. A radio message of duration 800 s is transmitted by the space rocket.

(i) Calculate the duration of the message when it is received at the Earth.

(ii) Calculate the distance moved by the rocket in the Earth's frame of reference in the time taken to send the message.

(4)

(b) A student claims that a twin who travels at a speed close to the speed of light from Earth to a distant star and back would, on return to Earth, be a different age to the twin who stayed on Earth. Discuss whether or not this claim is correct.

(3)

(Total 7 marks)

12 (i) Calculate the kinetic energy, in J, of a proton accelerated in a straight line from rest through a potential difference of 1.1×10^9 V.

(ii) Show that the mass of a proton at this energy is 2.2 m_0, where m_0 is the proton rest mass.

(iii) Hence calculate the speed of a proton of mass 2.2 m_0.

(Total 7 marks)

13 (a) One of the two postulates of Einstein's theory of special relativity is that *physical laws have the same form in all inertial frames of reference*.

Explain, with the aid of a suitable example, what is meant by an inertial frame of reference.

(2)

(b) A certain type of sub-atomic particle has a half-life of 18 ns when at rest. A beam of these particles travelling at a speed of 0.995*c* is produced in an accelerator.

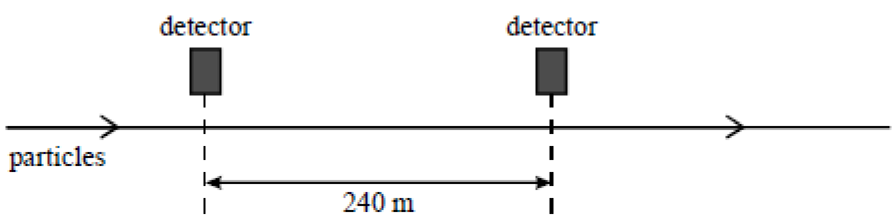
(i) Calculate the half-life of these particles in the laboratory frame of reference.

(ii) Calculate the time taken by these particles to travel a distance of 108 m in the laboratory at a speed of 0.995*c* and hence show that the intensity of the beam is reduced to 25% of its original value over this distance.

(5)

(Total 7 marks)

14 (a) In a particle beam experiment, a short pulse of 1 ns duration of particles moving at constant speed passed directly between 2 detectors at a fixed distance apart of 240 m.
The pulse took 0.84 μs to travel from one detector to the other.



(i) Calculate the speed of the particles.

(ii) Calculate the distance between the two detectors in the frame of reference of the particles.

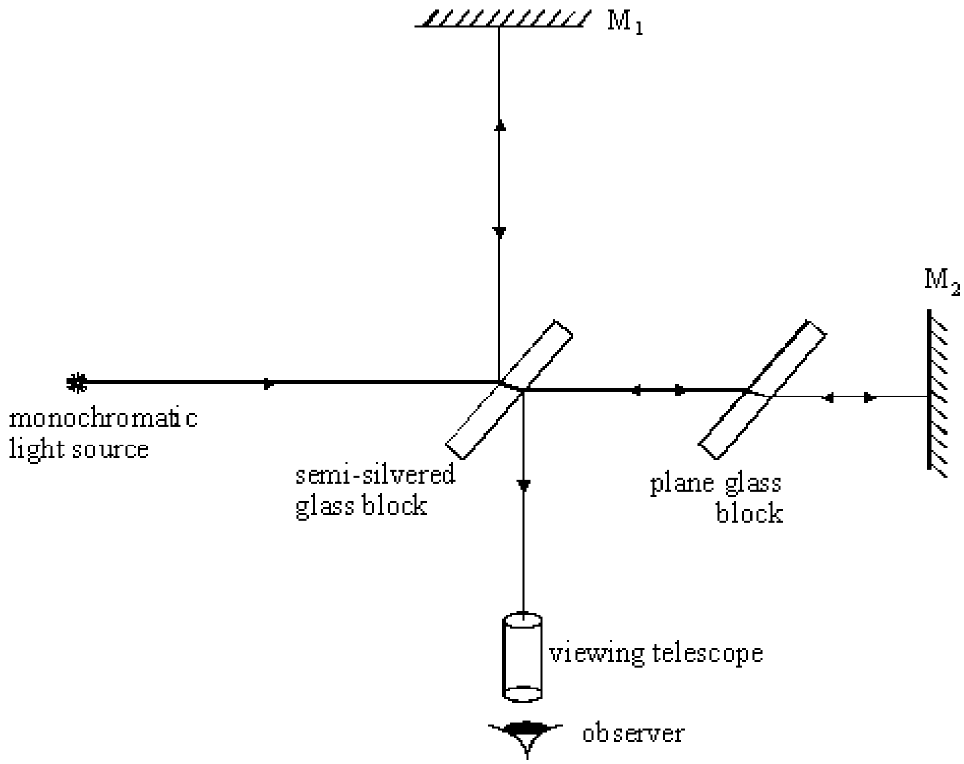
(4)

(b) In a 'thought experiment' about relativity, a student stated that a twin who travelled from the Earth to a distant planet and back at a speed close to the speed of light would be the same age on return as the twin who stayed on Earth. Explain why this statement is **not** correct.

(4)

(Total 8 marks)

15 The Michelson-Morley experiment represented in the diagram was designed to find out if the speed of light depended on its direction relative to the Earth's motion through space. Interference fringes were seen by the observer.



(a) (i) Explain why interference fringes were seen.

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(ii) The interference fringe pattern did not shift when the apparatus was rotated by 90°. Explain the significance of this null observation.

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(5)

(b) Einstein postulated that the speed of light in free space is invariant. Explain what is meant by this postulate.

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(2)

(Total 7 marks)

16 (a) The speed of an object cannot be greater than or equal to the speed of light yet its kinetic energy can be increased without limit. Explain the apparent contradiction that the speed of an object is limited whereas its kinetic energy is not limited.

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(3)

(b) Protons are accelerated from rest through a potential difference of 2.1×10^{10} V.

(i) Show that the kinetic energy of a proton after it has been accelerated from rest through this potential difference is 3.4×10^{-9} J.

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(ii) Show that the mass of a proton with the kinetic energy value calculated in part (a) is approximately $23m_0$, where m_0 is its rest mass.

(4)

(c) Calculate the speed of a proton which has a mass equal to $23m_0$.

(3)

(Total 10 marks)

17 (a) One of the two postulates of Einstein's theory of special relativity is that the speed of light in free space is invariant.

(i) Explain what is meant by this postulate.

(ii) State and explain the other postulate.

(4)

(b) A stationary muon has a rest mass of 1.88×10^{-28} kg and a half-life of 2.2×10^{-6} s.

Calculate

(i) the mass of a muon travelling at 0.996 c, where c is the speed of light in a vacuum,

(ii) the distance, in a laboratory frame of reference, travelled in one half-life by a muon moving at 0.996 c.

(6)

(Total 10 marks)

18 (a) Michelson and Morley attempted to detect absolute motion by investigating whether or not the speed of light in a direction parallel to the Earth's motion differs from the speed of light perpendicular to the Earth's motion.

Discuss what resulted from this experiment and what was concluded.

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(3)

(b) In a science fiction story, a space rocket left the Earth in 2066 and travelled out of the Solar System at a speed of $0.80c$, where c is the speed of light in vacuo, to a star 16 light years from the Earth.

(i) How many years, in the frame of reference of the Earth, did the spacecraft take to reach the star?

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(ii) What was the distance, in the frame of reference of the spacecraft, between the Earth and the star?

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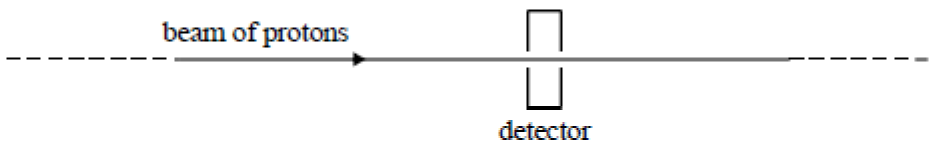
(iii) A member of the crew was 21 years old on leaving the Earth. How old was this person on arrival at the star?

(6)
(Total 9 marks)

19 In a particle beam experiment, a pulsed beam of protons at a speed of 1.00×10^8 m s^{-1} passed through a stationary detector in a time of 15.0 ns.



(a) Calculate the length of the pulsed beam in

(i) the frame of reference of the detector,

(ii) the frame of reference of the protons.

(3)

(b) (i) Calculate the kinetic energy of each proton in the beam, in J.

(ii) The beam consisted of 10^7 protons. It passed through the detector and was stopped by a stationary target. Calculate the average power which the proton beam delivered to the target during the pulse.

(5)

(Total 8 marks)

20 (a) One of the two postulates of Einstein's theory of special relativity is that the speed of light in free space is invariant.

(i) Explain what is meant by this statement.

(ii) What is the other postulate?

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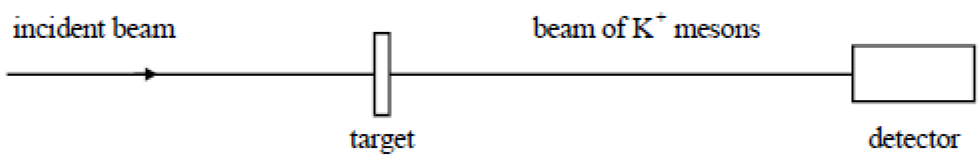
(3)

(b) K^+ mesons are sub-atomic particles of half-life 86 ns when at rest. In an accelerator experiment, a beam of K^+ mesons travelling at a speed of 0.95*c* is created, where *c* is the speed of light.



(i) Calculate the half-life of the K^+ mesons in the beam measured in the laboratory frame of reference.

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(ii) What is the greatest distance that a detector could be sited from the point of production of the K^+ mesons to detect at least 25% of the K^+ mesons produced?

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(6)
(Total 9 marks)

Mark schemes

1 (a) speed of light in free space independent of motion of source and / or the observer✓ and of motion of observer

1

(b) laws of physics have the same form in all inertial frames
laws of physics unchanged from one inertial frame to another ✓

1

(c) time taken$\left(= \frac{\text{distance}}{\text{speed}} = \frac{34 \text{ m}}{0.95 \times 3.0 \times 10^8 \text{ m s}^{-1}}\right) = 1.2 \times 10^{-7}$ s✓

1

(d) $t = \frac{18 \text{ ns}}{(1 - 0.95^2 c^2 / c^2)^{1/2}}$ ✓

Allow substitution for this mark

1

time taken for π meson to pass from one detector to the other = 58 ns ✓

1

2 half-lives (approximately) in the detectors' frame of reference. ✓

1

two half-lives corresponds to a reduction to 25 % so 75% of the π mesons passing the first detector do not reach the second detector. ✓
OR
Appreciation that in the lab frame of reference the time is about 6 half-lives had passed✓

1

In 6 half-lives 1 / 64 left so about 90% should have decayed✓

Clear conclusion made
Either Using special relativity gives agreement with experiment
or Failure to use relativity gives too many decaying (WTTE)

1

[8]

2 (a) The laws of physics are the <u>same</u> in <u>all inertial</u> frames of reference OWTTE

Allow specified laws eg Newton's laws applies in all inertial frames of reference

Do not allow laws of physics are ***obeyed or apply***

Allow any / every inertial frame of reference

1

(b) (i) Converts 24 GeV to J $24 \times 10^{9} \times 1.6 \times 10^{-19}$ or 3.84×10^{-9} (J) seen ✓

3.84×10^{-9} or $24 \times 10^{9} = \frac{9.11 \times 10^{-31} \times (3 \times 10^{8})^{2}}{\gamma} (-9.11 \times 10^{-31} \times (3 \times 10^{8})^{2})$ ✓

2.14 or 2.13×10^{-5} ✓ from correct working
at least 3 sf needed

Many convert to equivalent mass

4.27×10^{-26} and then work in masses throughout

May include the bracketed term. Depending on whether they assume 24 GeV to be the total energy or the kinetic energy

Allow incorrect powers of 10

May use given γ and find energy and compare with 24 GeV

3

(ii) $3000 \times 2.1 \times 10^{-5} = 0.063$ or 0.064 m ✓

1

(c) Starts at m_o ✓

Shallow increase to

- no more than $2\,m_o$ at $0.7c$
- curves (sharply) upwards becoming greater than $0.9\,c$ at $6m_o$
- never greater than $1.0\ c$
- within ½ square of $1.0\,c$ at $12m_o$ ✓

Never greater than 1.0c

Allow statement of asymptote

2

[7]

3 (a) (i) Only Box Ticked: Mesons

1

(ii) (Muon) anti−neutrino symbol

Not electron anti−neutrino

Penalise incorrect subscript

1

(b) (i) Use of Speed = distance / time by rearrangement and 3.75×10^{-5} (s) seen
Or
$10.7 \times 10^3 \div 2.85 \times 10^8 = 3.75 \times 10^{-5}$ (s) seen
Or substitution **and** 3.75×10^{-5} (s) seen

No. of half-lives = $3.75 \times 10^{-5} \div 2.2 \times 10^{-6} = 17.065$ or 17.07
not 17.05 not 17.06
At least 3 sf for answer 17.1

$3.75 \times 10^{-5} \div 17 = 2.208$ or 2.21 μs

At least 3 sf

Or

$17 \times 2.2 \times 10^{-6} = 37.4 \times 10^{-5}$ ***with comparison***

2

(ii) $2.5 \times 10^8 \times (1/2)^{17}$ or equivalent

1900 to 1910 (1910 maximum to 4 sf)

Answer consistent with any working seen

Use of $N = N_0\, e^{-\lambda t}$ and $\lambda = \frac{\ln 2}{2.2 \times 10^{-6}}$ correct sub

Answer in range 1.8×10^3 to 1.91×10^3

(1820 minimum and 1910 maximum to 4 sf)

2

(iii) (Theory of special) relativity

Time dilation / length contraction treat as neutral

Not general relativity

1

(iv)
- Travelling close to speed of light less time passes in muon's reference frame for the journey (so fewer decay)
- Travelling close to speed of light so journey is shorter in length for the muon's frame of reference (so fewer decay)
- Travelling close to speed of light so muons are observed to travel further in a half−life (on Earth) than expected (so fewer decay during journey)
- Travelling close to speed of light so muon's half−life is observed to be longer (on Earth) (so fewer decay)

Allow:

• travelling close to speed of light so time is slower (for muons) so fewer decay

• travelling close to speed of light so time dilates so fewer decay

1

(v) Attempted use of $L = L_o (1-v^2/c^2)^{1/2}$ or $t = t_o /(1-v^2/c^2)^{1/2}$

Correct use of $L = L_o (1-v^2/c^2)^{1/2}$ **and** $(t_o = L/v) = 3341/2.85 \times 10^8$

or correctly makes t_o subject of $t = t_o /(1-v^2/c^2)^{1/2}$

(t_o =) 1.17×10^{-5} or 1.2×10^{-5} (s)

Condone mix up on L / L_o or t / t_o

1.2×10^{-4} s gets 1 mark

Sub for L_o as 10.7×10^3

Or sub for $t = 3.75 \times 10^{-5}$

3

(vi) Use of $T_{½} = ln2/\lambda$ seen with sub for $T_{½}$

allow if seen in partial sub in $N = N_o e^{-\lambda t}$

Use of $N = N_o e^{-\lambda t}$ with $\lambda = 3.15 \times 10^5$ (or equivalent) and t = answer from b(v)

5.7×10^6 to 6.3×10^6
no ecf on answer

Or use of no half−lives = $\frac{b(v)}{2.2 \times 10^{-6}}$

And $\frac{2.5 \times 10^8}{2^{\frac{b(v)}{2.2 \times 10^{-6}}}}$

Only accept answers in this range

No ecf on answer

3

[16]

4 (a) They expected the time taken for the light to travel in one direction to be different from the other ✓
or
Expected light to travel at different speeds in the two directions

However expressed e.g. in terms of the different times taken parallel and at right angles to the Earth's motion (through the Aether)

There would be a phase shift / change in the phase relationship

Not
longer / different paths
or
path difference

2

(b) (i) Speed through aether

$$= \frac{\text{circumference of Earth orbit around the Sun}}{\text{time for one orbit (1 year)}}$$

Need to be clear about the distance and time

or $v = (GM / r)^{1/2}$ with M and r defined

Watch out for confusion between Earth's orbit around the Sun and Earth's rotation on its axis

1

(ii) 11 m

1

(c) Experiment showed speed of light from moving object is same as that from stationary object or
Speed of light in direction of motion is same as in perpendicular direction or
Speed of light does not depend on speed of source or observer
Speed of light being invariant
or
Aether theory incorrect / no aether / no absolute motion

Allow is always 3×10^8 m s^{-1} in air or vacuum instead of invariant

It was a postulate / assumption of the theory of special relativity
Or this supports the theory ✓

Second mark is for explicitly linking the observation to Einstein's theory

2

[6]

5 (a) (i) Distance travelled in muons' frame of reference

$= 10700(1\text{-}0.996^2)^{1/2} = 956$ m ✓

Time taken in muons' frame of reference = 3.2 μs ✓

This is 2 half-lives so number reaching Earth = 250 ✓

OR

Time in Earth frame of reference

$= 10700 / (0.996 \times 3 \times 10^8) = 3.581 \times 10^{-5}$ s ✓

Time taken in muons' frame of reference = 3.2 μs ✓

This is 2 half-lives so number reaching Earth = 250 ✓

OR

Half-life in Earth frame of reference

$=1.6 \times 10^{-6} / (1\text{-}0.996^2)^{1/2} = 17.9 \times 10^{-6}$ s ✓

Time taken = 35.8×10^{-6} s ✓

This is 2 half lives so number reaching Earth = 250 ✓

OR

Distance travelled in muons' frame of reference

$= 10700(1\text{-}0.996^2)^{1/2} = 956$ m ✓

Distance the muon travels in one half-life in muons reference frame

$= 0.996 \times 3 \times 10^8 \times 1.6 \times 10^{-6} = 478$ m ✓

Therefore 2 half-lives elapse to travel 956 m so number = 250 ✓

OR

Decay constant in muon frame of reference

Or decay constant in the Earth frame of reference ✓

Uses the corresponding elapsed time and decay constant in

$N = N_0 e^{-\lambda t}$ ✓

Arrives at 250 ✓

All steps in the working must be seen

Award marks according to which route they appear to be taking

The number left must be deduced from the correct time that has elapsed in the frame of reference they are using

3

(ii)

	✓ if correct
For an observer in a laboratory on Earth the distance travelled by a muon is greater than the distance travelled by the muon in its frame of reference	✓
For an observer in a laboratory on Earth time passes more slowly than for a muon in its frame of reference	
For an observer in a laboratory on Earth, the probability of a muon decaying each second is lower than it is for a muon in its frame of reference	

1

(b) (i) Total energy = $9.11 \times 10^{-31} \times (3 \times 10^8)^2 / (1-0.98^2)^{1/2}$ ✓
4.12×10^{-13} J seen to 2 or more sf ✓
Show that so working must be seen
2

(ii) Change = 7.5×10^{-14} J
V = 469 (470) kV allow ecf using their answer to (i) ✓
ecf is their ((i) −3.37) × 10^{-13}) / 1.6 × 10^{-19}
Using 4 × 10^{-13} gives 394 (390) kV
Using 3.9 × 10^{-13} gives 331(330) kV
Do not allow 1 sf answer
1

[7]

6 (a) (A frame of reference) that has a constant velocity ✓
Accept no acceleration
1

(b) (i) Distance = 4.3 c light years (or 4.1×10^{16} m)
Correct answer only gets the mark

Speed (= $\frac{4.3\,c}{5.0}$) = 2.6×10^8 m s^{-1} (or 0.86 c)
Accept 2.58
1

(ii) $t = \frac{t_o}{(1 - v^2/c^2)^{1/2}}$ where t = 5.0 years (or 1.58 × 10^8 s)

and v = 0.86 c (or 2.58 × 10^8 m s^{-1})

CF from bi to bii provided answer to bi < c

1st mark for correct substitution of either t or v into the above eqn ✓
$t_o = 5.0 \times (1 - (0.86c)^2 / c^2)^{1/2}$ ✓ = 2.6 years ✓

Accept t or v in alternative units

Accept 1.58 (or 1.6) × 10^8 s in place of 5.0 yr in 3rd mark point

Alt scheme
$l = l_o (1 - v^2 / c^2)^{1/2}$ where t = 5.0 years (or 1.58 × 10^8 s) and
v = 0.86 c (or 2.58 × 10^8 m s^{-1})

Accept 2.5 to 2.6 to any number of sfs

1st mark for correct substitution of either t or v into the above eqn ✓
($l_o = 4.3 \times 365 \times 24 \times 3600 \times 3.0 \times 10^8 = 4.07 \times 10^{16}$ m)
$l = 4.07 \times 10^{16} (1 - (0.86c)^2/ c^2)^{1/2}$ or 2.08×10^{16} m✓

$t_o = \frac{l}{v}$ $\left(= \frac{2.08 \times 10^{16} \text{ m}}{2.6 \times 10^8 \text{ m/s}}\right.$ = 8.05×10^7s) = 2.6 years ✓

Alternative for last 2 marks in Alt scheme (l_o = 4.3 l yr)

$l = 4.3 (1 - (0.86c)^2/ c^2)^{1/2}$ = 2.2 l yr✓

$t_o = \frac{l}{v}$ $\left(= \frac{2.2}{0.86}\right)$ = *2.6 years ✓*

3

[5]

7 (a)

(Using $m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$ gives)

$$2 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{or} \quad \sqrt{1 - \frac{v^2}{c^2}} = 0.5 \quad ✓$$

(Rearranging gives)

$v \left(= \sqrt{1 - 0.5^2}\ c\right) = 0.866\ c$ or 2.6×10^8 m s^{-1} ✓

Accept either answer.

2

(b) curve starts at v=0, $m = m_o$ and rises smoothly ✓

2nd mark; ecf from a if plotted correctly

curve passes through $2m_o$ at v = 0.87 c (± 0.03*c* or in 2nd half of x-scale div containing 0.87c) ✓

3rd mark; There must be visible white space between the curve and the v = c line; also, the curve must reach $7m_o$ at least.

curve is asymptotic at $v = c$ (and does not cross or touch $v = c$ or curve back) ✓

3

(c) Energy = mc^2 so (as $v \to c$) energy of particle increases as mass increases ✓

Alternative scheme for 1 mark only; mass infinite at v = c which is (physically) impossible ✓

mass -> infinity as *v* -> *c* so energy -> infinity which is (physically) impossible ✓

[OR for one mark only

force = *ma* so force increases as mass increases

Mass -> infinity as v->c so force -> infinity which is (physically) impossible ✓]

2

[7]

8 (a) bright (or dark) fringe is seen where the two beams are in phase (or out of phase by 180°) ✓

changing the distance to either mirror changes the path (or phase) difference (between the two beams) so fringes shift ✓

2

(b) (i) speed of light was thought to depend on the speed of the light source (or the speed of the observer) ✓ (or on the motion of the Earth (through the aether))

distance travelled by each beam unchanged (by rotation) ✓

time difference between the two beams would change on rotation ✓

phase difference would therefore change (so fringes would shift) ✓

3

(ii) speed of light is independent of the speed (or motion) of the light source (or the observer) ✓

(or ‘aether’ hypothesis incorrect (owtte)) or absolute motion does not exist)

1

[6]

9 (a) (i) d_0 =(speed × time = $1.8 \times 10^8 \times 95 \times 10^{-9}$) = 17(.1) m ✓

1

(ii) d (= $d_0 (1 - v^2/c^2)^{½}$)

= $17.1 \times (1 - (1.8 \times 10^8/3.0 \times 10^8)^2))^{½}$ ✓

= 14 m ✓ (or 13.7 m or 13.68 m)

or

$t = t_0 (1 - v^2/c^2)^{-½}$

$95 = t_0 \times (1 - (1.8 \times 10^8/3.0 \times 10^8)^2)^{-½}$ gives t_0 = 76 ns ✓

$d = vt_0 = 1.8 \times 10^8 \times 76 \times 10^{-9}$ = 14 m ✓ (or 13.7 m or 13.68 m)

2

(b) m (= $m_0 (1 - v^2/c^2)^{-½}$)

= $1.67(3) \times 10^{-27} \times (1 - (1.8 \times 10^8/3.0 \times 10^8)^2)^{-½}$) ✓

= 2.09×10^{-27} kg ✓

kinetic energy = $(m - m_0)\, c^2$

or correct calculation of $E = mc^2$ (= 1.88×10^{-10} J)

or correct calculation of $E_0 = m_0c^2$ (= 1.50×10^{-10} J) ✓

$$\frac{\text{kinetic energy}}{\text{rest energy}} = \frac{(m - m_0)c^2}{m_0c^2} = \frac{(2.09 - 1.67) \times 10^{-27}}{1.67 \times 10^{-27}}$$ ✓

= 0.25 (allow 0.245 to 0.255 or ¼ or 1:4) ✓

5

[8]

10 (a) $10m_0 = m_0\left(1 - \frac{v_2}{c^2}\right)^{-\frac{1}{2}}$ **(1)**

gives $\frac{v^2}{c^2}$ = 1 – 0.01 = 0.99 **(1)**

v (= 0.995c) = $2.98(5) \times 10^8$ m s^{-1} **(1)**

3

(b) $m = m_0\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}$ **(1)**

$m \to$ infinity as $v \to c$ **(1)**

[or m increases as v increases]

$E_k(= mc^2 - m_0c^2) \to$ infinity as $v \to c$ **(1)**

$v = c$ would require infinite E_k (or mass) which is (physically)

impossible **(1)**

Max 3

[6]

11 (a) (i) $t_0 = 800$ (s) **(1)**

(use of $t = t_0\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}$ gives) $t = 800(1 - 0.994^2)^{-1/2}$ **(1)**

$= 7300$ s **(1)**

(ii) distance ($= 0.994ct = 0.994 \times 3 \times 10^8 \times 7300$)
$= 2.2 \times 10^{12}$m **(1)** (2.18×10^{12}m)
(allow C.E. for value of t from (i))

4

(b) space twin's travel time = proper time (or t_0) **(1)**

time on Earth, $t = t_0\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}$ **(1)**

$t > t_0$
[or time for traveller slows down compared with Earth twin] **(1)**
space twin ages less than Earth twin **(1)**
travelling in non-inertial frame of reference **(1)**

max 3

[7]

12 (i) E_k (= eV) ($= 1.6 \times 10^{-19} \times 1.1 \times 10^9$)
$= 1.8 \times 10^{-10}$ (J) **(1)** (1.76×10^{-10} (J))

(ii) (use of $E = mc^2$ gives) $\Delta m = \left(\frac{1.8 \times 10^{-10}}{(3 \times 10^8)^2}\right) = 2.0 \times 10^{-27}$ (kg) **(1)**

$= \frac{2.0 \times 10^{-27}}{1.67 \times 10^{-27}} m_0 = 1.2 m_0$ **(1)**

(allow C.E. for value of E_k from (i), but not 3rd mark)

$\therefore$ $m = m_0 + \Delta m$ **(1)** ($= 2.2\, m_0$)

(iii) (use of $m = m_0\left(1-\frac{v^2}{c^2}\right)^{-1/2}$ gives) $2.2m_0 = \left(1-\frac{v^2}{c^2}\right)^{-1/2}$ **(1)**

$v = \left(1-\frac{1}{2.2^2}\right)^{1/2} c$ **(1)**

$= 2.7 \times 10^8$ m s^{-1} **(1)**

[7]

13 (a) Newton's laws obeyed in an inertial frame
[or inertial frames move at constant velocity relative to each other] **(1)**
suitable example (e.g. object moving at constant velocity) **(1)**

2

(b) (i) (use of $t = t_0\left(1-\frac{v^2}{c^2}\right)^{-1/2}$ gives) t_0 = 18 (ns) **(1)**

$t = 18 \times 10^{-9}\left(1-\frac{(0.995c)^2}{c^2}\right)^{-1/2}$ **(1)**

$= 1.8 \times 10^{-7}$ s **(1)**

(ii) time taken $\left(= \frac{\text{distance}}{\text{speed}}\right) = \left(\frac{108}{0.995 \times 3.0 \times 10^8}\right) = 3.6 \times 10^{-7}$ s **(1)**

time taken = 2 half-lives, which is time to decrease to 25% intensity **(1)**

[alternative scheme: (use of $l = l_0\left(1-\frac{v^2}{c^2}\right)^{1/2}$ gives) l_0 = 108 (m)

$l = 108\left(1-\frac{(0.995c)^2}{c_2}\right)^{1/2}$ = 10.8 m **(1)**

time taken $\left(=\frac{10.8}{0.995c}\right) = 3.6 \times 10^{-8}$ s

= 2 half-lives, which is time to decrease to 25% intensity **(1)**]

5

[7]

14

(a) (i) (use of $v = \frac{d}{t}$ gives) $v = \frac{240}{0.84 \times 10^{-6}}$ = 2.8(6) × 10^8 m s^{-1} **(1)**

(ii) actual length = 240 m **(1)**

(use of $l = l_0\left(1 - \frac{v^2}{c^2}\right)^{1/2}$ gives)

length in particle frame, $l = 240\left(1 - \frac{2.86^2}{3^2}\right)^{1/2}$ **(1)**

(allow C.E. for value of v)

l = (240 × 0.30) = 72(.5) m **(1)**

4

(b) time between two events depends on speed of observer

[or $t = t_0\left(1 - \frac{v^2}{c^2}\right)^{1/2}$ or rocket time depends on speed of traveller] **(1)**

traveller's journey time is the proper time between start and stop

[or t_0 is the proper time or t is the time on Earth] **(1)**

journey time measured on Earth > journey time measured by traveller

[or $t > t_0$ or rocket time slower / less than Earth time] **(1)**

traveller younger than twin on return to Earth **(1)**

4

[8]

15

(a) (i) two beams (or rays) reach the observer **(1)**

interference takes place between the two beams **(1)**

bright fringe formed if/where (optical) path difference = whole number of wavelengths

(or two beams in phase)

[or dark fringe formed if/where (optical) path difference = whole number + 0.5 wavelengths]

(or two beams out of phase by 180 °C/ π/2 /½ cycle) **(1)**

(ii) rotation by 90° realigns beams relative to direction of Earth's motion **(1)**

no shift means no change in optical path difference between the two beams **(1)**

(∴) time taken by light to travel to each mirror unchanged by rotation **(1)**

distance to mirrors is unchanged by rotation **(1)**

(∴) no shift means that the speed of light is unaffected [or disproves other theory] **(1)**

max 5

(b) the speed of light does not depend on the motion of the light source **(1)** or that of the observer **(1)**

2

[7]

16 (a) as speed → c, mass → infinite **(1)**
gain of E_k causes large gain of mass when speed is close to c **(1)**
gain of E_k causes small gain of speed when speed is close to c **(1)**
$E_k = \frac{1}{2}mv^2$ valid at speeds << c **(1)**

max 3

The Quality of Written Communication marks are awarded for the quality of answers to this question.

(b) (i) $E_k = eV = 1.6 \times 10^{-19} \times 2.1 \times 10^{10}$ **(1)** (= $3.3(6) \times 10^{-9}$J)

(ii) (use of $m = \frac{E_k}{c^2}$ gives) gain of mass = $\frac{3.36 \times 10^{-9}}{(3 \times 10^8)^2}$ = 3.7×10^{-26}(kg) **(1)**

$$= \frac{3.37 \times 10^{-26}}{1.67 \times 10^{-27}}\ m_0 = 22\ m_0$$ **(1)**

mass of proton = 22 m_0 +m_0 **(1)** (=23 m_0)

(using $E_k = 3.4 \times 10^{-9}$ gives gain of mass = 3.8×10^{-26} (kg) ≡ 23 m_0
mass of proton = 24 m_0

4

(c) $23 = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$ **(1)**

$\frac{v^2}{c_2} = \left(1 - \frac{1}{25}\right) = 0.998$ **(1)**

$v = 0.999\ c = 2.99(7) \times 10^8$ m s^{-1}

3

[10]

17 (a) (i) speed of light (in free space) independent of motion of source **(1)**
and of motion of observer **(1)**
[*alternative (i)*
speed of light is same in all frames of reference **(1)**]

(ii) laws of physics have same form in all inertial frames **(1)**
inertial frame is one in which Newton's 1 st law of motion obeyed **(1)**
laws of physics unchanged in coordinate transformation
from one inertial frame of reference to any other inertial frame **(1)**

(max 4)

(b) (i) $m\left(=m_0\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}\right)$ = 1.88 × 10^{-28} (1 – $(0.996)^2)^{-\frac{1}{2}}$ **(1)**

= 2.10 × 10^{-27} kg **(1)**

(ii) t_0 = 2.2 × 10^{-6} s **(1)**

$t\left(=t_0\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}\right)$ = 2.2 × 10^{-6} (1 – $(0.996)^2$) $^{-\frac{1}{2}}$ (s) **(1)**

= 2.46 × 10^{-5} (s) **(1)**

s(= vt = 3.00 × 10^8 × 0.996 × 2.46 × 10^{-5}) = 7360 m **(1)**

[*alternative (ii)*
l (= vt = 0.996 × 3.0 × 10^8 × 2.2 × 10^6) = 657 (m) **(1)**

correct substitution of l in $l = l_0\sqrt{1-\frac{v^2}{c^2}}$ **(1)**

$l_0\left(=\frac{l}{\sqrt{1-\frac{v^2}{c^2}}}\right)=\frac{657}{\sqrt{1-0.996^2}}$ **(1)**

l_0 = 7360 m **(1)**

(6)

[10]

18 (a) no change in the fringe pattern on rotation **(1)**

the speed of light is the same in the two directions **(1)**

the speed of light from a light source on Earth is

unaffected by the motion of the Earth **(1)**

[or the speed of light is invariant

or independent of the motion of the source or observer] **(1)**

the laws of dynamics cannot be applied to light **(1)**

no ether **(1)**

(max 3)

(b) (i) time $\left(= \frac{\text{distance}}{\text{speed}} = \frac{16cT_{\text{one year}}}{0.8c}\right)$ = 20 yr **(1)**

(ii) $L_0 = 16c$ [or 16 light years] **(1)**

$$L\left(= L_0\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}\right) = 16(1 - 0.8^2)^{\frac{1}{2}} (= 0.6 \times 16c) = 9.6c \ \textbf{(1)}$$

(iii) Δt = 20 years **(1)**

$$\Delta t_0 = \Delta t\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}} = 20(1 - 0.8^2)^{\frac{1}{2}} \ \textbf{(1)}$$

$= 0.6 \times 20 = 12$ yr $\therefore$ age = 21 + 12 = 33 yr **(1)**

(6)

[9]

19 (a) (i) $l = (vt = 1.00 \times 10^8 \times 15 \times 10^{-9}) = 1.50$m **(1)**

(ii) $\left(l = l_0\sqrt{1 - \frac{v^2}{c^2}}\right)$

$$1.50 = l_0\sqrt{1 - \frac{(1.00 \times 10^8)^2}{(3.00 \times 10^8)^2}} \ \textbf{(1)}$$

$l_0\left(= \frac{1.50}{0.943}\right)$ = 1.59 m **(1)**

3

(b) (i) $m = \dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$ **(1)** $\left[\text{or } \dfrac{m_0}{\sqrt{1-\dfrac{(1.00\times10^8)^2}{(3.00\times10^8)^2}}}\right]$

$$m\left(\text{or } \frac{m_0}{\sqrt{1-\dfrac{(1.00\times10^8)^2}{(3.00\times10^8)^2}}}\right) = 1.06m_0$$

[or = $1.06 \times 1.67 \times 10^{-27}$ or 1.77×10^{-27} kg] **(1)**
kinetic energy = $(m - m_0)c^2$ **(1)**
[or = $0.06m_0c^2$ or $0.06 \times 1.67 \times 10^{-27} \times (3 \times 10^8)^2$]
= 9.1×10^{-12} (J) **(1)**

(ii) total k.e. = $(10^7 \times 9.1 \times 10^{-12}) = 9.1 \times 10^{-5}$ (J) **(1)**

k.e. per second $\left(= \dfrac{9.1\times10^{-5}}{1.5\times10^{-9}}\right)$ = 6080W

max 5

[8]

20 (a) (i) the same or constant **(1)**
regardless of the speed of the observer or source **(1)**

(ii) physical laws have the same form in all frames **(1)**

(3)

(b) (i) $T_{\frac{1}{2}}$ or beams of mesons $= 8.6\text{ ns} \times \left(1-\dfrac{v^2}{c^2}\right)^{-\frac{1}{2}}$ **(1)**

= $8.6 \times (1 - 0.95^2)^{-\frac{1}{2}}$ = 27.5 ns **(1)**

(ii) beam reduces to 25% in 2 half-lives **(1)**
$v(= 0.95\,c) = 2.85 \times 10^8$ m s^{-1} **(1)**
distance = 2×27.5 ns $\times 2.85 \times 10^8$ m s^{-1} **(1)**
= 15.6 m **(1)**

(6)

[9]